An Update on Helicity Evolution at Small x

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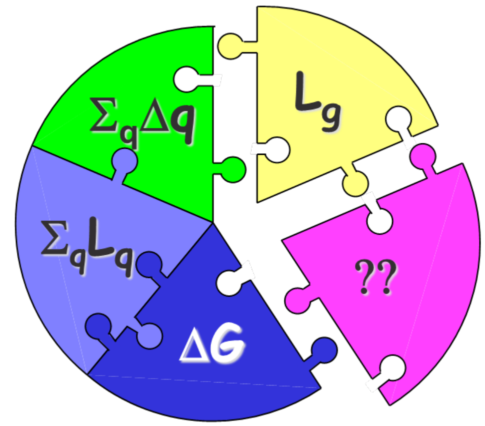
BASED ON WORK DONE WITH DAN PITONYAK AND MATT SIEVERT (2015-2018), WITH FLORIAN COUGOULIC (2019-2020), AND WITH JOSH TAWABUTR (2020)

Proton Spin Puzzle



- Helicity sum rule (Jaffe-Manohar form):

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

with the net quark and gluon spin

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

- The helicity parton distributions are

$$\Delta f(x, Q^2) \equiv f^+(x, Q^2) - f^-(x, Q^2)$$

with the net quark helicity distribution

$$\Delta\Sigma \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

- L_q and L_g are the quark and gluon orbital angular momenta

Our goal

- The goal is to constrain theoretically the amount of proton spin and OAM coming from small x.

- Any existing and future experiment probes the helicity distributions and OAM down to some x_{min} .

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx \, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx \, \Delta G(x, Q^2)$$

$$L_{q+\bar{q}}(Q^2) = \int_0^1 dx \, L_{q+\bar{q}}(x, Q^2) \qquad L_G(Q^2) = \int_0^1 dx \, L_G(x, Q^2)$$

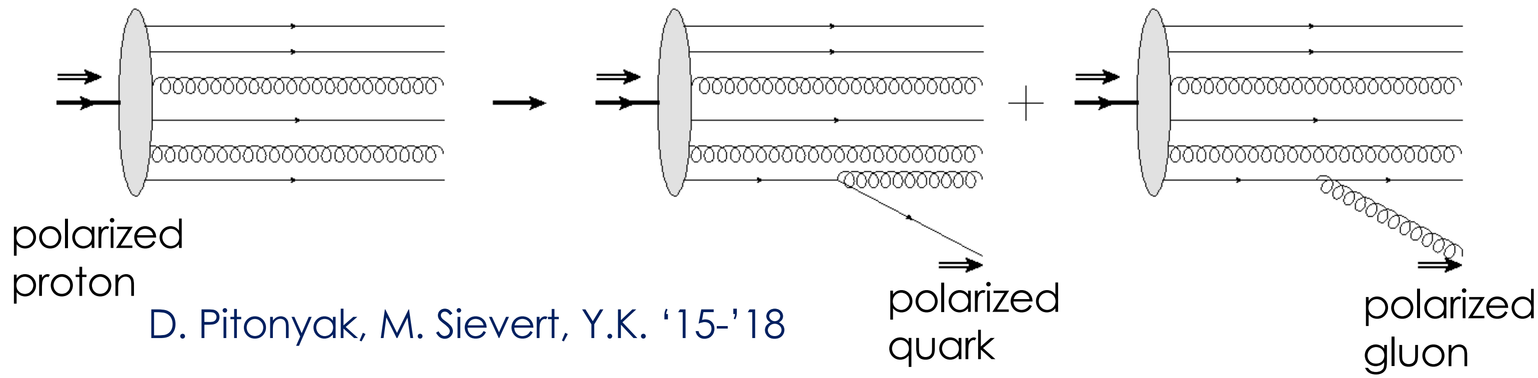
- At very small x (for the proton), saturation sets in: that region likely carries a negligible amount of proton spin. But what happens at larger (but still small) x?

Philosophy of our approach

- DGLAP equation evolves in Q^2, it does not evolve in x.
- Hence, DGLAP-based analyses (DSSV, NNPDF, standard JAM) cannot predict the x-dependence of PDFs.
- If we want to predict helicity PDFs at small x, we need a different evolution equation evolving in x.
- Such equations were constructed by D. Pitonyak, M. Sievert, and YK, 2015-2018 using an approach similar to the BK/JIMWLK evolution.

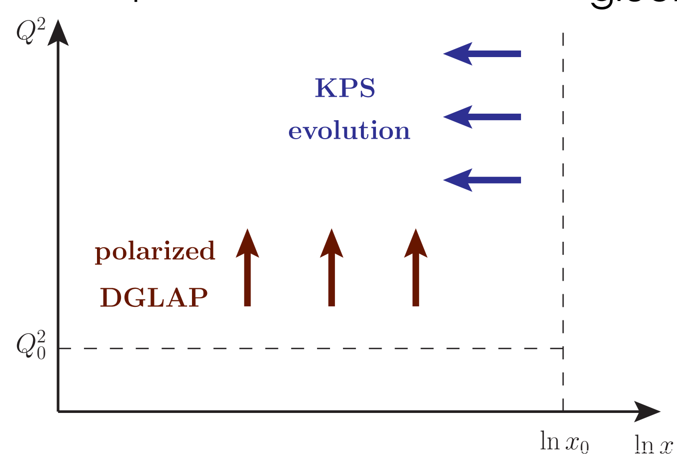
Helicity Evolution at Small x

- To understand how much of the proton's spin is at small x one can construct a helicity analogue of the BFKL equation:

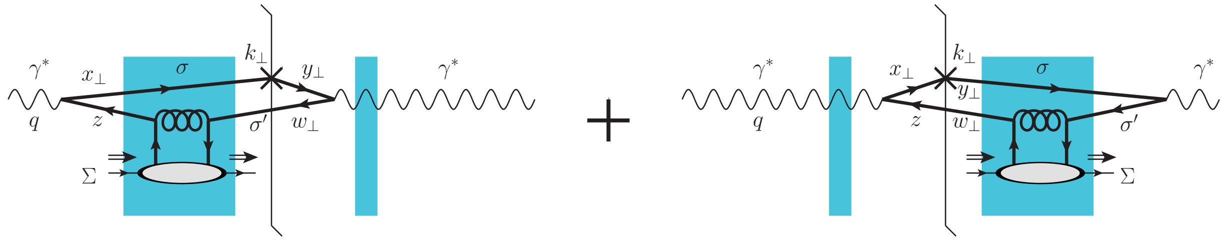


D. Pitonyak, M. Sievert, Y.K. '15-'18

- This new helicity evolution equation is subtle, since it must keep track of both quark and gluon helicities. (BFKL/BK/JIMWLK only have gluons at leading order.)



