High-energy OPE for polarized DIS

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- Review of high-energy OPE
- High-energy evolution equation in the shock-wave background
- High-energy OPE with sub-eikonal corrections
- Evolution equations of sub-eikonal corrections
- Conclusions and Outlook

Based on:

G.A.C. JHEP 01 (2019) 118 arXiv: 1807.11435 [hep-ph]

G.A.C. arXiv: 2101.12744 [hep-ph] (under review for JHEP publication)

DIS structure functions

DIS differential cross-section

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{Mq^4}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu}$$

Hadronic tensor $\qquad$ S^μ spin of the target

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) + \left(P_\mu - q_\mu\frac{q\cdot P}{q^2}\right)\left(P_\nu - q_\nu\frac{q\cdot P}{q^2}\right)\frac{F_2(x,Q^2)}{P\cdot q}$$
$$+ i\,\varepsilon_{\mu\nu\lambda\sigma}\, q^\lambda S^\sigma \frac{M}{P\cdot q}\, g_1(x,Q^2) + i\,\varepsilon_{\mu\nu\lambda\sigma} q^\lambda \left(S^\sigma - P^\sigma\frac{q\cdot S}{q\cdot P}\right)\frac{M}{P\cdot q}\, g_2(x,Q^2)$$

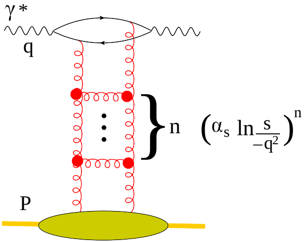
$$W_{\mu\nu} = \frac{1}{\pi}\mathrm{Im}T_{\mu\nu}$$

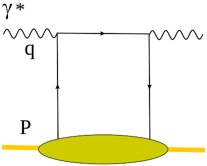
$$T_{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle P,S|\mathrm{T}\{j_\mu(x)j_\nu(0)\}|P,S\rangle$$

Leading Log Approximation in scatt. process at high energy

electron-proton/nucleus Deep Inelastic Scattering (DIS) $s = (q+P)^2$

$$\langle P|\mathrm{T}\, j^{\mu}(x) j^{\nu}(y)|P\rangle$$

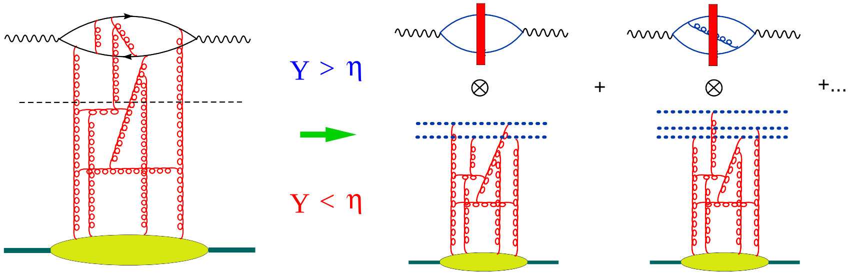




energy suppressed

- BFKL resum $\left(\alpha_s \ln \frac{s}{-q^2}\right)^n$
- Dynamics is linear and it describes proliferation of gluons
  $\Rightarrow$ Violation of Unitarity

remind: High-energy OPE for DIS



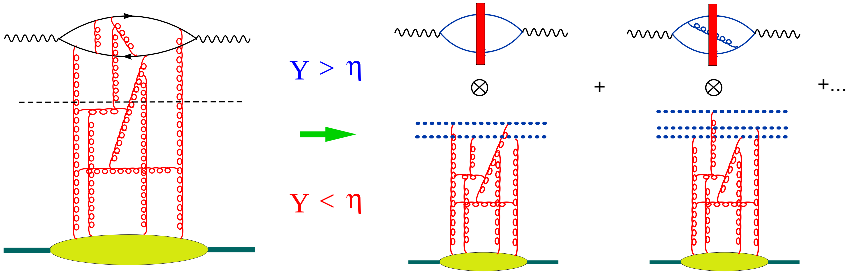
factorization scale: rapidity η

Rapidity Y > η - coefficient function (“impact factor”)

Rapidity Y < η - matrix elements of (light-like) Wilson lines with rapidity divergence cut by η

$$U_x^\eta \;=\; \mathrm{Pexp}\Big[ig\int_{-\infty}^{\infty} dx^+ A_+^\eta(x_+, x_\perp)\Big]$$

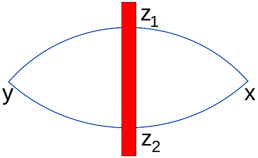
remind: High-energy OPE for DIS



The high-energy operator product expansion is

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\, \mathcal{I}^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$
$$+ \int d^2z_1 d^2z_2 d^2z_3\, \mathcal{I}^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

DIS at Leading Log Approximation at high-energy



$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\, \mathcal{I}^{\rm LO}_{\mu\nu}(z_1,z_2,x,y)\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

- Calculate LO Imapct factor: $\mathcal{I}^{\rm LO}_{\mu\nu}(z_1,z_2,x,y)$
- Calculate evolution of matrix element $\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$: BK/JIMWLK equation
- Solve the evolution equation with initial condition: GBW/MV model
- Convolute the solution of the evolution equation with the impact factor

Symmetries of the Impact factors

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\, \mathcal{I}^{\rm LO}_{\mu\nu}(z_1,z_2,x,y)\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

Electromagnetic gauge invariance

$$\partial^x_\mu\, \mathrm{T}\{j^\mu(x)j^\nu(y)\} = 0 \quad \Rightarrow \quad \partial^x_\mu\, \mathcal{I}^{\mu\nu}(x,y;z_1,z_2) = 0$$

$SL(2,C)$ Möbius invariance (inversion $x^\mu \to \frac{x^\mu}{x^2}$)

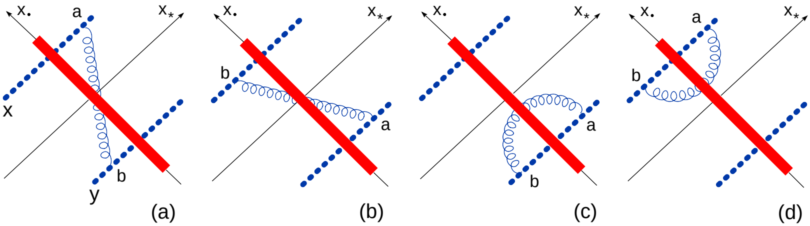
$$\int d^2z_1 d^2z_2\, \mathcal{I}^{\mu\nu}(x,y;z_1,z_2) \overset{\rm inv.}{=} \int d^2z_1 d^2z_2\, \mathcal{I}^{\mu\nu}(x,y;z_1,z_2)$$

At NLO conformal invariance is restored through the composite conformal Wilson-line operators (I. Balitsky and G.A.C. 2010)

Leading order evolution equation

$$\frac{d}{d\eta}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} = K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \ldots \quad \Rightarrow$$

$$\frac{d}{d\eta}\langle\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}} = \langle K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}}$$



$x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad$ $x_* = \sqrt{\frac{s}{2}}x^+$ $\qquad$ $x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

Non-linear evolution equation

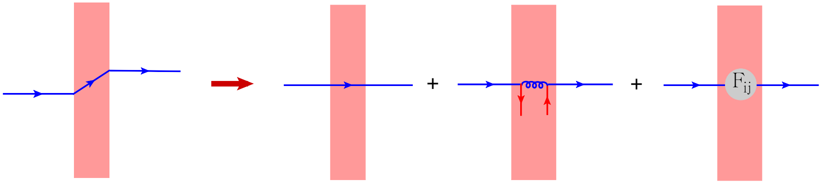
$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) = \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\,(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z)+\hat{\mathcal{U}}(z,y)-\hat{\mathcal{U}}(x,y)-\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

- LLA for DIS in pQCD $\Rightarrow$ BFKL
  - (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$): Ladder type of diagrams: proliferation of gluons.

