ITMD and its recent applications

The Henryk Niewodniczański
Institute of Nuclear Physics
Polish Academy of Sciences

Krzysztof Kutak

NCN

HORIZON 2020

Based on results obtained with:
M. Bury, P. Kotko, C. Marquet, E. Petreska
S. Sapeta, A. van Hameren, E. Zarow

ITMD

ITMD = small x Improved Transverse Momentum Dependent factorization

- accounts for saturation
- correct gauge structure i.e. uses gauge links to define TMD's
- takes into account kinematical effects – the whole phase space is available at LO
- is implemented in MC event generator KaTie, LxJet
- valid in region $p_T > Q_s$, k_T can by any. p_T is hard final state momentum, k_T is inbalance

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1+\delta_{cd}} \sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(P_t, k_t)\Phi^{(i)}_{ag\to cd}(x_2, k_t)$$

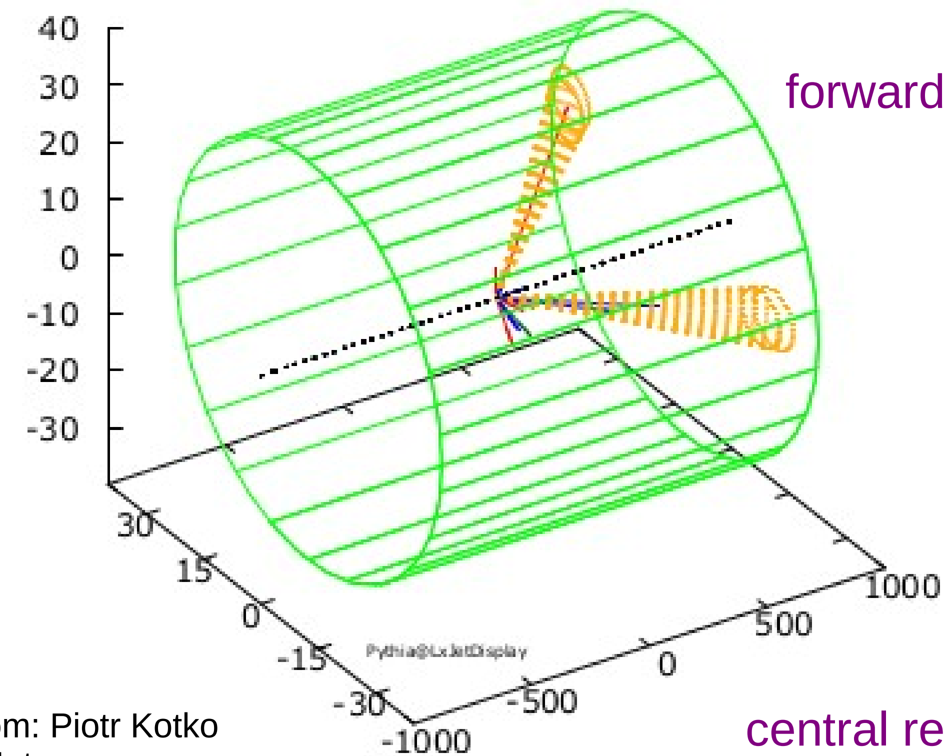
(one reprezentations of the ITMD formula)

Generic structure: transverse momentum enters hard factors and gluon distributions gluon distribution depends on color flow amplitude topology

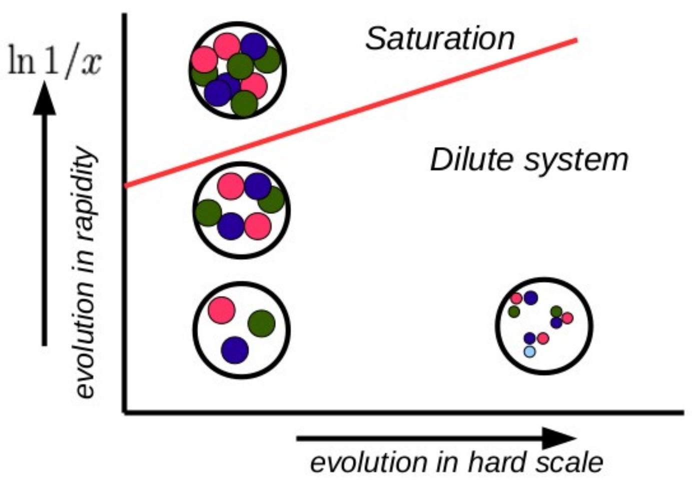
P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106

P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren JHEP 12 (2016) 034

p – A (dilute-dense) forward-forward di-jets



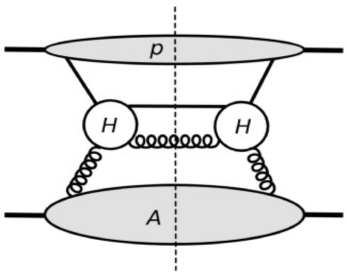
From: Piotr Kotko
LxJet

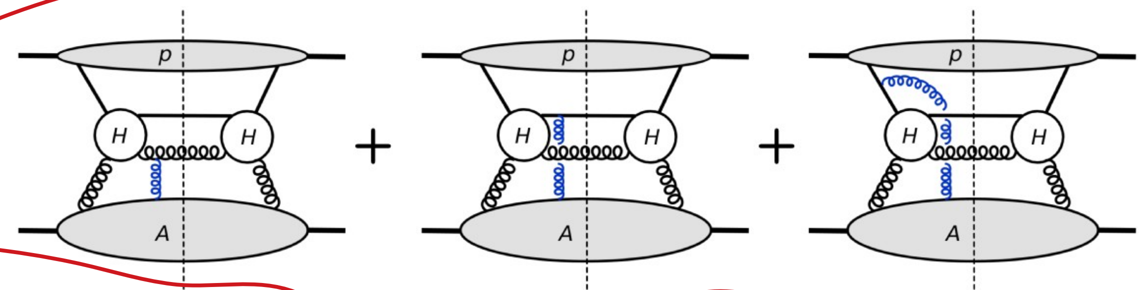


It originated from the aim to provide predictions for forward-forward jet production at the LHC

Formula for TMD gluons and gauge links

$$\mathcal{F}\left(x,k_T\right)=2\int\frac{d\xi^- d^2\xi_T}{(2\pi)^3\,P^+}\,e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\,\langle P|\,\mathrm{Tr}\left\{\hat{F}^{i+}(0)\,\hat{F}^{i+}\left(\xi^+=0,\xi^-,\vec{\xi}_T\right)\right\}|P\rangle$$





