Revisiting GPD evolution

Valerio Bertone
IRFU, CEA, Université Paris-Saclay
In collaboration with Cedric Mezrag

May 11, 2021, QCD Evolution Workshop 2021

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement № 824093

Introduction

- Generalised parton distributions (GPDs) are a "byproduct" of leading-power factorisation of the amplitudes for exclusive processes such as deeply-virtual Compton scattering (DVCS) at high energies:

$$\mathcal{H}(x,\xi,t,Q) = \sum_i \int_{-1}^{1} \frac{dy}{y} C_i\left(\frac{x}{y},\frac{\xi}{x},\alpha_s(\mu),\frac{\mu}{Q}\right) F_i(y,\xi,t,\mu)$$

- The GPD F_i has an operator definition affected by UV divergencies that need to be **renormalised**.

- Renormalisation introduces the scale μ whose dependence is determined by the evolution equation:

$$\frac{dF_i(x,\xi,t,\mu)}{d\ln\mu^2} = \sum_i \int_{-1}^{1} \frac{dy}{|2\xi|} \mathbb{V}_{ij}\left(\frac{x}{\xi},\frac{y}{\xi},\alpha_s(\mu)\right) F_i(y,\xi,t,\mu)$$

- The evolution kernels are currently known up to two/three loops.
[D. Müller et al., *Fortsch.Phys.* 42 (1994) 101-141] [A.V. Belitsky et al.,*Nucl.Phys.B* 574 (2000) 347-406] [V.M.Braun et al., e-Print:2101.01471 [hep-ph]]

- We are concerned with the **full understanding of the kernels**.

Analytics

- Let us focus on LO evolution for a non-singlet GPD combination:

$$\frac{dF^{(-)}(x,\xi)}{d\ln\mu^2} = \frac{\alpha_s(\mu)}{4\pi}\int_{-1}^{1}\frac{dy}{|2\xi|}V_{\rm NS}^{(0)}\left(\frac{x}{\xi},\frac{y}{\xi}\right)F^{(-)}(y,\xi) \qquad F^{(-)}(x,\xi)\equiv F_q(x,\xi)+F_q(-x,\xi)$$

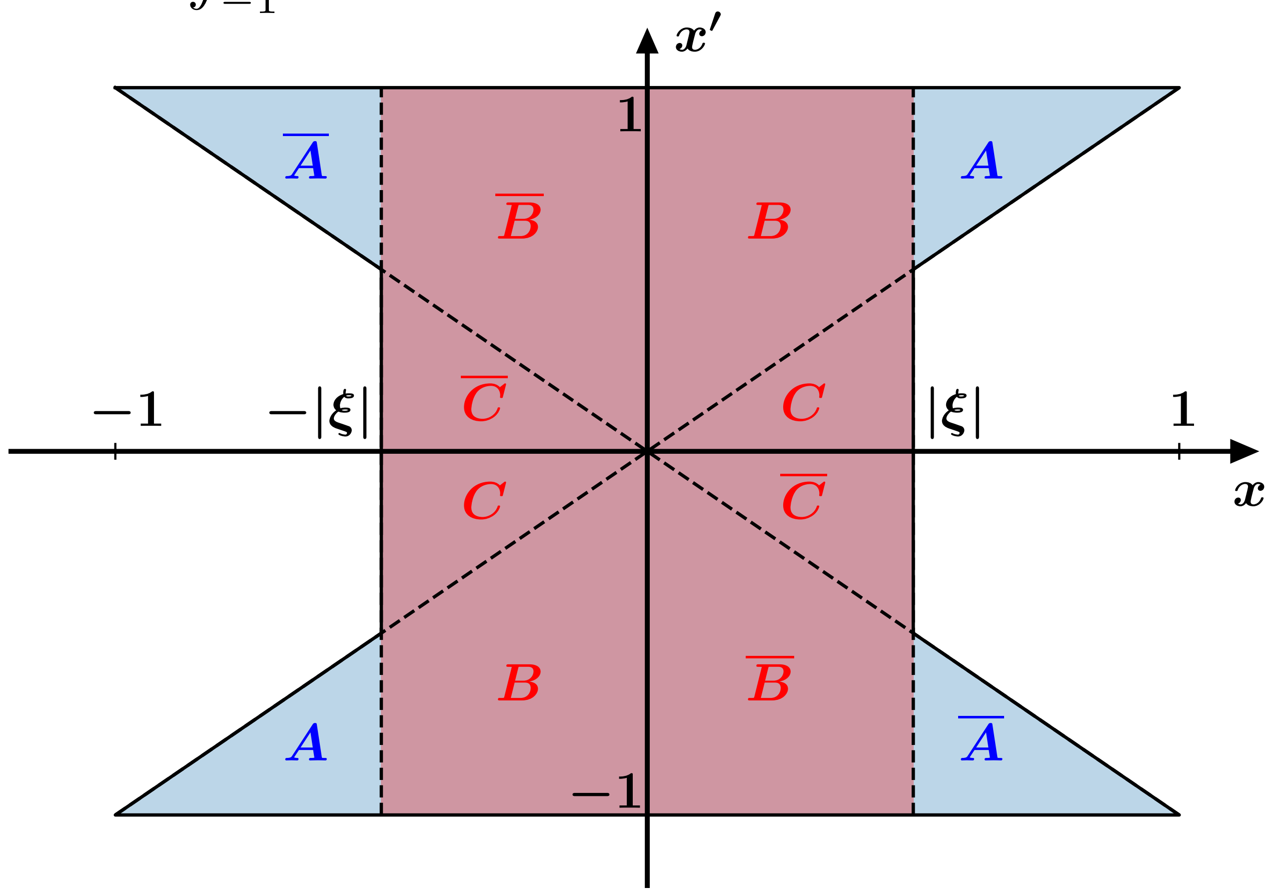
- In the literature the non-singlet LO kernel is often presented as:

$$V_{\rm NS}^{(0)}(x,x') = 2C_F\left[\rho(x,x')\left\{\frac{1+x}{1+x'}\left(1+\frac{2}{x'-x}\right)\right\}+(x\to -x, x'\to -x')\right]_+$$

$$\rho(x,x') = \theta(-x+x')\theta(1+x) - \theta(x-x')\theta(1-x)$$

$$[f(x,x')]_+ = f(x,x') - \delta(x-x')\int_{-1}^{1}dx'' f(x'',x')$$

- [M. Diehl, *Phys.Rept.* 388 (2003) 41-277] Eq. (101)
- [A.V. Belitsky, A.V. Radyushkin, *Phys.Rept.* 418 (2005) 1-387] Eq. (4.42)
- [D. Müller et al., *Fortsch.Phys.* 42 (1994) 101-141] Eq. (5.15)



Analytics: a new calculation

- We redid the one-loop calculation:
  - direct calculation based on Feynman diagrams in light-cone gauge,
  - amenable to numerical implementation,
  - correct DGLAP and ERBL limits,
  - a number of additional interesting properties,
  - the calculation of Ji [Phys. Rev. D55, 7114-7125 (1997)] for unpolarised kernels exactly reproduced.
- The LO evolution equations valid in both the DGLAP and the ERBL regions can be recasted in a **DGLAP-like compact form**:

$$\frac{dF^{(-)}(x,\xi,\mu)}{d\ln\mu^2} = \frac{\alpha_s(\mu)}{4\pi}\int_x^{\infty}\frac{dy}{y}\mathcal{P}^{-,(0)}\left(\frac{x}{y},\kappa\right)F^{(-)}(y,\xi,\mu)\,,\qquad \kappa=\frac{\xi}{x}$$

$$\mathcal{P}^{-,(0)}(y,\kappa) = \theta(1-y)\mathcal{P}_1^{-,(0)}(y,\kappa) + \theta(\kappa-1)\mathcal{P}_2^{-,(0)}(y,\kappa)$$

Analytics: a new calculation

- We redid the one-loop calculation:
    - direct calculation based on Feynman diagrams in light-cone gauge,
    - amenable to numerical implementation,
    - correct DGLAP and ERBL limits,
    - a number of additional interesting properties,
    - the calculation of Ji [Phys. Rev. D55, 7114-7125 (1997)] for unpolarised kernels exactly reproduced.
- The LO evolution equations valid in both the DGLAP and the ERBL regions can be recasted in a **DGLAP-like compact form**:

$$\frac{dF^{(-)}(x,\xi,\mu)}{d\ln\mu^2} = \frac{\alpha_s(\mu)}{4\pi}\int_x^{\infty}\frac{dy}{y}\mathcal{P}^{-,(0)}\left(\frac{x}{y},\kappa\right)F^{(-)}(y,\xi,\mu)\,, \qquad \kappa=\frac{\xi}{x}$$

$$\mathcal{P}^{-,(0)}(y,\kappa) = \boxed{\theta(1-y)\mathcal{P}_1^{-,(0)}(y,\kappa)} + \boxed{\theta(\kappa-1)\mathcal{P}_2^{-,(0)}(y,\kappa)}$$

Generalisation of the DGLAP kernels

Contribution from the ERBL region

Properties of the kernels

$$\mathcal{P}^{-,(0)}(y,\kappa) = \theta(1-y)\mathcal{P}_1^{-,(0)}(y,\kappa) + \theta(\kappa-1)\mathcal{P}_2^{-,(0)}(y,\kappa)$$

- Interesting properties:
  - **DGLAP** splitting function recovered:

$$\lim_{\kappa\to 0} \mathcal{P}^{-,(0)}(y,\kappa) = 2C_F \left[\frac{1+y^2}{1-y}\right]_+$$

  - **ERBL** kernel recovered:

$$\frac{1}{2u-1}\mathcal{P}^{-,(0)}\left(\frac{2t-1}{2u-1}, \frac{1}{2t-1}\right) = C_F\left[\theta(u-t)\left(\frac{t-1}{u}+\frac{1}{u-t}\right) - \theta(t-u)\left(\frac{t}{1-u}+\frac{1}{u-t}\right)\right]_+$$