Quark Helicity Observables at Small x

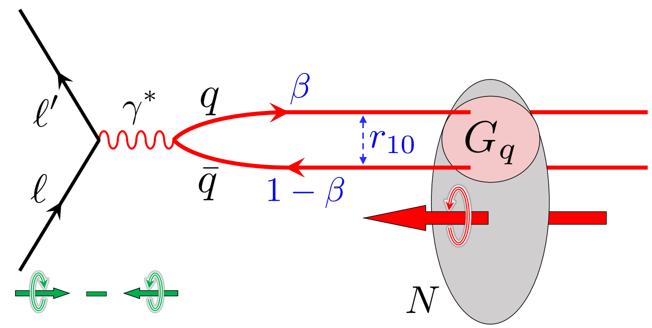


- One can show that the g_1 structure function and quark helicity PDF (Δq) and TMD at small-x can be expressed in terms of the polarized dipole amplitude (flavor singlet case):

$$g_1^S(x,Q^2) = \frac{N_c\,N_f}{2\,\pi^2\alpha_{EM}} \int\limits_{z_i}^{1} \frac{dz}{z^2(1-z)} \int dx_{01}^2 \left[\frac{1}{2} \sum_{\lambda\sigma\sigma'} |\psi^T_{\lambda\sigma\sigma'}|^2_{(x_{01}^2,z)} + \sum_{\sigma\sigma'} |\psi^L_{\sigma\sigma'}|^2_{(x_{01}^2,z)} \right] G(x_{01}^2,z),$$

$$\Delta q^S(x,Q^2) = \frac{N_c\,N_f}{2\pi^3} \int\limits_{z_i}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\frac{1}{zQ^2}} \frac{dx_{01}^2}{x_{01}^2}\, G(x_{01}^2,z), \;= \Delta\Sigma(x,Q^2)$$

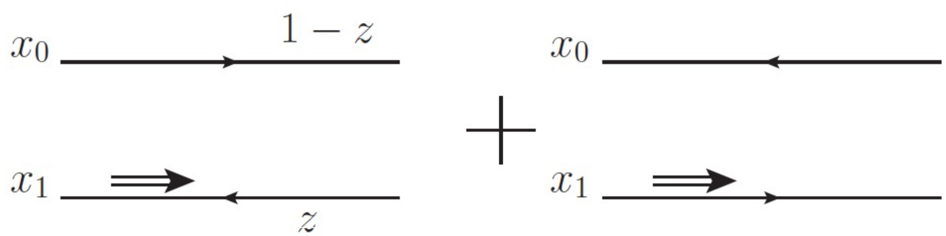
$$g_{1L}^S(x,k_T^2) = \frac{8\,N_c\,N_f}{(2\pi)^6} \int\limits_{z_i}^{1} \frac{dz}{z} \int d^2x_{01}\, d^2x_{0'1}\, e^{-i\underline{k}\cdot(\underline{x}_{01}-\underline{x}_{0'1})}\, \frac{\underline{x}_{01}\cdot\underline{x}_{0'1}}{x_{01}^2 x_{0'1}^2}\, G(x_{01}^2,z)$$



- Here s is cms energy squared, $z_i=\Lambda^2/s$, $\quad G(x_{01}^2,z) \equiv \int d^2b\, G_{10}(z)$

Polarized Dipole: non-eikonal small-x physics

- All flavor-singlet small-x helicity observables depend on one object, "polarized dipole amplitude":



$$G_{10}(z) \equiv \frac{1}{2N_c} \mathrm{Re} \left\langle\!\!\left\langle \mathrm{T\,tr}\left[V_{\underline{0}} V_{\underline{1}}^{pol\,\dagger}\right] + \mathrm{T\,tr}\left[V_{\underline{1}}^{pol} V_{\underline{0}}^{\dagger}\right] \right\rangle\!\!\right\rangle(z)$$

unpolarized quark

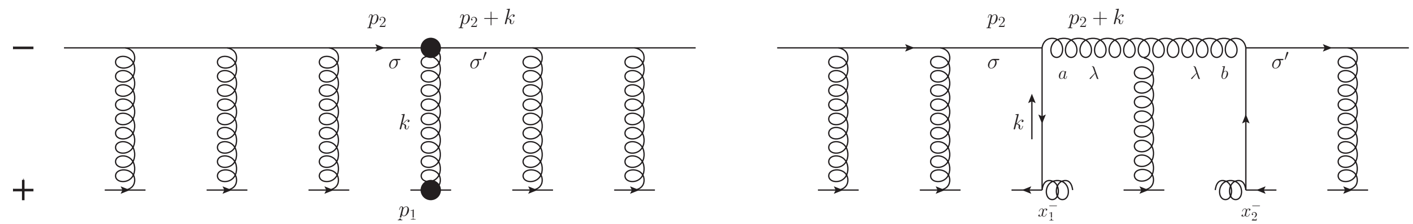
polarized quark: eikonal propagation,
non-eikonal spin-dependent interaction

$$V_{\underline{x}} \equiv \mathcal{P} \exp \left[ig \int\limits_{-\infty}^{\infty} dx^+ \, A^-(x^+, 0^-, \underline{x}) \right]$$

- Double brackets denote an object with energy suppression scaled out:

$$\left\langle\!\!\left\langle \mathcal{O} \right\rangle\!\!\right\rangle(z) \equiv zs \left\langle \mathcal{O} \right\rangle(z)$$

Polarized fundamental "Wilson line"