Valid for large transversal momentum and was obtained in a specific gauge

From S. Sapeta

similar diagrams with 2,3,….gluon exchanges. All this need to be resummed

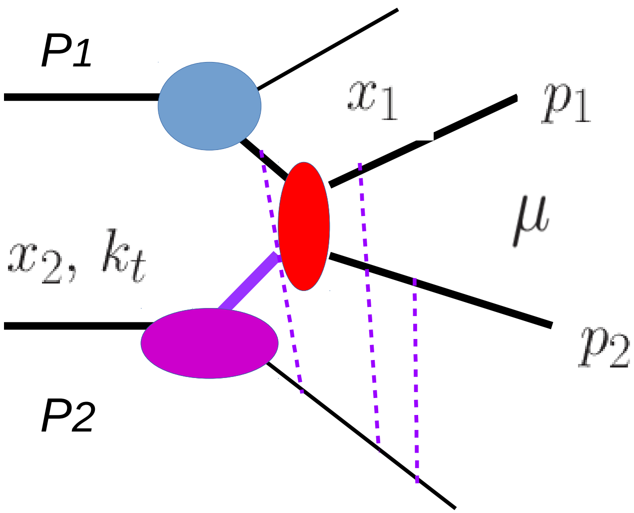
C.J. Bomhof, P.J. Mulders, F. Pijlman
Eur.Phys.J. C47 (2006) 147-162

$$\mathcal{F}\left(x,k_T\right)=2\int\frac{d\xi^- d^2\xi_T}{(2\pi)^3\,P^+}\,e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\,\langle P|\,\mathrm{Tr}\left\{\hat{F}^{i+}(0)\,\mathcal{U}_{C_1}\hat{F}^{i+}(\xi)\,\mathcal{U}_{C_2}\right\}|P\rangle$$

Hard part defines the path of the gauge link

$$\mathcal{U}^{[C]}(\eta;\xi)=\mathcal{P}\exp\Big[-ig\int_C \mathrm{d}z\cdot A(z)\Big]$$

The ITMD factorization for di-jets



- The color structure is separated from kinematic part of the amplitude by means of the color decomposition.
- The TMD gluon distributions are derived for the color structures following

P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106

A. van Hameren, P. Kotko, K. Kutak, C. Marquet, E. Petreska JHEP 12 (2016) 034

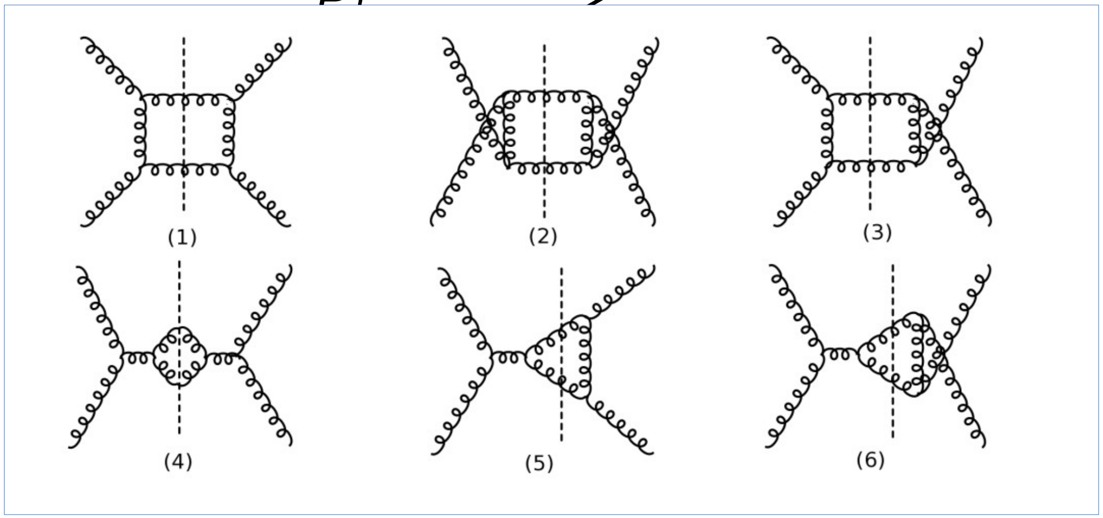
Formalism implemented in Monte Carlo programs KaTie by A. van Hameren and LxJet by P. Kotko

gauge invariant amplitudes with k_t and TMDs

Example for g g → g g*

$$\frac{d\sigma^{pA\to ggX}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\, x_1 f_{g/p}(x_1,\mu^2) \sum_{i=1}^{6} \mathcal{F}_{gg}^{(i)} H_{gg\to gg}^{(i)}$$

Improved Transverse Momentum Dependent Factorization



from

F. Dominguez, C. Marquet, Bo-Wen Xiao, F. Yuan Phys.Rev. D83 (2011) 105005

The same gauge link and as in TMD 's

Fabio Dominguez, Bo-Wen Xiao, Feng Yuan Phys.Rev.Lett. 106 (2011) 022301

F. Dominguez, C. Marquet, Bo-Wen Xiao, F. Yuan Phys.Rev. D83 (2011) 105005

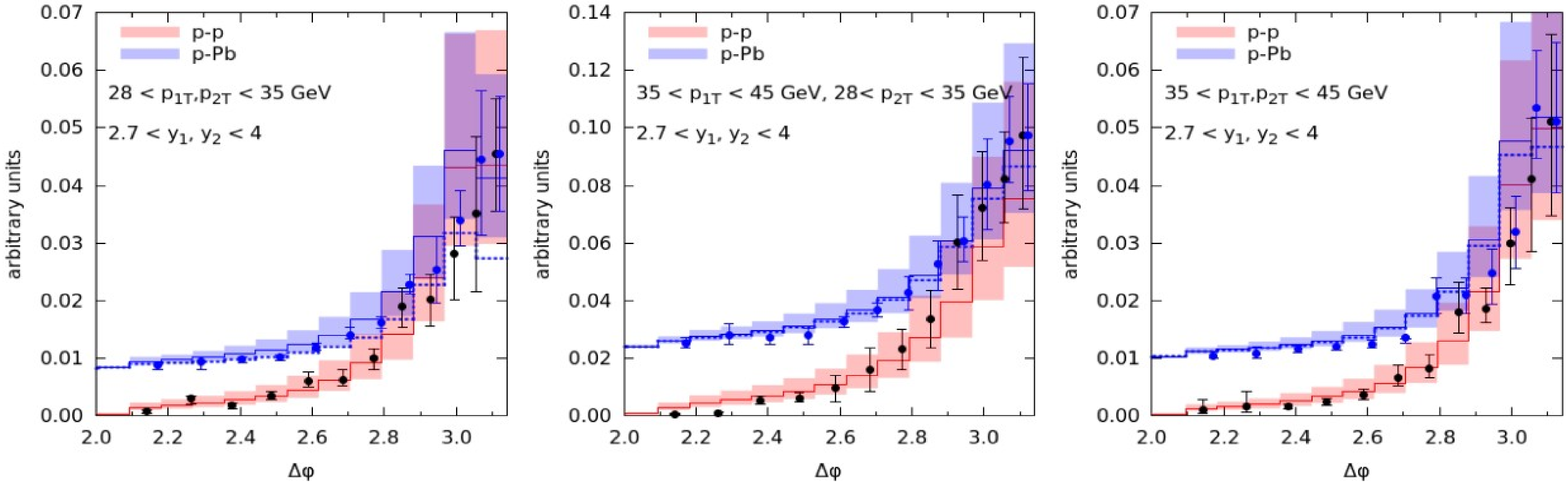
P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106