- LLA for DIS in semi-classical-QCD $\Rightarrow$ BK/JIMWLK eqn
  - background field method: describes recombination process.

Quark propagator with sub-eikonal corrections



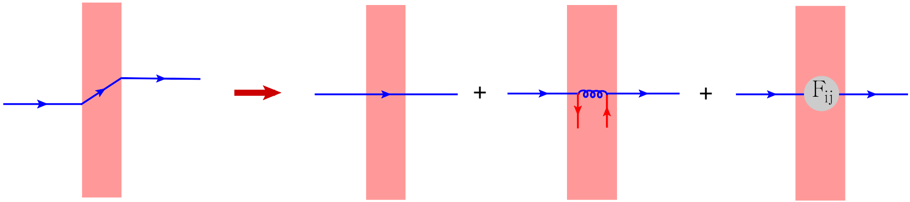
Quark propagator for g_1 structure function

$$\langle \mathrm{T}\{\psi(x)\bar\psi(y)\}\rangle_{A,\psi,\bar\psi} = -\frac{1}{2\pi^3 x_*^2 y_*^2}\int \frac{d^2z}{(\mathcal{Z}+i\epsilon)^3}\left(\frac{2}{s}x_*\not{p}_1 + \not{X}_\perp\right)$$
$$\times\left\{ i\not{p}_2 U(z_\perp) + \frac{\mathcal{Z}}{8}\left(\frac{1}{s}\not{p}_2\gamma^5\mathcal{F}(z_\perp) + \gamma^\mu_\perp Q(z_\perp)\gamma^\perp_\mu\right)\right\}\left(\frac{2}{s}y_*\not{p}_1 + \not{Y}_\perp\right)$$

$$\mathcal{Z} \equiv \frac{(x-z)^2_\perp}{x_*} - \frac{(y-z)^2_\perp}{y_*} - \frac{4}{s}(x_\bullet - y_\bullet)$$

$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^- \qquad\qquad \not{p}_1 = \sqrt{\frac{s}{2}}\gamma^- = \sqrt{\frac{s}{2}}\gamma_+$

Quark propagator with sub-eikonal corrections



Quark propagator for g_1 structure function

$$\langle \mathrm{T}\{\psi(x)\bar{\psi}(y)\}\rangle_{A,\psi,\bar{\psi}} = -\frac{1}{2\pi^3 x_*^2 y_*^2}\int \frac{d^2 z}{(\mathcal{Z}+i\epsilon)^3}\Big(\frac{2}{s}x_*\not{p}_1 + \not{X}_\perp\Big)$$
$$\times\Big\{ i\not{p}_2 U(z_\perp) + \frac{\mathcal{Z}}{8}\Big(\frac{1}{s}\not{p}_2\gamma^5\mathcal{F}(z_\perp) + \gamma_\perp^\mu Q(z_\perp)\gamma_\mu^\perp\Big)\Big\}\Big(\frac{2}{s}y_*\not{p}_1 + \not{Y}_\perp\Big)$$

- Eikonal interaction
- Sub-eikonal interaction

Quark propagator with sub-eikonal corrections

$$\langle \mathrm{T}\{\psi(x)\bar{\psi}(y)\}\rangle_{A,\psi,\bar{\psi}} = -\frac{1}{2\pi^3 x_*^2 y_*^2}\int \frac{d^2z}{(\mathcal{Z}+i\epsilon)^3}\Big(\frac{2}{s}x_*\not{p}_1 + \not{X}_\perp\Big)$$
$$\times\Big\{i\not{p}_2 U(z_\perp) + \frac{\mathcal{Z}}{8}\Big(\frac{1}{s}\not{p}_2\gamma^5\mathcal{F}(z_\perp) + \gamma^\mu_\perp Q(z_\perp)\gamma^\perp_\mu\Big)\Big\}\Big(\frac{2}{s}y_*\not{p}_1 + \not{Y}_\perp\Big)$$

$$Q^{\alpha\beta}_{ij}(x_\perp) \equiv g^2\int_{-\infty}^{+\infty}dz_*\int_{-\infty}^{z_*}dz'_*\Big([\infty p_1, z_*]_x t^a\psi^\alpha(z_*, x_\perp)[z_*, z'_*]^{ab}_x\bar{\psi}^\beta(z'_*, x_\perp)t^b[z'_*, -\infty p_1]_x\Big)_{ij}$$

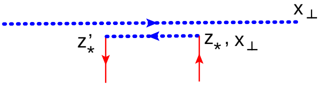
$$\mathcal{F}(z_\perp) \equiv ig\frac{s}{2}\int_{-\infty}^{+\infty}dz_*[\infty p_1, z_*]_z\,\epsilon^{ij}F_{ij}\big(z_*, z_{1\perp}\big)[z_*, -\infty p_1]_z\,.$$

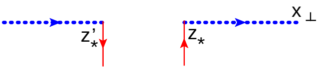
$x_* = \sqrt{\frac{s}{2}}x^+\,,\quad x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad$ $\not{p}_1 = \sqrt{\frac{s}{2}}\gamma^-\,,\quad \not{p}_2 = \sqrt{\frac{s}{2}}\gamma^+$

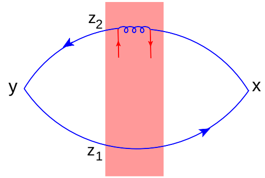
Quark propagator with sub-eikonal corrections

$$Q_{ij}^{\alpha\beta}(x_\perp) \equiv g^2\int_{-\infty}^{+\infty}dz_*\int_{-\infty}^{z_*}dz'_*\Big([\infty p_1, z_*]_x t^a\psi^\alpha(z_*,x_\perp)[z_*,z'_*]_x^{ab}\bar\psi^\beta(z'_*,x_\perp)t^b[z'_*,-\infty p_1]_x\Big)_{ij}$$

$$Q_{ij}^{\alpha\beta}(x_\perp) = -g^2\int_{-\infty}^{+\infty}dz_*\int_{-\infty}^{z_*}dz'_*\Big[\frac{1}{2}\,U_x^{ij}\,\bar\psi^\alpha(z'_*,x_\perp)[z'_*,z_*]_x\psi^\beta(z_*,x_\perp) + \frac{1}{2N_c}\big([\infty p_1,z_*]_x\psi^\beta(z_*,x_\perp)\bar\psi^\alpha(z'_*,x_\perp)[z'_*,-\infty]\big)_{ij}\Big]$$







$$i\,\bar{\psi}(z'_*, z_{2\perp})\gamma^{\perp}_{\rho}\hat{Y}_2\gamma^{\nu}\hat{Y}_1\not{p}_2\hat{X}_1\gamma^{\mu}\hat{X}_2\gamma^{\rho}_{\perp}\psi(z_*, z_{2\perp})$$
$$= \frac{8}{s}\bar{\psi}(z'_*, z_{2\perp})\, i\not{p}_1\psi(z_*, z_{2\perp})I^{\mu\nu}_1(x_*, y_*; z_{1\perp}, z_{2\perp})$$
$$-\frac{8}{s}\bar{\psi}(z'_*, z_{2\perp})\gamma^5\not{p}_1\psi(z_*, z_{2\perp})I^{\mu\nu}_5(x_*, y_*; z_{1\perp}, z_{2\perp}) + O(\lambda^{-1})$$

$X^{\mu}_i = \frac{2}{s}x_* p^{\mu}_1 + X^{\mu}_{i\perp}$ $\quad X^{\mu}_{i\perp} = x^{\mu}_{\perp} - z^{\mu}_{i\perp}$ $\quad i = 1, 2$

$x_* = \sqrt{\frac{s}{2}}x^+\,,$ $\quad x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad \not{p}_1 = \sqrt{\frac{s}{2}}\gamma^-\,,$ $\quad \not{p}_2 = \sqrt{\frac{s}{2}}\gamma^+$

