Evolution of leading-twist B-meson distribution amplitude

Yao Ji

University of Siegen

based on: Vladimir M. Braun, YJ, and Alexander N. Manashov, PRD 100 (2019) 1, 014023

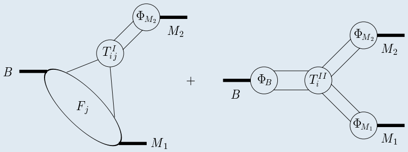
QCD evolution workshop

Exclusive B–Decays

- Heavy quark expansion methods ($m_b \gg \Lambda_{\rm QCD}$)
- Soft-collinear factorization (final state particle energies $\gg \Lambda_{\rm QCD}$)

Factorization Theorem: [M. Beneke, G. Buchalla, M. Neubert and Sachrajda (1999)]

$$\langle M_1 M_2|O_i|B\rangle = F^{B\to M_1}(0)\int_0^1 du\, T^{(1)}(u)\Phi_{M_2}(u)$$
$$+\int_0^\infty d\omega \int_0^1 du\, dv\, T^{(2)}(\omega,u,v)\Phi_B(\omega)\Phi_{M_1}(u)\Phi_{M_2}(v) + \dots$$



u, v — *momentum fractions*
ω — *light quark energy in B-meson*
$\Phi_{M,B}$ — *distribution amplitudes*

$B \to \gamma \ell \nu_\ell$ *provides the cleanest probe for unraveling the B-meson DAs*

Leading-twist distribution amplitude

Definition

[A. Grozin, M. Neubert (1997)]

$$\langle 0|\left[\bar{q}(zn)\,\not{n}[zn,0]\gamma_5 h_v(0)\right]_R|\bar{B}(v)\rangle = iF_B(\mu)\ \Phi_+(z,\mu)$$

- v_μ is the heavy quark velocity, $n^2 = 0$, and $n \cdot v = 1$

Fourier transform

$$\phi_+(\omega,\mu) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} dz\, e^{i\omega z}\Phi_+(z - i0,\mu)\,,$$

$$\Phi_+(z,\mu) = \int\limits_{0}^{\infty} d\omega\, e^{-i\omega z}\phi_+(\omega,\mu)\,.$$

- $\omega > 0$ is the $(2\times)$ light quark energy in the b–quark rest frame
- $\Phi_+(z - i0,\mu)$ is an analytic function of z in the lower half-plane

One-loop evolution of leading twist DA

- **RGE**

$$\Big(\mu\frac{\partial}{\partial\mu} + \beta(a)\frac{\partial}{\partial a} + \mathcal{H}(a)\Big)\Phi_+(z,\mu) = 0\,,$$

[B. Lange, M. Neubert (2003)]

with $\mathcal{H}$ being the evolution kernel, usually presented as an integral operator.

- **One-loop evolution kernel** [A. Grozin and M. Neubert, (1997); V. Braun, D. Ivanov and G. Korchemsky, (2004)]

$$\mathcal{H}^{(1)}\Phi_+(z,\mu) = 4C_F\left\{[\ln(i\widetilde{\mu}z)+1/2]\Phi_+(z,\mu) + \int_0^1 du\frac{\bar{u}}{u}\left[\Phi_+(z,\mu)-\Phi_+(\bar{u}z,\mu)\right]\right\}$$

- **Solution to one-loop RGE** [G. Bell, T. Feldmann, Y.-M. Wang and M. W. Y. Yip, (2013); V. Braun and A. Manashov (2014)]

$$\Phi_+(z,\mu) = -\frac{1}{z^2}\int_0^\infty ds\, s\, e^{is/z}\,\eta_+(s,\mu)\,,$$

$$\eta_+(s,\mu) = R(s,\mu,\mu_0)\eta_+(s,\mu_0)\,, \qquad R(s,\mu,\mu_0) \propto s^{\frac{2C_F}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}}$$

