Gauge-invariant TMD factorization for Drell-Yan hadronic tensor at small x

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Outline

1. Introduction: DY process at small $q_\perp^2/Q^2$:
   - Hadronic tensors from TMD factorization.
   - (No) EM gauge invariance of leading-twist hadronic tensor.
2. TMD factorization from rapidity factorization:
   - Rapidity factorization for particle production.
   - Classical fields from retarded propagators at $\frac{p_\perp^2}{p_\parallel^2} \ll 1$.
   - Leading-N_c power corrections.
   - Gauge-invariant result for EM hadronic tensor.
3. Results for Z-boson production:
   - Hadronic tensor for Z-boson production
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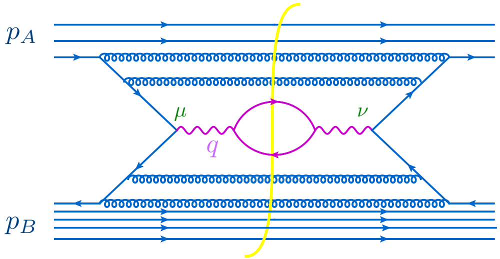
DY hadronic tensor

The hadronic tensor $W_{\mu\nu}$ is defined as

$$
\begin{aligned}
W_{\mu\nu}(p_A, p_B, q) \quad &\stackrel{\text{def}}{=} \quad \frac{1}{(2\pi)^4}\sum_X \int d^4x\; e^{-iqx}\langle p_A, p_B|J_\mu(x)|X\rangle\langle X|J_\nu(0)|p_A, p_B\rangle \\
&= \quad \frac{1}{(2\pi)^4}\int d^4x\; e^{-iqx}\langle p_A, p_B|J_\mu(x)J_\nu(0)|p_A, p_B\rangle
\end{aligned}
$$



p_A, p_B = hadron momenta, q = the momentum of DY pair, $\sum_X$ = the sum over full set of "out" states and J_μ is the electromagnetic current.

DY hadronic tensor

For unpolarized hadrons, the hadronic tensor $W_{\mu\nu}$ is parametrized by 4 functions, for example in Collins-Soper frame

$$W_{\mu\nu} = -\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big)(W_T + W_{\Delta\Delta}) - 2X_\mu X_\nu W_{\Delta\Delta} + Z_\mu Z_\nu(W_L - W_T - W_{\Delta\Delta}) - (X_\mu Z_\nu + X_\nu Z_\mu)W_\Delta$$

where X, Z are unit vectors orthogonal to q and to each other

TMD representation for W_i

The hadronic tensor in the Sudakov region $q^2 \equiv Q^2 \gg q_\perp^2$ can be studied by TMD factorization. For example, functions W_T and $W_{\Delta\Delta}$ can be represented as

$$W_i \;=\; \sum_{\text{flavors}} e_f^2 \int d^2k_\perp \mathcal{D}^{(i)}_{f/A}(x_A, k_\perp)\mathcal{D}^{(i)}_{f/B}(x_B, q_\perp - k_\perp) C_i(q, k_\perp)$$
$$+ \text{ power corrections } + \text{ Y} - \text{terms} \tag{1}$$

- $\mathcal{D}_{f/A}(x_A, k_\perp)$ is the TMD density of a parton f in hadron A with fraction of momentum x_A and transverse momentum $k_\perp$,
- $\mathcal{D}_{f/B}(x, q_\perp - k_\perp)$ is a similar quantity for hadron B,
- $C_i(q, k)$ are determined by the cross section $\sigma(f\!f \to \mu^+\mu^-)$ of production of DY pair of invariant mass q^2 in the scattering of two partons.

TMD representation for W_i

There is, however, a problem with Eq. (1) for the functions W_L and W_Δ.

W_T and $W_{\Delta\Delta}$ are determined by leading-twist quark TMDs,
but W_Δ and W_L start from terms $\sim \frac{q_\perp}{Q}$ and $\sim \frac{q_\perp^2}{Q^2}$ determined by quark-quark-gluon TMDs.

