

Effects of Non-Gaussian in-medium transverse momentum broadening on di-jet observables

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Polish Academy of Sciences

based on:

[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876; JHEP 04 (2021) 014] (k_T broadening/entropy in jets)

[v. Hameren, Kutak, Płaczek, MR, Tywoniuk, Phys. Rev. C 102, 044910] (dijet production)

[Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317] (Monte Carlo for gluon fragmentation functions)

Jet Production(1/3)

Cross section =

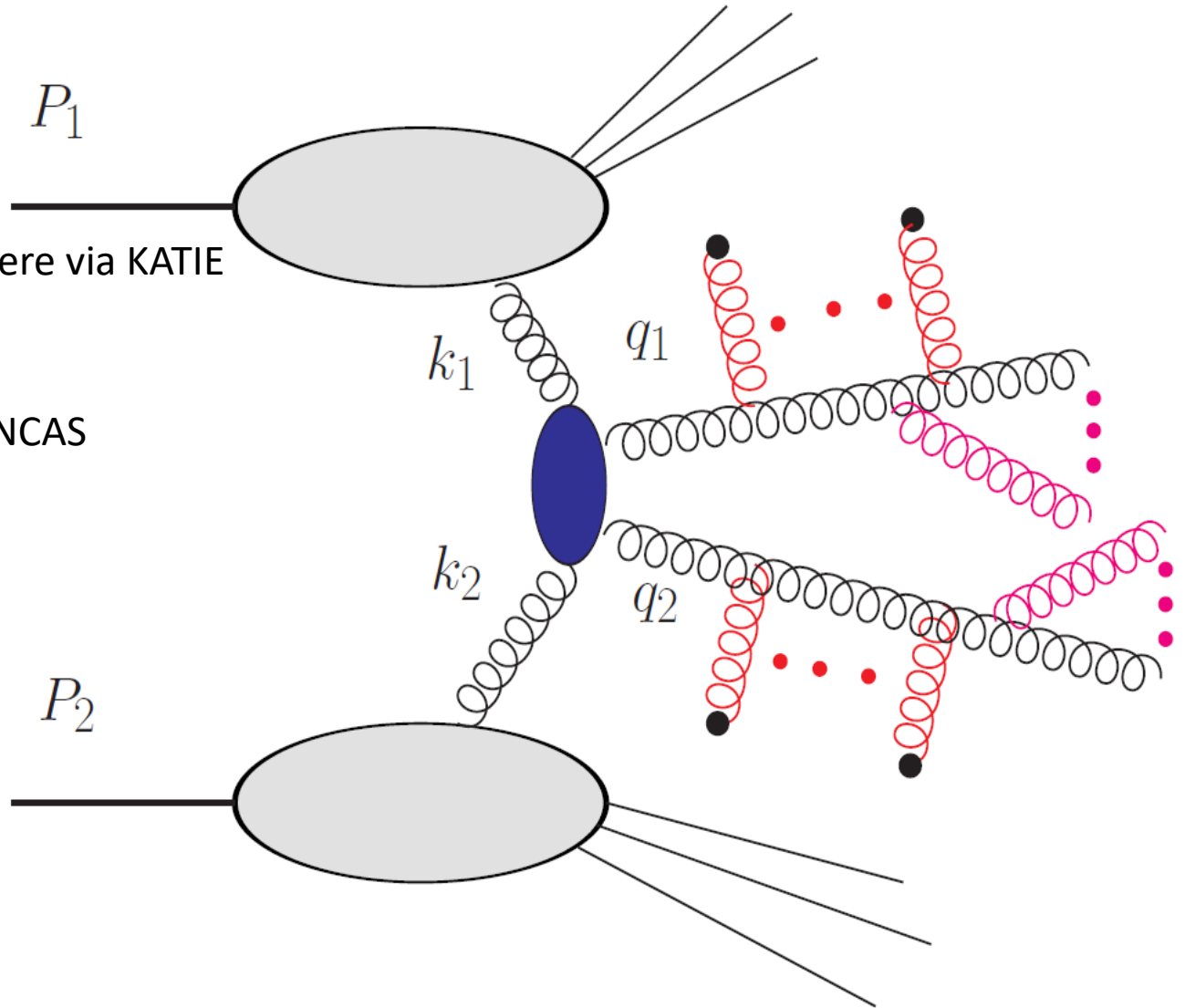
$(u)PDF1 * (u)PDF2 * \text{hard cross section}$

$* \text{fragmentation of jet1}$

$* \text{fragmentation of jet2}$

Here via MINCAS

Here via KATIE



[van Hameren: Comput.Phys.Commun. 224 (2018) 371-380] (KATIE)

[Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317] (MINCAS)

Jet Production (2/3)

k_T factorization:

$$\frac{d\sigma_{pp}}{dy_1 dy_2 d^2q_{1T} d^2q_{2T}} = \int \frac{d^2k_{1T}}{\pi} \frac{d^2k_{2T}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} \overline{|\mathcal{M}_{g^* g^* \rightarrow gg}^{\text{off-shell}}|^2} \\ \times \delta^{(2)}(\vec{k}_{1T} + \vec{k}_{2T} - \vec{q}_{1T} - \vec{q}_{2T}) \mathcal{F}_g(x_1, k_{1T}^2, \mu_F^2) \mathcal{F}_g(x_2, k_{2T}^2, \mu_F^2)$$

$\mathcal{F}_g(x, k_T^2, \mu_F^2)$...unintegrated parton distribution function (uPDF)

→ full phase space access at LO

Numerical simulations by **KATIE** algorithm.

[van Hameren: Comput.Phys.Commun. 224 (2018) 371-380]

Jet Production (3/3)

Factorization for AA collisions:

$$\frac{d\sigma_{AA}}{d\Omega_p} = \int d\Omega_q \int d^2\mathbf{l} \int_0^1 \frac{d\tilde{x}}{\tilde{x}} \delta(p^+ - \tilde{x}q^+) \delta^{(2)}(\mathbf{p} - \mathbf{l} - \mathbf{q}) D(\tilde{x}, \mathbf{l}, \tau(q^+)) \frac{d\sigma_{pp}}{d\Omega_q}$$

$$d\Omega_q = dq^+ d^2\mathbf{q} \quad \tau(q^+) = \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\hat{q}}{q^+}} L$$



$$\begin{aligned} \frac{d^2\sigma_{AA}}{d\Omega_{p_1} d\Omega_{p_2}} &= \int d\Omega_{q_1} \int d\Omega_{q_2} \int d^2\mathbf{l}_1 \int d^2\mathbf{l}_2 \int_0^1 \frac{d\tilde{x}_1}{\tilde{x}_1} \delta(p_1^+ - \tilde{x}_1 q_1^+) \int_0^1 \frac{d\tilde{x}_2}{\tilde{x}_2} \delta(p_2^+ - \tilde{x}_2 q_2^+) \\ &\quad \delta^{(2)}(\mathbf{p}_1 - \mathbf{l}_1 - \mathbf{q}_1) \delta^{(2)}(\mathbf{p}_2 - \mathbf{l}_2 - \mathbf{q}_2) D(\tilde{x}_1, \mathbf{l}_1, \tau(q_1^+)) D(\tilde{x}_2, \mathbf{l}_2, \tau(q_2^+)) \frac{d^2\sigma_{pp}}{d\Omega_{q_1} d\Omega_{q_2}} \end{aligned}$$

Coherent emission

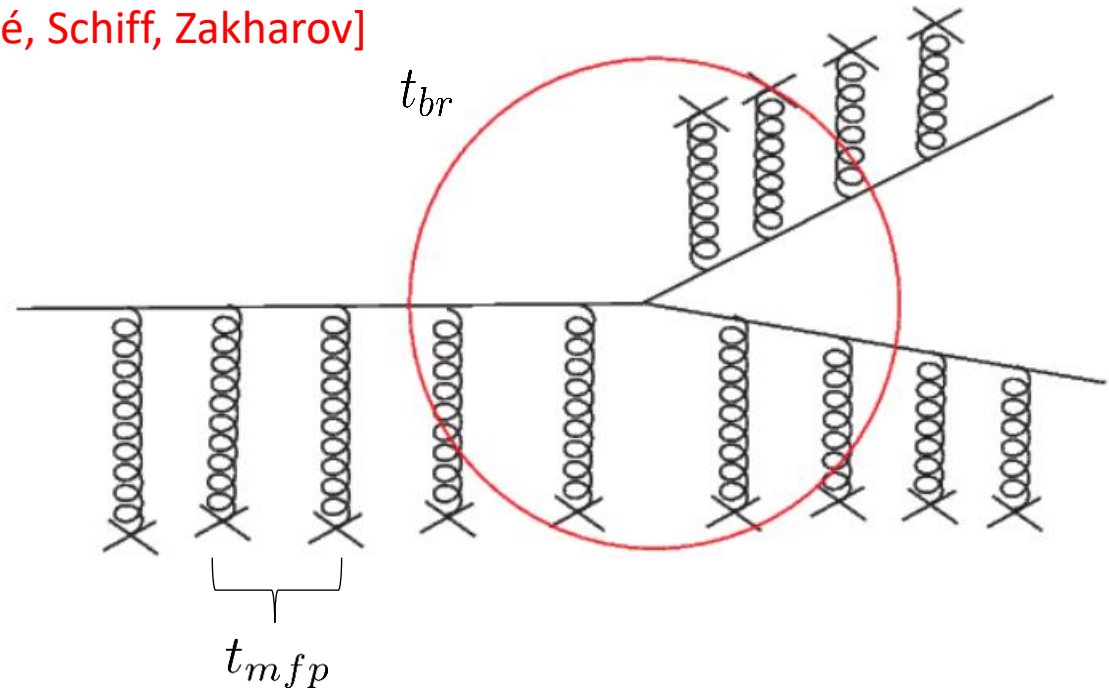
...à la BDMPS-Z [Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov]

$$t_{br} \sim \sqrt{\frac{2\omega}{\hat{q}}}$$

$t_{br} \sim t_{mfp}$: one scattering + radiation
...Bethe-Heitler spectrum

$t_{br} \gg t_{mfp}$: coherent radiation

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{br}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$