- **Eigenfunctions for higher twists to $\mathcal{O}(1/N_c^2)$ also found** [V. M. Braun, YJ, A. N. Manashov (2017)]
  integrable spin chains [V. Braun, YJ, and A. Manashov (2018)]

Two-loop evolution of twist-2 DA

- **Motivations**
  - Last missing piece for a complete NNLL resummation for charmless B-decays
  - Theoretically interesting in its own right
    - Conformal symmetry in light-heavy systems (*new method*!)
    - Test for light-heavy relation at two-loop [V. Braun, YJ. and A. Manashov (2018)]

Conformal symmetry of light kernels

- **One-loop light kernels are $SL(2)$ invariant** [Bukhvostov, Frolov, Kuraev, Lipatov (1985)]

generators
$$S_+^{(i)} = z_i^2\partial_{z_i} + 2j_i z_i, \quad S_0^{(i)} = z_i\partial_{z_i} + j_i, \quad S_-^{(i)} = -\partial_{z_i}$$
j : conformal spin

algebra
$$[S_+^{(i)}, S_-^{(i)}] = 2S_0^{(i)}, \quad [S_0^{(i)}, S_+^{(i)}] = S_+^{(i)}, \quad [S_0^{(i)}, S_-^{(i)}] = -S_-^{(i)}$$

Example $\mathcal{O}(z_1, z_2) = \bar{q}(nz_1)\not{n}q(nz_2)$ $(j_{1,2} = 1)$ $SL(2)$ generators $S_{0,\pm}^{(12)} = S_{0,\pm}^{(1)} + S_{0,\pm}^{(2)}$

$SL(2)$ invariance $\Longrightarrow [S_{0,\pm}^{(12)}, \mathcal{H}_{\bar{q}q}^{(1)}] = 0 \Longrightarrow \mathcal{H}_{\bar{q}q}^{(1)} = h(S_{12}^2)$

$$S_{12}^2 = S_+^{(12)}S_-^{(12)} + S_0^{(12)}(S_0^{(12)} - 1)$$
quadratic Casimir operator

explicitly,

$$\mathcal{H}_{\bar{q}q}^{(1)} = 2C_F\left[\psi(\widehat{J}+1) + \psi(\widehat{J}-1) - 2\gamma_E - \frac{3}{2}\right], \qquad S_{12}^2 = \widehat{J}(\widehat{J}-1)$$

- **Light kernels up to three-loops available** [V. Braun, A. Manashov, S. Moch and M. Strohmaier (2016)-(2019)]

Conformal symmetry of heavy kernels

What about heavy kernels??

- $\mathcal{H}_h^{(1)}$ commute with special conformal generator of light field $S_+ \sim v^\mu \mathbf{K}_\mu$ but *not* S_0 (cusp)

$$[S_+, \mathcal{H}_h^{(1)}] = 0\,, \quad [S_0, \mathcal{H}_h^{(1)}] = 4C_F = \Gamma_{\text{cusp}}^{(1)}$$

[M. Knödlseder and N. Offen (2011)]

Solution: $$\mathcal{H}_h^{(1)} = \Gamma_{\text{cusp}}^{(1)} \ln(i\mu S_+) + \text{const}$$ (for $\mathcal{O}_{\bar{q}h} = \bar{q}(nz)\gamma^+ h(0)$, $j = 1$)

- **Light $\mapsto$ heavy reduction**

$$S_+^{(h)} \mapsto \lambda^{-1} S_+^{(h)}\,, \qquad S_-^{(h)} \mapsto \lambda S_-^{(h)} \mapsto \mu\,, \qquad S_0^{(h)} \mapsto S_0^{(h)}\,, \qquad \lambda \sim m_b \to \infty$$

$$\Longrightarrow \quad \mathcal{H}_{\bar{q}q}^{(1)} \mapsto \mathcal{H}_{\bar{q}h} = \Gamma_{\text{cusp}}^{(1)} \ln(i\mu S_+) + \text{const}$$