  - **Continuity** at the crossover point $x=\xi$ $(\kappa = 1)$ guaranteed:

$$\lim_{\kappa\to 1} \mathcal{P}_1^{-,(0)}(y,\kappa) = 2C_F\left\{\left[\frac{1}{1-y}\right]_+ + \delta(1-y)\left[\frac{3}{2}-\ln(2)\right]\right\} \qquad \mathcal{P}_2^{(0),-}(y,\kappa) \propto (1-\kappa)$$

  - Cancellation of **spurious divergencies**:

$$\lim_{y\to\kappa^{-1}} (1-\kappa^2y^2)\mathcal{P}_1^{-,(0)}(y,\kappa) = -2C_F\frac{1+\kappa}{\kappa}\,,$$

$$\lim_{y\to\kappa^{-1}} (1-\kappa^2y^2)\mathcal{P}_2^{-,(0)}(y,\kappa) = 2C_F\frac{1+\kappa}{\kappa}\,.$$

Properties of the kernels

$$\mathcal{P}^{-,(0)}(y,\kappa) = \theta(1-y)\mathcal{P}_1^{-,(0)}(y,\kappa) + \theta(\kappa-1)\mathcal{P}_2^{-,(0)}(y,\kappa)$$

- Interesting properties:
  - Valence **sum rule** (polynomiality for the first moment) conserved:

$$\int_0^1 dx\, F^{(-)}(x,\xi) = \mathrm{FF} \quad \Longrightarrow \quad \int_0^1 dz\, \mathcal{P}_1^{-,(0)}\left(z, \frac{\xi}{yz}\right) + \int_0^{\xi/y} dz\, \mathcal{P}_2^{-,(0)}\left(z, \frac{\xi}{yz}\right) = 0$$

  - Direct computation of conformal moments reveals that Gegenbauer polynomials do diagonalise the LO evolution kernels with **ξ-independent kernels**:

$$\int_{-1}^1 \frac{dx}{\xi} C_n^{(3/2)}\left(\frac{x}{\xi}\right) \mathbb{V}_{\mathrm{NS}}^{(0)}\left(\frac{x}{\xi}, \frac{y}{\xi}\right) = V_n^{(0)} C_n^{(3/2)}\left(\frac{y}{\xi}\right)$$

  - **Singlet sector** (non-trivially) shares the same properties: $F^{(+)}(y,\xi) = \begin{pmatrix} F_q(y,\xi) - F_q(-y,\xi) \\ F_g(y,\xi) \end{pmatrix}$

$$\frac{dF^{(+)}(x,\xi,\mu)}{d\ln\mu^2} = \frac{\alpha_s(\mu)}{4\pi} \int_x^\infty \frac{dy}{y} \mathcal{P}^{+,(0)}\left(\frac{x}{y}, \kappa\right) F^{(+)}(y,\xi,\mu)$$

$$\mathcal{P}^{+,(0)}(y,\kappa) = \theta(1-y)\mathcal{P}_1^{+,(0)}(y,\kappa) + \theta(\kappa-1)\mathcal{P}_2^{+,(0)}(y,\kappa)$$

Numerics

- The evolution kernels for *unpolarised* evolution that we have recomputed are now implemented in **APFEL++** and available through **PARTONS** allowing for LO GPD evolution in momentum space.