Look at range: $\omega_{BH} < \omega < \omega_c$

need effective kernel: $\mathcal{K}(z, k_T)$

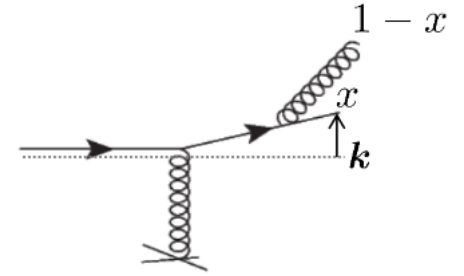
cf. [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]

BDIM Equation

[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]

Generalizes BDMPS-Z approach

Includes transverse momentum broadening



Momentum distribution:

$$p \rightarrow xp$$

Momentum transfer:

$$p \rightarrow p + \mathbf{k}$$

For gluon-jets:

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \alpha_s \int_0^1 dz \int \frac{d^2 q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x, \mathbf{k}, t) \right] + \int \frac{d^2 \mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t).$$

Induced Radiation:

$$\mathcal{K}(\mathbf{Q}, z, p_0^+) = \frac{2}{p_0^+} \frac{P_{gg}(z)}{z(1-z)} \sin \left[\frac{\mathbf{Q}^2}{2k_{br}^2} \right] \exp \left[-\frac{\mathbf{Q}^2}{2k_{br}^2} \right]$$

$$\omega = x p_0^+, \quad k_{br}^2 = \sqrt{\omega_0 \hat{q}_0}, \quad \mathbf{Q} = \mathbf{k} - z \mathbf{q}, \quad \omega_0 = z(1-z) p_0^+$$

$$\hat{q}_0 = \hat{q} f(z), \quad f(z) = 1 - z(1-z), \quad P_{gg}(z) = N_c \frac{[1 - z(1-z)]^2}{z(1-z)}$$

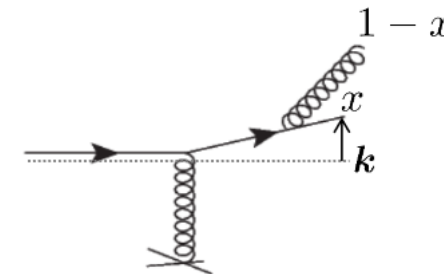
Scattering:

$$C(\mathbf{q}) = w(\mathbf{q}) - \delta(\mathbf{q}) \int d^2 \mathbf{q}' w(\mathbf{q}')$$

BDIM Equation

[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]

Generalizes BDMPS-Z approach ?



Momentum distribution:

$$p \rightarrow xp$$

Momentum transfer:

$$p \rightarrow p + \mathbf{k}$$

$$\int_0^\infty d^2\mathbf{Q} \mathcal{K}(z, \mathbf{Q}, p_0^+) = 2\pi \sqrt{\frac{\hat{q}}{p_0^+}} N_c \mathcal{K}(z) \xrightarrow{\text{k}_T \text{ averaged Kernel:}} \mathcal{K}(z) = \frac{(1 - z + z^2)^{\frac{5}{2}}}{[z(1 - z)]^{\frac{3}{2}}}$$

$$\frac{1}{t^*} = \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\hat{q}}{p_0^+}} \propto \frac{1}{t_{\text{br}}}$$

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] + \int \frac{d^2\mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t)$$

Integrate over $\mathbf{k}$

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

Different models

➤ Broadening in branching:
$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \alpha_s \int_0^1 dz \int \frac{d^2 q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x, \mathbf{k}, t) \right]$$

- No scattering
- Scattering: $w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\mathbf{q}^4}$
- Scattering: $w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\mathbf{q}^2(\mathbf{q}^2 + m_D^2)}$

All models yield the same k_T averaged splitting kernel $\mathcal{K}(z)$!

➤ No broadening in branching:
$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] + \int \frac{d^2 \mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t)$$

- Scattering: $w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\mathbf{q}^4}$
- Scattering: $w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\mathbf{q}^2(\mathbf{q}^2 + m_D^2)}$

Numerical simulations by **MINCAS** algorithm.

[Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

➤ Gaussian broadening:

x given by
$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

$\mathbf{k}$ given by Gaussian distribution with variance $\sigma^2 \sim \hat{q}L$

BDIM Equation as Integral Equation

[Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, t) \right]$$



$$D(x, \tau) = e^{-\phi(x)(\tau - \tau_0)} D(x, \tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_0^{1-\epsilon} dz \int_0^1 dy \delta(x - zy) \sqrt{\frac{z}{x}} z \mathcal{K}(z) e^{-\phi(x)(\tau - \tau')} D(y, \tau')$$

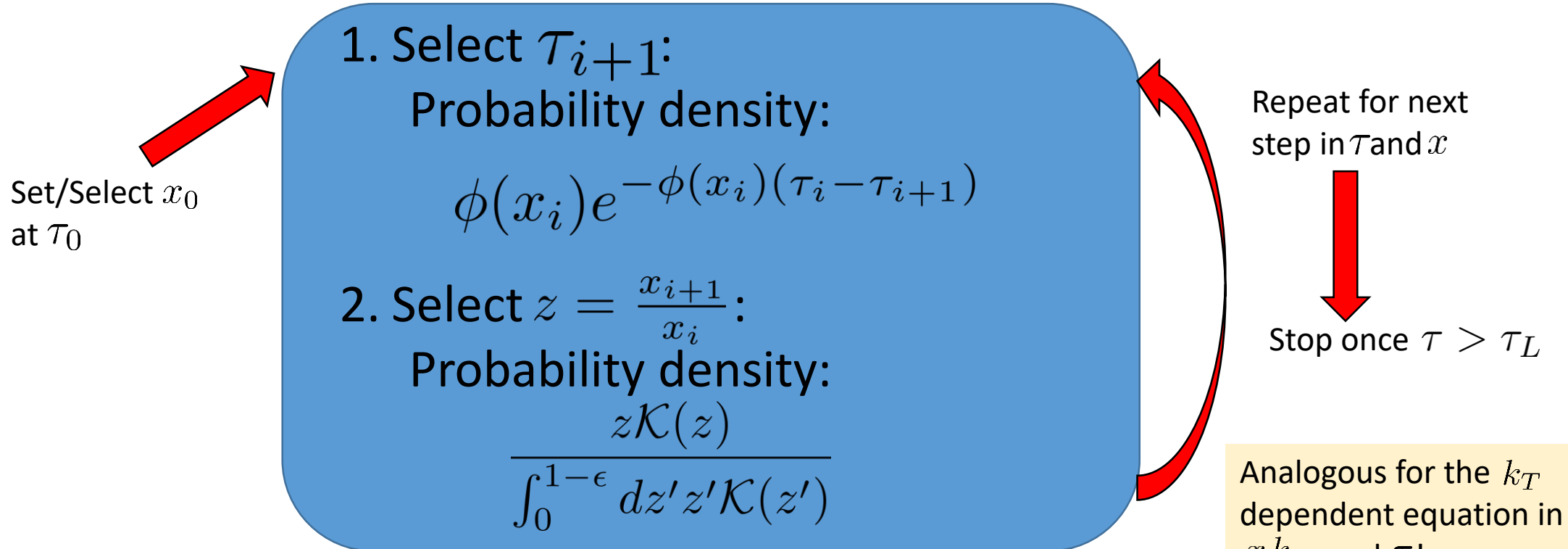
$$\tau = \frac{t}{t^*}$$

$$\phi(x) = \frac{1}{\sqrt{x}} \int_0^{1-\epsilon} dz z \mathcal{K}(z)$$

Monte-Carlo algorithm

$$D(x, \tau) = e^{-\phi(x)(\tau - \tau_0)} D(x, \tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_0^{1-\epsilon} dz \int_0^1 dy \delta(x - zy) \sqrt{\frac{z}{x}} z \mathcal{K}(z) e^{-\phi(x)(\tau - \tau')} D(y, \tau')$$

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Departure from Gaussian broadening

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \int \frac{d^2 \mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t) + \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right]$$

always same distribution for changes $p \rightarrow p + q$
 $\rightarrow$ central limit theorem

Splitting à la $p \rightarrow zp$
 $\rightarrow$ perturbations of different sizes
 $\rightarrow$ non Gaussian behavior

Virtual emissions

For example:
 $p \rightarrow z_1 p \rightarrow z_1 p + \mathbf{q}_1$
 $\rightarrow z_1 p + \mathbf{q}_1 + \mathbf{q}_2$
 $\rightarrow z_2(z_1 + \mathbf{q}_1 + \mathbf{q}_2) \rightarrow \dots$