- **Eigenfunction of $\mathcal{H}_{\bar{q}h}^{(1)}$ coincides with that of S_+:**

$$Q_s(z) = -\frac{e^{is/z}}{z^2}$$

[V. Braun and A. Manashov (2014)]

Conformal symmetry of heavy kernels

- **Proposition:** $\mathcal{H}_h^{t=2} = \Gamma_{\text{cusp}}(a)\ln(i\bar{\mu}S_+) + \Gamma_+(a)$ **to all orders** **why?**

Evolution kernels in the MS-like schemes are ϵ-independent

Exact conformal symmetry in $d = 4 - 2\epsilon$ at the critical point $\beta(a_*) = 0$

$$(1) \quad \left[S_+^{\text{full}}, \mathcal{H}_h^{t=2}(a_*)\right] = 0$$

$SL(2)$ generators receive quantum corrections:

$$S_+^{(0)} = z^2\partial_z + 2z \mapsto S_+^{\text{full}}(a_*) = S_+^{(0)} + z\left[-\epsilon + \Delta(a_*)\right],$$
$$S_0^{(0)} = z\partial_z + 1 \mapsto S_0^{\text{full}}(a_*) = S_0^{(0)} - \epsilon + \mathcal{H}_h^{t=2}(a_*)$$

$\Delta(a_*) = a_*\Delta^{(1)} + a_*^2\Delta^{(2)} + \ldots$ is called conformal anomaly satisfying

from (1) and $SL(2)$ algebra $\Longrightarrow$ $(2) \quad [z\partial_z, S_+^{\text{full}}(a_*)] = S_+^{\text{full}}(a_*)$

$\ln\mu z$ enters $\mathcal{H}_h^{t=2}$ only linearly with coefficient Γ_{cusp} [G. Korchemsky, A. Radyushkin (1992)]

$$(3) \quad [z\partial_z, \mathcal{H}_h^{t=2}(a_*)] = \Gamma_{\text{cusp}}(a_*)$$

$$(1) \Longrightarrow \mathcal{H}_h^{t=2}(a_*) = f(S_+^{\text{full}}(a_*)) \overset{(2),(3)}{\Longrightarrow} zf'(z) = \Gamma_{\text{cusp}}(a_*) \Longrightarrow \textbf{Proposition}$$

Conformal generators at one-loop

- **Two-loop evolution of twist-2 DA** [V. Braun, YJ and A. Manashov (2019)]

$$\mathcal{H}_h^{(2)}(a_*) = \Gamma_{\text{cusp}}^{(2)}(a_*)\ln(i\bar{\mu}S_+^{(1)}(a_*)) + \Gamma_+^{(2)}(a_*)\,,$$
$$S_+^{(1)}(a_*) = S_+^{(0)} + z\big(-\epsilon(a_*) + a_*\Delta^{(1)}\big)$$

$$\bar{\mu} = \widetilde{\mu}e^{\gamma_E} = \mu_{\overline{\text{MS}}}\, e^{2\gamma_E}$$

$$\epsilon(a_*) = -\beta_0 a_* + O(a_*^2)$$

One-loop conformal anomaly *four one-loop diagrams*

$$\Delta^{(1)}\mathcal{O}(z) = C_F\left\{3\mathcal{O}(z) + 2\int_0^1 d\alpha\left(\frac{2\bar{\alpha}}{\alpha} + \ln\alpha\right)[\mathcal{O}(z) - \mathcal{O}(\bar{\alpha}z)]\right\}$$

- **The scheme-dependent constant $\Gamma_+^{(2)}(a)$ is found from Feynman diagrams**

Two-loop kernel in integral representation

- **Integral representation for $\mathcal{H}_h^{t=2}$ is usually preferred**

Ansatz

$$\mathcal{H}(a)\mathcal{O}(z) = \Gamma_{\text{cusp}}(a)\left[\ln(i\widetilde{\mu}z)\mathcal{O}(z) + \int_0^1 d\alpha\,\frac{\bar{\alpha}}{\alpha}\big(1+h(a,\alpha)\big)\big(\mathcal{O}(z)-\mathcal{O}(\bar{\alpha}z)\big)\right] + \gamma_+(a)\mathcal{O}(z)$$