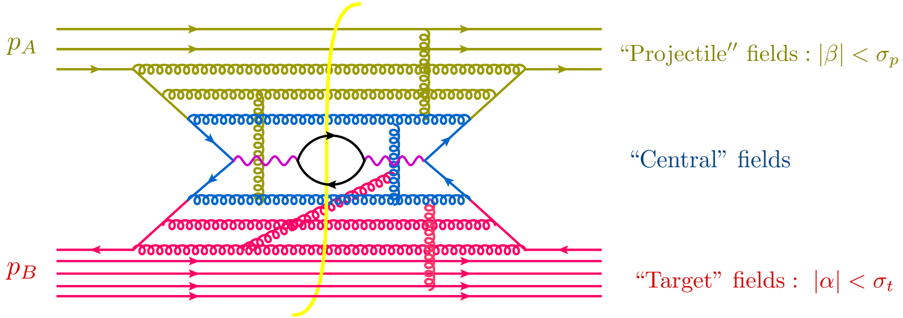
The power corrections $\sim \frac{q_\perp}{Q}$ were found more than two decades ago but there was no calculation of power corrections $\sim \frac{q_\perp^2}{Q^2}$ until recently.

Also, the leading-twist contribution is not EM gauge invariant.

Rapidity factorization for particle production

Sudakov variables:

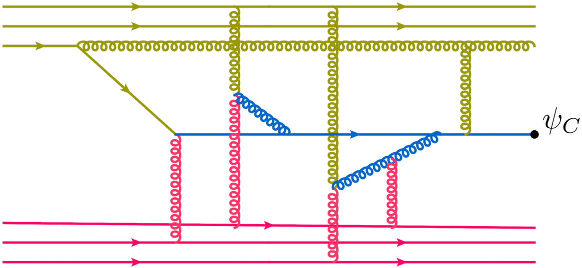
$$p = \alpha p_1 + \beta p_2 + p_\perp, \qquad p_1 \simeq p_A, \;\; p_2 \simeq p_B, \;\; p_1^2 = p_2^2 = 0$$
$$x_* \equiv p_2 \cdot x = \sqrt{\frac{s}{2}} x^+, \qquad x_\bullet \equiv p_1 \cdot x = \sqrt{\frac{s}{2}} x^-$$



We integrate over "central" fields in the background of projectile and target fields.

ψ_C in the tree approximation

Projectile fields: $\beta = 0 \Rightarrow A(x_\bullet, x_\perp),\ \psi_A(x_\bullet, x_\perp)$
Target fields: $\alpha = 0 \Rightarrow A(x_*, x_\perp),\ \psi_B(x_\bullet, x_\perp)$



Tree approximation for central fields:
ψ_C = sum of tree diagrams with *retarded* propagators in external A, ψ_A and A, ψ_B fields with sources
$J_\psi = (\not{P} + m)(\psi_A + \psi_B)$, $\quad J_\nu = D^\mu F^{\mu\nu}(A + A)$

Classical fields in the leading order in $p_\perp^2/p_\parallel^2 \sim q_\perp^2/Q^2$

The solution of YM equations in general case (scattering of two "color glass condensates") is yet unsolved problem.

Fortunately, for our case of particle production with $\frac{q_\perp}{Q} \ll 1$ we can use this small parameter and construct the approximate solution.

At the tree level transverse momenta are $\sim q_\perp^2$ and longitudinal are $\sim Q^2 \Rightarrow$

$$\psi, A \;=\; \text{series in } \frac{q_\perp}{Q}: \qquad \psi = \psi^{(0)} + \psi^{(1)} + \ldots, \qquad A = A^{(0)} + A^{(1)} + \ldots$$

NB: After the expansion

$$\frac{1}{p^2 + i\epsilon p_0} \;=\; \frac{1}{p_\parallel^2 - p_\perp^2 + i\epsilon p_0} \;=\; \frac{1}{p_\parallel^2} - \frac{1}{p_\parallel^2 + i\epsilon p_0} p_\perp^2 \frac{1}{p_\parallel^2 + i\epsilon p_0} \;+\ldots$$

the dynamics in transverse space is trivial.