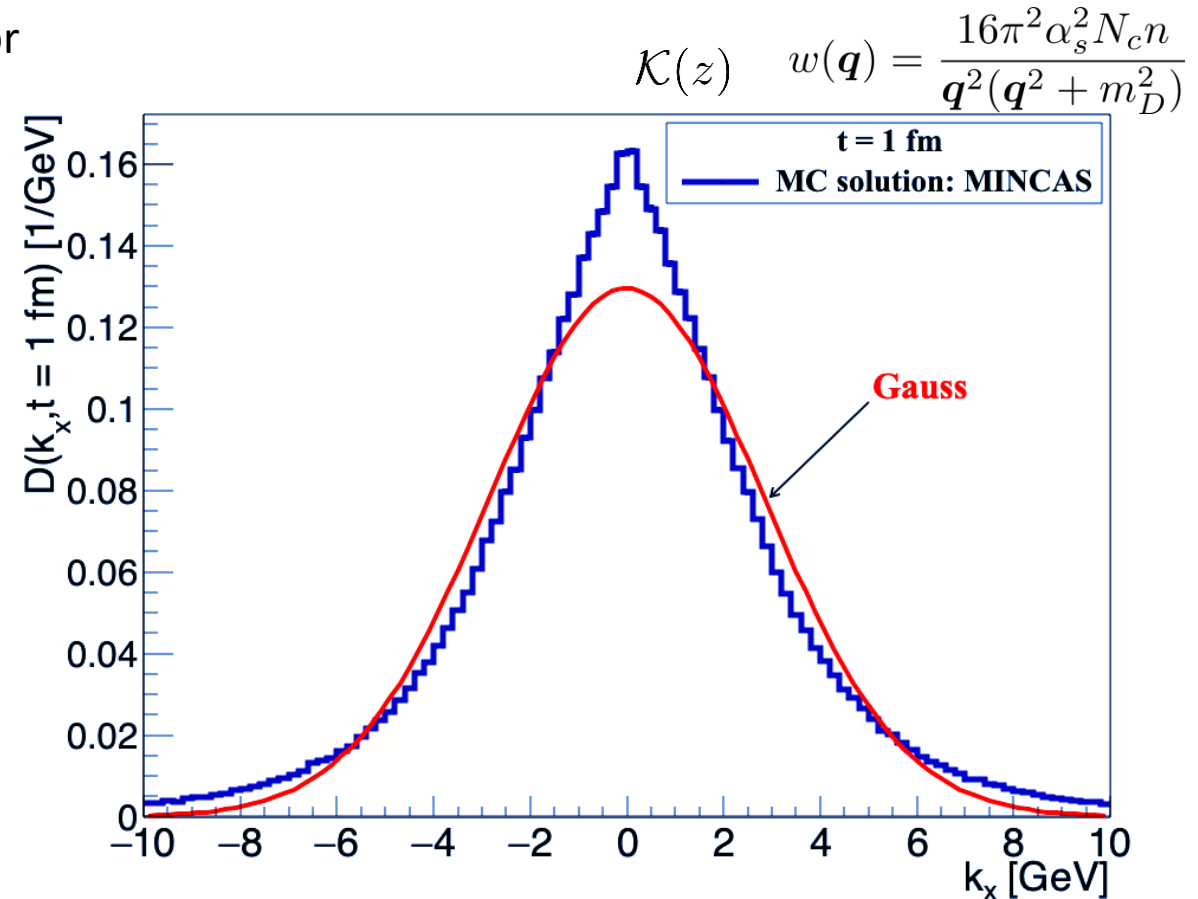
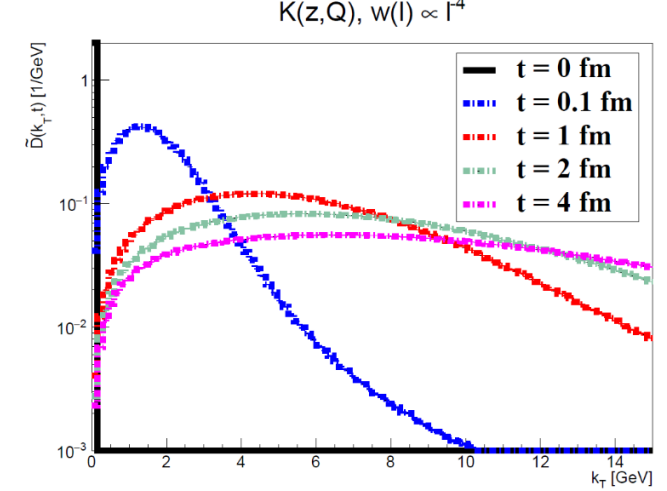
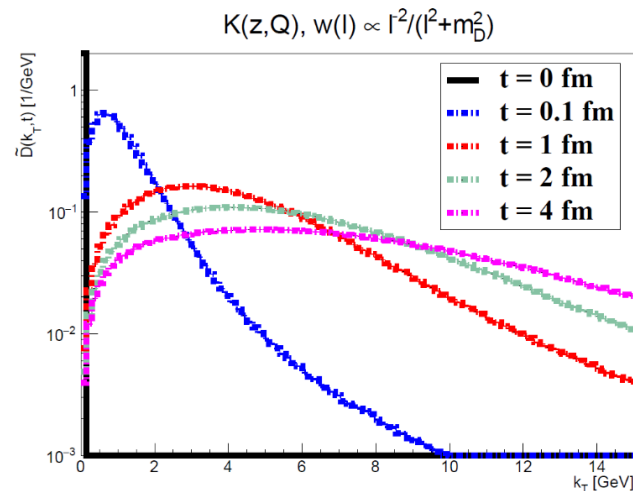
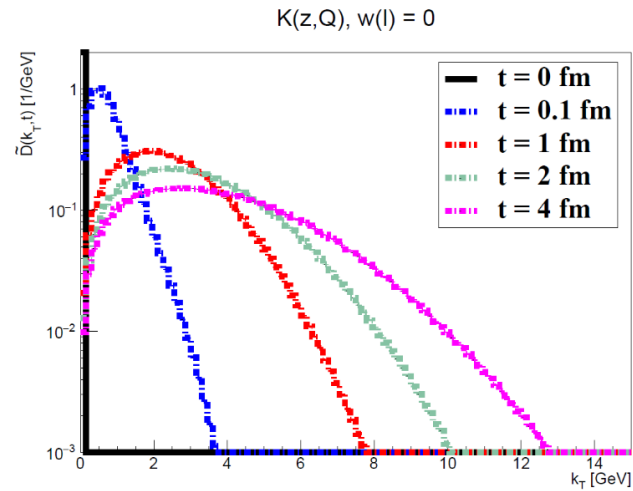
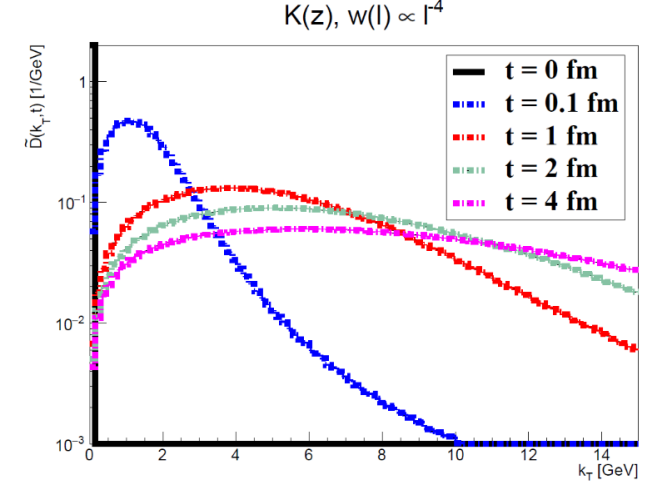
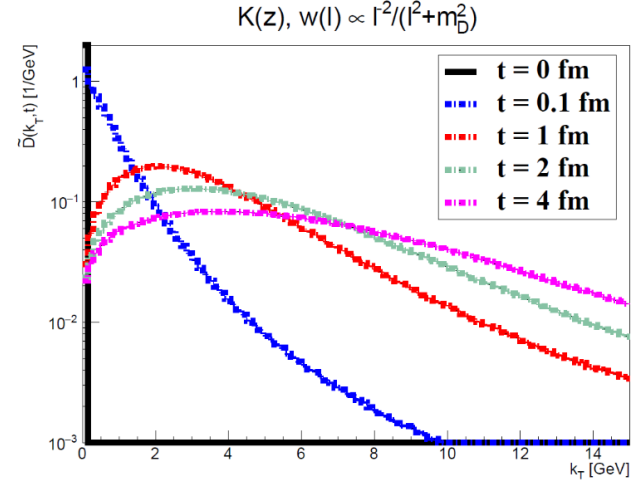
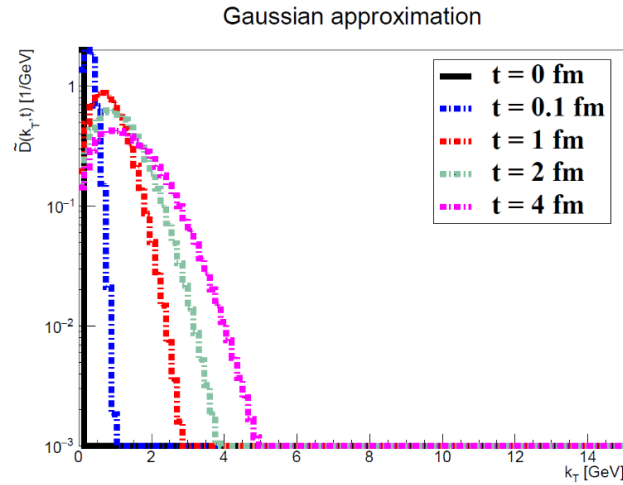