- $\Delta^{(1)}$ **and** $\epsilon(a_*) = -\beta_0 a_* + O(a_*^2)$ **dictate** $h(a,\alpha)$ *going to Mellin space*

$$h(a,\alpha) = a\ln\bar{\alpha}\left\{\beta_0 - 2C_F\left(\frac{3}{2} + \ln\frac{\alpha}{\bar{\alpha}} + \frac{\ln\alpha}{\bar{\alpha}}\right)\right\} + O(a^2)$$

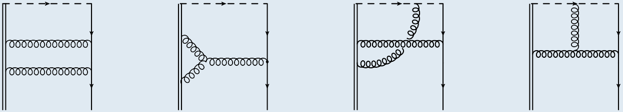
- γ_+ **requires additional calculation** *scheme-dependent*,

$$\gamma_+^{\overline{\text{MS}}}(a) = -aC_F + a^2 C_F\left\{4C_F\left[\frac{21}{8}+\frac{\pi^2}{3}-6\zeta_3\right] + C_A\left[\frac{83}{9}-\frac{2\pi^2}{3}-6\zeta_3\right] + \beta_0\left[\frac{35}{18}-\frac{\pi^2}{6}\right]\right\} + \dots$$

Two-loop kernel from Feynman diagrams

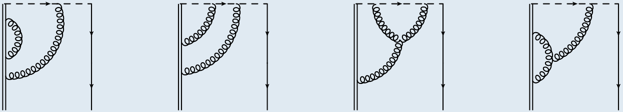
There are ~ 30 diagrams in three categories:

- **Exchange diagrams**



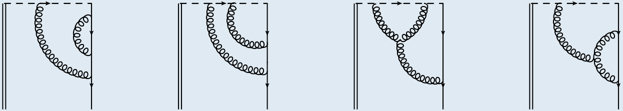
...

- **Cusp diagrams**



...

- **Light vertices**



...

Two-loop kernels from Feynman diagrams

- **Exchange diagrams contribute to both $h(a,\alpha)$ and γ_+** *(many are UV-finite)*
- **Cusp diagrams generate $\sim \ln z$ and contribute to γ_+**
- **Light vertices contribute to $h(a,\alpha)$ only, known**

[V. Braun, A. Manashov, S. Moch, and M. Strohmaier (2016)]

- $h(a,\alpha)$ **confirmed by explicit Feynman diagram calculation!**

Light-heavy reduction

- **Evolution kernel of $\mathcal{O} = \bar{q}(nz_1)\gamma^+ q(nz_2)$ in integral form**

$$[\mathcal{H}_l\varphi](z_1,z_2) \propto \int_0^1 du\, h(u)\Big[2\varphi(z_1,z_2) - \varphi(z_{12}^u, z_2) - \boxed{\varphi(z_1, z_{21}^u)}\Big] + \boxed{\int_0^1 du \int_0^{\bar{u}} dv\, \chi(u,v)[\varphi(z_{12}^u, z_{21}^v) + \varphi(z_{12}^v, z_{21}^u)]} + c\varphi(z_1,z_2)$$

- **drop terms in boxes and $z_2 \to 0$ to obtain $\mathcal{H}_h^{\mathrm{ex}} + \mathcal{H}_h^{\mathrm{lv}}$.**
  ↰ **Location of the heavy quark is fixed!**

  Explicit expressions for $\mathcal{H}_l^{(2)}$ available [V. Braun, A. Manashov, S. Moch and M. Strohmaier (2016)]
- **Adding contribution of cusp diagrams again gives us $\mathcal{H}_h^{(2)}$.**