Fields are either at the point $x_\perp$ or at the point $0_\perp \Rightarrow$ TMDs

Gauge:

$$A_* = 0 \quad \Rightarrow \quad A_i = \frac{2}{s}\int_{-\infty}^{x_\bullet} dx'_\bullet\, F_{*i}(x'_\bullet, x_\perp)$$

$$A_\bullet = 0 \quad \Rightarrow \quad A_i = \frac{2}{s}\int_{-\infty}^{x_*} dx'_*\, F_{*i}(x'_*, x_\perp)$$

Expansion of quark fields

Expanding it in powers of $p_\perp^2/p_\parallel^2$ gives:

$$\Psi(x) \; = \; \Psi_1^{(0)} + \Psi_2^{(0)} + \Psi^{(1)} + \Psi^{(2)} + \ldots,$$

where

$$\begin{aligned}
\Psi_A^{(0)} &= \psi_A + \Xi_1, \quad & \Xi_1 &= -\frac{g\not{p}_2}{s}\gamma^i A_i \frac{1}{\alpha + i\epsilon}\psi_A, \\
\bar\Psi_A^{(0)} &= \bar\psi_A + \bar\Xi_1, \quad & \bar\Xi_1 &= -\Big(\bar\psi_A \frac{1}{\alpha - i\epsilon}\Big)\gamma^i A_i \frac{g\not{p}_2}{s}, \\
\Psi_B^{(0)} &= \psi_B + \Xi_2, \quad & \Xi_{1B} &= -\frac{g\not{p}_1}{s}\gamma^i A_i \frac{1}{\beta + i\epsilon}\psi_B, \\
\bar\Psi_B^{(0)} &= \bar\psi_B + \bar\Xi_2, \quad & \bar\Xi_2 &= -\Big(\bar\psi_B \frac{1}{\beta - i\epsilon}\Big)\gamma^i A_i \frac{g\not{p}_1}{s}.
\end{aligned}$$

$\Psi^{(1)}$, $\Psi_B^{(2)}$,.. lead to next terms in series in $\frac{q_\perp^2}{Q^2}$

Leading-N_c power corrections

Power corrections are $\sim$ leading twist $\times\left(\frac{q_\perp}{Q} \text{ or } \frac{q_\perp^2}{Q^2}\right) \times \left(1 + \frac{1}{N_c} + \frac{1}{N_c^2}\right)$.

(Pleasant) surprise: most of the terms not suppressed by $\frac{1}{N_c}$ are determined by the leading-twist TMDs due to QCD equations of motion

Leading twist:

$$\frac{1}{8\pi^3 s}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet + i(k,x)_\perp}\ \langle p_A|\hat{\bar{\psi}}_f(x_\bullet, x_\perp)\ \not{p}_2 \hat{\psi}_f(0)|p_A\rangle\ =\ f_{1f}(\alpha, k_\perp^2)$$

Power correction:

$$\frac{1}{8\pi^3 s}\int dx_\bullet dx_\perp\ e^{-i\alpha_q x_\bullet + i(k,x)_\perp}$$

$$\times\ \langle p_A|\hat{\bar{\psi}}^f(x_\bullet, x_\perp)\ \not{p}_2[\hat{A}_i(x_\bullet, x_\perp) - i\gamma_5 \hat{\tilde{A}}_i(x_\bullet, x_\perp)]\hat{\psi}^f(0)|p_A A\rangle$$

$$=\ -k_i f_1(\alpha_q, k_\perp) + \alpha_q k_i\big[f_\perp(\alpha_q, k_\perp) + g^\perp(\alpha_q, k_\perp)\big],$$

(Mulders & Tangerman, 1996)

At small $\alpha_q \equiv x_A$ one can drop the second term

Result:

$$W_{\mu\nu}(q) \;=\; W^1_{\mu\nu}(q) + W^2_{\mu\nu}(q)$$

The first, gauge-invariant, part is given by

$$W^1_{\mu\nu}(q) \;=\; W^{1F}_{\mu\nu}(q) + W^{1H}_{\mu\nu}(q),$$

$$W^{1F}_{\mu\nu}(q) \;=\; \sum_f e_f^2 W^{fF}_{\mu\nu}(q), \quad W^{fF}_{\mu\nu}(q) \;=\; \frac{1}{N_c}\int d^2k_\perp F^f(q,k_\perp)\mathcal{W}^F_{\mu\nu}(q,k_\perp),$$

$$W^{1H}_{\mu\nu}(q) \;=\; \sum_f e_f^2 W^{fH}_{\mu\nu}(q), \quad W^{fH}_{\mu\nu}(q) \;=\; \frac{1}{N_c}\int d^2k_\perp H^f(q,k_\perp)\mathcal{W}^H_{\mu\nu}(q,k_\perp)$$

where F^f and H^f are ($\alpha_q \equiv x_A$, $\beta_q \equiv x_B$)

$$F^f(q,k_\perp) \;=\; f_1^f(\alpha_q,k_\perp)\bar{f}_1^f(\beta_q,(q-k)_\perp) \;+\; f_1^f \leftrightarrow \bar{f}_1^f$$
$$H^f(q,k_\perp) \;=\; h_{1f}^\perp(\alpha_q,k_\perp)\bar{h}_{1f}^\perp(\beta_q,(q-k)_\perp) \;+\; h_{1f}^\perp \leftrightarrow \bar{h}_{1f}^\perp$$