- In the end one arrives at (KPS '17; YK, Sievert, '18; cf. Chirilli '18; Altinoluk et al, '20)

$$V_{\underline{x}}^{pol} = \frac{igp_1^+}{s} \int\limits_{-\infty}^{\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, F^{12}(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$

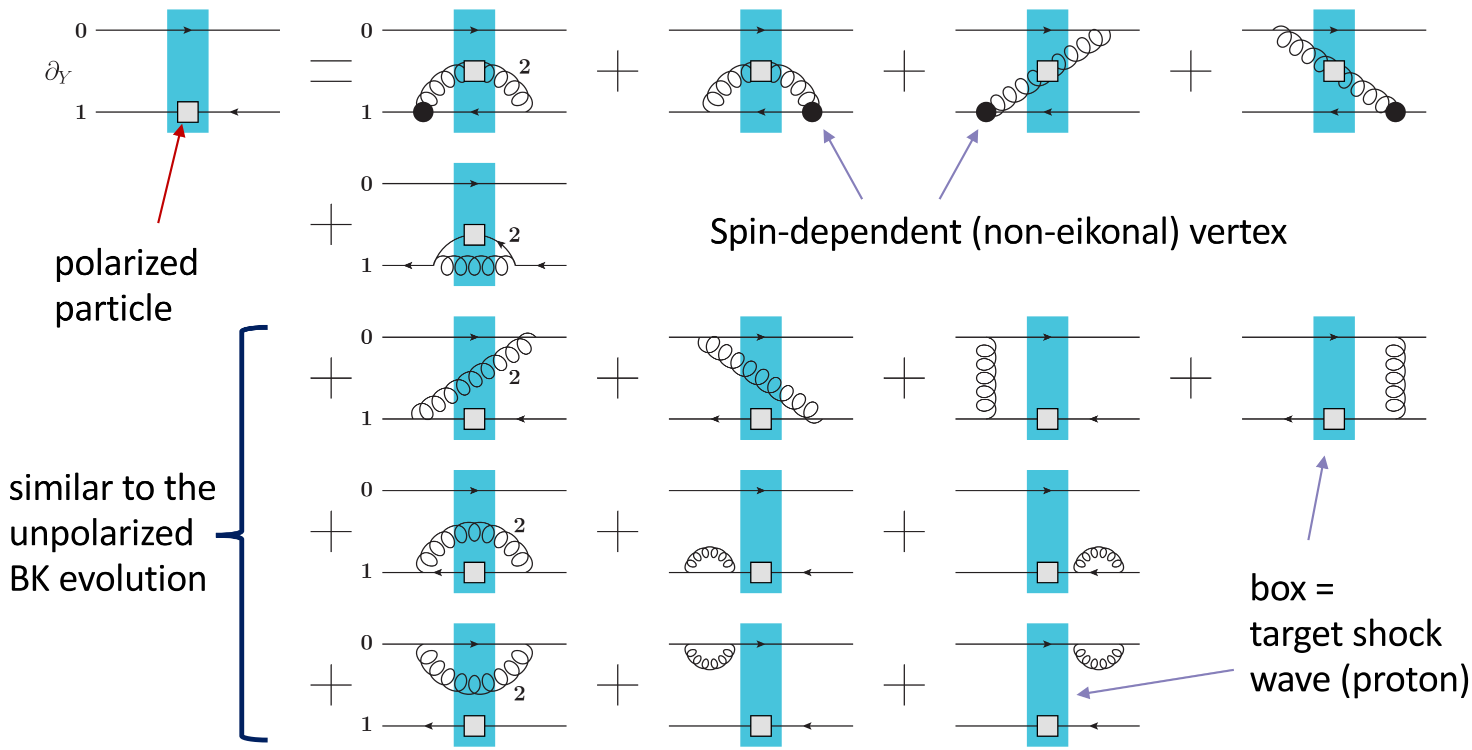
$$- \frac{g^2 \, p_1^+}{s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[+\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\frac{1}{2} \, \gamma^+ \, \gamma^5\right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty].$$

- We have employed an adjoint light-cone Wilson line $U_{\underline{x}}[b^-, a^-] = \mathcal{P} \exp \left[ig \int\limits_{a^-}^{b^-} dx^- \, \mathcal{A}^+(x^+ = 0, x^-, \underline{x}) \right]$
- Note the simple physical meaning of the first term:

$$-\vec{\mu} \cdot \vec{B} = -\mu_z \, B_z = \mu_z \, F^{12}$$

Evolution for Polarized Quark Dipole

One can construct an evolution equation for the polarized dipole:



Large-N_c Evolution

Resummation parameter: $\alpha_s \ln^2 \frac{1}{x}$

Double-logarithmic approximation (DLA)

- In the strict DLA limit and at large N_c, we get (here Γ is an auxiliary function we call the 'neighbor dipole amplitude') (KPS '15)

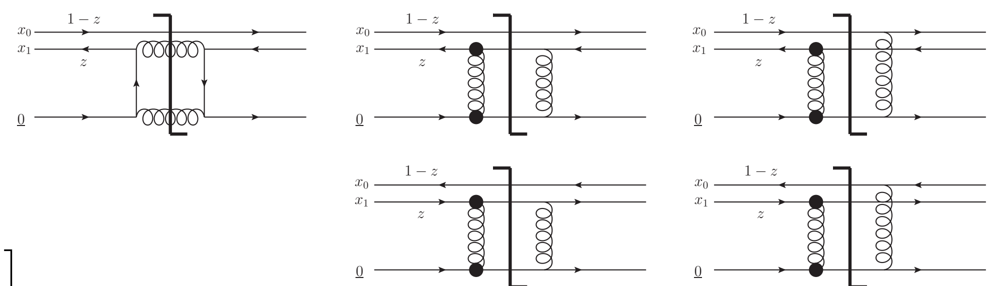
$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3\, G(x_{21}^2, z')\right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = \Gamma^{(0)}(x_{10}^2, x_{21}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left\{x_{10}^2, x_{21}^2 \frac{z'}{z''}\right\}} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3\, G(x_{32}^2, z'')\right]$$