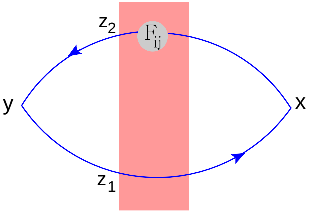
$$
\begin{aligned}
& i\,\bar\psi(z'_*, z_{2\perp})\gamma^\perp_\rho \hat Y_2\gamma^\nu \hat Y_1 \not{p}_2 \hat X_1\gamma^\mu \hat X_2 \gamma^\rho_\perp \psi(z_*, z_{2\perp}) \\
&= \frac{8}{s}\bar\psi(z'_*, z_{2\perp})\, i\not{p}_1 \psi(z_*, z_{2\perp}) I_1^{\mu\nu}(x_*, y_*; z_{1\perp}, z_{2\perp}) \\
&\quad -\frac{8}{s}\bar\psi(z'_*, z_{2\perp})\gamma^5 \not{p}_1 \psi(z_*, z_{2\perp}) I_5^{\mu\nu}(x_*, y_*; z_{1\perp}, z_{2\perp}) + O(\lambda^{-1})
\end{aligned}
$$

$$
I_1^{\mu\nu}(x,y;z_1,z_2) = \frac{1}{2}x_*^2 y_*^2 \frac{\partial^2}{\partial x_\mu \partial y_\nu}\left(\mathcal{Z}_1\mathcal{Z}_2 - z^2_{12\perp}\frac{(x-y)^2}{x_* y_*}\right)
$$

$$
I_5^{\mu\nu}(x,y;z_1,z_2) = \left(x_*\partial_x^\mu - p_2^\mu\right)\left(y_*\partial_y^\nu - p_2^\nu\right)\left[(\vec Y_1\times\vec Y_2)X_1\cdot X_2 - (\vec X_1\times\vec X_2)Y_1\cdot Y_2\right]
$$

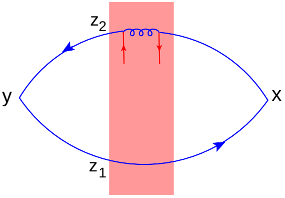
$X_i^\mu = \frac{2}{s}x_* p_1^\mu + X_{i\perp}^\mu \qquad X_{i\perp}^\mu = x_\perp^\mu - z_{i\perp}^\mu \qquad i = 1,2$

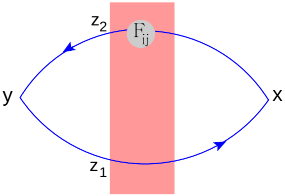
$\vec x\times\vec y = \epsilon^{ij}x_i y_j$



$$
\begin{aligned}
&\mathrm{tr}\{\not{X}_1 \not{p}_2 \not{Y}_1 \gamma^\nu \not{Y}_2 \not{p}_2 \sigma_\perp^{\alpha\beta} \not{X}_2 \gamma^\mu\} \\
&= 8i\epsilon^{\alpha\beta}\left(x_* \partial_x^\mu - p_2^\mu\right)\left(y_* \partial_y^\nu - p_2^\nu\right)\left[(\vec{X}_1 \times \vec{X}_2) Y_1 \cdot Y_2 - (\vec{Y}_1 \times \vec{Y}_2) X_1 \cdot X_2\right] \\
&= 8\, i\, \epsilon^{\alpha\beta}\, I_{\mathcal{F}}^{\mu\nu}
\end{aligned}
$$

$$I_{\mathcal{F}}^{\mu\nu} = -I_5^{\mu\nu}$$





$$
\begin{aligned}
&\mathrm{T}\{\bar{\hat{\psi}}(x)\gamma^\mu\psi(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} \\
&= \int dz_1 dz_2\, \mathcal{I}^{\mu\nu}_{LO}(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\} + \frac{\mathcal{Z}_2}{8\,s}\Big(\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{\mathcal{Q}}_{1z_2}\} + \mathrm{Tr}\{\hat{U}_{z_1}\hat{\mathcal{Q}}^\dagger_{1z_2}\}\Big)\Big] \\
&+\frac{1}{s}\int d^2z_1 d^2z_2\, \mathcal{I}^{\mu\nu}_5(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\big(\hat{\mathcal{Q}}_{5z_2} + \hat{\mathcal{F}}_{z_2}\big)\hat{U}^\dagger_{z_1}\} + \mathrm{Tr}\{\big(\hat{\mathcal{Q}}^\dagger_{5z_2} + \hat{\mathcal{F}}^\dagger_{z_2}\big)\hat{U}_{z_1}\}\Big] \\
&+\mathcal{O}(\alpha_s) + \mathcal{O}(\lambda^{-2})
\end{aligned}
$$

Sub-eikonal Impact Factors

$$\mathcal{I}_1^{\mu\nu}(x,y;z_1,z_2) = \frac{\mathcal{Z}_2}{8}\mathcal{I}_{LO}^{\mu\nu} = \frac{1}{4\pi^6 x_*^4 y_*^4}\frac{I_1^{\mu\nu}(x,y;z_1,z_2)}{[\mathcal{Z}_1+i\epsilon]^3[\mathcal{Z}_2+i\epsilon]^2}$$

$$\mathcal{I}_5^{\mu\nu}(x,y;z_1,z_2) = -\frac{1}{4\pi^6 x_*^4 y_*^4}\frac{I_5^{\mu\nu}(x,y;z_1,z_2)}{[\mathcal{Z}_1+i\epsilon]^3[\mathcal{Z}_2+i\epsilon]^2}$$

$$I_1^{\mu\nu}(x,y;z_1,z_2) = \frac{1}{2}x_*^2 y_*^2 \frac{\partial^2}{\partial x_\mu \partial y_\nu}\left(\mathcal{Z}_1\mathcal{Z}_2 - z_{12\perp}^2\frac{(x-y)^2}{x_* y_*}\right)$$

$$I_5^{\mu\nu}(x,y;z_1,z_2) = \left(x_*\partial_x^\mu - p_2^\mu\right)\left(y_*\partial_y^\nu - p_2^\nu\right)\left[(\vec{Y}_1\times\vec{Y}_2)X_1\cdot X_2 - (\vec{X}_1\times\vec{X}_2)Y_1\cdot Y_2\right]$$

$X_i^\mu = \frac{2}{s}x_* p_1^\mu + X_{i\perp}^\mu \qquad X_{i\perp}^\mu = x_\perp^\mu - z_{i\perp}^\mu \qquad i=1,2$

$\vec{x}\times\vec{y} = \epsilon^{ij}x_i y_j$

Symmetry of the sub-eikonal Impact Factors

Sub-eikonal Impact Factors are electromagnetic gauge invariant

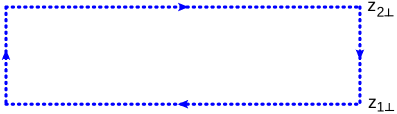
$$\partial_\mu\,\mathcal{I}_1^{\mu\nu}(x,y;z_1,z_2)=0$$
$$\partial_\mu\,\mathcal{I}_5^{\mu\nu}(x,y;z_1,z_2)=0$$

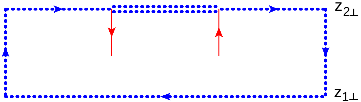
and $SL(2,C)$ Möbius invariant (inv. $x^\mu \to \frac{x^\mu}{x^2}$)

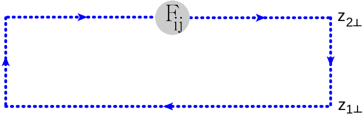
$$\int d^2z_2 d^2z_2\,\mathcal{I}_1^{\mu\nu}(x,y;z_1,z_2) \stackrel{\text{inv.}}{=} \int d^2z_2 d^2z_2\,\mathcal{I}_1^{\mu\nu}(x,y;z_1,z_2)$$
$$\int d^2z_2 d^2z_2\,\mathcal{I}_5^{\mu\nu}(x,y;z_1,z_2) \stackrel{\text{inv.}}{=} \int d^2z_2 d^2z_2\,\mathcal{I}_5^{\mu\nu}(x,y;z_1,z_2)$$

