

Exclusive Double Drell-Yan factorization and GTMDs

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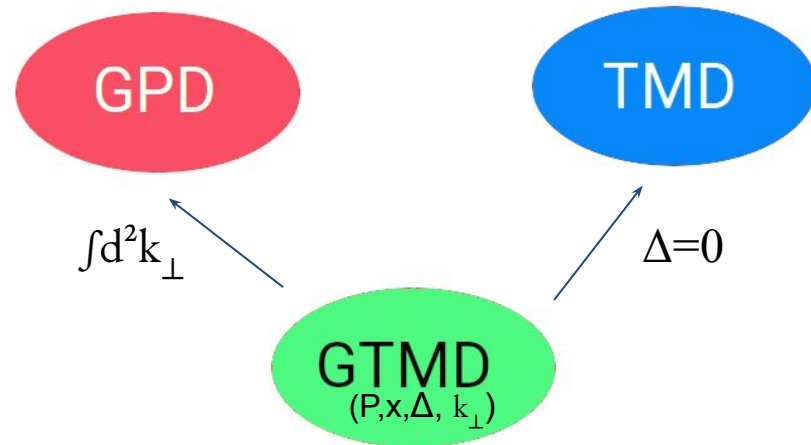


Outline

- GTMDs
- Exclusive Double Drell-Yan process
- Factorization Theorem
 - Current operator
 - Multipole expansion
 - Soft factor, 2TMD, GTMDs
- Color
- Zero-bin subtraction
- Conclusions

Goal of the study

- Multi dimensional imaging of strongly interacting systems
- GTMDs absorb both GPDs and TMDs
- *How are GTMDs defined?*
- *How to obtain GTMDs in a cross-section?*



GTMDs

- General correlator for GTMDs:

$$W_{\lambda\lambda'}^{[\Gamma],0}(P, \Delta, x, \vec{k}_\perp) = \frac{1}{2} \int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} e^{i(z^- k^+ / 2 - \vec{z}_\perp \cdot \vec{k}_\perp)} \underbrace{(p', \lambda') \bar{q} W_n(-z/2) \Gamma W_n^\dagger(z/2) q(p, \lambda)}_{\text{Wilson line}} \Big|_{z^+=0}$$

$$P = (p + p')/2 \quad \Delta = p - p'$$

Wilson line:

$$W_{n;\alpha\beta}(z) = \left\{ \mathcal{P} \exp \left[-ig \int_0^\infty ds \bar{n} \cdot A(z + s\bar{n}) \right] \right\}_{\alpha\beta}$$

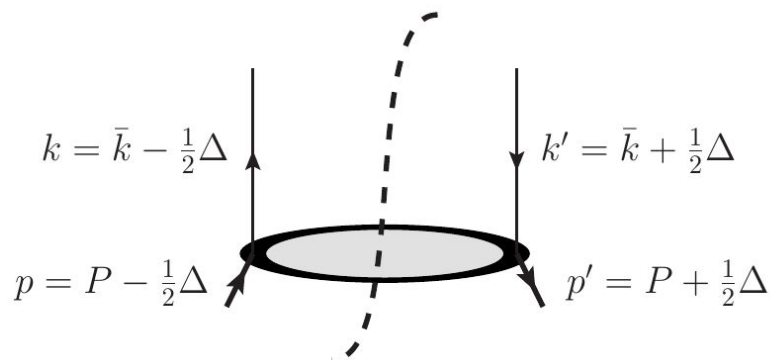


Figure 1: Kinematics of a GTMD. Echevarría Et. al. Physics Letters B, 759, 336-341 [1]

GTMDs

- General correlator for GTMDs:

$$W_{\lambda\lambda'}^{[\Gamma],0}(P, \Delta, x, \vec{k}_\perp) = \frac{1}{2} \int \frac{dz^- d^2 \vec{z}_\perp}{(2\pi)^3} e^{i(z^- k^+ / 2 - \vec{z}_\perp \cdot \vec{k}_\perp)} \underbrace{\langle p', \lambda' | \bar{q} W_n(-z/2) \Gamma W_n^\dagger q(z/2) | p, \lambda \rangle}_{z^\pm=0}$$