- The initial conditions are given by the Born-level graphs

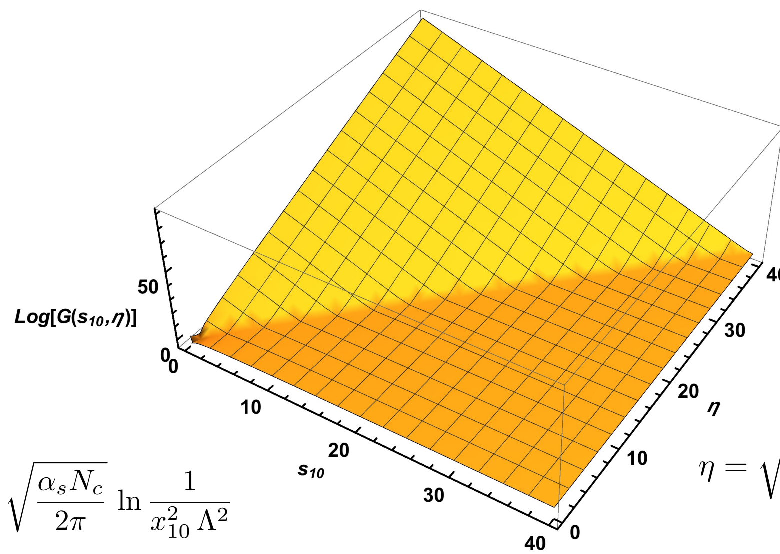
$$\Gamma^{(0)}(x_{10}^2, x_{21}^2, z) = G^{(0)}(x_{10}^2, z)$$

$$G^{(0)}(x_{10}^2, z) = \frac{\alpha_s^2 C_F}{N_c} \pi \left[C_F \ln \frac{zs}{\Lambda^2} - 2\, \ln(zs\, x_{10}^2)\right]$$



Quark Helicity at Small x

- These equations can be solved both numerically and analytically. (KPS '16-'17)



$$s_{10} = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{1}{x_{10}^2 \Lambda^2}$$

$$\eta = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{zs}{\Lambda^2}$$

- The small-x asymptotics of quark helicity is (at large N_c)

$$\Delta q(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \approx 2.31 \sqrt{\frac{\alpha_s \, N_c}{2\pi}}$$

Small-x Helicity Evolution at large N_c&N_f

- The resulting equations are (KPS ‘15)

$$
\begin{aligned}
Q_{10}(zs) = Q_{10}^{(0)}(zs) + & \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{1/(z's)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left\{ \frac{1}{2}\, \Gamma_{02,21}^{adj}(z') + \frac{1}{2}\, G_{21}^{adj}(z') + Q_{12}(z') - \bar{\Gamma}_{01,21}(z') \right\} \\
& + \frac{\alpha_s\, N_c}{4\, \pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z's)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2}\, Q_{21}(z'),
\end{aligned}
$$

$$
\begin{aligned}
G_{10}^{adj}(z) = G_{10}^{adj\,(0)}(z) + & \frac{\alpha_s\, N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z'\,s)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma_{10,21}^{adj}(z') + 3\, G_{21}^{adj}(z') \right] \\
& - \frac{\alpha_s\, N_f}{2\, \pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z'\,s)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2}\, \bar{\Gamma}_{02;21}(z'),
\end{aligned}
$$

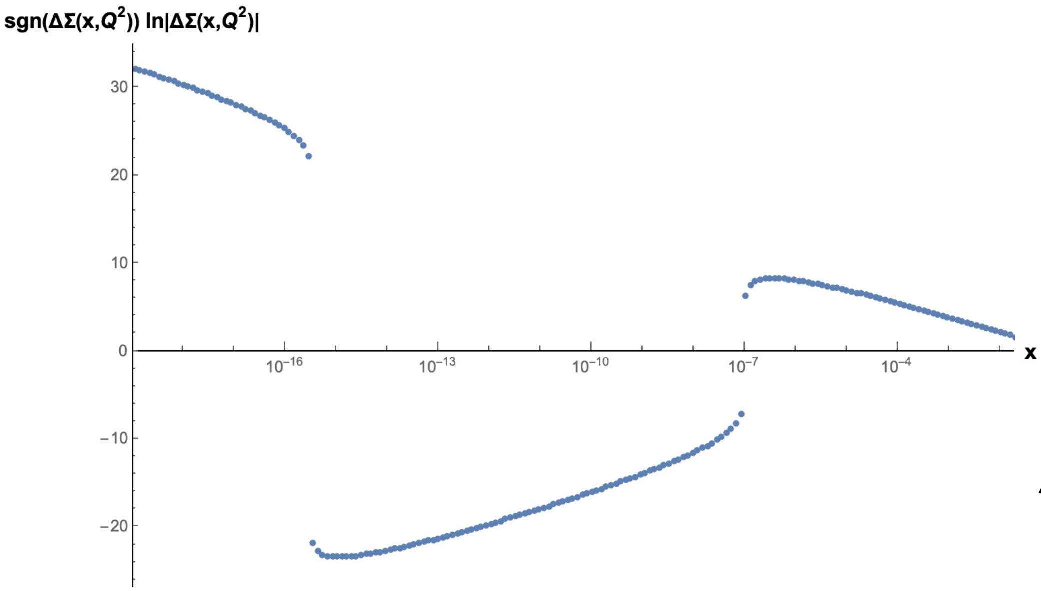
$$
\begin{aligned}
\Gamma_{10,21}^{adj}(z') = \Gamma_{10,21}^{adj\,(0)}(z') + & \frac{\alpha_s\, N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''\,s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma_{10,32}^{adj}(z'') + 3\, G_{32}^{adj}(z'') \right] \\
& - \frac{\alpha_s\, N_f}{2\, \pi} \int\limits_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''\,s)}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2}\, \bar{\Gamma}_{03;32}(z''),
\end{aligned}
$$

$$
\begin{aligned}
\bar{\Gamma}_{10,21}(z') = \bar{\Gamma}_{10,21}^{(0)}(z') + & \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left\{ \frac{1}{2}\, \Gamma_{03,32}^{adj}(z'') + \frac{1}{2}\, G_{32}^{adj}(z'') + Q_{32}(z'') - \bar{\Gamma}_{01,32}(z \right. \\
& + \frac{\alpha_s\, N_c}{4\, \pi} \int\limits_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''s)}^{x_{21}^2 z/z'} \frac{dx_{32}^2}{x_{32}^2}\, Q_{32}(z'').
\end{aligned}
$$