gauge invariant amplitudes with k_t and TMDs

example for $g^* g \to g g$

$$\frac{d\sigma^{pA\to ggX}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\; x_1 f_{g/p}(x_1,\mu^2) \sum_{i=1}^{6} \mathcal{F}_{gg}^{(i)} H_{gg\to gg}^{(i)}$$

The ITMD factorization for jets



A. Hameren, P. Kotko, K. Kutak, S. Sapeta
Phys.Lett. B795 (2019) 511-515

Formalism implemented in Monte Carlo programs KaTie by A. van Hameren and LxJet by P. Kotko

gauge invariant amplitudes with k_t and TMDs

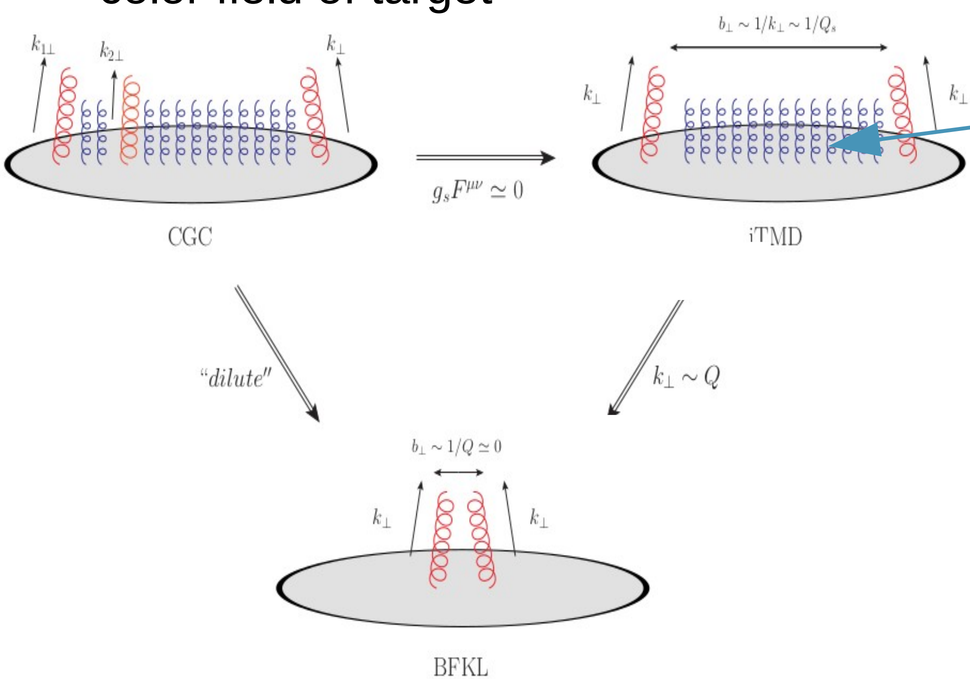
Example for g g → g g*

$$\frac{d\sigma^{pA\to ggX}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \, x_1 f_{g/p}(x_1,\mu^2) \sum_{i=1}^{6} \mathcal{F}_{gg}^{(i)} H_{gg\to gg}^{(i)}$$

ITMD from CGC

T. Altinoluk, R. Boussarie, P. Kotko JHEP 1905 (2019) 156
T. Altinoluk, R. Boussarie, JHEP10(2019)208

Expansion in distance - parameter entering as argument Wilson lines appearing in generic CGC amplitude i.e. amplitude for propagation in strong color field of target



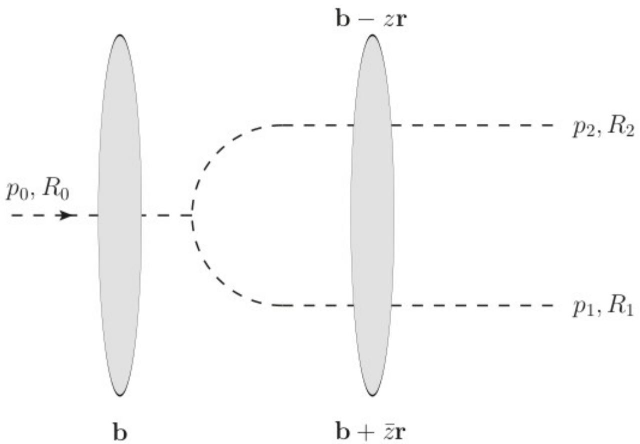
Multiple soft emissions

from R. Boussarie
Initial Stages 2019

Wandzura-Wilczek approximmation ITMD neglects higher genuine twist contributions i.e. hard gluon exchanges between the target and the amplitude, while it resums all kinematic twist

ITMD from CGC

T. Altinoluk, R. Boussarie, P. Kotko JHEP 1905 (2019) 156



Generic CGC amplitude

Color reprezentations

$$\mathcal{A}_{0\to 12} = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right)\int d^2\boldsymbol{b}\, d^2\boldsymbol{r}\, e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})-i(\boldsymbol{k}\cdot\boldsymbol{b})}\,\frac{r_\perp^\mu}{\boldsymbol{r}^2}\left[\left(U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}}T^{R_0}U^{R_2}_{\boldsymbol{b}-z\boldsymbol{r}}\right) - \left(U^{R_1}_{\boldsymbol{b}}T^{R_0}U^{R_2}_{\boldsymbol{b}}\right)\right]\phi_\mu$$

Dirac structure for the process

Expand to n'th order in r

$$\mathcal{A}^{(n)}_{0\to 12} = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right)\int d^2\boldsymbol{b}\, d^2\boldsymbol{r}\, e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})-i(\boldsymbol{k}\cdot\boldsymbol{b})}\,\frac{r_\perp^\mu\phi_\mu}{\boldsymbol{r}^2}$$

$$\times\frac{1}{n!}r_\perp^{\alpha_1}\dots r_\perp^{\alpha_n}\sum_{m=0}^{n}\binom{n}{m}\bar{z}^m(-z)^{n-m}\left(\partial_{\alpha_1}\dots\partial_{\alpha_m}U^{R_1}_{\boldsymbol{b}}\right)T^{R_0}\left(\partial_{\alpha_{m+1}}\dots\partial_{\alpha_n}U^{R_2}_{\boldsymbol{b}}\right)$$