Figure: [Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

k_T Broadening

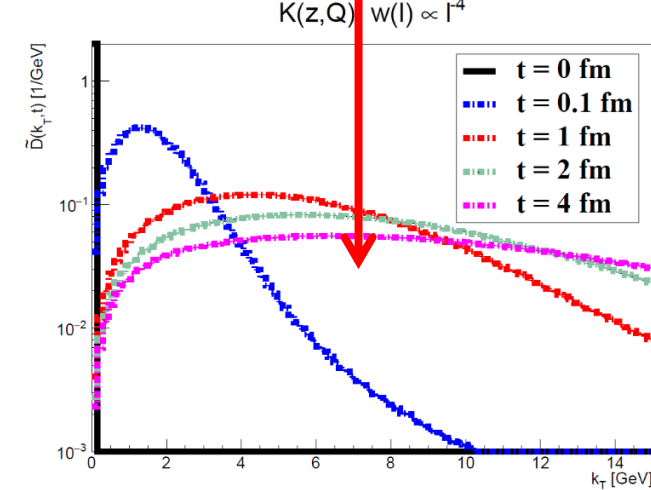
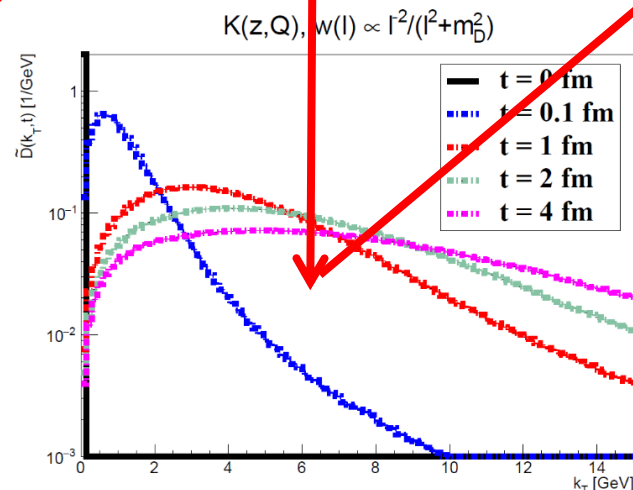
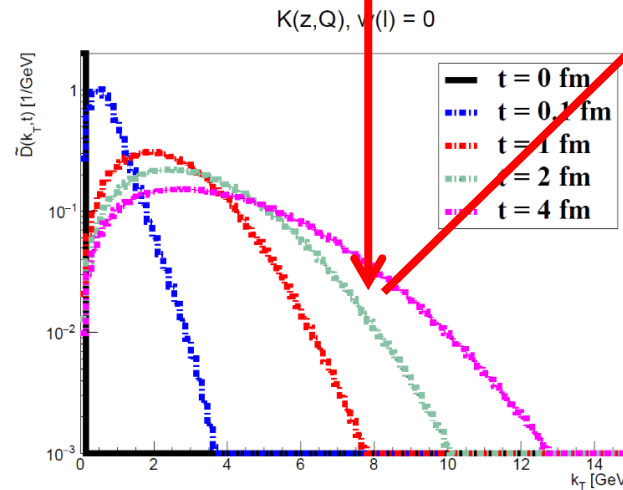
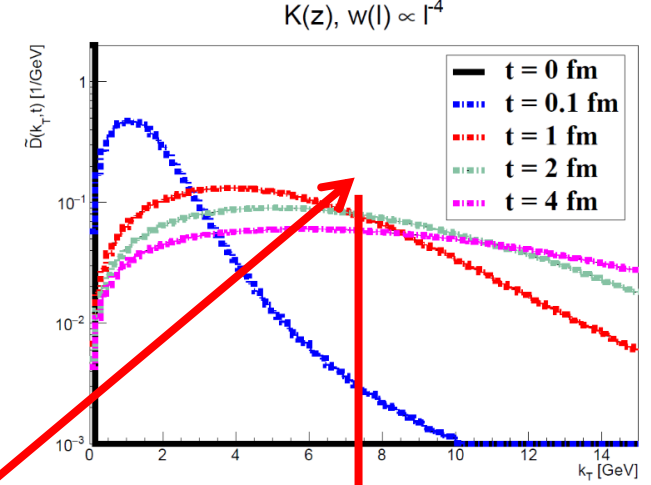
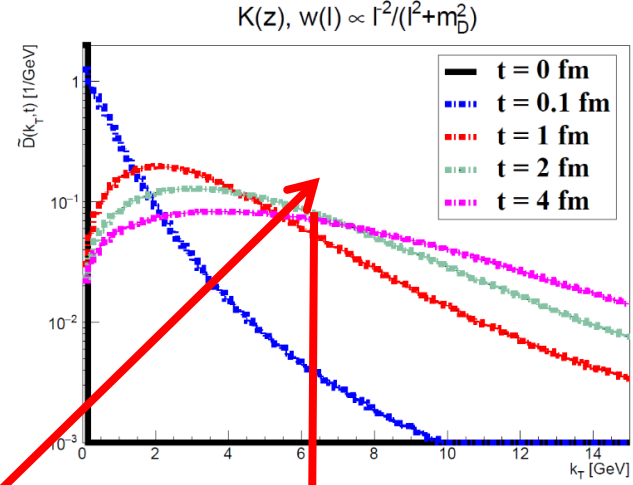
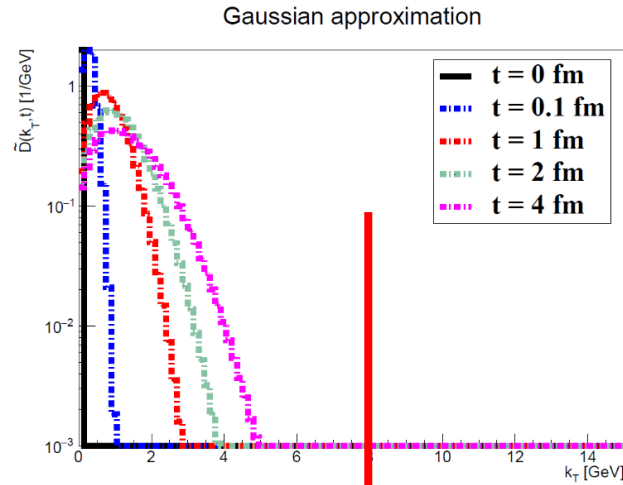
$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$



[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

k_T Broadening

$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$



[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

Approach: KATIE+MINCAS

- Use KATIE for hard initial collisions:
 - PDFs/TMDs for colliding nucleons
 - Hard collision cross-section (Monte-Carlo simulation)
 - Resulting particles → initial particles of jets

[van Hameren: *Comput.Phys.Comm.* 224 (2018) 371-380]

- Jets: by MINCAS
 - Monte-Carlo simulation of BDIM equation
 - Time-evolution of jets in medium

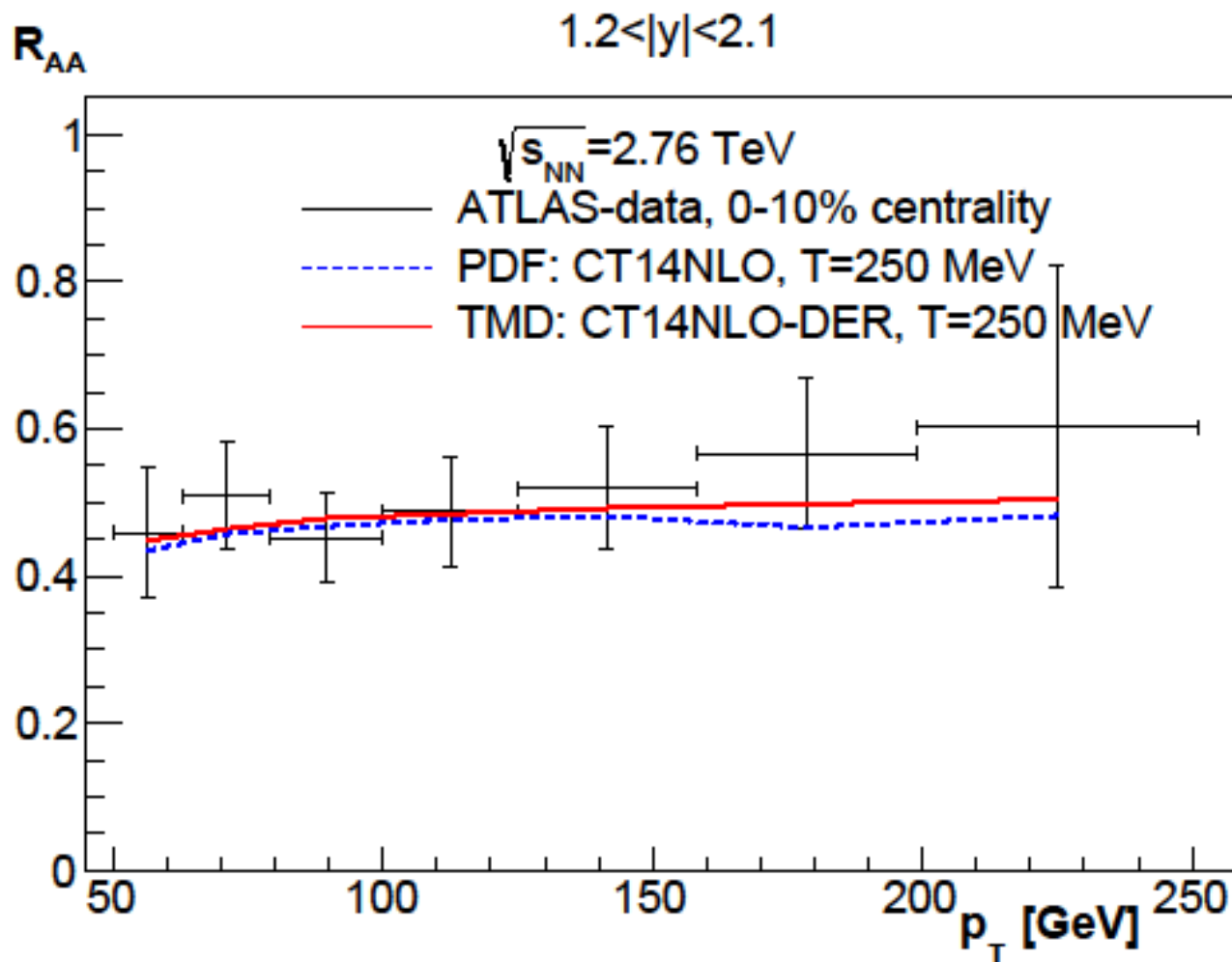
[Kutak, Płaczek, Straka: *Eur.Phys.J.* C79 (2019) no.4, 317]

Other codes implementing
BDMPS-Z:

MARTINI, JEWEL, QPYTHIA, ...