New since QCD Evolution 2019

Small-x Helicity Asymptotics at large N_c&N_f

- Large N_c&N_f equations can be solved only numerically, due to their complexity. This was done by Y. Tawabutr & YK in arXiv:2005.07285 [hep-ph].
- The solution exhibits an interesting qualitative change compared to large-N_c: it oscillates with ln(1/x)!



$$\alpha_h^q \approx 2.3 \sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

$$\omega_q \approx \frac{0.66(N_f/N_c)}{1 + 0.3795(N_f/N_c)} \sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

$$\Delta\Sigma(x, Q^2)\Big|_{\text{large-}N_c\&N_f} \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \cos\left[\omega_q \ln\frac{1}{x} + \varphi_q\right]$$

New since QCD Evolution 2019

Helicity JIMWLK

$$\alpha = A^+, \quad \beta = F^{12}$$

$$\langle \mathcal{O}_{\alpha,\beta,\psi,\bar\psi}\rangle_Y = \frac{\int \mathcal{D}\alpha \mathcal{D}\beta \mathcal{D}\psi \mathcal{D}\bar\psi \; \mathcal{O}_{\alpha,\beta,\psi,\bar\psi} \, \mathcal{W}_Y[\alpha,\beta,\psi,\bar\psi]}{\int \mathcal{D}\alpha \mathcal{D}\beta \mathcal{D}\psi \mathcal{D}\bar\psi \; \mathcal{W}_Y[\alpha,\beta,\psi,\bar\psi]}$$

- To go beyond the large-N_c and large-N_c&N_f limits need to write down a helicity analogue of JIMWLK evolution.
- This has been done recently (F. Cougoulic, YK, arXiv:1910.04268 [hep-ph], arXiv:2005.14688 [hep-ph]):

$$W_\tau[\alpha,\beta,\psi,\bar\psi] = W_\tau^{(0)}[\alpha,\beta,\psi,\bar\psi] + \int d^3\tau' \; \mathcal{K}_h[\tau,\tau'] \cdot W_{\tau'}[\alpha,\beta,\psi,\bar\psi]$$

with $\tau \equiv \{z, z\,X_\perp^2, z\,Y_\perp^2\}$ and the kernel

Life-time ordered!

$$\mathcal{K}_h[\tau,\tau'] = \frac{\alpha_s}{\pi^2}\int d^2 w_\perp \frac{\underline{X}'\cdot\underline{Y}'}{X'^2\,Y'^2}\;\theta^{(3)}(\tau-\tau')\theta\left(z'-\frac{\Lambda^2}{s}\right)\theta\left(X'^2-\frac{1}{z'\,s}\right)\theta\left(Y'^2-\frac{1}{z'\,s}\right)$$

$$\times\Big\{U_w^{ba}\,D^+_{x,a,<}\,D^+_{y,b,>} - \frac{1}{2}\,(D^+_{x,a,<}D^+_{y,a,<} + D^+_{x,a,>}\,D^+_{y,a,>})$$

Regular (spin-averaged) JIMWLK

$$+\frac{1}{2}U_w^{pol,ba}(D^+_{x,a,<}D^\perp_{y,b,>} + D^\perp_{x,a,<}D^+_{y,b,>})$$

Polarized gluon emissions

$$+\left(\frac{1}{2}\gamma^5\gamma^-\right)_{\beta\alpha}\frac{1}{2}\left((V_w^{pol})_{ij}D^{\bar\psi}_{x,j,\alpha,<}D^{\psi}_{y,i,\beta,>} + (V_w^{pol\,\dagger})_{ij}D^{\bar\psi}_{x,j,\alpha,>}D^{\psi}_{y,i,\beta,<}\right)\Big\}$$

Polarized quark emissions

New since QCD Evolution 2019

Helicity McLerran-Venugopalan Model

- The initial conditions for helicity JIMWLK are given by the helicity MV model, with the weight functional (F. Cougoulic, YK, 2005.14688 [hep-ph])

Standard MV

$$\mathcal{W}^{(0)}[\alpha,\beta,\psi,\bar{\psi}] \propto \exp\left\{-\int \mathrm{d}^2x_\perp dx^- \ \mathrm{tr}\left[(\nabla_\perp^2\alpha)^2\,\frac{\mu_+^2+\mu_-^2}{8\mu_+^2\,\mu_-^2} + \left(\langle p^+\rangle\beta\right)^2\frac{\mu_+^2+\mu_-^2}{2\mu_+^2\,\mu_-^2} + (\nabla_\perp^2\alpha)\langle p^+\rangle\beta\,\frac{\mu_+^2-\mu_-^2}{2\mu_+^2\,\mu_-^2}\right]\right\}$$

$$\times \exp\left\{\int \mathrm{d}^2x_\perp dx^- \ \langle p^+\rangle \left[\frac{\nu_+^2+\nu_-^2}{\nu_+^2\,\nu_-^2}\,\bar{\psi}\frac{1}{2}\gamma^+\nabla_\perp^2\psi - \frac{\nu_+^2-\nu_-^2}{\nu_+^2\,\nu_-^2}\,\bar{\psi}\frac{1}{2}\gamma^+\gamma^5\nabla_\perp^2\psi\right]\right\}.$$