Single out and resummation of contributions of type

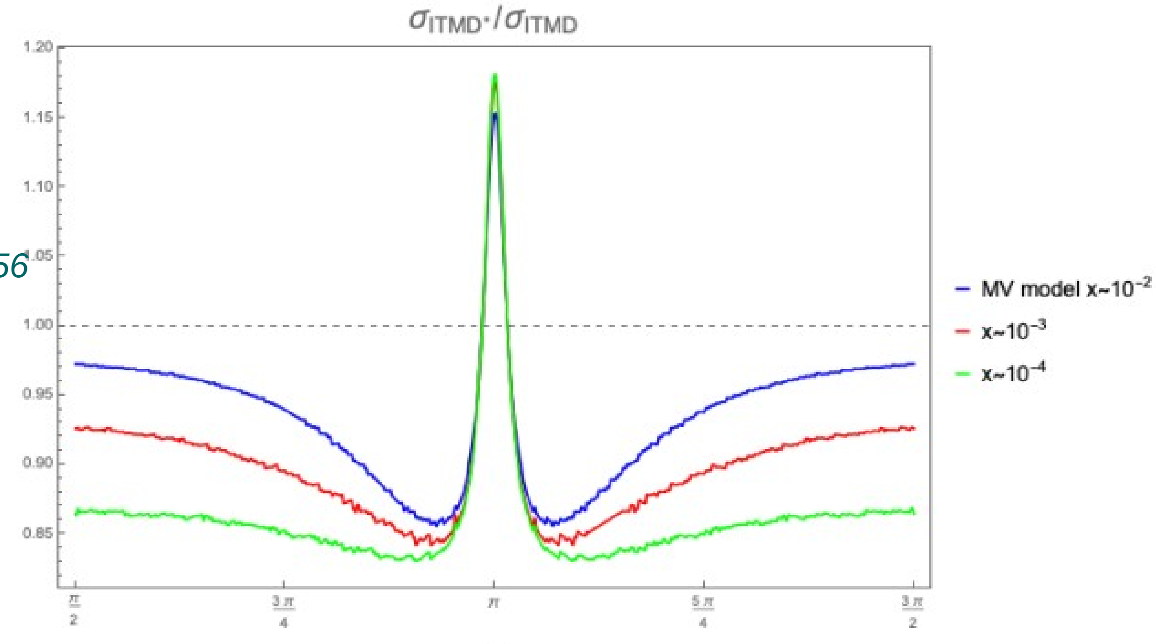
$$U^{R_1}_{\boldsymbol{b}}T^{R_0}\left(\partial_{\alpha_1}U^{R_2}_{\boldsymbol{b}}\right) \qquad \left(\partial_{\alpha_1}U^{R_1}_{\boldsymbol{b}}\right)T^{R_0}U^{R_2}_{\boldsymbol{b}}$$

Symmetric structure. Physical gluon and gauge restoring term. Terms without this symmetry do not contribute to kinematic twist

ITMD for massive final states

Application of the resummation method from
T. Altinoluk, R. Boussarie, P. Kotko JHEP 1905 (2019) 156
allowed to generalize ITMD to address production of hadrons.

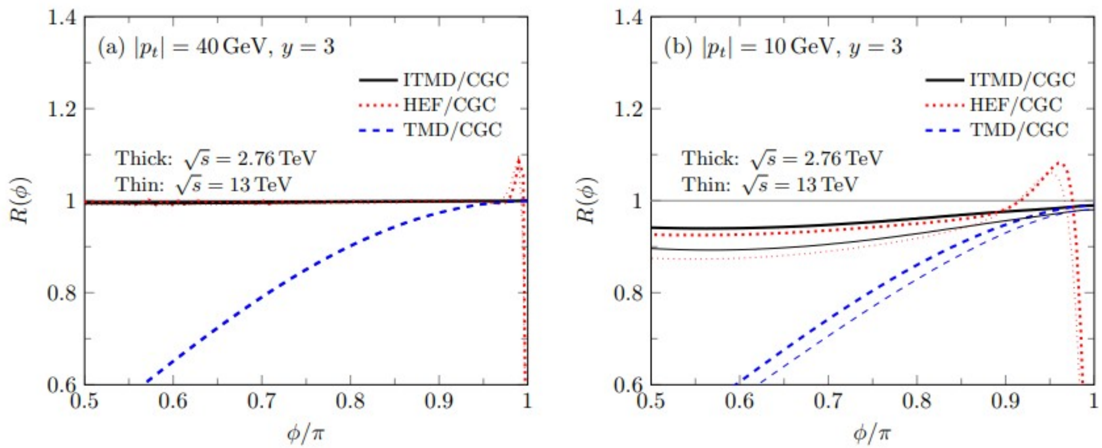
In particular one could obtain the contribution from longitudinally polarized gluon distributions
T. Altinoluk, C. Marquet, P. Taels, 2103.14495

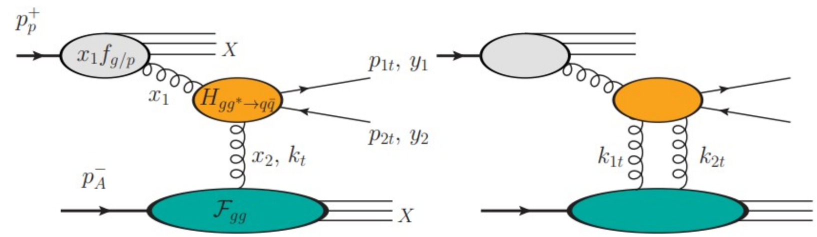


$$d\sigma \propto f(x_p) \sum_c H^{ij}_{(c)}(\mathbf{P}, \mathbf{k}) \left[\frac{1}{2}\delta^{ij} \mathcal{F}_{(c)}(x_A, \mathbf{k}) + \left(\frac{k^i k^j}{\mathbf{k}^2} - \frac{1}{2}\delta^{ij} \right) \mathcal{H}_{(c)}(x_A, \mathbf{k}) \right]$$