High-Energy OPE with sub-eikonal corrections

$$
\begin{aligned}
&\mathrm{T}\{\bar{\hat{\psi}}(x)\gamma^\mu\psi(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}\\
&=\int dz_1 dz_2\,\mathcal{I}^{\mu\nu}_{LO}(z_{1\perp},z_{2\perp};x,y)\Big[\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}+\frac{\mathcal{Z}_2}{8\,s}\Big(\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{\mathcal{Q}}_{1z_2}\}+\mathrm{Tr}\{\hat{U}_{z_1}\hat{\mathcal{Q}}^\dagger_{1z_2}\}\Big)\Big]\\
&+\frac{1}{s}\int d^2z_1 d^2z_2\,\mathcal{I}^{\mu\nu}_5(z_{1\perp},z_{2\perp};x,y)\Big[\mathrm{Tr}\{\big(\hat{\mathcal{Q}}_{5z_2}+\hat{\mathcal{F}}_{z_2}\big)\hat{U}^\dagger_{z_1}\}+\mathrm{Tr}\{\big(\hat{\mathcal{Q}}^\dagger_{5z_2}+\hat{\mathcal{F}}^\dagger_{z_2}\big)\hat{U}_{z_1}\}\Big]\\
&+\mathcal{O}(\alpha_s)+\mathcal{O}(\lambda^{-2})
\end{aligned}
$$







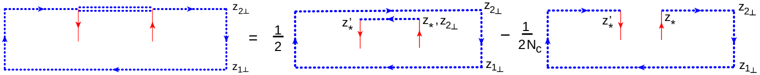
Fierz identity

$\not{p}_1 = \sqrt{\frac{s}{2}}\gamma_+ = \sqrt{\frac{s}{2}}\gamma^-$

$$Q_{1x} = -g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_*\, \bar\psi(z'_*, x_\perp)\, i\not{p}_1 [z'_*, z_*]_x \psi(z_*, x_\perp)$$
$$Q_{5x} \equiv -g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_*\, \bar\psi(z'_*, x_\perp)\, \gamma^5\not{p}_1 [z'_*, z_*]_x \psi(z_*, x_\perp)$$

$$\tilde{Q}_{1ij}(x_\perp) \equiv g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_* \Big([\infty p_1, z_*]_x \mathrm{tr}\{\psi(z_*, x_\perp)\bar\psi(z'_*, x_\perp)\, i\not{p}_1\}[z'_*, -\infty p_1]\Big)_{ij}$$
$$\tilde{Q}_{5ij}(x_\perp) \equiv g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_* \Big([\infty p_1, z_*]_x \mathrm{tr}\{\psi(z_*, x_\perp)\bar\psi(z'_*, x_\perp)\, \gamma^5\not{p}_1\}[z'_*, -\infty p_1]\Big)_{ij}$$

$$\mathrm{Tr}\{\hat{\mathcal{Q}}_{1x}\hat{U}^\dagger_y\} = \frac{1}{2}\,\mathrm{Tr}\{\hat{U}^\dagger_y\hat{U}_x\}\hat{Q}_{1\,x} - \frac{1}{2N_c}\,\mathrm{Tr}\{\hat{U}^\dagger_y\hat{\tilde{Q}}_{1x}\}$$
$$\mathrm{Tr}\{\hat{\mathcal{Q}}_{5x}\hat{U}^\dagger_y\} = \frac{1}{2}\,\mathrm{Tr}\{\hat{U}^\dagger_y\hat{U}_x\}\hat{Q}_{5\,x} - \frac{1}{2N_c}\,\mathrm{Tr}\{\hat{U}^\dagger_y\hat{\tilde{Q}}_{5x}\}$$



High-Energy OPE with sub-eikonal corrections

$$
\begin{aligned}
&\mathrm{T}\{\bar{\hat{\psi}}(x)\gamma^\mu\psi(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}\\
&= \int dz_1 dz_2\, \mathcal{I}^{\mu\nu}_{LO}(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{U}^\dagger_{z_2}\}\\
&\quad + \frac{\mathcal{Z}_2}{16\,s}\Big(\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{U}_{z_2}\}\hat{Q}_{1z_2} + \mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}\hat{Q}^\dagger_{1z_2} - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{\tilde{Q}}_{1z_2}\} - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}_{z_1}\hat{\tilde{Q}}^\dagger_{1z_2}\}\Big)\Big]\\
&\quad + \frac{1}{s}\int d^2z_1 d^2z_2\, \mathcal{I}^{\mu\nu}_{5}(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{U}_{z_2}\}\hat{Q}_{5z_2} + \mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}\hat{Q}^\dagger_{5z_2}\\
&\quad - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}^\dagger_{z_1}(\hat{\tilde{Q}}_{5z_2} - 2N_c\hat{\mathcal{F}}_{z_2})\} - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}_{z_1}(\hat{\tilde{Q}}^\dagger_{5z_2} - 2N_c\hat{\mathcal{F}}^\dagger_{z_2})\}\Big]\\
&\quad + \mathcal{O}(\alpha_s) + \mathcal{O}(\lambda^{-2})
\end{aligned}
$$

Parametrization of the forward matrix elements

Quark distributions

$S^\mu \simeq \frac{\lambda}{M}P^\mu + S_\perp^\mu \qquad \Delta_\perp^\mu = (x-y)_\perp^\mu$

$$\int d^2\Delta\, e^{i(\Delta,k)}\langle\langle P,S|\Big[Q_1(x_\perp)\,\mathrm{Tr}\{U_x U_y^\dagger\} + \mathrm{a.c.}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big(q_1(k_\perp^2,x) + \frac{\vec{S}\times\vec{k}}{M}q_{1T}(k_\perp^2,x)\Big)$$

$$\int d^2\Delta\, e^{i(\Delta,k)}\langle\langle P,S|\Big[\,\mathrm{Tr}\{\tilde{Q}_1(x_\perp)U_y^\dagger\} + \mathrm{a.c.}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big(\tilde{q}_1(k_\perp^2,x) + \frac{\vec{S}\times\vec{k}}{M}\tilde{q}_{1T}(k_\perp^2,x)\Big)$$

$$\int d^2\Delta e^{i(\Delta,k)}\langle\langle P,S|\Big[Q_5(x_\perp)\mathrm{Tr}\{U_x U_y^\dagger\} + \mathrm{a.c}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big(\lambda q_{5L}(k_\perp^2,x) - \frac{(S,k)_\perp}{M}q_{5T}(k_\perp^2,x)\Big)$$

$$\int d^2\Delta\, e^{i(\Delta,k)}\langle\langle P,S|\Big[\,\mathrm{Tr}\{\tilde{Q}_5(x_\perp)U_y^\dagger\} + \mathrm{a.c}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big(\lambda\tilde{q}_{5L}(k_\perp^2,x) - \frac{(S,k)_\perp}{M}\tilde{q}_{5T}(k_\perp^2,x)\Big)$$

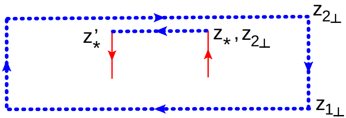
Parametrization of the forward matrix elements

Gluon distributions

$S^\mu \simeq \frac{\lambda}{M}P^\mu + S^\mu_\perp \qquad \Delta^\mu_\perp = (x-y)^\mu_\perp$

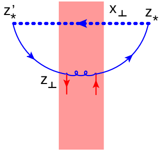
$$\int d^2\Delta e^{i(\Delta,k)_\perp}\langle\langle P,S|\Big[\mathrm{Tr}\{\mathcal{F}(x_\perp)U^\dagger_y\} + \mathrm{a.c}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big[\lambda\frac{k^2_\perp}{M^2}G_L(k^2_\perp,x) + \frac{(S,k)_\perp}{M}G_T(k^2_\perp,x)\Big]$$

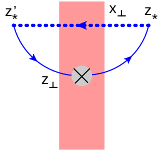
Evolution equation of sub-eikonal corrections



Diagrams at one loop: quantum quark field

Figure : Diagrams with $\hat{Q}_{1x}$ and $\hat{Q}_{5x}$ quantum.