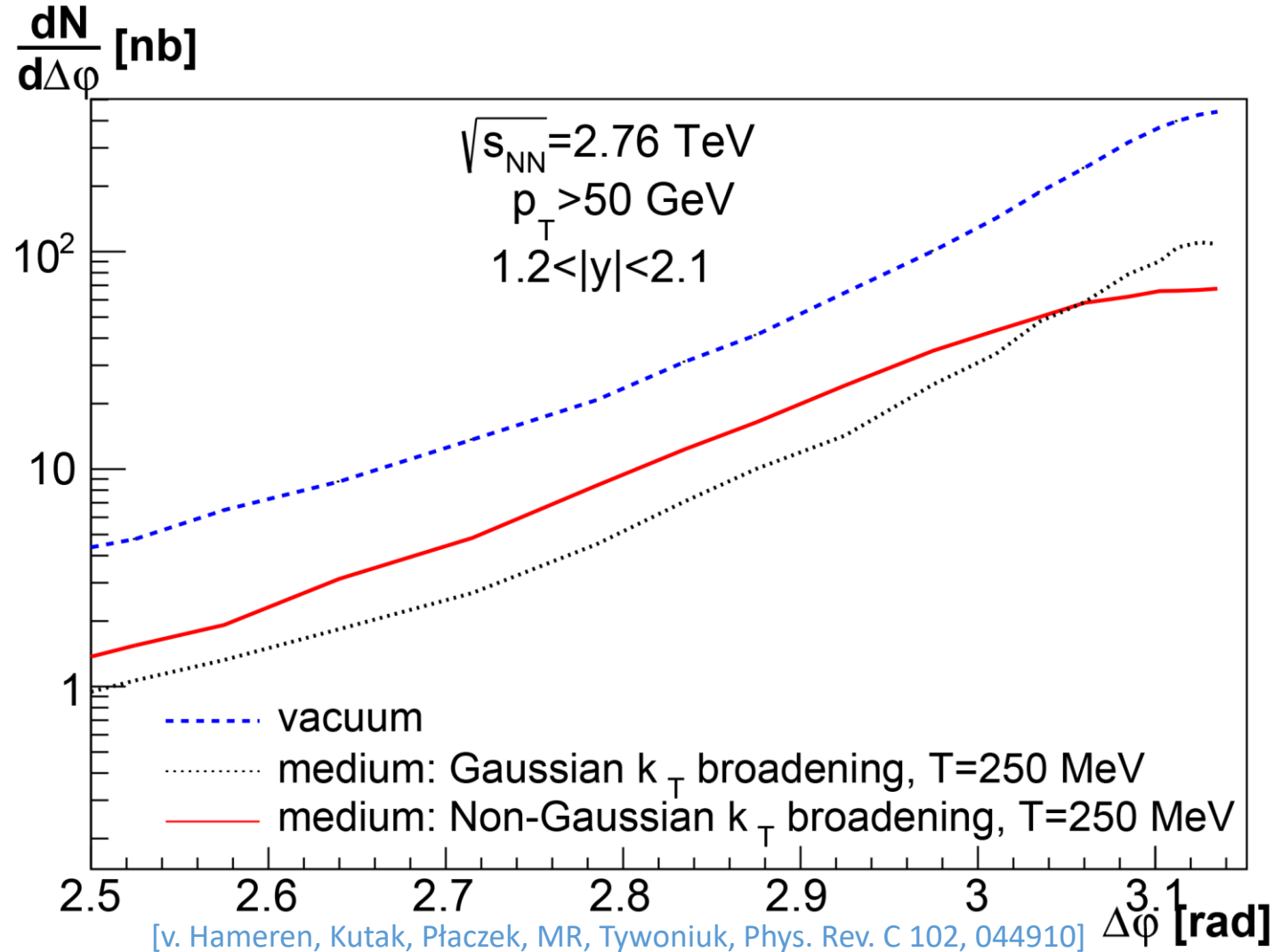
R_{AA}

$$R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle T_{AA} \rangle \frac{d\sigma_{pp}}{dp_T}} \approx \frac{\frac{d\sigma_{AA}}{dp_T}}{\frac{d\sigma_{pp}}{dp_T}}$$



$\mathcal{K}(z) \quad w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{q^2 (q^2 + m_D^2)}$

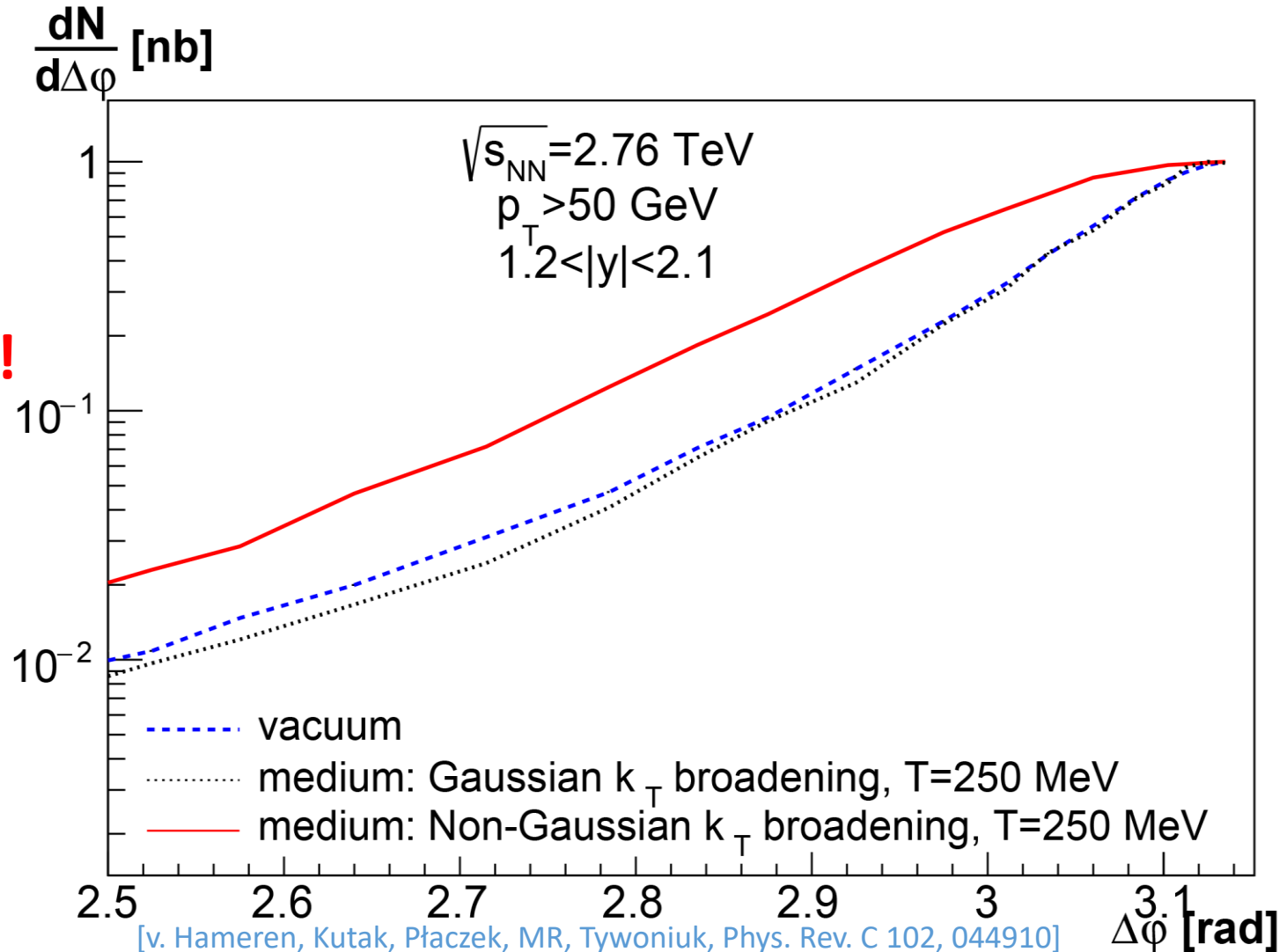
Azimuthal Decorrelations



$$\mathcal{K}(z) \quad w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\mathbf{q}^2 (\mathbf{q}^2 + m_D^2)}$$

Azimuthal Decorrelations

Normalized
to maximum!



$$\mathcal{K}(z) \quad w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\mathbf{q}^2 (\mathbf{q}^2 + m_D^2)}$$

Summary

- **MINCAS**: jet evolution based on coherent emission and scattering
- ...solves BDIM equations for: collinear or noncollinear splitting, with or without scattering
- Combination with **KATIE**: allows for calculation of jet-observables
- Transverse momentum broadening differs from Gaussian distribution
- Gaussian distribution: smallest k_T broadening
- ...leads to broadening in angular dijet decorrelations