Gauge-invariant structures $\quad q^\mu W^F_{\mu\nu} = q^\mu W^H_{\mu\nu} = 0$

$$\mathcal{W}^F_{\mu\nu}(q,k_\perp)$$

$$= -g^\perp_{\mu\nu} + \frac{1}{Q_\parallel^2}(q^\parallel_\mu q^\perp_\nu + q^\parallel_\nu q^\perp_\mu) + \frac{q_\perp^2}{Q_\parallel^4} q^\parallel_\mu q^\parallel_\nu + \frac{\tilde{q}_\mu \tilde{q}_\nu}{Q_\parallel^2}[q_\perp^2 - 4(k,q-k)_\perp]$$

$$- \Big[\frac{\tilde{q}_\mu}{Q_\parallel^2}\Big(g^\perp_{\nu i} - \frac{q^\parallel_\nu q_i}{Q_\parallel^2}\Big)(q-2k)^i_\perp + \mu \leftrightarrow \nu\Big]$$

$$\tilde{q} \equiv \alpha_q p_1 - \beta_q p_2$$

$$m^2 \mathcal{W}^H_{\mu\nu}(q,k_\perp)$$

$$= -k^\perp_\mu (q-k)^\perp_\nu - k^\perp_\nu (q-k)^\perp_\mu - g^\perp_{\mu\nu}(k,q-k)_\perp + 2\frac{\tilde{q}_\mu\tilde{q}_\nu - q^\parallel_\mu q^\parallel_\nu}{Q_\parallel^4} k_\perp^2 (q-k)_\perp^2$$

$$- \Big(\frac{q^\parallel_\mu}{Q_\parallel^2}\big[k_\perp^2 (q-k)^\perp_\nu + k^\perp_\nu (q-k)^2_\perp\big] + \frac{\tilde{q}_\mu}{Q_\parallel^2}\big[k_\perp^2 (q-k)^\perp_\nu - k^\perp_\nu (q-k)^2_\perp\big] + \mu \leftrightarrow \nu\Big)$$

$$- \frac{\tilde{q}_\mu\tilde{q}_\nu + q^\parallel_\mu q^\parallel_\nu}{Q_\parallel^4}\big[q_\perp^2 - 2(k,q-k)_\perp\big](k,q-k)_\perp - \frac{q^\parallel_\mu \tilde{q}_\nu + \tilde{q}_\mu q^\parallel_\nu}{Q_\parallel^4}(2k-q,q)_\perp (k,q-k)_\perp$$

W^F part coincides with parton Reggeization result (Nefedov, Saleev, 2019)

Non-gauge-invariant corrections

$$W^2_{\mu\nu}(q) \;=\; \frac{1}{N_c}\sum_f e_f^2 \frac{1}{Q^2}\int d^2k_\perp \Big[\frac{1}{m^2}H_A^f(q,k_\perp)\big\{[k^\perp_\mu(q-k)^\perp_\nu + \mu\leftrightarrow\nu](k,q-k)_\perp$$
$$- k_\perp^2(q-k)^\perp_\mu(q-k)^\perp_\nu - (q-k_\perp)^2 k^\perp_\mu k^\perp_\nu + g^\perp_{\mu\nu}(k,q-k)^2_\perp - g^\perp_{\mu\nu}k_\perp^2(q-k_\perp)^2\big]\big\}$$
$$+ \frac{N_c}{N_c^2-1}\Big\{[k^\perp_\mu(q-k)^\perp_\nu + \mu\leftrightarrow\nu + g^\perp_{\mu\nu}(k,q-k)_\perp]J_1^f(q,k_\perp)$$
$$- g^\perp_{\mu\nu}(k,q-k)_\perp J_2^f(q,k_\perp)\Big\} \;+\; O\Big(\frac{1}{N_c^2}\Big)\Big] \;+\; O\Big(\frac{Q_\perp^4}{Q^4}\Big)$$

H_A and $J_{1,2}$ terms involve twist-3 quark-quark-gluon TMDs which do not reduce to leading-twist distributions.