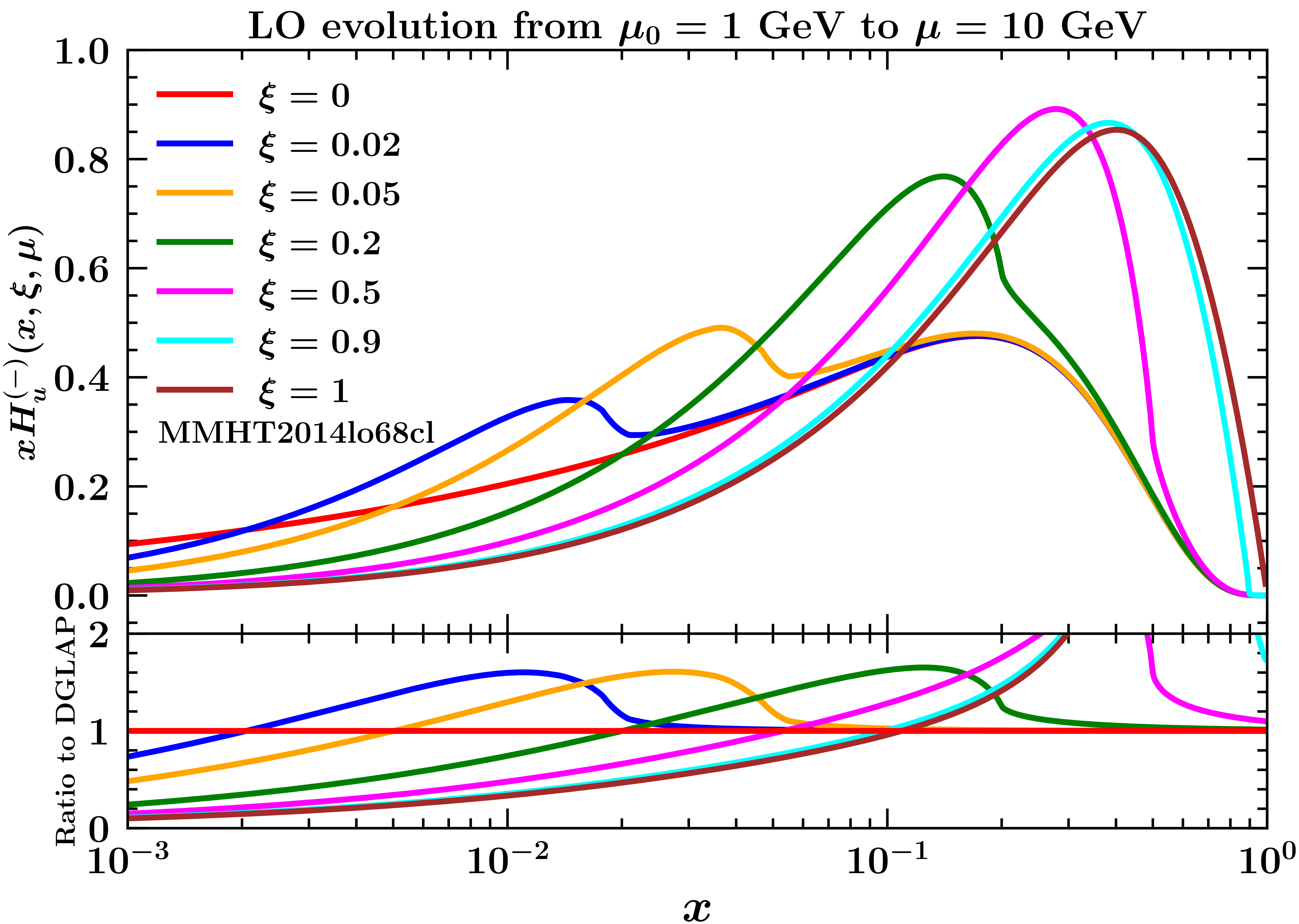
- The remarkable properties of the evolution kernels allow for a stable numerical implementation over the full range $\mathbf{0 \leq \xi \leq 1}$.

- Possibility to check numerically that both the DGLAP and ERBL limits are recovered.

- Numerical tests using the MMHT14 PDF set at LO as an initial-scale set of distributions evolved from 1 to 10 GeV in the **variable-flavour-number scheme**, *i.e.* accounting for heavy-quark threshold crossing.

- Yet to be done: direct comparison to Vinnikov's code:
  [A.V. Vinnikov, ePrint: hep-ph/0604248]
  - impossible to cover the full (ξ, x) plane,
  - numerical instabilities.

Numerics: evolution



- **DGLAP limit** reproduced within 10^{-5} relative accuracy,
- the evolution generates a cusp at $x = \xi$ leaving the distribution continuous.

Numerics: the ERBL limit

- The limit $\xi \to \mathbf{1}$ should reproduce the **ERBL equation**.