Outlook

- to account for quarks
- to study more forward processes
- and vacuum-like emissions

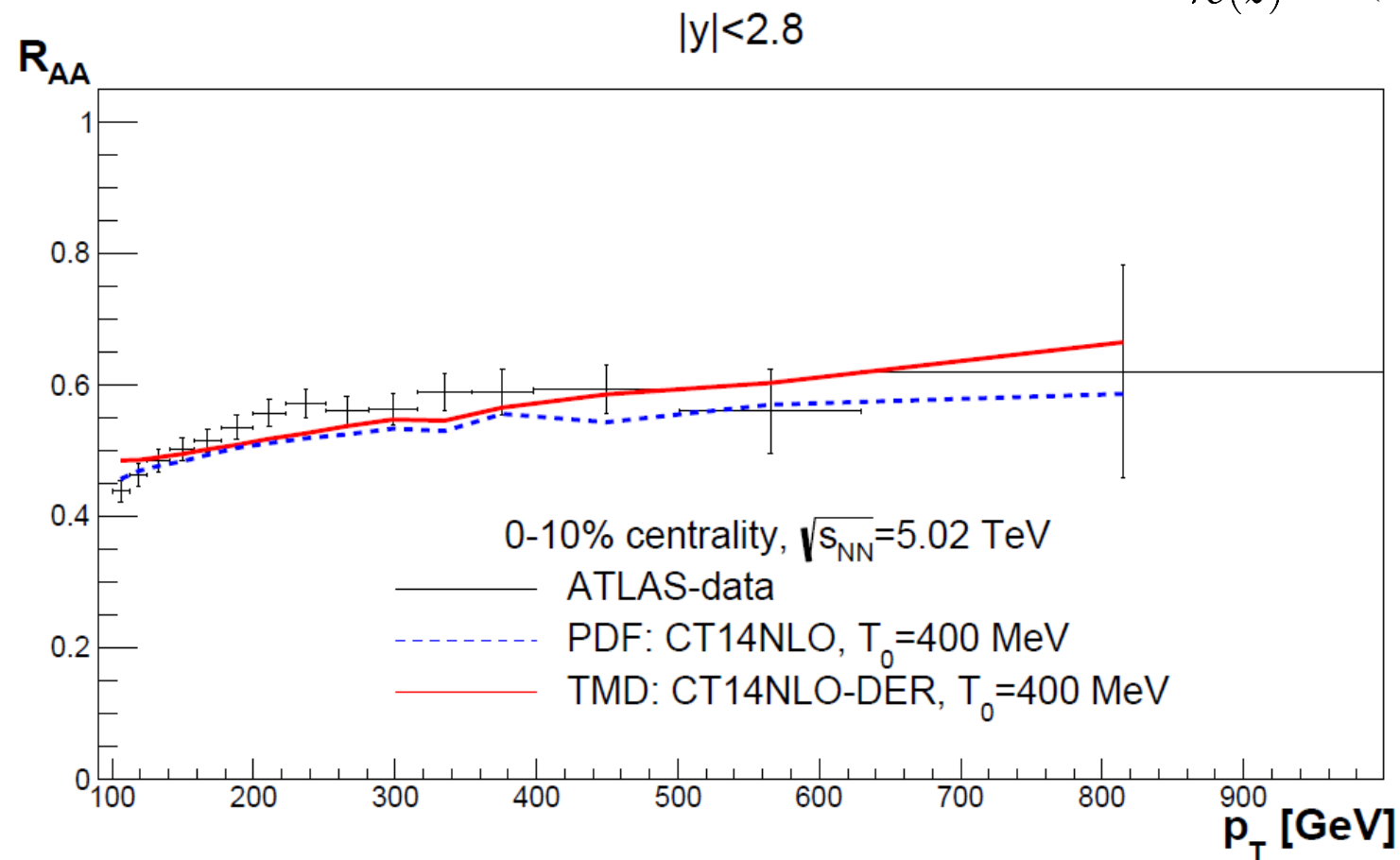
Thank you for your
attention!

Back-up slides

R_{AA}

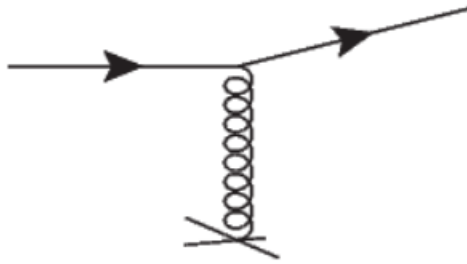
$$R_{AA}(p_T) = \frac{\frac{dN_{AA}}{dp_T}}{\langle T_{AA} \rangle \frac{d\sigma_{pp}}{dp_T}} \approx \frac{\frac{d\sigma_{AA}}{dp_T}}{\frac{d\sigma_{pp}}{dp_T}}$$

$$\mathcal{K}(z) \quad w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{q^2(q^2 + m_D^2)}$$



Processes in Jets

scattering...



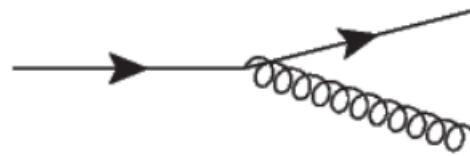
Transverse momentum transfer!

$$p \rightarrow p + k_T$$

Scattering Kernel: $C(k_T)$

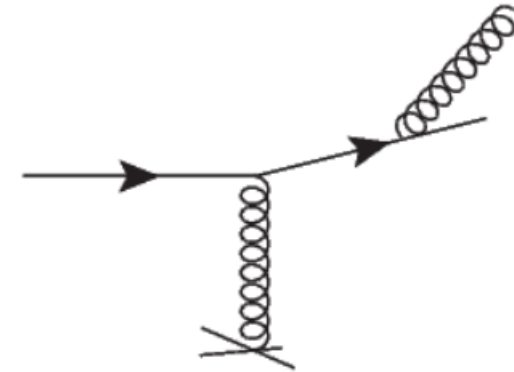
Average transfer: $\hat{q}$

...splitting...



Bremsstrahlung as in vacuum.

...induced radiation



Momentum distribution:

$$p \rightarrow zp$$

+Momentum transfer:

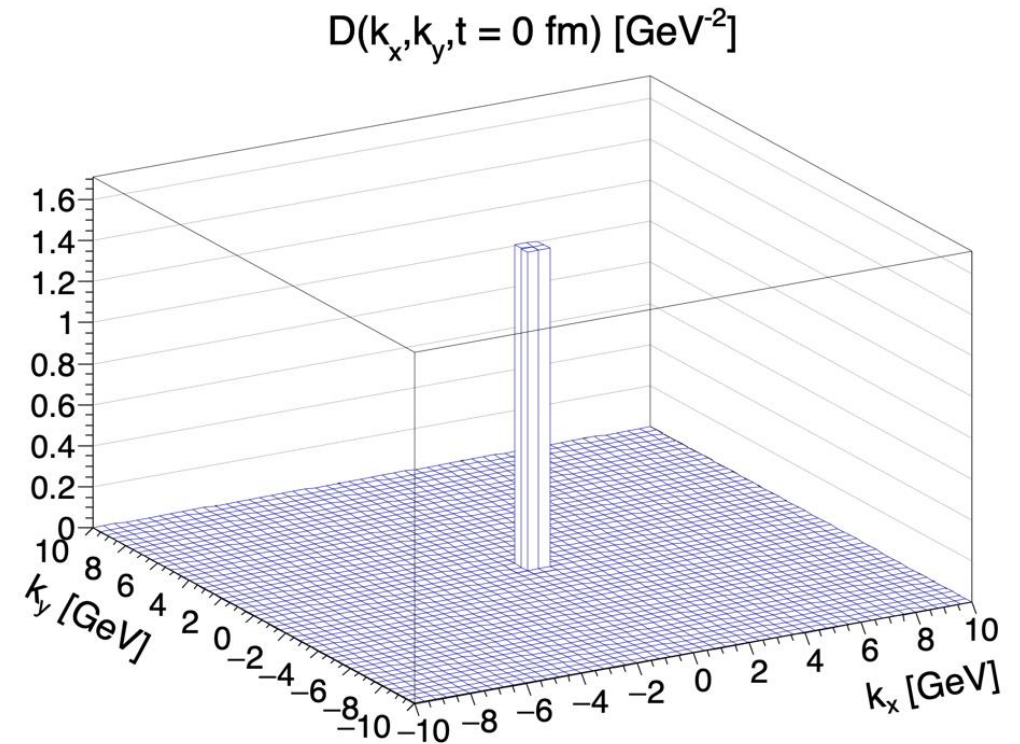
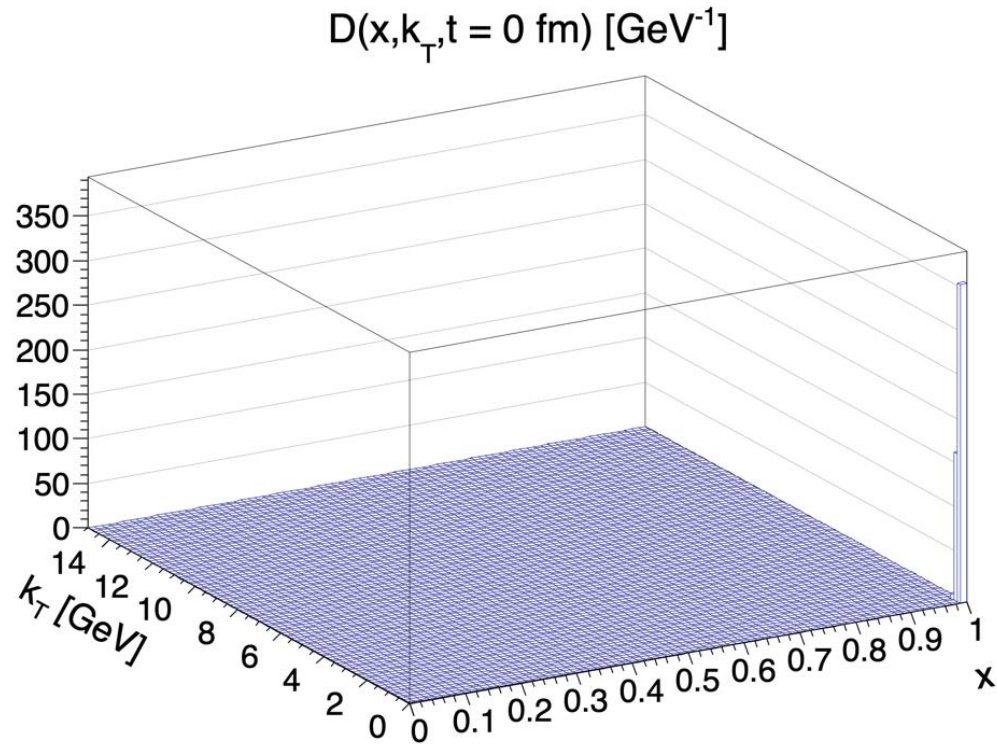
$$p \rightarrow zp + k_T$$