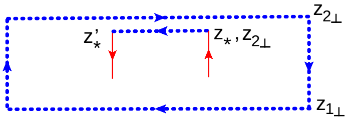
$$\langle \mathrm{Tr}\{U_y^\dagger U_x\}Q_{1x}\rangle = \frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\frac{\mathrm{Tr}\{U_y^\dagger U_x\}}{(x-z)_\perp^2}\left[\mathrm{Tr}\{U_x^\dagger U_z\}\,Q_{1z} - \frac{1}{N_c}\mathrm{Tr}\{U_x^\dagger \tilde{Q}_{1z}\}\right]$$

and

$$\begin{aligned}\langle \mathrm{Tr}\{U_y^\dagger U_x\}Q_{5x}\rangle \;=\; &\frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\frac{\mathrm{Tr}\{U_y^\dagger U_x\}}{(x-z)_\perp^2}\\ &\times\left[\mathrm{Tr}\{U_x^\dagger U_z\}\,Q_{5z} - \frac{1}{N_c}\mathrm{Tr}\{U_x^\dagger\big(\tilde{Q}_{5z} - 2N_c\mathcal{F}_z\big)\}\right]\end{aligned}$$

Sanity check: operators of different parity do not mix

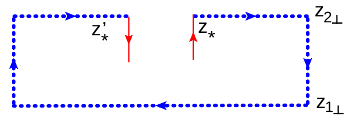
Evolution equation of sub-eikonal corrections



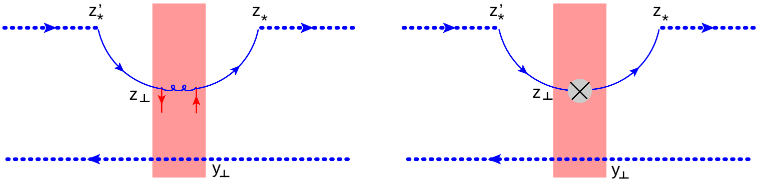
Diagrams at one loop: classical quark field (BK diagrams)

$$
\begin{aligned}
Q_{1x}\langle \mathrm{Tr}\{U_x U_y^\dagger\}\rangle \;=\; & \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \frac{(x-y)_\perp^2}{(x-z)_\perp^2\,(y-z)_\perp^2} \\
& \times\Big[\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_y^\dagger\} - N_c \mathrm{Tr}\{U_x U_y^\dagger\}\Big] Q_{1x}
\end{aligned}
$$

It gives automatically Leading-Log resummation contribution.



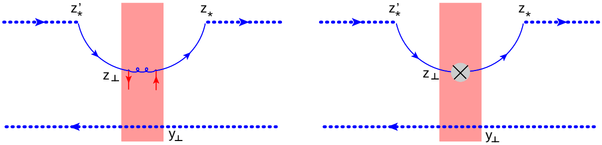
Diagrams at one loop: quantum quark field



a) b)

Figure : Diagrams with $\tilde{Q}_{1x}$ and $\tilde{Q}_{5x}$ quantum.

Evolution equation of sub-eikonal corrections



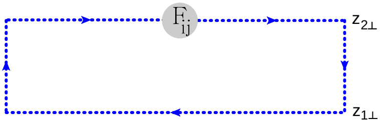
$$\langle \mathrm{Tr}\{U^\dagger_y \tilde{Q}_{1x}\}\rangle = \frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int\frac{d^2z}{(x-z)^2_\perp}\Big[\mathrm{Tr}\{U^\dagger_y U_z\}Q_{1z} - \frac{1}{N_c}\mathrm{Tr}\{U^\dagger_y \tilde{Q}_{1z}\}\Big]$$

and

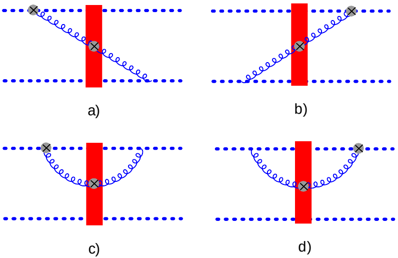
$$\begin{aligned}\langle \mathrm{Tr}\{U^\dagger_y \tilde{Q}_{5x}\}\rangle \;=\; &\frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int\frac{d^2z}{(x-z)^2_\perp}\\ &\times\Big[\mathrm{Tr}\{U^\dagger_y U_z\}Q_{5z} - \frac{1}{N_c}\mathrm{Tr}\{U^\dagger_y\big(\tilde{Q}_{5z} - 2N_c\mathcal{F}_z\big)\}\Big]\end{aligned}$$

Sanity check: operators of different parity do not mix

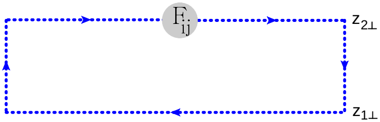
Evolution equation of sub-eikonal corrections



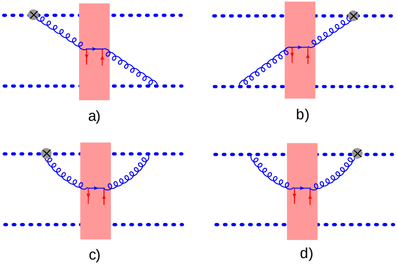
Diagrams with F_{ij} quantum



Evolution equation of sub-eikonal corrections



Diagrams with F_{ij} quantum



Evolution equation of sub-eikonal corrections

Diagrams with F_{ij} quantum

$$
\begin{aligned}
\langle \mathrm{Tr}\{\mathcal{F}_x\, U_y^\dagger\}\rangle \;=\; & -\frac{\alpha_s}{\pi^2}\,\mathrm{Tr}\{U_x t^a U_y^\dagger t^b\}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \\
& \times\Bigg[\frac{(\vec{x}-\vec{z})\times(\vec{z}-\vec{y})}{(x-z)_\perp^2\,(y-z)_\perp^2}\left(\mathcal{Q}_{1z}^{ba}-\mathcal{Q}_{1z}^{ba\,\dagger}\right) \\
& -\left(\frac{(x-z,z-y)}{(x-z)_\perp^2\,(y-z)_\perp^2}+\frac{1}{(x-z)_\perp^2}\right)\left(\mathcal{Q}_{5z}^{ba}+\mathcal{Q}_{5z}^{ba\,\dagger}+\mathcal{F}_z^{ba}\right) \\
& -4\pi^2\int\frac{d\!\!{}^-{}^2q}{q_\perp^2}\left(e^{i(q,y-z)}-e^{i(q,x-z)}\right)\delta^{(2)}(z-x)\mathcal{F}_z^{ba}\Bigg]
\end{aligned}
$$

$$
\mathcal{Q}_5^{ab}(z_\perp)\equiv g^2\int_{-\infty}^{+\infty}dz_{1*}\int_{-\infty}^{z_{1*}}dz_{2*}\bar\psi(z_{1*},z_\perp)\,\gamma^5\not{p}_1\,[z_{1*},\infty p_1]_z\,t^aU_z\,t^b\,[-\infty p_1,z_{2*}]_z\,\psi(z_{2*},z_\perp)
$$