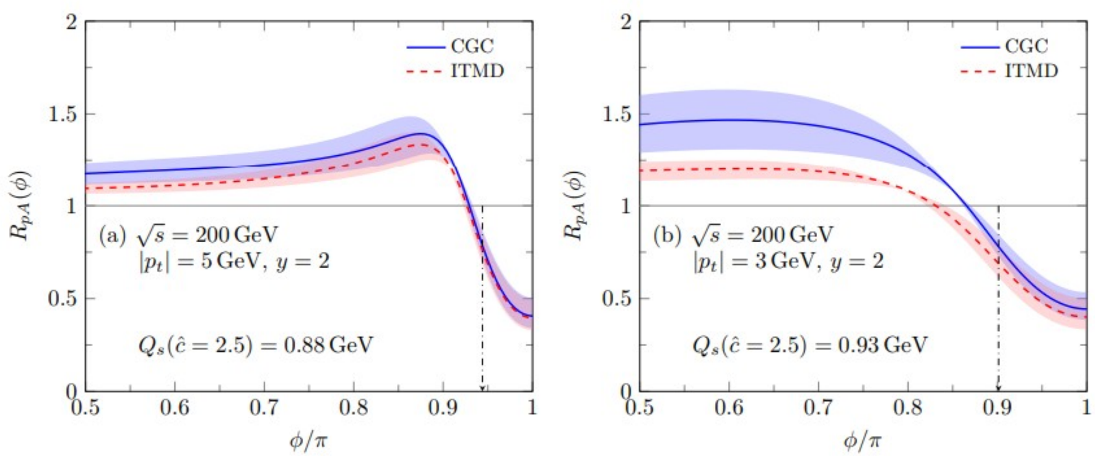
Linearly polarized gluons. It drops if m → 0. But reappears for massless 3 jets as shown in the correlation limit in T. Altinoluk, R. Boussarie, C. Marquet, P. Taels JHEP07(2020)143. Subleading contribution for di-jets i.e. gg→ gg, qg* →qg are leading.*

ITMD vs CGC





H. Fujii, C. Marquet, K. Watanabe, JHEP12(2020)181



The ITMD formula is a good approximation to the CGC formula in a wide range of azimuthal angle.

Set of basic TMD's for 2, 3 and 4 jets

KK, Bury, Kotko '18

$$\mathcal{F}_{qg}^{(1)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(2)}(x,k_T) = \text{F.T.}\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x,k_T) = \text{F.T.}\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\Box]\dagger}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x,k_T) = \text{F.T.}\,\frac{1}{N_c}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\Box]\dagger}\right]\text{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\Box]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(4)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x,k_T) = \text{F.T.}\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\frac{\text{Tr}\left[\mathcal{U}^{[\Box]\dagger}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x,k_T) = \text{F.T.}\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

Set of basic TMD's for 2, 3 and 4 jets

KK, Bury, Kotko '18

$$\mathcal{F}_{qg}^{(1)}(x,k_T) = \text{F.T.} \left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

dipole gluon

$$\mathcal{F}_{qg}^{(2)}(x,k_T) = \text{F.T.} \left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x,k_T) = \text{F.T.} \left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x,k_T) = \text{F.T.} \left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x,k_T) = \text{F.T.}\,\frac{1}{N_c} \left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\text{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x,k_T) = \text{F.T.} \left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

WW gluon

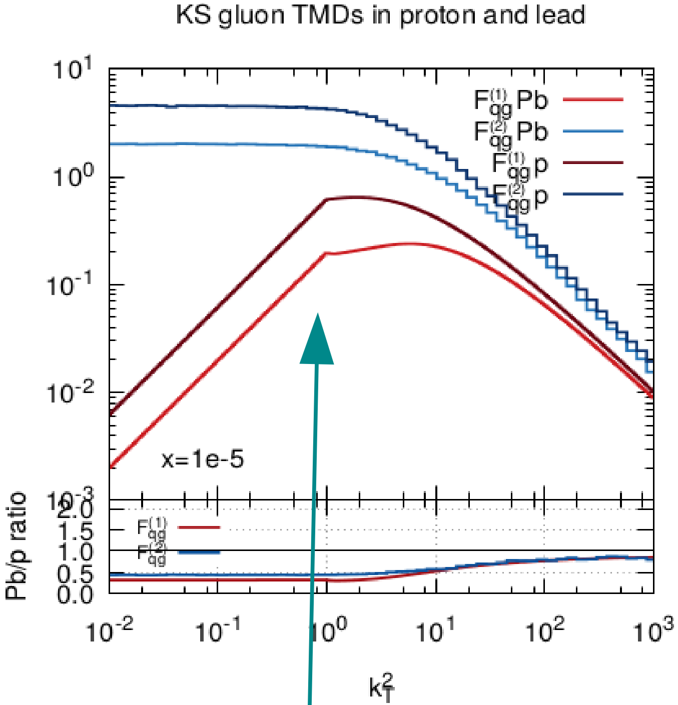
$$\mathcal{F}_{gg}^{(4)}(x,k_T) = \text{F.T.} \left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x,k_T) = \text{F.T.} \left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x,k_T) = \text{F.T.} \left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\text{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

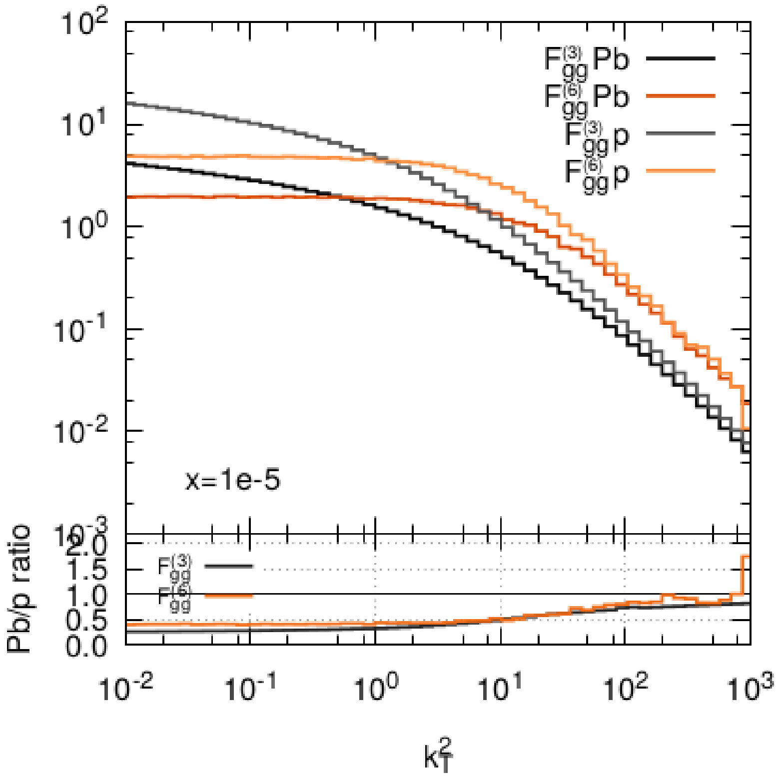
$$\mathcal{F}_{gg}^{(7)}(x,k_T) = \text{F.T.} \left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

Plots of ITMD gluons



Calculation – in large Nc approximation with analitic model for dipole gluon density – all gluons can be calculated from the dipole one. KS gluon used.