- Subtracted correlator: Echevarría Et. al. Physics Letters B, 759, 336-341 [1] $\phi_{\lambda\lambda'}^{[\Gamma],q}$

Free from rapidity divergences

$$W_{\lambda\lambda'}^{[\Gamma],q} = \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{+i(\frac{1}{2}z^- \bar{k}^+ - \mathbf{z}_\perp \cdot \bar{\mathbf{k}}_\perp)} \phi_{\lambda\lambda'}^{[\Gamma],q}(0, z^-, \mathbf{z}_\perp) S^{\frac{1}{2}}(z_T)$$

- Soft factor:

$$S(z_T) = \frac{\text{Tr}_c}{N_c} \langle 0 | \mathcal{S}_n^\dagger \left(-\frac{z}{2} \right) \mathcal{S}_{\bar{n}} \left(-\frac{z}{2} \right) \mathcal{S}_{\bar{n}}^\dagger \left(\frac{z}{2} \right) \mathcal{S}_n \left(\frac{z}{2} \right) | 0 \rangle \Big|_{z^\pm=0}$$

Exclusive DDY

- GTMDs: Exclusive Double Drell-Yan

$$\pi(p_b) + N(p_a, \lambda_a) \rightarrow \gamma_1^*(q_1, \lambda_1) + \gamma_2^*(q_2, \lambda_2) + N'(p'_a, \lambda'_a)$$

- Amplitude calculation at LO by Bhattacharya et.al.

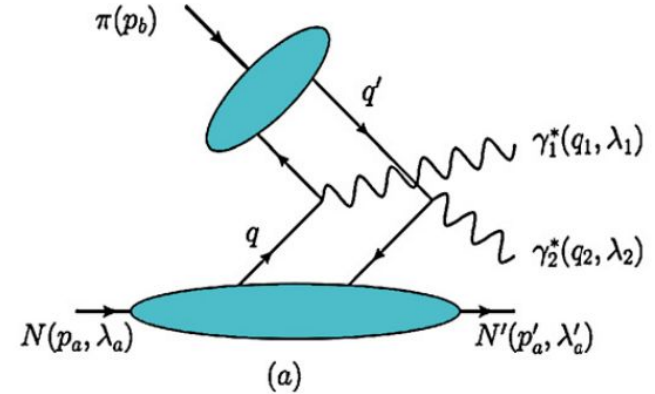


Figure 2: Exclusive DDY. Bhattacharya, S., Metz, A., & Zhou, J. (2017) Physics Letters B, 771, 396-400 [2]

$$d\sigma_{\lambda_a, \lambda'_a}^{\lambda_1, \lambda_2} = \frac{\pi}{2s^{3/2}} \frac{1 + \xi_a}{1 - \xi_a} |\mathcal{T}_{\lambda_a, \lambda'_a}^{\lambda_1, \lambda_2}|^2 \delta(p_a'^0 + q_1^0 + q_2^0 - \sqrt{s})$$

$$\times \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4},$$

Exclusive DDY

- GTMDs: Exclusive Double Drell-Yan

$$\pi(p_b) + N(p_a, \lambda_a) \rightarrow \gamma_1^*(q_1, \lambda_1) + \gamma_2^*(q_2, \lambda_2) + N'(p'_a, \lambda'_a)$$

- Amplitude calculation at LO by Bhattacharya et.al.

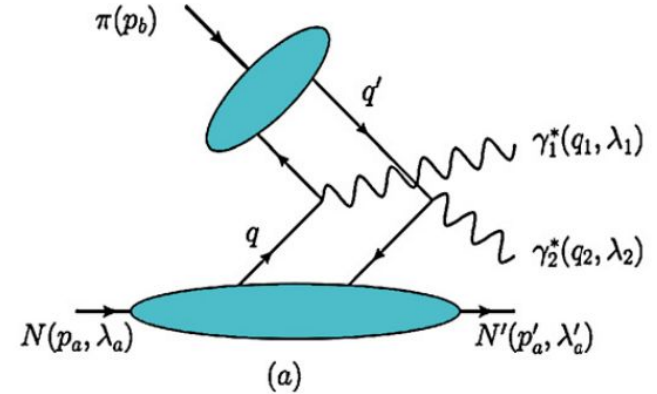


Figure 2: Exclusive DDY. Bhattacharya, S., Metz, A., & Zhou, J. (2017) Physics Letters B, 771, 396-400 [2]