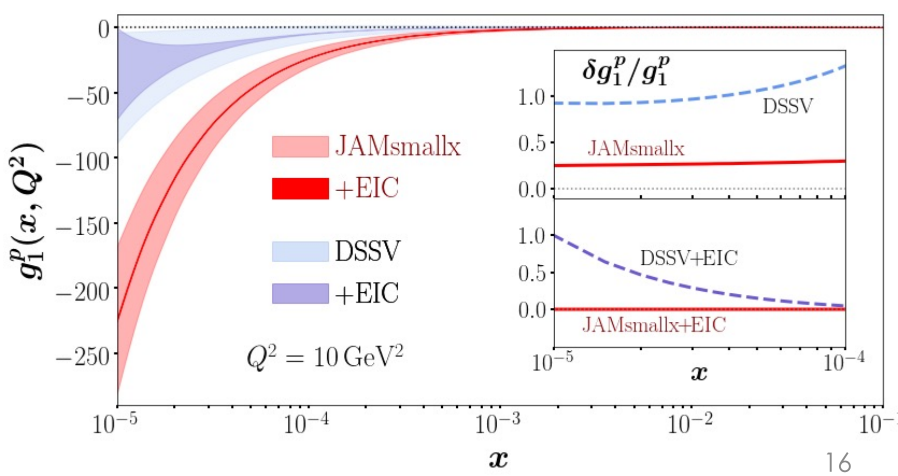
- Here $\alpha = A^+, \quad \beta = F^{12}$

- The parameters $\mu_\pm, \quad \nu_\pm$ are ~ "saturation scales" of helicity-plus and helicity-minus partons.

New results to be presented at this QCD Evolution

- Small-x helicity evolution equations have been constructed to the single-logarithmic order (SLA) + running coupling (Tarasov, Tawabutr, YK) – see the talk by Josh Tawabutr next.
- DLA: $\alpha_s \ln^2 \frac{1}{x}$
- SLA: $\alpha_s \ln \frac{1}{x}$
- Include exact LO DGLAP splitting functions. Unprecedented precision for small-x helicity evolution.

- DLA large-N_c equations with Born-inspired initial conditions have been used to describe the world polarized DIS data for x<0.1 (Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, YK, 2102.06159 [hep-ph] = JAMsmallx) – see the talk by Daniel Adamiak on Thursday.



Musings on gluon helicity

- Small-x evolution for helicity mixes two operators (in the DLA):

$$\vec{\mu} \cdot \vec{B} \sim F^{12} \quad \text{and} \quad \bar{\psi}\frac{1}{2}\gamma^+\gamma^5\psi \sim \Delta q \sim \Delta\Sigma(x, Q^2)$$

- See for instance the polarized fundamental Wilson line:

$$V_{\underline{x}}^{pol} = \frac{igp_1^+}{s} \int\limits_{-\infty}^{\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, F^{12}(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$

$$- \frac{g^2 \, p_1^+}{s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[+\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\frac{1}{2}\gamma^+ \gamma^5\right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty].$$

- This is <u>different</u> from polarized DGLAP evolution in Q^2, which mixes

$$\Delta G \sim \epsilon^{ij} F^{+i} F^{+j} \quad \text{and} \quad \bar{\psi}\frac{1}{2}\gamma^+\gamma^5\psi \sim \Delta q \sim \Delta\Sigma(x, Q^2)$$

- This difference may be related to the difficulties in defining the gluon helicity PDF. Or small-x evolution simply avoids it. Any thoughts from the experts?

Musings on gluon helicity

- At SLA (subleading to DLA), the two gluon operators

$$\vec{\mu} \cdot \vec{B} \sim F^{12} \quad \text{and} \quad \Delta G \sim \epsilon^{ij} F^{+i} F^{+j}$$

appear to mix with each other (KPS '17; Cougoulic, YK, Tarasov, Tawabutr, in preparation).

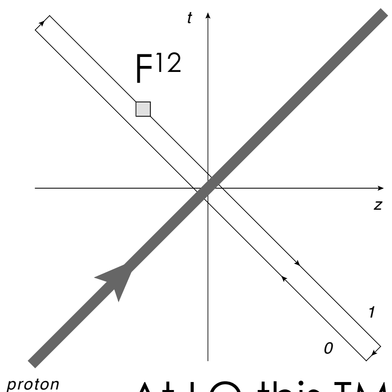
- The above operators involved in small-x evolution,

$$\vec{\mu} \cdot \vec{B} \sim F^{12} \text{ and } \bar{\psi} \frac{1}{2} \gamma^+ \gamma^5 \psi \sim \Delta q \sim \Delta\Sigma(x, Q^2)$$

were independently confirmed by Chirilli (2018, 2020), Altinoluk et al (2020), and by Tarasov (unpublished).

Musings on gluon helicity

- Certainly, $\vec{\mu}\cdot\vec{B}\sim F^{12}$ is a local operator and has no collinear limit…
- However, one can make a TMD out of it (cf. Chirilli '20):



$$g_{1L}^{G\,true\,dip}(x,k_T^2)=\frac{-4}{g(2\pi)^3}\,P^+\int d^2\xi\,d^2\zeta\;e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})}$$
$$\times\left\{\left\langle \mathrm{tr}\left[i\left(\int d\zeta^-\,V_{\underline{\zeta}}[-\infty,\zeta^-]\,F^{12}(\zeta)\,V_{\underline{\zeta}}[\zeta^-,+\infty]\right)\;V_{\underline{\xi}}[+\infty,-\infty]\right]\right\rangle\right.$$
$$\left.+\left\langle \mathrm{tr}\left[V_{\underline{\xi}}[-\infty,+\infty]\;(-i)\left(\int d\zeta^-\,V_{\underline{\zeta}}[+\infty,\zeta^-]\,F^{12}(\zeta)\,V_{\underline{\zeta}}[\zeta^-,-\infty]\right)\right]\right\rangle\right\}$$

- At LO this TMD scales as $g_{1L}^{G\,true\,dip}(x,k_T^2)\propto(2\pi)^2\,\delta^2(\underline{k})\,\ln\frac{s}{\Lambda^2}-\frac{4\pi}{k_T^2}$
- The k_T integral of this TMD is zero, but only due to the delta-function, the TMD itself looks "healthy" for non-zero k_T. Easy to confuse with the standard gluon helicity TMD. $\sim\epsilon^{ij}F^{+i}F^{+j}$

Beyond Helicity

- Similar small-x analysis was applied to transversity (Sievert, YK, 2018), yielding

$$h_{1T}^{NS}(x, k_T^2) \sim h_{1T}^{\perp\,NS}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_t^q} \quad \text{with} \quad \alpha_t^q = -1 + 2\sqrt{\frac{\alpha_s\, C_F}{\pi}}$$

- and to the quark Sivers function, obtaining the spin-dependent odderon contribution (see the talk by M. Gabriel Santiago later this morning), such that

$$f_{1T}^{\perp q}(x, k_T^2) \sim \frac{1}{x}$$

with the non-perturbative accuracy, it seems (cf. gluon Sivers by Boer et al, 2016).