K. Kotko, K.Kutak, Marquet, Petreska, Sapeta, van Hameren JHEP 1612 (2016) 034



C. Marquet, E. Petreska, C. Roiesnel JHEP 1610 (2016) 065

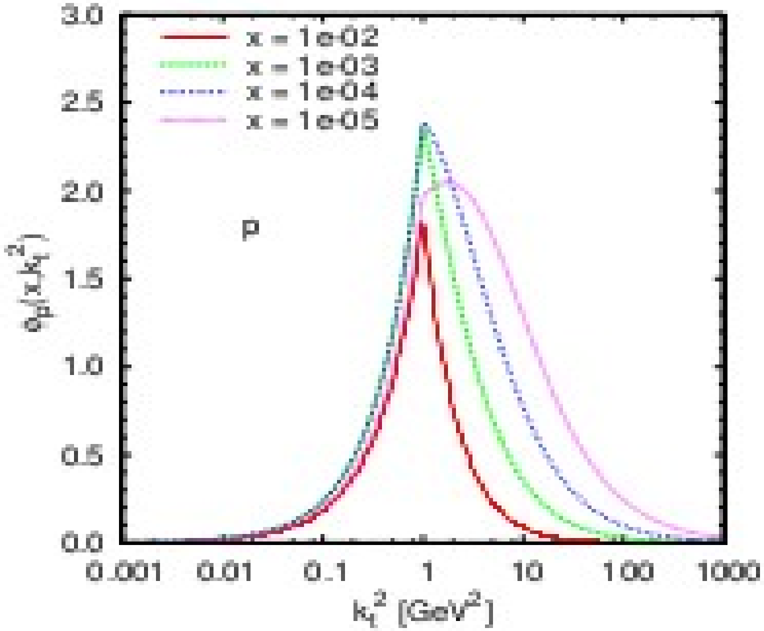
The distributions can be also obtained relxing large Nc approximationby solving JIMWLK

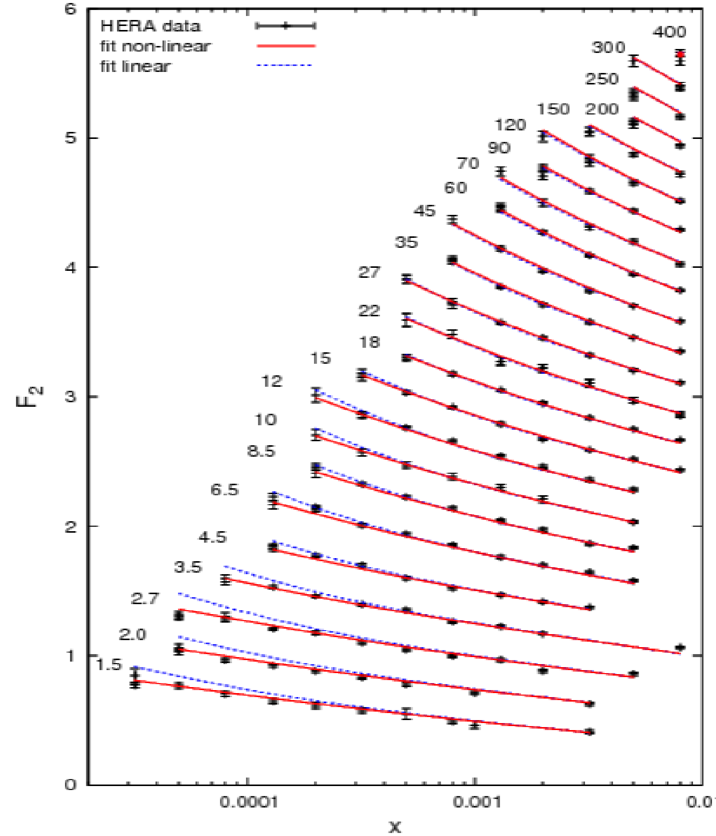
Standard HEF gluon density

The other densities are flat at low k_t → less saturation

Not negligible differences at large k_t → differences at small angles

Dipole gluon density from BK with corrections





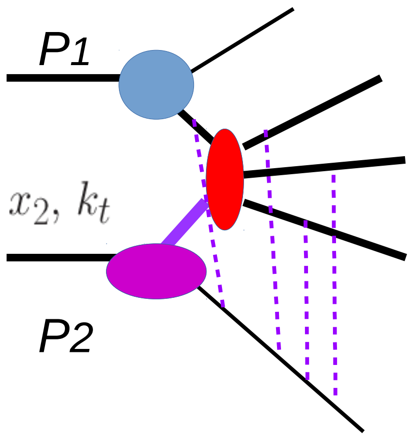
$$
\begin{aligned}
\mathcal{F}_p(x,k^2) \;=\; & \mathcal{F}_p^{(0)}(x,k^2) \\
& + \frac{\alpha_s(k^2)N_c}{\pi}\int_x^1 \frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}\left\{\frac{l^2\mathcal{F}_p(\frac{x}{z},l^2)\,\theta(\frac{k^2}{z}-l^2)-k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|l^2-k^2|}+\frac{k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\} \\
& + \frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\left(P_{gg}(z)-\frac{2N_c}{z}\right)\int_{k_0^2}^{k^2} dl^2\,\mathcal{F}_p(\frac{x}{z},l^2) \\
& - \frac{2\alpha_s^2(k^2)}{R^2}\left[\left(\int_{k^2}^{\infty}\frac{dl^2}{l^2}\mathcal{F}_p(x,l^2)\right)^2+\mathcal{F}_p(x,k^2)\int_{k^2}^{\infty}\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}_p(x,l^2)\right]
\end{aligned}
$$

Kwiecinski, KK '03

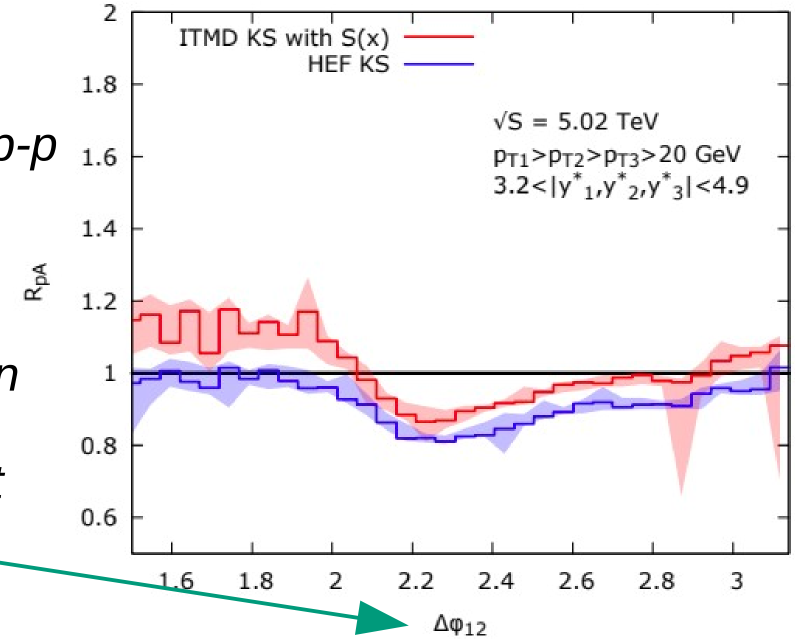
Corrections of higher orders Included.
Kin. Constr
DGLAP spf