$$\begin{aligned} \mathcal{T}_{\lambda_a, \lambda'_a}^{\mu\nu} = & i \sum_{q, q'} e_q e'_q e^2 \frac{(2\pi)^4}{N_c} \int d^2 \vec{k}_{a\perp} \int d^2 \vec{k}_{b\perp} \delta^{(2)} \left(\frac{\Delta \vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp} \right) \Phi_\pi^{q'q}(x_b, \vec{k}_{b\perp}^2) \\ & \left[-i \varepsilon_\perp^{\mu\nu} \left(W_{\lambda_a, \lambda'_a}^{qq'}[\gamma^+] (x_a, \vec{k}_{a\perp}) - W_{\lambda_a, \lambda'_a}^{qq'}[\gamma^+] (-x_a, -\vec{k}_{a\perp}) \right) \right. \\ & \left. - g_\perp^{\mu\nu} \left(W_{\lambda_a, \lambda'_a}^{qq'}[\gamma^+ \gamma_5] (x_a, \vec{k}_{a\perp}) + W_{\lambda_a, \lambda'_a}^{qq'}[\gamma^+ \gamma_5] (-x_a, -\vec{k}_{a\perp}) \right) \right], \end{aligned}$$

SCET factorization

- Matching to current operator:

$$J_{\mu QCD} = \sum_{q_1} e_{q_1} \bar{\psi} \gamma^\mu \psi$$

$$J_{SCET}^\mu = \sum_q e_q C(Q^2/\mu^2) \bar{\chi}_{\bar{n}}^q S_{\bar{n}}^{T\dagger} \gamma^\mu S_n^T \chi_n^q$$

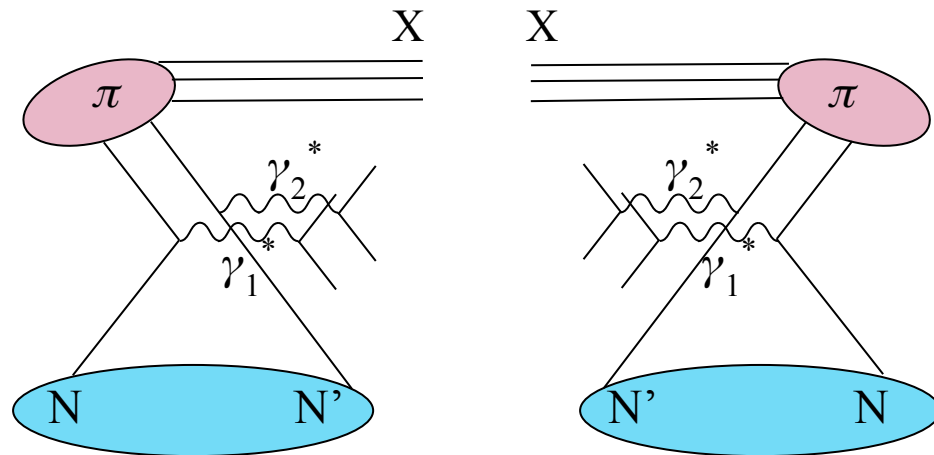
where:

$$\chi_{n(\bar{n})} = W_{n(\bar{n})}^{T\dagger} \xi_{n(\bar{n})} = W_{n(\bar{n})}^{T\dagger} P_{+(-)} \psi_{n(\bar{n})}$$

and:

$$S_n^T = T_{sn} S_n$$

$$S_n(x) = P \exp \left[ig \int_{-\infty}^0 ds n \cdot A_s^a(x + sn) t^a \right]$$



$$\pi(p_b) + N(p_a, \lambda_a) \rightarrow \gamma_1^*(q_1, \lambda_1) + \gamma_2^*(q_2, \lambda_2) + N'(p'_a, \lambda'_a)$$