Gauge invariance should be restored after adding sub-leading power corrections. For example,

$$q^\mu W^2_{\mu\nu}(q) \;\sim\; \frac{q^\perp_\nu q^2_\perp}{Q^2} \quad \text{and} \quad q^\mu \times \Big(\frac{q^\parallel_\mu q^\perp_\nu}{Q^2}\times\frac{q^2_\perp}{Q^2}\Big) \;=\; \frac{q^\perp_\nu q^2_\perp}{Q^2}$$

They are of the same order so one should expect that gauge invariance is restored after calculation of all such terms.

Power corrections for total cross section (A. Tarasov and I.B.,2019)

If $Q^2 \gg k_\perp^2 \gg m_N^2$ we can approximate TMDs by perturbative tails:

$$f_1(\alpha_z, k_\perp^2) \simeq \frac{f(\alpha_z)}{k_\perp^2}, \quad h_1^\perp(\alpha_z, k_\perp^2) \simeq \frac{m_N^2 h(\alpha_z)}{k_\perp^4}$$

For the total DY cross section

$$\begin{aligned}
W_\mu^\mu(q) &= -\frac{2}{N_c}\sum e_f^2 \int d^2k_\perp \Big\{\Big[1 - 2\frac{(k,q-k)_\perp}{Q^2}\Big] F^f(q,k_\perp) + 2\frac{k_\perp^2 (q-k)_\perp^2}{m_N^2 Q^2} H^f(q,k_\perp)\Big\} \\
&\simeq -\frac{2}{N_c}\sum e_f^2 \int d^2k_\perp \Big\{\Big[1 - 2\frac{(k,q-k)_\perp}{Q^2}\Big] \frac{F^f(\alpha_q,\beta_q)}{k_\perp^2 (q-k)_\perp^2} + \frac{2m^2}{Q^2}\frac{H^f(\alpha_q,\beta_q)}{k_\perp^2 (q-k)_\perp^2}\Big\} \\
&\simeq -\frac{2}{N_c}\sum e_f^2 \int d^2k_\perp \Big[1 - 2\frac{(k,q-k)_\perp}{Q^2}\Big] \frac{F^f(\alpha_q,\beta_q)}{k_\perp^2 (q-k)_\perp^2}
\end{aligned}$$

With logarithmic accuracy

$$W_\mu^\mu(q) = -\frac{4\pi}{N_c}\sum e_f^2 \Big[\frac{1}{q_\perp^2}\ln\frac{q_\perp^2}{m_N^2} + \frac{1}{Q^2}\ln\frac{Q^2}{q_\perp^2}\Big]\sum_f [f^f(\alpha_z)\bar{f}^f(\beta_z) + \bar{f}^f(\alpha_z) f^f(\beta_z)]$$

$\Rightarrow$ power corrections reach 10% level at $q_\perp \sim \frac{1}{4}Q$

Z-boson production at LHC

The relevant terms of the Lagrangian for quark fields ψ^f are

$$\mathcal{L}_Z = e\int d^4x\, \mathcal{J}_\mu Z^\mu(x), \qquad \mathcal{J}_\mu = c_e\bar{e}(a_e-\gamma_5)e - \sum_{\text{flavors}} c_f\bar{\psi}^f\gamma_\mu(a_f-\gamma_5)\psi^f$$

where

$$c_{u,c} = \frac{1}{4c_Ws_W}, \quad a_{u,c} = 1-\frac{8}{3}s_W^2, \quad c_{d,s} = -\frac{1}{4c_Ws_W}, \quad a_{d,s} = 1-\frac{4}{3}s_W^2,$$
$$c_e = \frac{1}{4c_Ws_W}, \quad a_e = 1-4s_W^2, \qquad c_W\equiv\cos\theta_W, \quad s_W\equiv\sin\theta_W.$$

$$d\sigma = \frac{e^4}{16\pi^2 sN_c}\frac{dQ^2}{Q^2}dYd^2q_\perp d\Omega_l\,\mathbb{W}(q,l,l')$$

l, l' - lepton momenta

In CMS and ATLAS experiments $s = 8$ TeV, $Q = 80 - 100$ GeV and $Q_\perp$ varies from 0 to 120 GeV.