Andersson, Gustafson,Sammuelsson '96
Kwiecinski, Martin, Stasto '96,

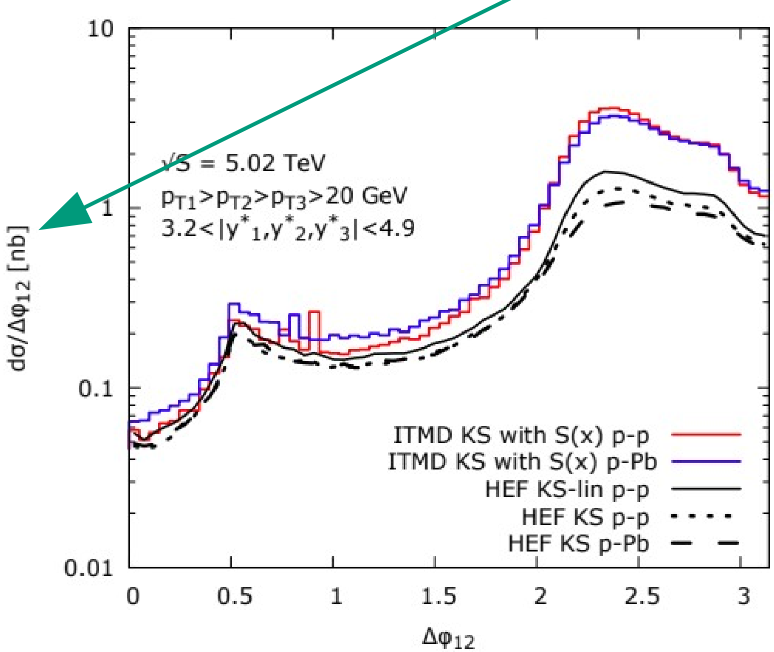
ITMD for three jets*



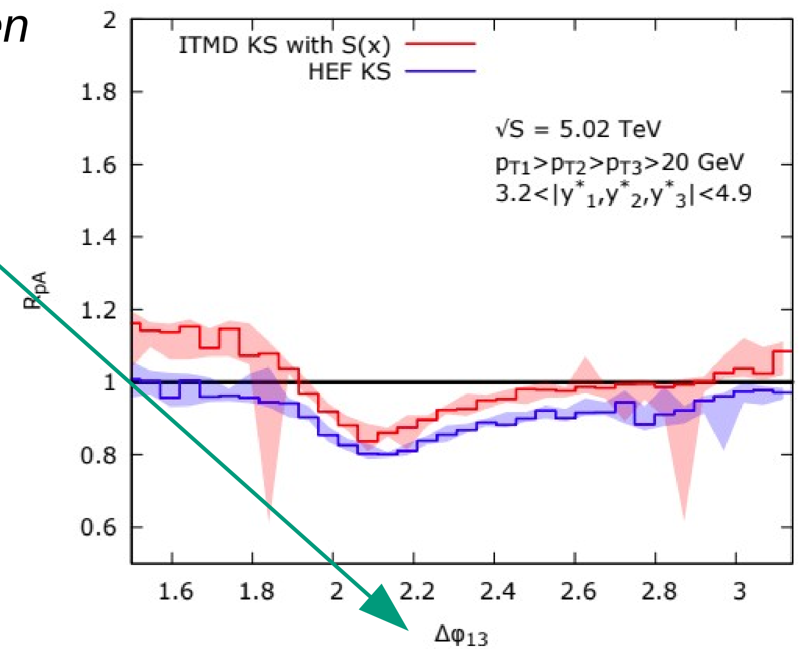
ratio p-A/p-p



angle between leading and subleading jet



angle between leading and softest jet

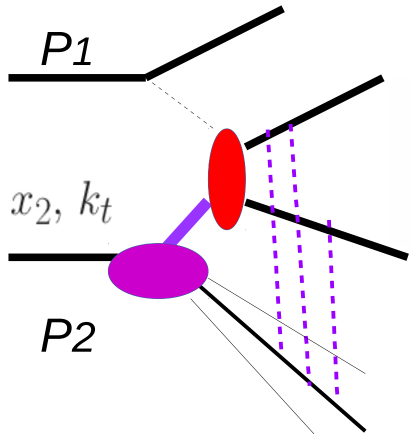


Bury, van Hameren, Kotko, Kutak
10.1007/JHEP09(2020)175

The lingitudinal polarisation contribution is not addressed here.

ITMD with Sudakov for dijets in DIS

P. Kotko, KK. S. Sapeta, A. van Hameren, E. Zarow



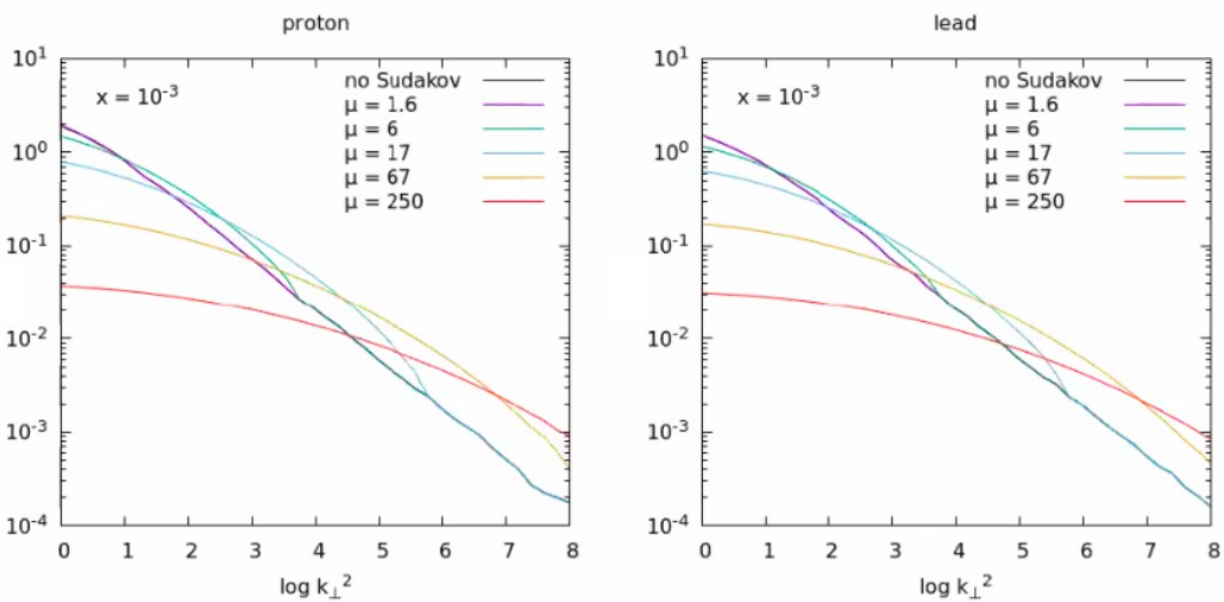
$$d\sigma_{\gamma^* A\to 2j+X} \propto \int \frac{dx}{x} d^2k_T\, \mathcal{F}_{gg}^{(3)}(x,k_T,\mu)\, \mathcal{M}_{\gamma^* g^*\to 2j}$$