Hard

$$q^\mu \sim Q(1, 1, \lambda)$$

Collinear

$$k^\mu \sim Q(\lambda^2, 1, \lambda)$$

Anti-collinear

$$k^\mu \sim Q(1, \lambda^2, \lambda)$$

Soft

$$k^\mu \sim Q(\lambda, \lambda, \lambda)$$

$$\lambda = \frac{q_T}{p_a^+}$$

SCET factorization

- Double parton scattering

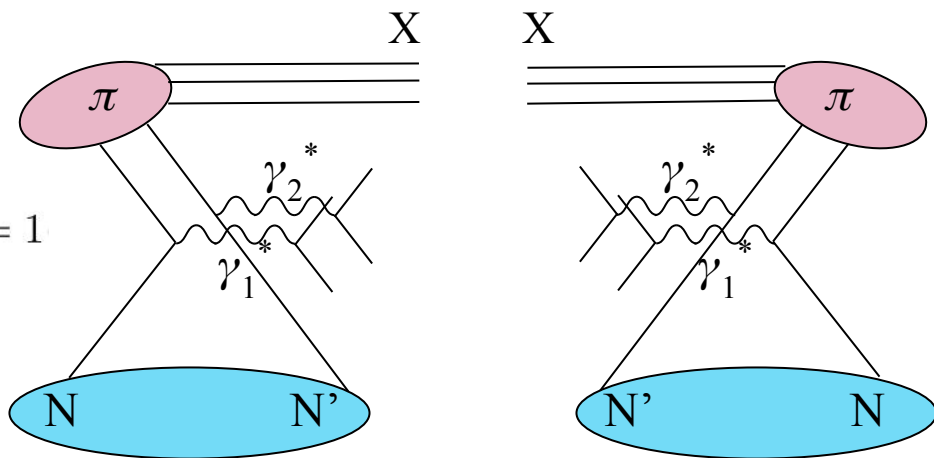
$$\frac{d\sigma}{d^4q_1 d^4q_2} \propto \sum_X \int dz_{1,2,3} e^{-iq_1 z_1 - iq_2 z_2 + iq_1 z_3} \langle \Pi N | \bar{T} \{ J^{\dagger\alpha}(z_1) J^{\dagger\beta}(z_2) \} | X N' \rangle \times \langle X N' | T \{ J^\mu(z_3) J^\nu(0) \} | \Pi N \rangle$$

- Particles assigned to sectors

$$N \rightarrow n, \Pi \rightarrow \bar{n}, X \rightarrow s, \bar{n}$$

$$\sum_X |X\rangle \langle X| = \sum_{X_{\bar{n}}} |X_{\bar{n}}\rangle \langle X_{\bar{n}}| \times \sum_{X_s} |X_s\rangle \langle X_s| = 1$$

- After fierzing we factorize:



$$\frac{d\sigma}{d^4q_1 d^4q_2} = \int d^4z_{1,2,3} e^{-iq_1 z_1 - iq_2 z_2 + iq_1 z_3} \sum_{\Gamma} \sum_{\Gamma'} C_{\Gamma} C_{\Gamma'} H(Q^2/\mu^2) \Phi_{DDY}(z) f_{pion}(z) f_{NN'}(0, z_3) f_{N'N}(z_1, z_2)$$

SCET factorization

$$\frac{d\sigma}{d^4q_1 d^4q_2} = \int d^4z_{1,2,3} e^{-iq_1 z_1 - iq_2 z_2 + iq_1 z_3} \sum_{\Gamma} \sum_{\Gamma'} C_{\Gamma} C_{\Gamma'} H(Q^2/\mu^2) \Phi_{DDY}(z) f_{pion}(z) f_{NN'}(0, z_3) f_{N'N}(z_1, z_2)$$