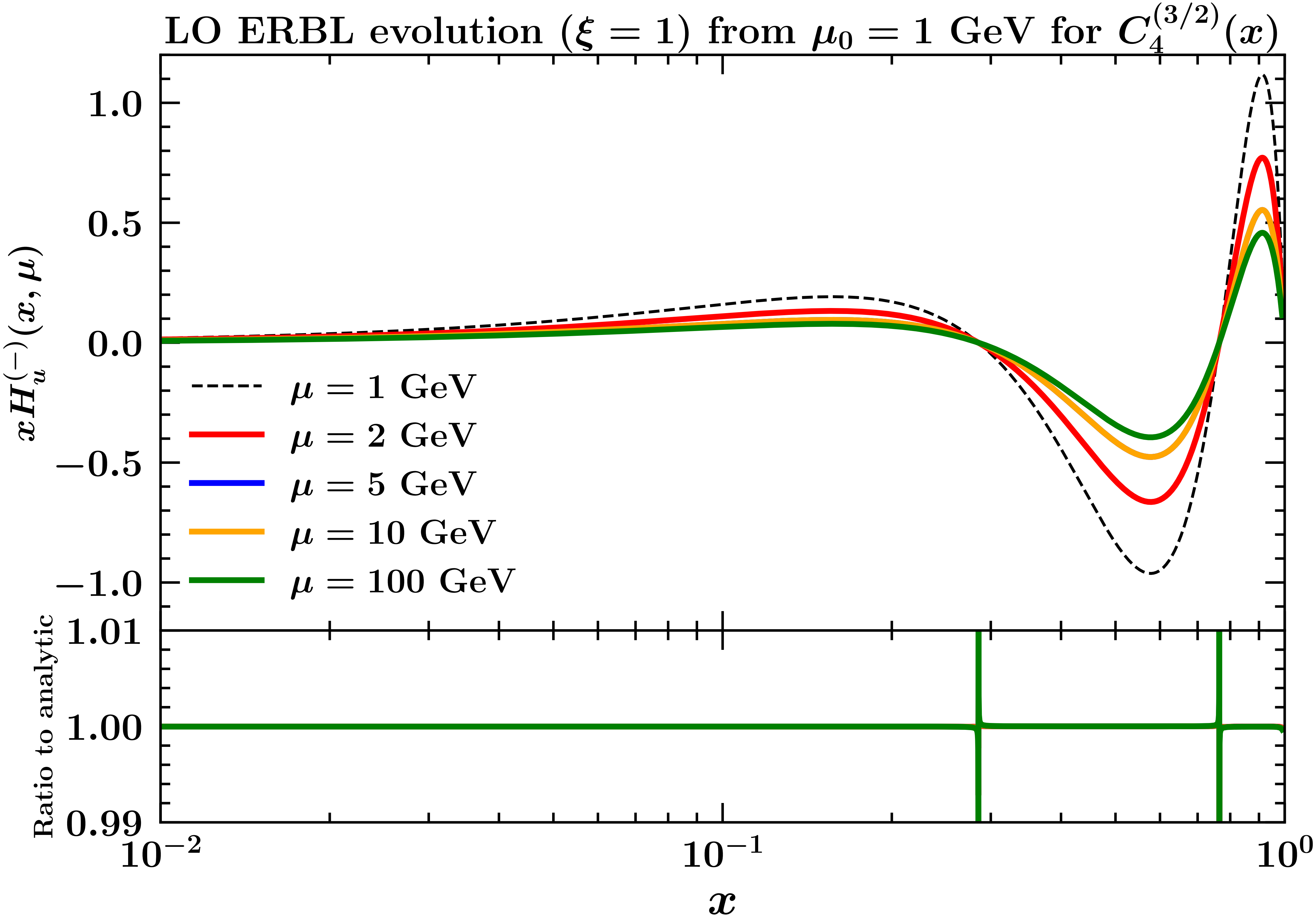
- It is well known that in this limit **Gegenbauer polynomials** decouple upon LO evolution, such that:

$$H^{(-)}(x,\mu_0) = (1-x^2)C_{2n}^{(3/2)}(x) \quad \Rightarrow \quad H^{(-)}(x,\mu) = \exp\left[\frac{V_{2n}}{4\pi}\int_{\mu_0}^{\mu} d\ln\mu^2\alpha_s(\mu)\right] H^{(-)}(x,\mu_0)$$

- where the kernels V_{2n} can be read off, for example, from [Brodsky, Lepage, *Phys.Rev.D* 22 (1980) 2157] or [Efremov, Radyushkin, *Phys.Lett.B* 94 (1980) 245-250].

- We have compared this expectation with the numerical results for GPD evolution with $\xi = 1$ using a Gegenbauer polynomial as an initial scale GPD.

Numerics: the ERBL limit



- **ERBL limit** reproduced within less than 10^{-5} relative accuracy,
- Same accuracy for **higher-degree** Gegenbauer polynomials.