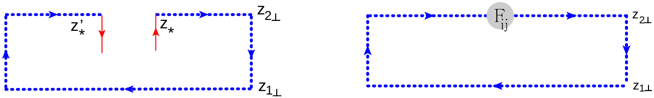
$$
\mathcal{Q}_1^{ab}(z_\perp)\equiv g^2\int_{-\infty}^{+\infty}dz_{1*}\int_{-\infty}^{z_{1*}}dz_{2*}\bar\psi(z_{1*},z_\perp)\,i\not{p}_1\,[z_{1*},\infty p_1]_z\,t^aU_z\,t^b\,[-\infty p_1,z_{2*}]_z\,\psi(z_{2*},z_\perp)
$$

Evolution equation of sub-eikonal corrections

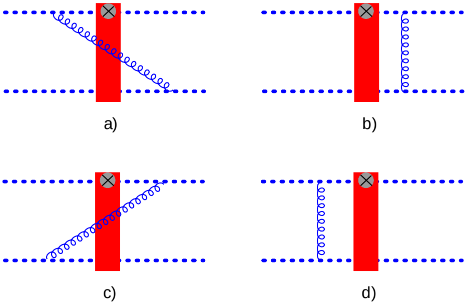
Diagrams with F_{ij} quantum

$$
\begin{aligned}
\langle \mathrm{Tr}\{\mathcal{F}_x\, U_y^\dagger\}\rangle =& \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \\
&\times\Bigg\{\frac{1}{2}\frac{(\vec{x}-\vec{z})\times(\vec{z}-\vec{y})}{(x-z)_\perp^2(y-z)_\perp^2}\Big[\mathrm{Tr}\{U_y^\dagger\tilde{Q}_{1\,z}\}\mathrm{Tr}\{U_z^\dagger U_x\} - \mathrm{Tr}\{U_x\tilde{Q}^\dagger_{1z}\}\mathrm{Tr}\{U_y^\dagger U_z\} \\
&+\frac{1}{N_c}\Big(\mathrm{Tr}\{U_xU_y^\dagger\tilde{Q}_{1z}U_z^\dagger\} + \mathrm{Tr}\{U_y^\dagger U_xU_z^\dagger\tilde{Q}_{1z}\} - \mathrm{Tr}\{U_xU_y^\dagger U_z\tilde{Q}^\dagger_{1z}\} - \mathrm{Tr}\{U_y^\dagger U_x\tilde{Q}^\dagger_{1z}U_z\}\Big) \\
&+\frac{1}{N_c^2}\mathrm{Tr}\{U_y^\dagger U_x\}\Big(Q^\dagger_{1z} - Q_{1z}\Big)\Big] - \frac{1}{2}\Big[\frac{(x-z,z-y)}{(x-z)_\perp^2(y-z)_\perp^2} + \frac{1}{(x-z)_\perp^2}\Big] \\
&\times\Big[\mathrm{Tr}\{U_y^\dagger(\tilde{Q}_{5z} - 2\mathcal{F}_z)\}\mathrm{Tr}\{U_z^\dagger U_x\} + \mathrm{Tr}\{U_x(\tilde{Q}^\dagger_{5z} - 2\mathcal{F}_z^\dagger)\}\mathrm{Tr}\{U_y^\dagger U_z\} \\
&-\frac{1}{N_c}\Big(\mathrm{Tr}\{U_xU_y^\dagger U_z\tilde{Q}^\dagger_{5z}\} + \mathrm{Tr}\{U_y^\dagger U_x\tilde{Q}^\dagger_{5z}U_z\} + \mathrm{Tr}\{U_xU_y^\dagger\tilde{Q}_{5z}U_z^\dagger\} + \mathrm{Tr}\{U_y^\dagger U_xU_z^\dagger\tilde{Q}_{5z}\}\Big) \\
&+\frac{1}{N_c^2}\mathrm{Tr}\{U_y^\dagger U_x\}\Big(Q_{5z} + Q^\dagger_{5z}\Big)\Big]\Bigg\}
\end{aligned}
$$

Evolution equation of sub-eikonal corrections



Diagrams with F_{ij} or $\tilde{Q}_1$ (and $\tilde{Q}_5$) classical: BK-type diagrams



+ self-energy diagrams

Evolution equation of sub-eikonal corrections

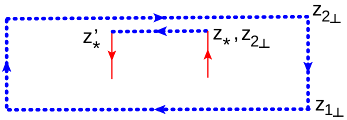
Diagrams with F_{ij} or $\tilde{Q}_1$ (and $\tilde{Q}_5$) classical: BK-type diagrams

$$\langle \mathrm{Tr}\{\tilde{Q}_{1x}\, U^\dagger_y\}\rangle \;=\; \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\,\frac{(x-y)^2_\perp}{(x-z)^2_\perp\,(y-z)^2_\perp}$$
$$\times\Big[\mathrm{Tr}\{U^\dagger_z\tilde{Q}_{1x}\}\,\mathrm{Tr}\{U^\dagger_y U_z\} - N_c\,\mathrm{Tr}\{U^\dagger_y\tilde{Q}_{1x}\}\Big]$$

$$\langle \mathrm{Tr}\{\tilde{Q}_{5x}\, U^\dagger_y\}\rangle \;=\; \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\,\frac{(x-y)^2_\perp}{(x-z)^2_\perp\,(y-z)^2_\perp}$$
$$\times\Big[\mathrm{Tr}\{U^\dagger_z\tilde{Q}_{5x}\}\,\mathrm{Tr}\{U^\dagger_y U_z\} - N_c\,\mathrm{Tr}\{U^\dagger_y\tilde{Q}_{5x}\}\Big]$$

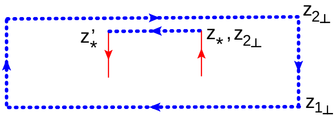
$$\langle \mathrm{Tr}\{\mathcal{F}_x\, U^\dagger_y\}\rangle \;=\; \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\,\frac{(x-y)^2_\perp}{(x-z)^2_\perp\,(y-z)^2_\perp}$$
$$\times\Big[\mathrm{Tr}\{U^\dagger_z\mathcal{F}_x\}\,\mathrm{Tr}\{U^\dagger_y U_z\} - N_c\,\mathrm{Tr}\{U^\dagger_y\mathcal{F}_x\}\Big]$$

Evolution equation of sub-eikonal corrections

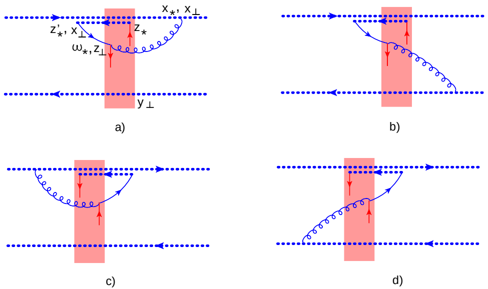


quark-to-gluon diagrams

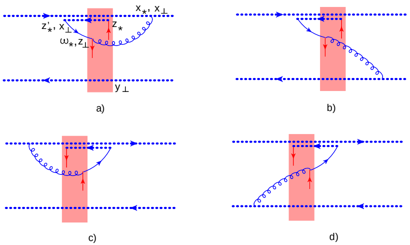
Evolution equation of sub-eikonal corrections