- Multipole expansion + FTs properties lead to:

$$\frac{d\sigma}{dx_{1,2} dy_{1,2} d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma} \sum_{\Gamma'} C_{\Gamma} C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp} - i\vec{q}_{2\perp}\vec{b}_{2\perp} + i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_{\perp}\}) \tilde{f}_{pion}(\{y, \vec{b}_{\perp}\}) \tilde{f}_{NN'} \tilde{f}_{N'N}(\{x, \vec{b}_{\perp}\})$$

Factorized cross-section in impact parameter space with:

$$\tilde{f}(x; \vec{b}_{\perp}) = \int d^2\vec{k}_{\perp} e^{i\vec{k}_{\perp}\cdot\vec{b}_{\perp}} f(x; \vec{k}_{\perp}) \quad \text{and}$$

Φ_{DDY}	- Soft factor with 8 Wilson lines
f_{pion}	- Naive Double TMD
$f_{NN'} f_{N'N}$	- Naive Two GTMDs

SCET factorization: Soft factor

- Factorized cross-section:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma} \sum_{\Gamma'} C_{\Gamma} C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp} - i\vec{q}_{2\perp}\vec{b}_{2\perp} + i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_{\perp}\}) \tilde{f}_{pion}(\{y, \vec{b}_{\perp}\}) \tilde{f}_{NN'} \tilde{f}_{N'N}(\{x, \vec{b}_{\perp}\})$$

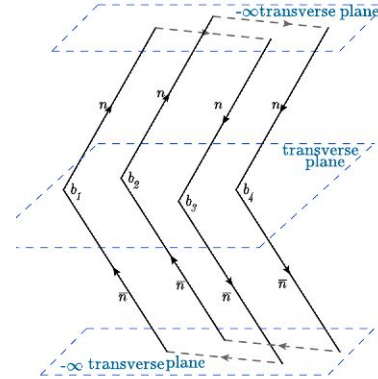


Figure 3: Double Wilson Loop Soft Factor.
Vladimirov, A. (2018)
Journal of High Energy Physics, 2018(4), 1-46 [3]

- Same Soft factor than in DPS

$$\tilde{\Phi}_{DDY}(\vec{b}_{1\perp}, \vec{b}_{2\perp}, \vec{b}_{3\perp}) = \langle 0 | S_n^{T\dagger}(\vec{b}_{1\perp}) S_{\bar{n}}^T(\vec{b}_{1\perp}) S_{\bar{n}}^{T\dagger}(\vec{b}_{2\perp}) S_n^T(\vec{b}_{2\perp}) S_{\bar{n}}^{T\dagger}(\vec{b}_{3\perp}) S_n^T(\vec{b}_{3\perp}) S_n^{T\dagger}(0) S_{\bar{n}}^T(0) | 0 \rangle$$

SCET factorization: 2TMD

- Factorized cross-section:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma} \sum_{\Gamma'} C_{\Gamma} C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp} - i\vec{q}_{2\perp}\vec{b}_{2\perp} + i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_{\perp}\}) \tilde{f}_{pion}(\{y, \vec{b}_{\perp}\}) \tilde{f}_{NN'} \tilde{f}_{N'N}(\{x, \vec{b}_{\perp}\})$$

- Pure Double TMDPDF (unsubtracted, with rapidity divergences):

Buffing, M. G., Diehl, M., & Kasemets, T. (2018). Journal of High Energy Physics, 2018(1), 1-112 [4]

$$\tilde{f}_{pion}(\{y, \vec{b}_{\perp}\}) = \prod_j \int \frac{dr_j^+}{2\pi} e^{-ir_j^+ y_j p_b^-} \left\langle \Pi | \bar{\chi}_{\bar{n}}(r_2^+, 0^-, \vec{b}_{2\perp}) \Gamma^{\mu} \chi_{\bar{n}}(r_1^+, 0^-, \vec{b}_{1\perp}) \bar{\chi}_{\bar{n}}(r_3^+, 0^-, \vec{b}_{3\perp}) \Gamma'^{\beta} \chi_{\bar{n}}(0) | \Pi \right\rangle$$

where $j=1, \dots, 3$ and $y_3^- = -y_1^-$

SCET factorization: GTMDs

- Factorized cross-section:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma} \sum_{\Gamma'} C_{\Gamma} C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp} - i\vec{q}_{2\perp}\vec{b}_{2\perp} + i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_{\perp}\}) \tilde{f}_{pion}(\{y, \vec{b}_{\perp}\}) \tilde{f}_{NN'} \tilde{f}_{N'N}(\{x, \vec{b}_{\perp}\})$$