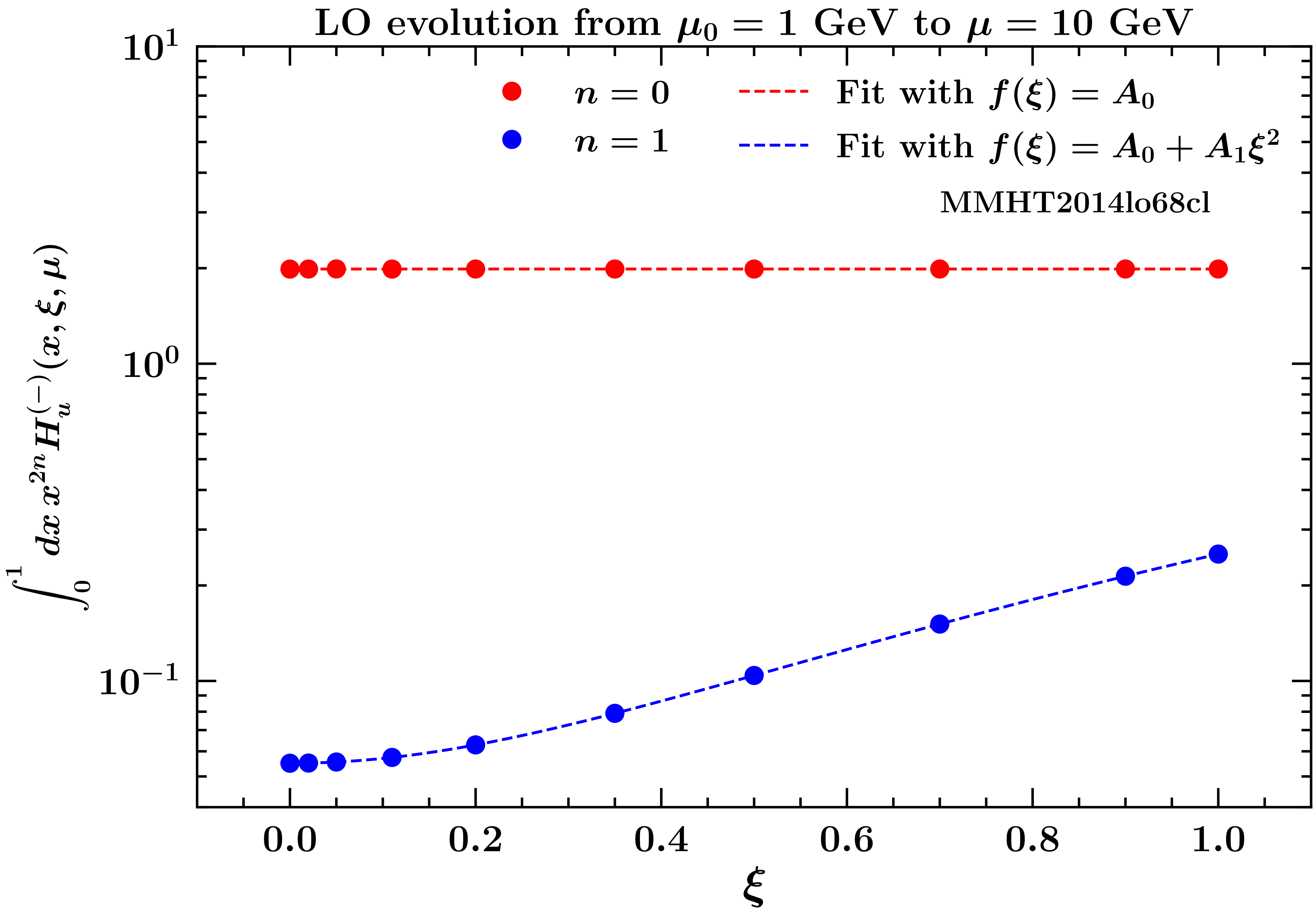
Numerics: polynomiality

- GPD evolution should preserve **polynomiality**.
  [Xiang-Dong Ji, *J.Phys.G* 24 (1998) 1181-1205] [A.V. Radyushkin, *Phys.Lett.B* 449 (1999) 81-88]
- Specifically, the following relation for the *even* Mellin moments must hold at all scales:

$$\int_0^1 dx\, x^{2n} H^{(-)}(x,\xi,\mu) = \sum_{k=0}^{n} \xi^{2k} A_k^{(n)}(\mu)$$

- For $n = 0$ polynomiality predicts that the integral is **constant** in ξ and since it is also connected with the FFs it has to be constant also in μ.
- For the other values of n one can just **fit** the behaviour in ξ and see if it follows the **expected law**.

Numerics: polynomiality



- First moment **constant** in ξ and equal to two (valence sum rule),
- Second moment **quadratic**:
  - including a linear term in the fit gives a coefficient that is very close to zero.