This process allows to probe the Weizsacker-Williams TMD

$$S_{\rm Sud}^{g\to q\bar{q}}(\mu, b_T) = \frac{\alpha_s N_c}{4\pi} \ln^2 \frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

The Weizsacker-Williams TMD with Sudakov resummation to account for soft emissions

A. Mueller, B-W. Xiao, F. Yuan, 2013



Related studies for dijet/dihadron at EIC

Back-to-back regime using MV model + Sudakov
L. Zheng, E.C. Aschenauer, J.H. Lee, B-W. Xiao, 2014

Full CGC calculations (no Sudakov)
A. Dumitru, V. Skokov, 2018

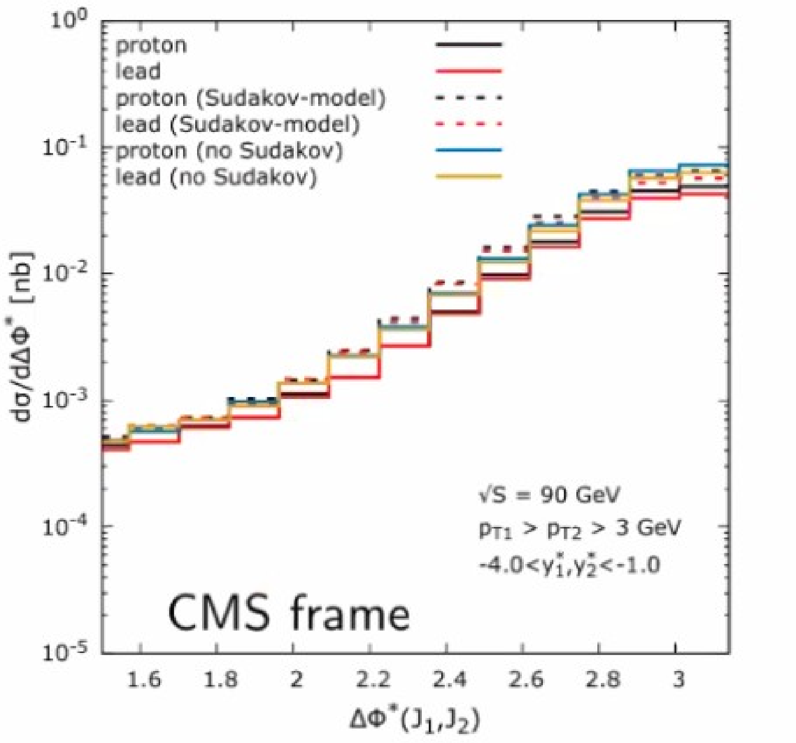
H. Mantysaari, N. Mueller, F. Salazar, B. Schenke, 2019
F. Salazar, B. Schenke, 2020

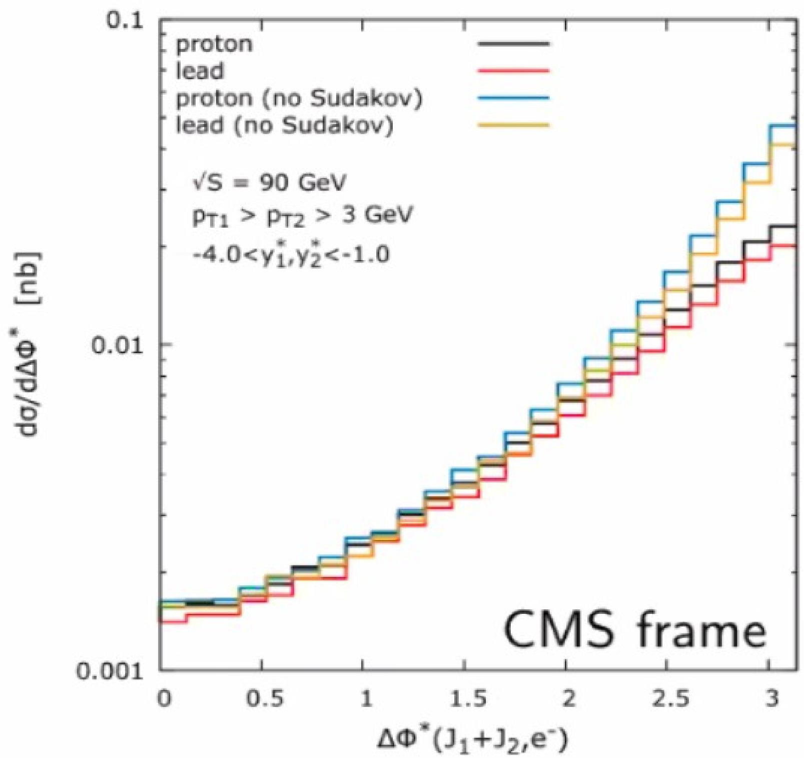
$$\mathcal{F}_{gg}^{(3)}(x,k_T,\mu) = \int db_T\, dk'_T\, b_T\, k'_T\, J_0(b_T k'_T)\, J_0(b_T k_T)\, \mathcal{F}_{gg}^{(3)}(x,k'_T)\, e^{-S_{\rm Sud}^{g\to q\bar{q}}(\mu,b_T)}$$

Azimuthal correlations EIC kinematics

Angle between dijets

New observable – angle between di-jet and electron





Large Sudakov effects

Comments: we use exact kinematics. We distinguish between x_g and x_{Bj}.
The WW gluon density comes from version of BK equation with kinematical constraint, accounts for large z parts of splitting function and gets contribution from quarks and is fitted to F_2 data preserving exact kinematics