Our analysis is valid at $Q_\perp = 10 - 30$ GeV and $Y \simeq 0$ ($x_A \sim x_B \sim 0.1$) so that power corrections are small but sizable.

Angular distribution of DY leptons in the Collins-Soper frame ($c_\phi \equiv \cos\phi$, $s_\phi \equiv \sin\phi$ etc.)

$$\frac{d\sigma}{dQ^2 dy d\Omega_l} = \frac{3}{16\pi}\frac{d\sigma}{dQ^2 dy}\Big[(1 + c_\theta^2) + \frac{A_0}{2}(1 - 3c_\theta^2) + A_1 s_{2\theta} c_\phi + \frac{A_2}{2} s_\theta^2 c_{2\phi} + A_3 s_\theta c_\phi + A_4 c_\theta + A_5 s_\theta^2 s_{2\phi} + A_6 s_{2\theta} s_\phi + A_7 s_\theta s_\phi\Big]$$

Result with $\frac{1}{Q^2}$, large-N_c and "f_1" accuracy

$$\mathbb{W}(q,l,l') = c_e^2 c_f^2 \frac{Q^4}{|m_Z^2 - Q^2|^2 + \Gamma_Z^2 m_Z^2}$$

$$\times \sum_f \Big\{(a_e^2+1)(a_f^2+1)\Big(\Big[\mathcal{W}^{\rm Ff} - \frac{Q_\perp^2}{2Q^2}(\mathcal{W}^{\rm Ff} - \mathcal{W}_L^{\rm Ff})\Big](1+\cos^2\theta)$$

$$+ \frac{Q_\perp^2}{2Q^2}\mathcal{W}_L^{\rm Ff}(1-3\cos^2\theta) + \frac{Q_\perp}{Q}\mathcal{W}_1^{\rm Ff}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2Q^2}\mathcal{W}^{\rm Ff}\sin^2\theta\cos 2\phi\Big]\Big)$$

$$+ 8a_e a_f\Big[\frac{Q_\perp}{Q}\mathcal{W}_3^{\rm Ff}\sin\theta\cos\phi + \mathcal{W}_4^{\rm Ff}\cos\theta\Big]\Big\}$$

$$\mathcal{W}^{\rm Ff}(q) = \int d^2k_\perp F^f(q,k_\perp), \qquad \mathcal{W}_L^{\rm Ff}(q) = \int dk_\perp \frac{(q-2k)_\perp^2}{q_\perp^2}F^f(q,k_\perp)$$

$$\mathcal{W}_1^{\rm Ff}(q) = \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2}F^f(q,k_\perp)$$

$$\mathcal{W}_3^{\rm Ff}(q) = \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2}\mathcal{F}^f(q,k_\perp), \qquad \mathcal{W}_4^{\rm Ff}(q) = \int d^2k_\perp \mathcal{F}^f(q,k_\perp),$$

$$\mathcal{F}^f(q,k_\perp) = f_1^f(\alpha_q,k_\perp)\bar{f}_1^f(\beta_q,(q-k)_\perp) - f_1^f \leftrightarrow \bar{f}_1^f$$

Comparison with LHC results

$$\mathbb{W} \sim \sum_f \mathcal{W}^{\mathrm{Ff}} \Big\{ (a_e^2+1)(a_f^2+1)\Big(\Big[1 - \frac{Q_\perp^2}{2m_Z^2} + \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}\Big](1+\cos^2\theta)$$
$$+ \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}(1-3\cos^2\theta) + \frac{Q_\perp}{m_Z}\frac{\mathcal{W}_1^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2m_Z^2}\sin^2\theta\cos 2\phi]\Big)$$
$$+ 8a_e a_f\Big[\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_3^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}\sin\theta\cos\phi + \frac{\mathcal{W}_4^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}\cos\theta\Big]\Big\}$$

We can easily estimate A_0 and A_2 which depend on $\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}$.