quark-to-gluon diagrams

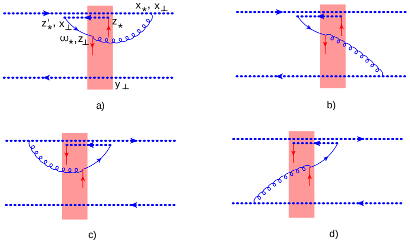


Evolution equation of sub-eikonal corrections



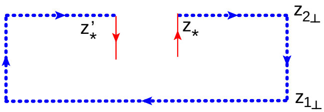
$$
\begin{aligned}
\langle \mathrm{Tr}\{U_x U_y^\dagger\} Q_{1x} \rangle &= \frac{\alpha_s}{4\pi^2} \int_0^{+\infty} \frac{d\alpha}{\alpha} \int d^2 z \Bigg\{ \frac{1}{(x-z)_\perp^2} \Big[\mathrm{Tr}\left\{ U_x U_y^\dagger U_z \mathcal{X}_{1zx}^\dagger \right\} \\
&+ \mathrm{Tr}\{U_z U_y^\dagger U_x \mathcal{X}_{1xz}^\dagger\} + \frac{1}{N_c} \mathrm{Tr}\{U_x U_y^\dagger\} \Big(\mathcal{H}_{1xz}^- + \mathcal{H}_{1zx}^+ \Big) \Big] + \frac{(x-z, z-y)_\perp}{(y-z)_\perp^2 (x-z)_\perp^2} \\
&\times \Big[\mathrm{Tr}\{U_x U_y^\dagger U_z \mathcal{X}_{1zx}^\dagger\} + \mathrm{Tr}\left\{ U_z U_y^\dagger U_x \mathcal{X}_{1xz}^\dagger \right\} + \frac{1}{N_c} \mathrm{Tr}\{U_x U_y^\dagger\} \Big(\mathcal{H}_{1xz}^- + \mathcal{H}_{1zx}^+ \Big) \Big] \\
&+ \frac{(\vec{x} - \vec{z}) \times (\vec{y} - \vec{z})}{(y-z)_\perp^2 (x-z)_\perp^2} \Big[\mathrm{Tr}\{U_x U_y^\dagger U_z \mathcal{X}_{5zx}^\dagger\} - \mathrm{Tr}\left\{ U_z U_y^\dagger U_x \mathcal{X}_{5xz}^\dagger \right\} - \frac{1}{N_c} \mathrm{Tr}\{U_x U_y^\dagger\} \Big(\mathcal{H}_{5xz}^- - \mathcal{H}_{5zx}^+ \Big) \Big] \Bigg\} .
\end{aligned}
$$

Evolution equation of sub-eikonal corrections

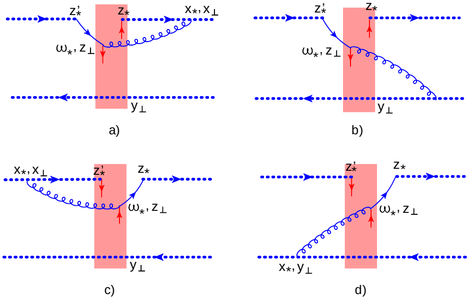


$$
\begin{aligned}
\langle \mathrm{Tr}\{U_x U_y^\dagger\} Q_{5x}\rangle = &-\frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\,\Bigg\{\frac{1}{(x-z)_\perp^2}\Big[\mathrm{Tr}\left\{U_x U_y^\dagger U_z \mathcal{X}^\dagger_{5zx}\right\} \\
&+\mathrm{Tr}\left\{U_z U_y^\dagger U_x \mathcal{X}^\dagger_{5xz}\right\} - \frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger U_x\}\Big(\mathcal{H}^-_{5xz}+\mathcal{H}^+_{5zx}\Big)\Big] + \frac{(x-z,z-y)_\perp}{(y-z)_\perp^2(x-z)_\perp^2} \\
&\times\Big[\mathrm{Tr}\{U_x U_y^\dagger U_z \mathcal{X}^\dagger_{5zx}\} + \mathrm{Tr}\left\{U_z U_y^\dagger U_x \mathcal{X}^\dagger_{5xz}\right\} - \frac{1}{N_c}\mathrm{Tr}\{U_x U_y^\dagger\}\Big(\mathcal{H}^-_{5xz}+\mathcal{H}^+_{5zx}\Big)\Big] \\
&+\frac{(\vec{x}-\vec{z})\times(\vec{y}-\vec{z})}{(y-z)_\perp^2(x-z)_\perp^2}\Big[\mathrm{Tr}\{U_z U_y^\dagger U_x \mathcal{X}^\dagger_{1xz}\} - \mathrm{Tr}\{U_x U_y^\dagger U_z \mathcal{X}^\dagger_{1zx}\} + \frac{1}{N_c}\mathrm{Tr}\{U_x U_y^\dagger\}\Big(\mathcal{H}^+_{1zx}-\mathcal{H}^-_{1xz}\Big)\Big]\Bigg\}.
\end{aligned}
$$

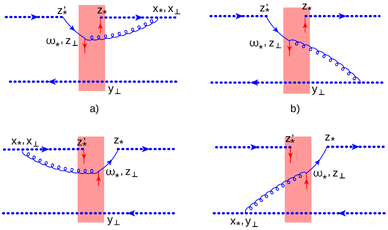
Evolution equation of sub-eikonal corrections



quark-to-gluon diagrams



Evolution equation of sub-eikonal corrections

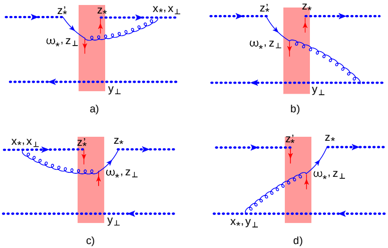


$$\langle \mathrm{Tr}\{U_y^\dagger \tilde{Q}_{1x}\}\rangle = -\frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z$$

$$\times\Bigg\{\frac{1}{(x-z)_\perp^2}\left[\mathrm{Tr}\left\{U_zU_y^\dagger\right\}\left(\mathcal{H}^+_{1xz}+\mathcal{H}^-_{1zx}\right)-\frac{1}{N_c}\mathrm{Tr}\left\{U_y^\dagger\left(\mathcal{X}_{1xz}+\mathcal{X}_{1zx}\right)\right\}\right]$$

$$+\frac{(x-z,z-y)}{(y-z)_\perp^2(z-x)_\perp^2}\left[\mathrm{Tr}\{U_zU_y^\dagger\}\left(\mathcal{H}^+_{1xz}+\mathcal{H}^-_{1zx}\right)-\frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger\left(\mathcal{X}_{1xz}+\mathcal{X}_{1zx}\right)\}\right]$$

$$+\frac{(\vec{x}-\vec{z})\times(\vec{y}-\vec{z})}{(y-z)_\perp^2(z-x)_\perp^2}\left[\mathrm{Tr}\{U_zU_y^\dagger\}\left(\mathcal{H}^-_{5zx}-\mathcal{H}^+_{5xz}\right)+\frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger\left(\mathcal{X}_{5xz}-\mathcal{X}_{5zx}\right)\}\right]\Bigg\}$$



$$
\begin{aligned}
\langle \mathrm{Tr}\{U_y^\dagger \tilde{Q}_{5x}\}\rangle &= -\frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \\
&\times\Bigg\{\frac{1}{(x-z)_\perp^2}\Big[\mathrm{Tr}\left\{U_zU_y^\dagger\right\}\Big(\mathcal{H}^+_{5xz}+\mathcal{H}^-_{5zx}\Big)-\frac{1}{N_c}\mathrm{Tr}\left\{U_y^\dagger\left(\mathcal{X}_{5xz}+\mathcal{X}_{5zx}\right)\right\}\Big] \\
&+\frac{(x-z,z-y)}{(y-z)_\perp^2(z-x)_\perp^2}\Big[\mathrm{Tr}\{U_zU_y^\dagger\}\Big(\mathcal{H}^+_{5xz}+\mathcal{H}^-_{5zx}\Big)-\frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger\Big(\mathcal{X}_{5xz}+\mathcal{X}_{5zx}\Big)\}\Big)\Big] \\
&+\frac{(\vec{x}-\vec{z})\times(\vec{y}-\vec{z})}{(y-z)_\perp^2(z-x)_\perp^2}\Big[\mathrm{Tr}\{U_zU_y^\dagger\}\Big(\mathcal{H}^+_{1xz}-\mathcal{H}^-_{1zx}\Big)+\frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger\left(\mathcal{X}_{1zx}-\mathcal{X}_{1xz}\right)\}\Big]\Bigg\}
\end{aligned}
$$