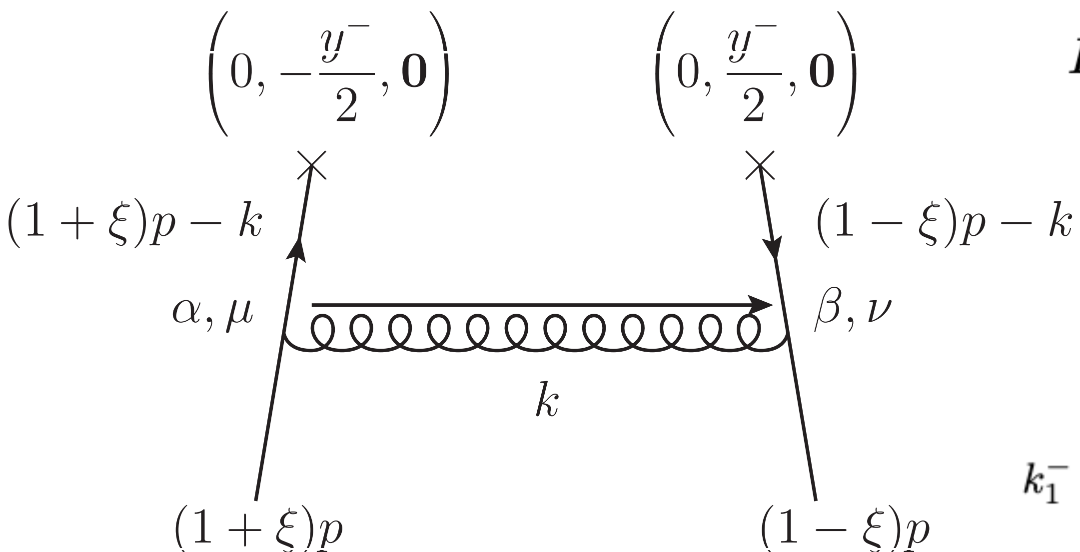
Conclusions and Outlook

- We have **revisited LO GPD evolution** in momentum space:
  - literature scrutinised.
  - *Ab-initio* calculation of the LO unpolarised splitting kernels based on Feynman diagrams.
  - Various analytical properties of the splitting kernels highlighted.
  - GPD evolution equation recasted in a DGLAP-like form convenient for implementation.
  - DGLAP and ERBL limits correctly recovered with excellent accuracy.
  - Evolution conserves polynomiality.
  - the code (APFEL++) is public and available through PARTONS
    https://github.com/vbertone/apfelxx
    http://partons.cea.fr/partons/doc/html/index.html
- **Next steps**:
  - **short term:** calculation/implementation of polarised (long. and trans. (?)) evolutions,
  - **longer term:** calculation and implementation of the NLO corrections.

Back up

On the calculation of $P_{\rm NS}$ at LO

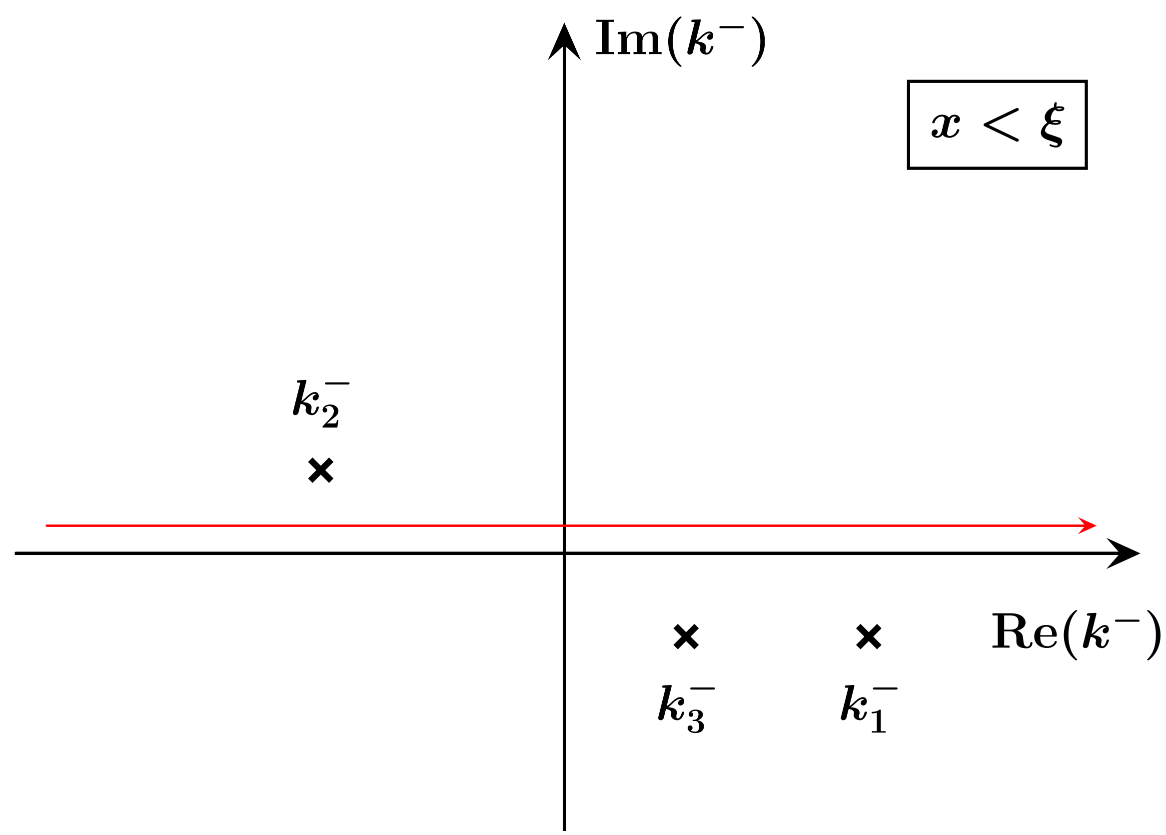
- In light cone gauge, there one single real diagram:

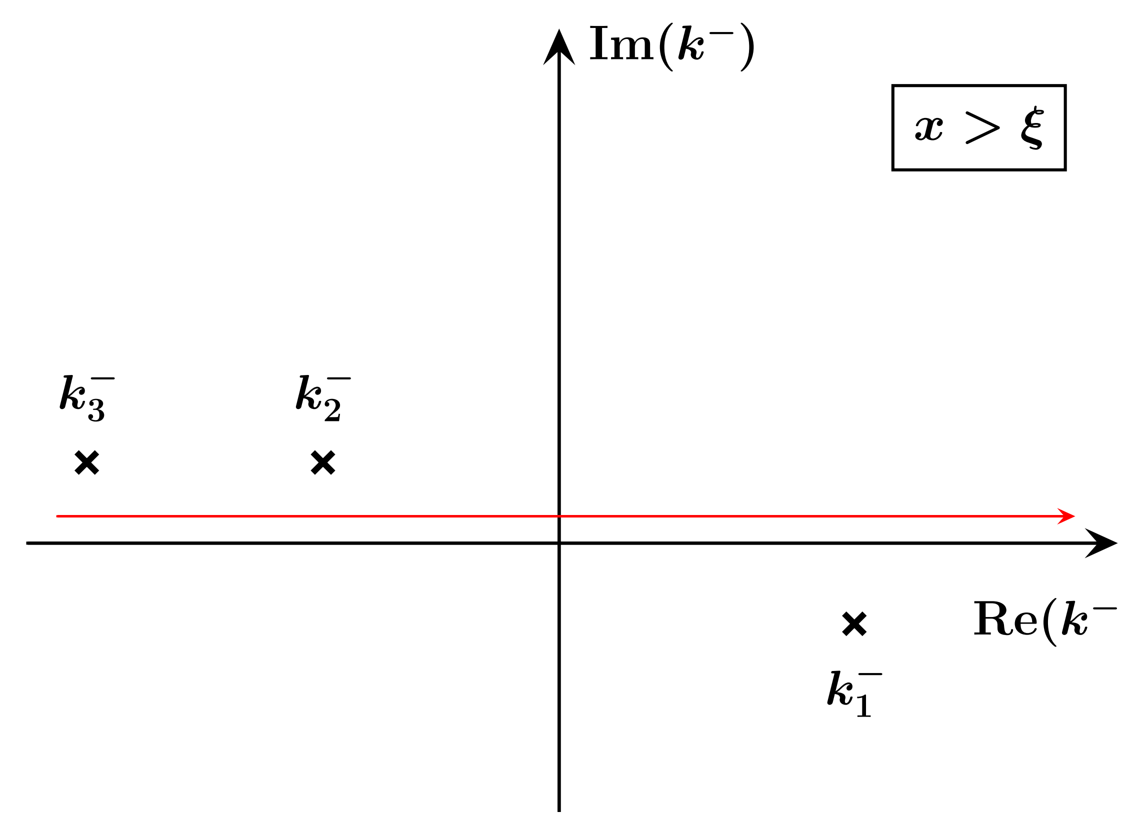


$$\hat{F}^{[1],(g^{\mu\nu})}_{(0),q/q}(x,\xi) = \sqrt{1-\xi^2}\,\frac{i}{2}C_F\frac{1}{(p^+)^2(1-x)(x^2-\xi^2)}\mu^{2\epsilon}\int\frac{d^{2-2\epsilon}\mathbf{k}_T}{(2\pi)^{2-2\epsilon}}\mathbf{k}_T^2 \times \int_{-\infty}^{+\infty}\frac{dk^-}{(k^- - k_1^-)(k^- - k_2^-)(k^- - k_3^-)}\,,$$

$$k_1^- = \frac{\mathbf{k}_T^2}{2(1-x)p^+} - i\varepsilon\,,\quad k_2^- = -\frac{\mathbf{k}_T^2}{2(x+\xi)p^+} + i(x+\xi)\varepsilon\,,\quad k_3^- = -\frac{\mathbf{k}_T^2}{2(x-\xi)p^+} + i(x-\xi)\varepsilon\,,$$

- Pole structure:





On the calculation of $P_{\rm NS}$ at LO

- The real diagram gives:

$$\hat{F}^{[1]}_{(0),q/q}(x,\xi) = \hat{F}^{[1],(n^\mu)}_{(0),q/q}(x,\xi) + \hat{F}^{[1],(g^{\mu\nu})}_{(0),q/q}(x,\xi)$$
$$= C_F \frac{\sqrt{1-\xi^2}}{\xi(1-x)}\left[\frac{(x+\xi)(1-x+2\xi)}{1+\xi} - \theta(x-\xi)\frac{(x-\xi)(1-x-2\xi)}{1-\xi}\right]\mu^{2\epsilon}S_\epsilon\int\frac{dk_T^2}{k_T^{2+2\epsilon}},$$

- The virtual contribution in light-cone gauge in off-forward kinematics was calculated by Curci, Furmansky, and Pertonzio back in 1980.

- Including the virtual diagram and isolating the UV divergence gives:

$$P^{[1]}_{qq}(x,\xi) = 2C_F\left\{\frac{1}{2\xi(1-x)}\left[\frac{(x+\xi)(1-x+2\xi)}{1+\xi} - \theta(x-\xi)\frac{(x-\xi)(1-x-2\xi)}{1-\xi}\right]\right.$$
$$\left. - \delta(1-x)\left[\int_0^1 dz\frac{1+z^2}{1-z} + \ln(|1-\xi^2|)\right]\right\}.$$

- No fully +-prescribed form because this expression *does not* integrate to zero.