Conclusions

Small-x helicity evolution has now been studied at large-N_c and large-N_c&N_f at DLA, yielding the small-x asymptotics of $\Delta\Sigma(x, Q^2)$ and $\Delta G(x, Q^2)$.

Beyond those limits: helicity JIMWLK is written down, yet to be solved.

Helicity evolution has been derived at DLA+SLA: $\alpha_s \ln^2(1/x) \quad + \quad \alpha_s \ln(1/x)$

First successful fit of polarized world DIS data for x<0.1 done using small-x helicity evolution (JAMsmallx).

Theoretical challenge: the interplay between F^{12} and $\epsilon^{ij} F^{+i} F^{+j}$ needs to be understood.

Backup Slides

Gluon Helicity at Small x

Yu.K., D. Pitonyak, M. Sievert, arXiv:1706.04236 [nucl-th]

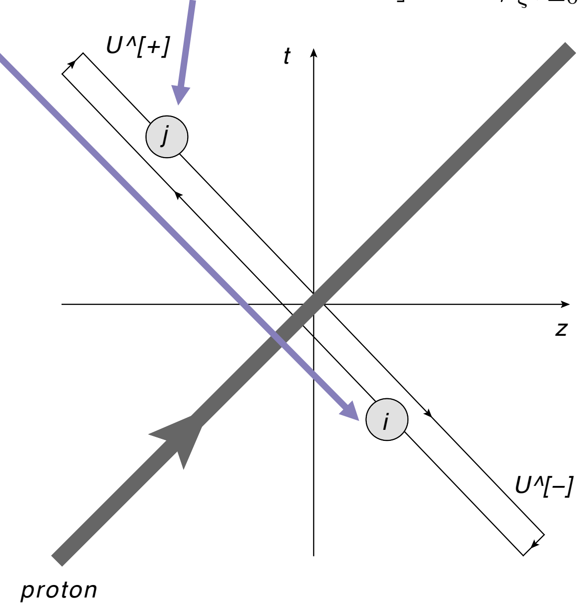
Dipole Gluon Helicity TMD

- Now let us repeat the calculation for gluon helicity TMDs.

- We start with the definition of the gluon dipole helicity TMD:

$$g_1^G(x, k_T^2) = \frac{-2i\, S_L}{x\, P^+} \int \frac{d\xi^- \, d^2\xi}{(2\pi)^3} \, e^{ixP^+ \xi^- - i\underline{k}\cdot\underline{\xi}} \left\langle P, S_L | \epsilon_T^{ij} \, \mathrm{tr} \left[F^{+i}(0)\, \mathcal{U}^{[+]\dagger}[0,\xi] \, F^{+j}(\xi)\, \mathcal{U}^{[-]}[\xi, 0] \right] | P, S_L \right\rangle_{\xi^+=0}$$

- Here $U^{[+]}$ and $U^{[-]}$ are future and past Wilson line staples (hence the name `dipole' TMD, F. Dominguez et al '11 – looks like a dipole scattering on a proton):



Dipole Gluon Helicity TMD

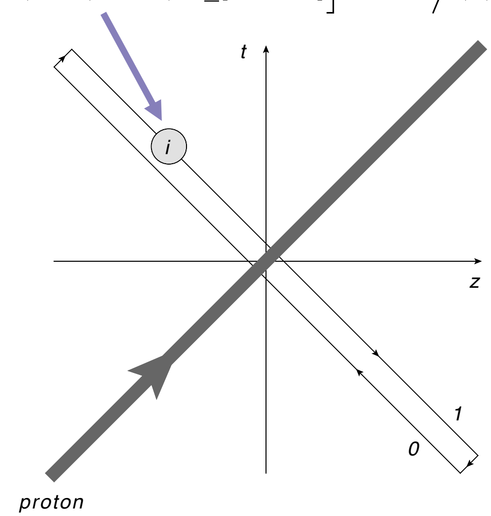
- At small x, the definition of dipole gluon helicity TMD can be massaged into

$$g_1^{G\,dip}(x, k_T^2) = \frac{8i\, N_c\, S_L}{g^2 (2\pi)^3} \int d^2 x_{10}\, e^{i \underline{k} \cdot \underline{x}_{10}}\, k_\perp^i\, \epsilon_T^{ij} \left[\int d^2 b_{10}\, G_{10}^j (zs = \frac{Q^2}{x}) \right]$$

- Here we obtain a new operator, which is a transverse vector (written here in A⁻=0 gauge, cf. Hatta et al, 2016):

$$G_{10}^i(z) \equiv \frac{1}{4N_c} \int\limits_{-\infty}^{\infty} dx^- \left\langle \mathrm{tr} \left[V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^-] \, (-ig) \, \tilde{A}^i(x^-, \underline{x}) \, V_{\underline{1}}[x^-, \infty] \right] + \mathrm{c.c.} \right\rangle (z)$$

- Note that $k_\perp^i \, \epsilon_T^{ij}$ can be thought of as a transverse curl acting on $G_{10}^i(z)$ and not just on $\tilde{A}^i(x^-, \underline{x})$ -- different from the polarized dipole amplitude!