- Two pure GTMDs evaluated in different positions (unsubtracted, with rapidity divergences)

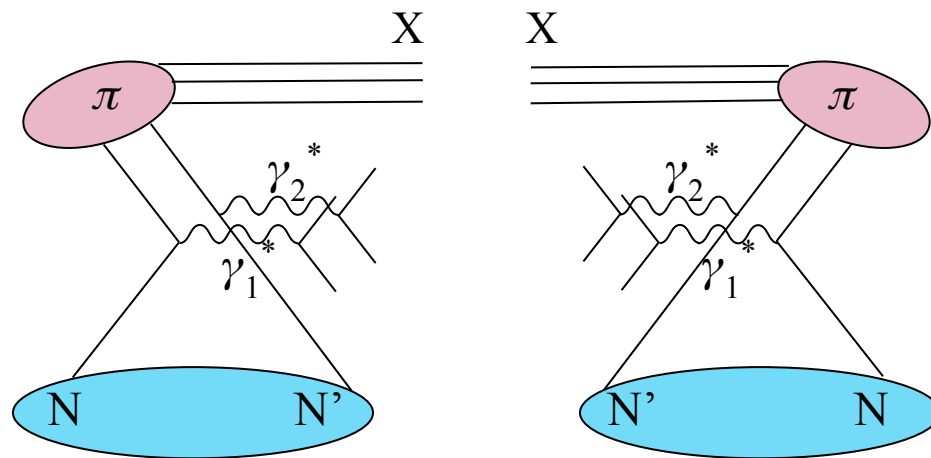
One-loop calculation in Echevarría et.al.[1]

$$\tilde{f}_{NN'} \tilde{f}_{N'N}(\{x, \vec{b}_{\perp}\}) =$$

$$\prod_j \int \frac{dr_j^-}{(2\pi)} e^{-ir_j^+ x_j p_a^+} \langle N | \bar{\chi}_n^{q_1}(0^+, r_1^-, \vec{b}_{1\perp}) \Gamma^\alpha \chi_n^{q'_1}(0^+, r_2^-, \vec{b}_{2\perp}) | N' \rangle \langle N' | \bar{\chi}_n^{q'_2}(0) \Gamma'^\nu \chi_n^{q_2}(0^+, r_3^-, \vec{b}_{3\perp}) | N \rangle$$

where $j=1, \dots, 3$ and $x_3 = -x_1$

Color



- Consider color structure

$$f_{pion}^{[d_4, \dots, d_1]} \propto \langle \Pi | \bar{\chi}_{\bar{n}}^{d_2}(r_2^+, 0^-, \vec{b}_{2\perp}) \Gamma^\mu \chi_{\bar{n}}^{d_1}(r_1^+, 0^-, \vec{b}_{1\perp}) \bar{\chi}_{\bar{n}}^{d_3}(r_3^+, 0^-, \vec{b}_{3\perp}) \Gamma'^\beta \chi_{\bar{n}}^{d_4}(0) | \Pi \rangle$$

$$f_{NN'} f_{N'N}^{[a_4, \dots, a_1]} \propto \langle N | \bar{\chi}_n^{a_1}(0^+, r_1^-, \vec{b}_{1\perp}) \Gamma^\alpha \chi_n^{a_2}(0^+, r_2^-, \vec{b}_{2\perp}) | N' \rangle \langle N' | \bar{\chi}_n^{a_4}(0) \Gamma'^\nu \chi_n^{a_3}(0^+, r_3^-, \vec{b}_{3\perp}) | N \rangle$$

$$\Phi_{DDY}^{a_1, \dots, a_4, b_4, \dots, b_1}(\{b\}) = \langle 0 | S_n^{T\dagger a_1} S_{\bar{n}}^{Td_1}(\vec{b}_{1\perp}) S_{\bar{n}}^{T\dagger d_2} S_n^{Ta_2}(\vec{b}_{2\perp}) S_{\bar{n}}^{T\dagger d_3} S_n^{Ta_3}(\vec{b}_{3\perp}) S_n^{T\dagger a_4} S_{\bar{n}}^{Td_4}(0) | 0 \rangle$$