$\mathcal{X}$-operators

$$\mathcal{X}_1(x_\perp,y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_* \bar\psi(z_*,y_\perp)[z_*,-\infty p_1]_y\, i\not{p}_1\, [\infty p_1,\omega_*]_x\psi(\omega_*,x_\perp)$$

$$\mathcal{X}_1^\dagger(x_\perp,y_\perp) = g^2\int_{-\infty}^{+\infty} dz_* d\omega_*\, \bar\psi(\omega_*,x_\perp)[\omega_*,\infty p_1]_x\, i\not{p}_1\, [-\infty p_1,z_*]_y\psi(z_*,y_\perp)$$

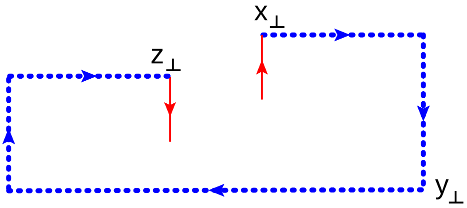
$$\mathcal{X}_5(x_\perp,y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_* \bar\psi(z_*,y_\perp)[z_*,-\infty p_1]_y\, \gamma^5\not{p}_1\, [\infty p_1,\omega_*]_x\psi(\omega_*,x_\perp)$$

$$\mathcal{X}_5^\dagger(x_\perp,y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_* \bar\psi(\omega_*,x_\perp)[\omega_*,\infty p_1]_x\, \gamma^5\not{p}_1\, [-\infty p_1,z_*]_y\psi(z_*,y_\perp)$$

$\mathcal{X}$-operators

$\mathcal{X}$-dipole operators

$\mathrm{Tr}\{U_y^\dagger \mathcal{X}_{1xz}\}$ or $\mathrm{Tr}\{U_y^\dagger \mathcal{X}_{5xz}\}$



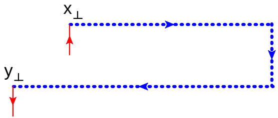
$$\mathcal{H}_1^+(x_\perp, y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_*\, \bar\psi(\omega_*, y_\perp)[\omega_*, \infty p_1]_y\, i\not p_1\, [\infty p_1, z_*]_x \psi(z_*, x_\perp)$$

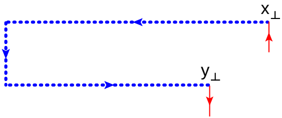
$$\mathcal{H}_5^+(x_\perp, y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_*\, \bar\psi(\omega_*, y_\perp)[\omega_*, \infty p_1]_y\, \gamma^5\not p_1\, [\infty p_1, z_*]_x \psi(z_*, x_\perp)$$

$$\mathcal{H}_1^-(x_\perp, y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_*\, \bar\psi(\omega_*, y_\perp)[\omega_*, -\infty p_1]_y\, i\not p_1\, [-\infty p_1, z_*]_x \psi(z_*, x_\perp)$$

$$\mathcal{H}_5^-(x_\perp, y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_*\, \bar\psi(\omega_*, y_\perp)[\omega_*, -\infty p_1]_y\, \gamma^5\not p_1\, [-\infty p_1, z_*]_x \psi(z_*, x_\perp)$$

TMD operators that usually appear in SIDIS and Drell-Yan processes.





Conclusions

- New Impact factors have been derived;
- New evolution equations which describe the high-energy spin dynamics;
- New quark and gluon distributions;
- TMD type of operators appeared in DIS process.
- Outlook
  - Disentangle the double from the single log of energy and compare with Bartels-Ermolaev-Ryskin (and the extension obtained by Boussarie, Hatta, and Yuan (2019));
  - Compare with NNLO calculation by Moch, Vermaseren and Vogt (2014);
  - Compare with results obtained by Kovchegov's group (20015-2021);
  - Include NLO and running coupling corrections.

Lesson from Unpolarized case

- Consider twist-two local operators
  - actually the full supermultiplet of multiplicatively renorm. operators

$$\mathcal{O}^j_\phi(x_\perp) = \int du\, \bar{\phi}^a_{AB} \nabla^j_- \phi^{ABa}(up_1 + x_\perp)$$
$$\mathcal{O}^j_\lambda(x_\perp) = \int du\, i\bar{\lambda}^a_A \nabla^{j-1}_- \lambda^a_A(up_1 + x_\perp)$$
$$\mathcal{O}^j_g(x_\perp) = \int du\, F^{a+}{}_i \nabla^{j-2}_- F^{a+i}(up_1 + x_\perp)$$

supermultiplet of local operators

$$S^j_1 = \mathcal{O}^j_g + \frac{1}{4}\mathcal{O}^j_\lambda - \frac{1}{2}\mathcal{O}^j_\phi$$
$$S^j_2 = \mathcal{O}^j_g - \frac{1}{4(j-1)}\mathcal{O}^j_\lambda + \frac{j+1}{6(j-1)}\mathcal{O}^j_\phi$$
$$S^j_3 = \mathcal{O}^j_g - \frac{j+2}{2(j-1)}\mathcal{O}^j_\lambda - \frac{(j+1)(j+2)}{2j(j-1)}\mathcal{O}^j_\phi$$

- Construct the analytic continuation to complex spin-j of light-ray operators

$$\mathcal{F}^j(x_\perp) = \int_0^\infty du\, u^{1-j}\mathcal{F}(up_1 + x_\perp)\,,$$
$$\Lambda^j(x_\perp) = \int_0^\infty du\, u^{-j}\Lambda(up_1 + x_\perp)\,,$$
$$\Phi^j(x_\perp) = \int_0^\infty du\, u^{-1-j}\Phi(up_1 + x_\perp)$$

with

$$\mathcal{F}^j(up_1, x_\perp) = \int dv\, F^{a+}{}_\mu(up_1 + vp_1 + x_\perp)[u+v, v]_x^{ab} F^{b+\mu}(vp_1 + x_\perp)\,,$$
$$\Lambda^j(up_1, x_\perp) = \frac{i}{2}\int dv\Big(-\bar{\lambda}^a_A(up_1 + vp_1 + x_\perp)[u+v, v]_x^{ab}\sigma_-\lambda^b_A(vp_1 + x_\perp)$$
$$+\bar{\lambda}^a_A(vp_1 + x_\perp)[v, u+v]_x^{ab}\sigma_-\lambda^b_A(up_1 + vp_1 + x_\perp)\Big)\,,$$
$$\Phi^j(u, x_\perp) = \int dv\, \phi^a_I(up_1 + vp_1 + x_\perp)[u+v, v]_x^{ab}\phi^b_I(vp_1 + x_\perp)$$

Lesson from Unpolarized case

Analytic continuation of the multiplicatively renormalizable light-ray operators to non-integer j

$$\mathcal{S}^j_1 = \mathcal{F}^j + \frac{j-1}{4}\Lambda^j - j(j-1)\frac{1}{2}\Phi^j\,,$$
$$\mathcal{S}^j_2 = \mathcal{F}^j - \frac{1}{4}\Lambda^j + \frac{j(j+1)}{6}\Phi^j\,,$$
$$\mathcal{S}^j_3 = \mathcal{F}^j - \frac{j+2}{2}\Lambda^j - \frac{(j+1)(j+2)}{2}\Phi^j\,.$$

Note: coefficients are different from the ones in the system of local multiplicatively renom. operators

Unpolarized case

Consider the 4-point function

- perform the high-energy limit in coord. space
  $x_1^- \to \lambda x_1^-, x_2^- \to \lambda x_2^-, x_3^+ \to \lambda' x_3^+$
  $x_4^+ \to \lambda' x_4^+$ with $\lambda\lambda' \to \infty$
- DGLAP limit $x_{12}^2 \to 0$

Balitsky (2013, 2019), Balitsky, Kazhakov, Sobko (20013-2018) Wilson frame

G.A.C. Quark and Gluon quasi-pdf at low-x (in preparation) quasi-pdf fram