Evolution of the coefficient function at two-loop

Reminder ($Q_s(z)$ form a complete orthonormal basis)

$$\Phi_+(z,\mu) = \int_0^\infty ds\, s\, Q_s(z)\, \eta_+(s,\mu) = -\frac{1}{z^2}\int_0^\infty ds\, s\, \mathrm{e}^{is/z}\, \eta_+(s,\mu)$$

- **RGE of $\phi_+(z,\mu) \mapsto$ integro-differential eq. over $\eta_+(s,\mu)$** [V. Braun, YJ and A. Manashov (2019)]

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(a)\frac{\partial}{\partial a} + \Gamma_{\mathrm{cusp}}(a)\ln(\widetilde{\mu}\mathrm{e}^{\gamma_E} s) + \gamma_\eta(a)\right)\eta_+(s,\mu) = 4C_F a\int_0^1 du\,\frac{\bar{u}}{u} h(a,u)\eta_+(\bar{u}s,\mu)$$

$$\gamma_\eta = \gamma_+^{\overline{\mathrm{MS}}} - \gamma_F - \Gamma_{\mathrm{cusp}}^{(2)}\left[1 - a\left(C_F\left(\frac{\pi^2}{6} - 3\right) + \beta_0\left(1 - \frac{\pi^2}{6}\right)\right)\right]$$

- **NNLL resummation requires Γ_{cusp} to $O(a^3)$ since numerically $\ln(s) \sim 1/a$**

Analytic solution of the two-loop RGE

- **Operator $\mathcal{O}(z)$ in Mellin space**

$$\mathcal{O}(z) = \int_{-i\infty}^{+i\infty} dj\, (i\mu_{\overline{\text{MS}}}\, e^{\gamma_E} z)^j \mathcal{O}(j)$$

gives rise to the Mellin-space RGE:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(a)\frac{\partial}{\partial a} - \Gamma_{\text{cusp}}(a)\frac{\partial}{\partial j} + V(j,a)\right)\mathcal{O}(j,a,\mu) = 0$$

explicit expression for $V(j,a)$ up to $\mathcal{O}(a^3)$ available in [V. Braun, YJ and A. Manashov, 1912.03210]

- **Mellin moment j as the second coupling, with Γ_{cusp} as the β-function**
- **Resembles RGE in TMD factorization** [I. Scimemi, A. Vladimirov, (2018)]

Numerical study

- **Numerically solving the integro-differential equation of $\eta_+(s,\mu)$**

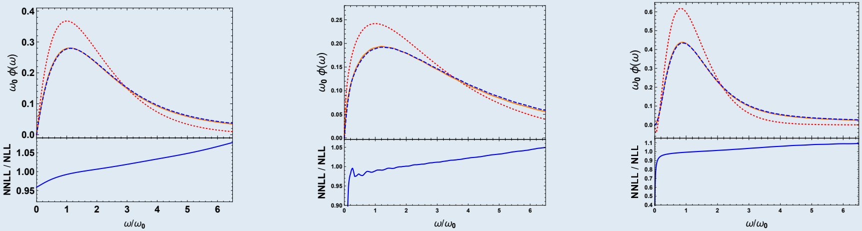


Figure : Models at $\mu_0^{\overline{\mathrm{MS}}} = 1$ GeV (dots) evolved to $\mu_1^{\overline{\mathrm{MS}}} = 2$ GeV at NLL (solid) and NNLL (dashed) for exponential model (left), Model II with σ_1^{max} (middle), and Model III with σ_1^{min} [M. Beneke, V. Braun, YJ and Y-B. Wei (2018)]

- **Two-loop evolution has a smaller effect than its one-loop counterpart**
- **Nonlinear behaviors generate larger NNLL corrections**

Conclusion and Outlook

Conclusion

- Integrability of RGEs at one-loop
- Two-loop kernel of twist-2 DA from conformal symmetry
- Light $\mapsto$ heavy kernel relation
- Mellin space solution for RGE

Outlook for future work

- Update charmless B-decays to the full NNLL accuracy
- γ_+ from local operator?
- coefficient functions?