Color

- Obtain valid color structures using projectors
- Cross-section:

$$d\sigma \propto \left(f_{pion}^{[1]}(\{y, \vec{b}_\perp\}), f_{pion}^{[8]}(\{y, \vec{b}_\perp\}) \right) \times \begin{pmatrix} \Phi_{11}(\{b\}) & \Phi_{18}(\{b\}) \\ \Phi_{81}(\{b\}) & \Phi_{88}(\{b\}) \end{pmatrix} \times \begin{pmatrix} f_{NN'}^{[1]}, f_{N'N}^{[1]}(\{x, \vec{b}_\perp\}) \\ 0 \end{pmatrix}$$

- Singlet term:

$$d\sigma^{[1,11,1]} \propto f_{pion}^{[1]}(\{y, \vec{b}_\perp\}) \Phi_{11}(\{b\}) f_{NN'}^{[1]} f_{N'N}^{[1]}(\{x, \vec{b}_\perp\})$$

- Up until now: un-subtracted terms

Zero bin subtraction

$$d\sigma^{[1,11,1]} \propto f_{pion}^{[1]}(\{y, \vec{b}_\perp\}) \Phi_{11}(\{b\}) f_{NN'}^{[1]} f_{N'N}^{[1]}(\{x, \vec{b}_\perp\})$$

- Remove overlapping between regions
- Anti-collinear term without overlap with soft region (pure):

$$f_{pion}(r_l^+, 0^-, \vec{b}_{j\perp}) = \frac{\hat{f}_{pion}(r_l^+, 0^-, \vec{b}_{j\perp})}{\Phi_{11}(0^+, 0^-, \vec{b}_{j\perp})}$$

- Collinear term without overlap with soft region (pure):

$$f_{NN'}(r_1^-, \vec{b}_{1\perp}, r_2^-, \vec{b}_{2\perp}) f_{N'N}(r_3^-, \vec{b}_{3\perp}) = \frac{\hat{f}_{NN'}(r_1^-, \vec{b}_{1\perp}, r_2^-, \vec{b}_{2\perp}) \hat{f}_{N'N}(r_3^-, \vec{b}_{3\perp})}{\Phi_{DY}(\vec{b}_{1\perp}, \vec{b}_{2\perp}) \Phi_{DY}(\vec{b}_{3\perp})}$$

where:

$$\Phi_{DY}(b_i, b_j) = \langle 0 | S_n^{T\dagger}(b_i) S_{\bar{n}}^T(b_i) S_{\bar{n}}^{T\dagger}(b_j) S_n^T(b_j) | 0 \rangle$$

Subtraction of rapidity divergences

$$d\sigma^{[1,11,1]} \propto f_{pion}^{[1]}(\{y, \vec{b}_\perp\}) \Phi_{11}(\{b\}) f_{NN'}^{[1]} f_{N'N}^{[1]}(\{x, \vec{b}_\perp\})$$

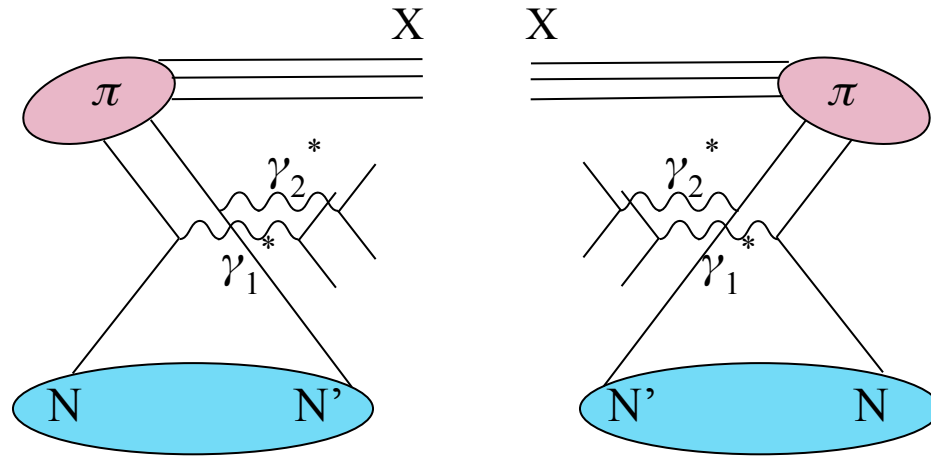
- Color summed cross-section term with **subtracted** functions:

$$\begin{aligned} d\sigma_{DDY}^{[1,11,1]}_{sub}(\{b\}) &\propto H(Q^2/\mu^2) \frac{\hat{f}_{pion}^{[1]}}{\Phi_{11}} \Phi_{11}(\{b\}) \frac{\hat{f}_{NN'}^{[1]} \hat{f}_{N'N}^{[1]}(\{x, \vec{b}_\perp\})}{\Phi_{DY}(\vec{b}_{1\perp}, \vec{b}_{2\perp}) \Phi_{DY}(0, \vec{b}_{3\perp})} = \\ &= H(Q^2/\mu^2) \frac{\hat{f}_{pion}^{[1]}}{\sqrt{\Phi_{11}}} \frac{\sqrt{\Phi_{11}}}{\sqrt{\Phi_{DY}(\vec{b}_{1\perp}, \vec{b}_{2\perp})} \sqrt{\Phi_{DY}(0, \vec{b}_{3\perp})}} \frac{\hat{f}_{NN'}^{[1]} \hat{f}_{N'N}^{[1]}(\{x, \vec{b}_\perp\})}{\sqrt{\Phi_{DY}(\vec{b}_{1\perp}, \vec{b}_{2\perp})} \sqrt{\Phi_{DY}(0, \vec{b}_{3\perp})}} = \\ &= H(Q^2/\mu^2) \times 2TMD \times \Phi_{New} \times GTMD * GTMD' \end{aligned}$$

- New ratio of soft factors term

Conclusions

- First factorization of Exclusive DDY cross-section into functions with different scales
- Exclusive DDY gives access to GTMDs (+ DTMD!)
- New ratio of soft factors not present before: important for pheno!
- Future work:
 - Keep studying color structures
 - Check explicitly cancellation of rapidity divergences
 - Polarizations
 - ...



THANK YOU FOR YOUR ATTENTION!

Extra slides

Color

- Consider DPS color structure
- DPS projectors to get singlet states:

$$I_1 = \frac{\delta_{a_1 a_4} \delta_{a_2 a_3}}{N_c^2} \qquad I_8 = \frac{2t_{a_1 a_4}^A t_{a_2 a_3}^A}{N_c \sqrt{N_c^2 - 1}}$$

- Soft factor:

$$\Phi_{MN}(\{b\}) = I_M \times \Phi_{DDY}^{[a_1, \dots, a_4][b_1, \dots, b_4]}(\{b\}) \times I_N$$

with:

$$\Phi_{DDY}^{a_1, \dots, a_4, b_1, \dots, b_4}(\{b\}) = \langle 0 | S_n^{T\dagger a_1} S_{\bar{n}}^{T d_1}(\vec{b}_{1\perp}) S_{\bar{n}}^{T\dagger d_2} S_n^{T a_2}(\vec{b}_{2\perp}) S_{\bar{n}}^{T\dagger d_3} S_n^{T a_3}(\vec{b}_{3\perp}) S_n^{T\dagger a_4} S_{\bar{n}}^{T d_4}(0) | 0 \rangle$$

$$\Rightarrow \Phi_{DDY}(\{b\}) = \begin{pmatrix} \Phi_{11}(\{b\}) & \Phi_{18}(\{b\}) \\ \Phi_{81}(\{b\}) & \Phi_{88}(\{b\}) \end{pmatrix}$$