Dipole TMD vs dipole amplitude

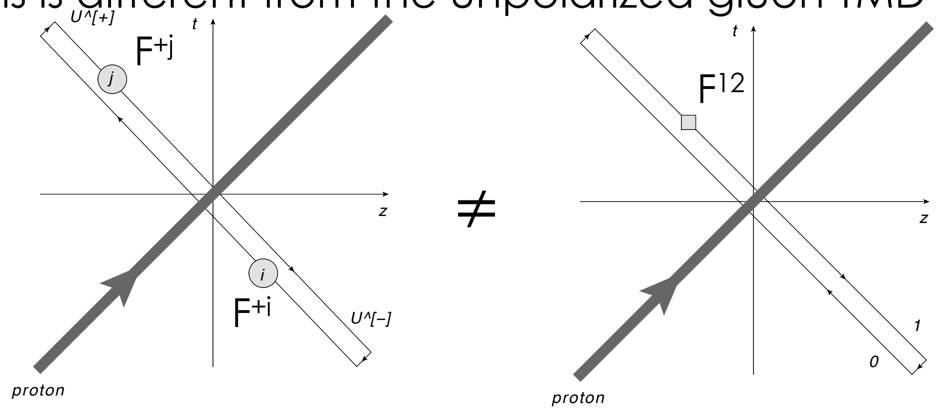
- Note that the operator for the dipole gluon helicity TMD

$$G^i_{10}(z) \equiv \frac{1}{4N_c} \int\limits_{-\infty}^{\infty} dx^- \left\langle \mathrm{tr}\left[V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^-] \, (-ig) \, \tilde{A}^i(x^-, \underline{x}) \, V_{\underline{1}}[x^-, \infty]\right] + \text{c.c.}\right\rangle (z)$$

is different from the polarized dipole amplitude

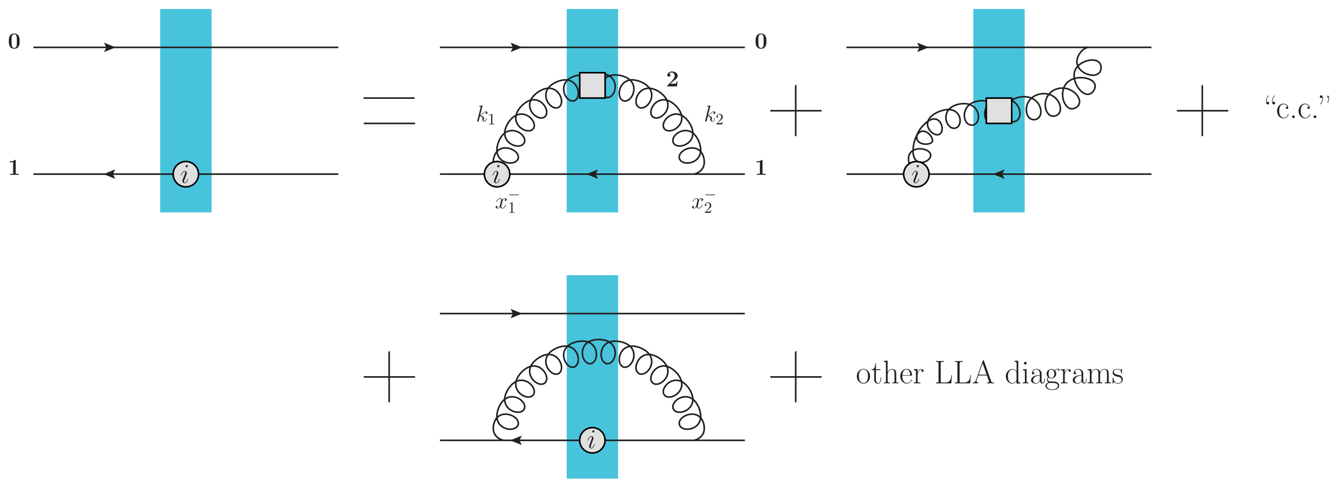
$$G_{10}(z) \equiv \frac{1}{4N_c} \int\limits_{-\infty}^{\infty} dx^- \left\langle \mathrm{tr}\left[V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^-] \, (-ig) \, \overbrace{\underline{\nabla} \times \underline{\tilde{A}}}^{\sim F^{12}}(x^-, \underline{x}) \, V_{\underline{1}}[x^-, \infty]\right] + \text{c.c.}\right\rangle (z)$$

- We conclude that the dipole gluon helicity TMD does not depend on the polarized dipole amplitude! (Hence the ‘dipole’ name may not even be valid for such TMDs.) This is different from the unpolarized gluon TMD case.



Evolution Equation

- To construct evolution equation for the operator G^i governing the gluon helicity TMD we resum similar (to the quark case) diagrams:



Large-N_c Evolution: Equations

- This results in the following evolution equations:

$$
\begin{aligned}
G^i_{10}(zs) = G^{i\,(0)}_{10}(zs) &+ \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{21})^j_\perp}{x_{21}^2} \left[\Gamma^{gen}_{20\,,\,21}(z's) + G_{21}(z's) \right] \\
&- \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{20})^j_\perp}{x_{20}^2} \left[\Gamma^{gen}_{20\,,\,21}(z's) + \Gamma^{gen}_{21\,,\,20}(z's) \right] \\
&+ \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[G^i_{12}(z's) - \Gamma^i_{10\,,\,21}(z's) \right]
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^i_{10\,21}(z's) = G^{i\,(0)}_{10}(z's) &+ \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int d^2 x_3 \, \ln \frac{1}{x_{31}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{31})^j_\perp}{x_{31}^2} \left[\Gamma^{gen}_{30\,,\,31}(z''s) + G_{31}(z''s) \right] \\
&- \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int d^2 x_3 \, \ln \frac{1}{x_{31}\Lambda} \, \frac{\epsilon_T^{ij} \, (x_{30})^j_\perp}{x_{30}^2} \left[\Gamma^{gen}_{30\,,\,31}(z''s) + \Gamma^{gen}_{31\,,\,30}(z''s) \right] \\
&+ \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min\left[x_{10}^2\,,\,x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{31}^2}{x_{31}^2} \left[G^i_{13}(z''s) - \Gamma^i_{10\,,\,31}(z''s) \right].
\end{aligned}
$$

Large-N_c Evolution Equations: Solution

- These equations can be solved in the asymptotic high-energy region yielding the small-x gluon helicity intercept

$$\alpha_h^G = \frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \approx 1.88 \sqrt{\frac{\alpha_s \, N_c}{2\pi}}$$

- We obtain the small-x asymptotics of the gluon helicity distributions:

$$\Delta G(x, Q^2) \sim g_{1L}^{G\,dip}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}}}$$