Logarithmic estimate of $\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}$: if $k_\perp^2 \gg m_N^2$ we can approximate

$$f_1(x,k_\perp^2) \simeq \frac{f(x)}{k_\perp^2} \quad \Rightarrow \quad F(q,k_\perp) \simeq \frac{f(\alpha_q)\bar{f}(\beta_q) + f \leftrightarrow \bar{f}}{k_\perp^2 (q-k)_\perp^2}$$

Performing integration over $k_\perp$ in logarithmical approximation, one obtains

$$\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}} \simeq 1 + 2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}$$

Comparison of A_0 with LHC results

Logarithmic estimate of A_0

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1 + 2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2} \quad \Rightarrow \quad A_0 = \frac{Q_\perp^2}{m_z^2}\frac{1 + 2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}}{1 + \frac{Q_\perp^2}{m_z^2}\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (*)$$

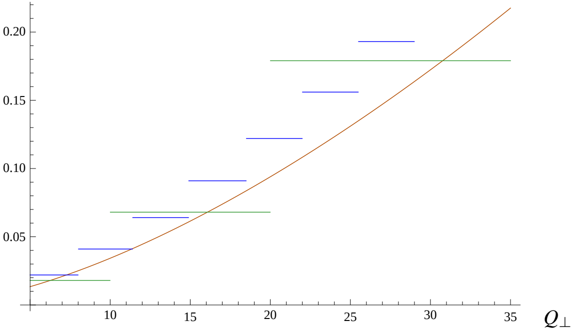


Figure: Comparison of prediction $(*)$ with lines depicting angular coefficient A_0 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Comparison of A_2 with LHC results

Logarithmic estimate of A_2

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1 + 2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2} \quad \Rightarrow \quad A_2 = \frac{Q_\perp^2}{m_z^2}\frac{1}{1 + \frac{Q_\perp^2}{m_z^2}\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (**)$$

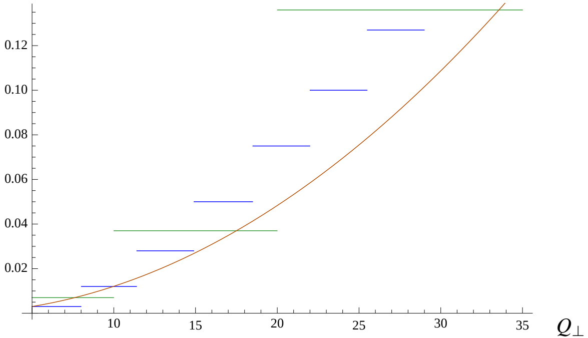


Figure: Comparison of prediction $(**)$ with lines depicting angular coefficient A_2 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Qualitative checks

$$\mathbb{W} \sim \sum_f r^f \mathcal{W}^{\rm Ff}\Big\{1+\cos^2\theta + \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}(1-3\cos^2\theta)$$
$$+\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_1^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2m_Z^2 r^f}\sin^2\theta\cos 2\phi]$$
$$+\frac{8a_e a_f}{(a_e^2+1)(a_f^2+1)}\Big[\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_3^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\sin\theta\cos\phi + \frac{\mathcal{W}_4^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\cos\theta\Big]\Big\}$$

$r^f \equiv 1 - \frac{Q_\perp^2}{2m_Z^2} + \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}$

Qualitative checks:

- Factorization of TMD $f_1(x,k_\perp^2) \simeq f(x)f(k_\perp^2) \Rightarrow \mathcal{W}_1^{\rm Ff}(q) = 0$
  $\Rightarrow A_1$ is smaller than A_2
- A_4 does not depend on $Q_\perp$ and increases with rapidity
- A_3 is smaller than A_4
- A_5, A_6, A_7 are order of magnitude smaller than A_0, A_2, A_4

Conclusions

1. Conclusions
   - The Drell-Yan hadronic tensor is calculated in the Sudakov region $s \gg Q^2 \gg q_\perp^2$ in the tree approximation with $\frac{1}{Q^2}$ accuracy.
   - In the leading order in N_c the higher-twist quark-quark-gluon TMDs reduce to leading-twist TMDs due to QCD equation of motion.
   - The resulting hadronic tensor for unpolarized hadrons is (EM) gauge-invariant and depends on two leading-twist TMDs: f_1 responsible for total DY cross section, and Boer-Mulders function $h_1^\perp$.
   - Results for angular coefficients of Z-boson production seem to agree with LHC measurements at corresponding kinematics.
2. Outlook
   - Rapidity factorization at the one-loop level.

Thank you for attention!