Kernel: $\mathcal{K}(z, k_T)$

Our results: combination of scattering and induced radiation processes!

Evolution of $D(x, k_T, t)$ (1/2)

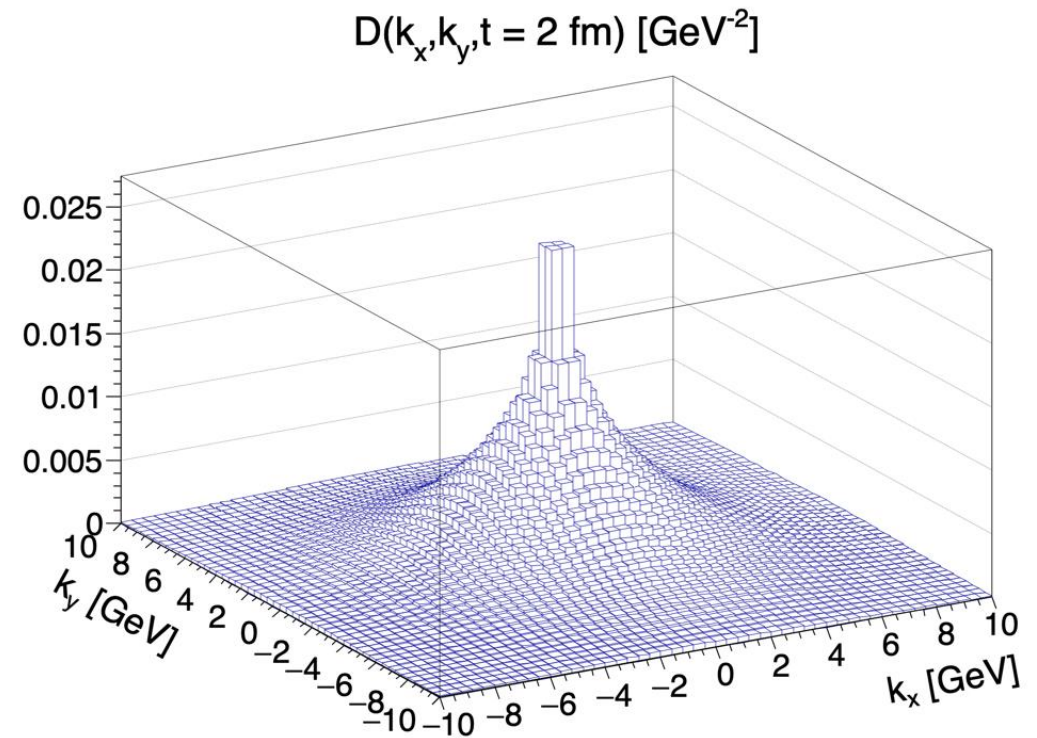
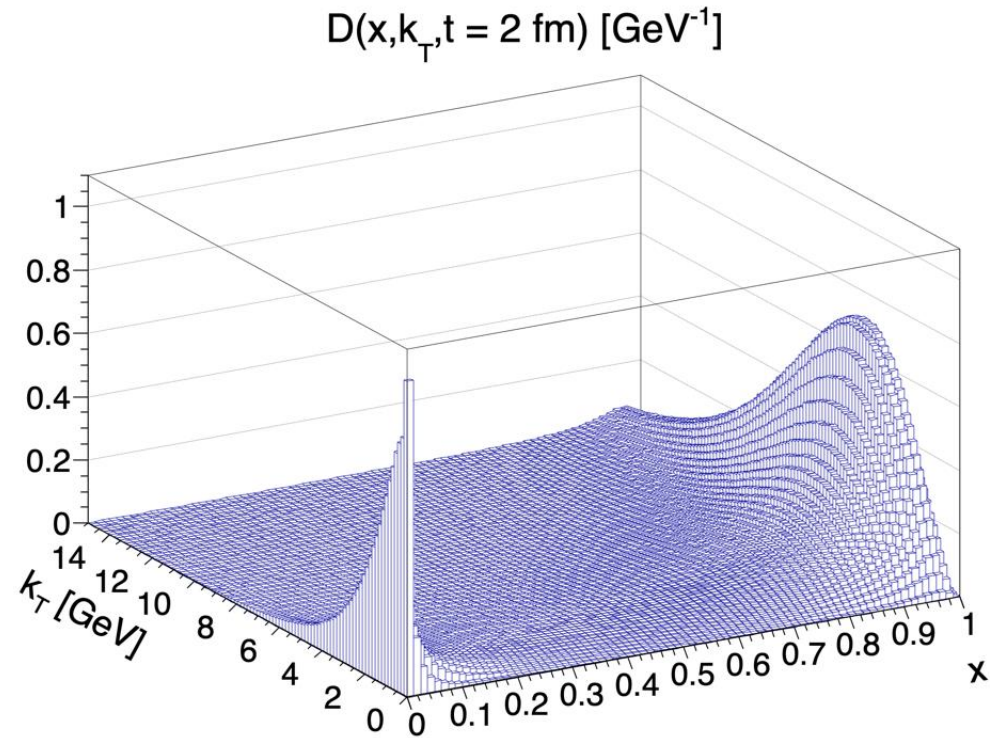
$$\mathcal{K}(z) \quad w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{q^2(q^2 + m_D^2)}$$



[Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

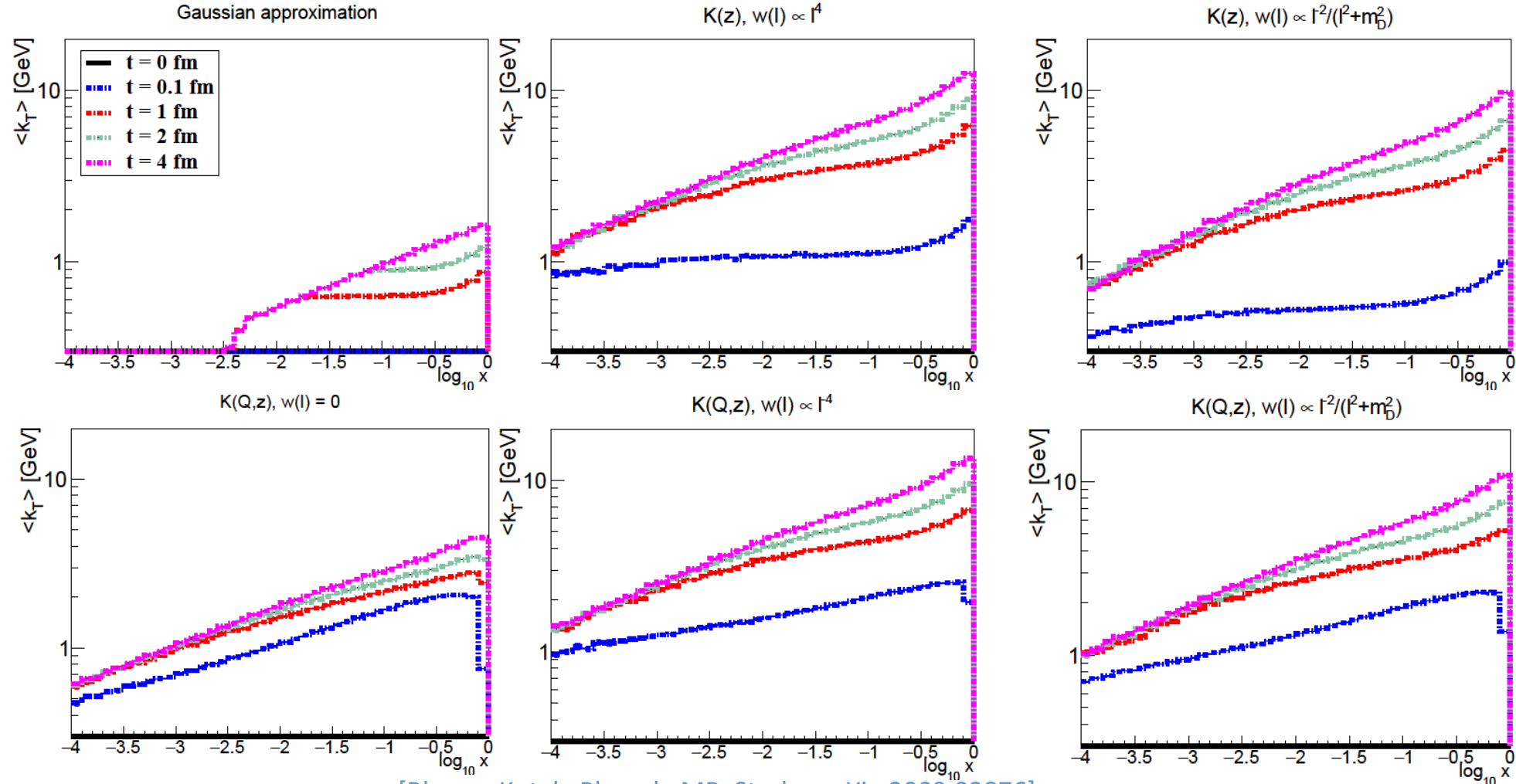
Evolution of $D(x, k_T, t)$ (2/2)

$$\mathcal{K}(z) \quad w(\mathbf{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\mathbf{q}^2 (\mathbf{q}^2 + m_D^2)}$$



[Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

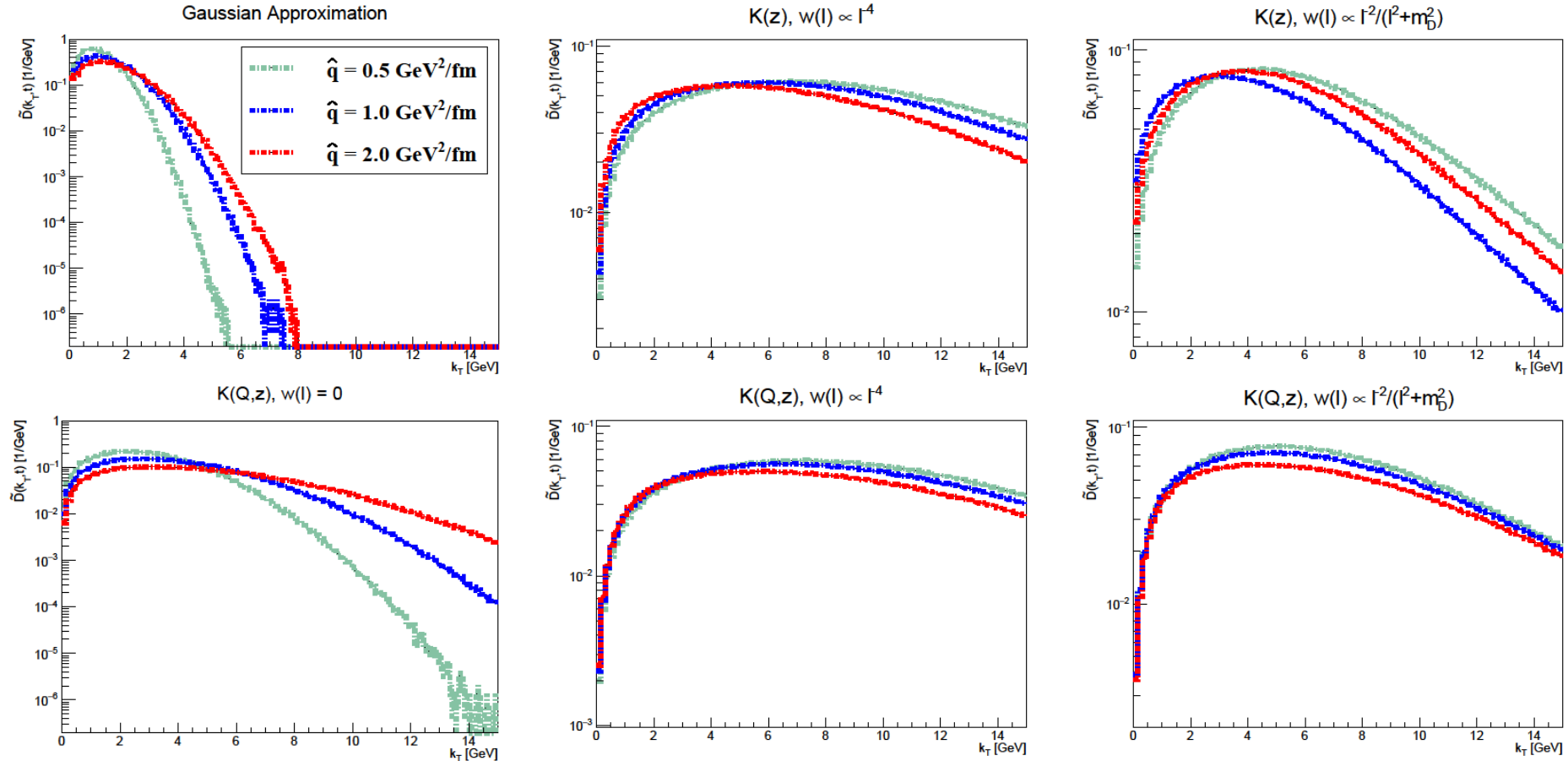
k_T Broadening (2/3)



[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

Non-Gaussian broadening effects on dijets

k_T Broadening (3/3)



[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

Entropy for leading particles

$$\hat{q} = 2 \text{ GeV}^2/\text{fm}$$

$$S_{\Delta}(t) = - \sum_{i=1}^N p_i(x, k_T, t) \ln p_i(x, k_T, t) + P(t) \ln [\Delta x \Delta k_T]$$

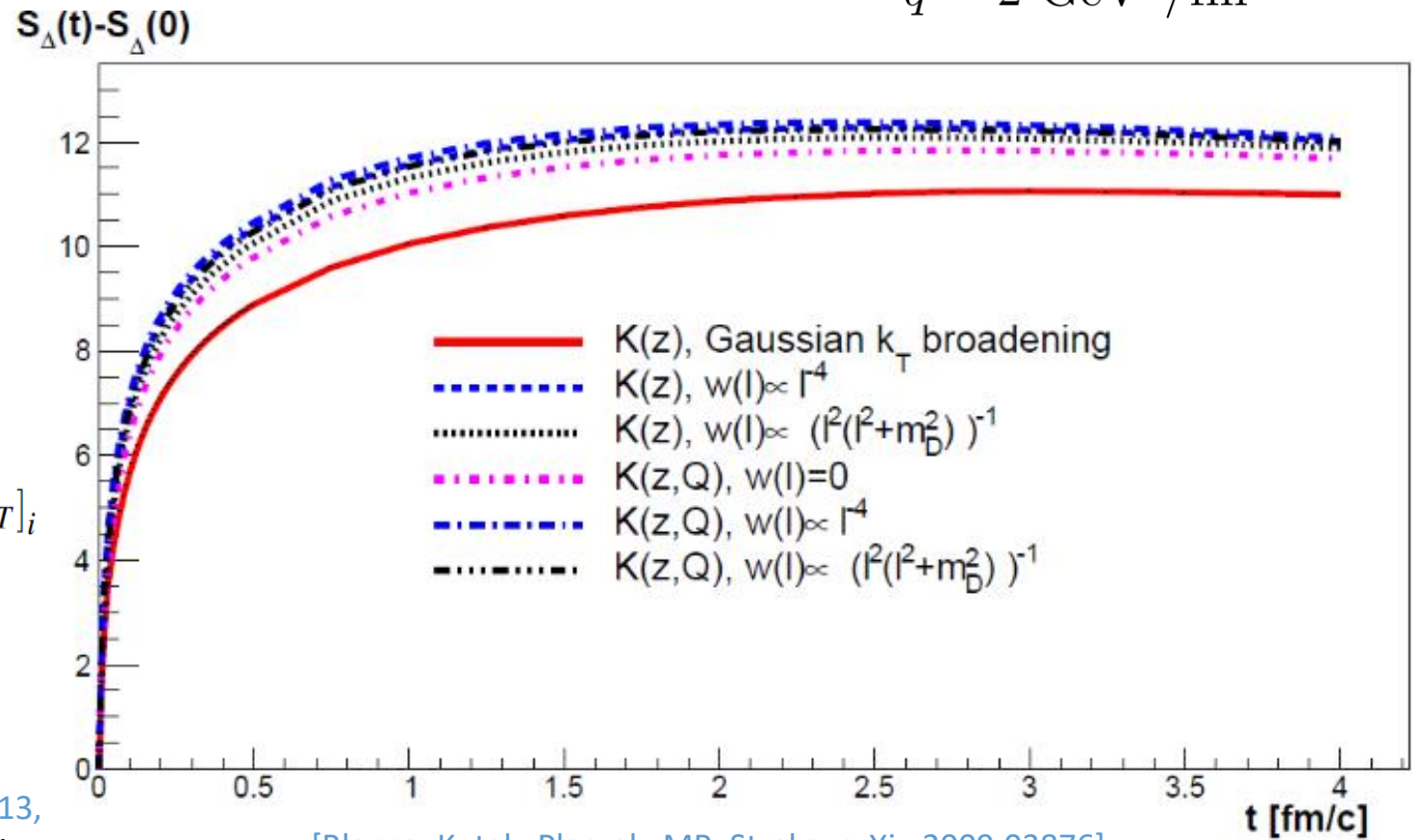
$$\Delta x \rightarrow 0, \Delta k_T \rightarrow 0$$

$$S_{\Delta}(t) \rightarrow - \int dx dk_T \tilde{D}(x, k_T, t) \ln \left(\tilde{D}(x, k_T, t) \right)$$

$$p_i(x, k_T, t) = [\tilde{D}(x, k_T, t) \Delta x \Delta k_T]_i = [2\pi k_T D(x, k_T, t) \Delta x \Delta k_T]_i$$

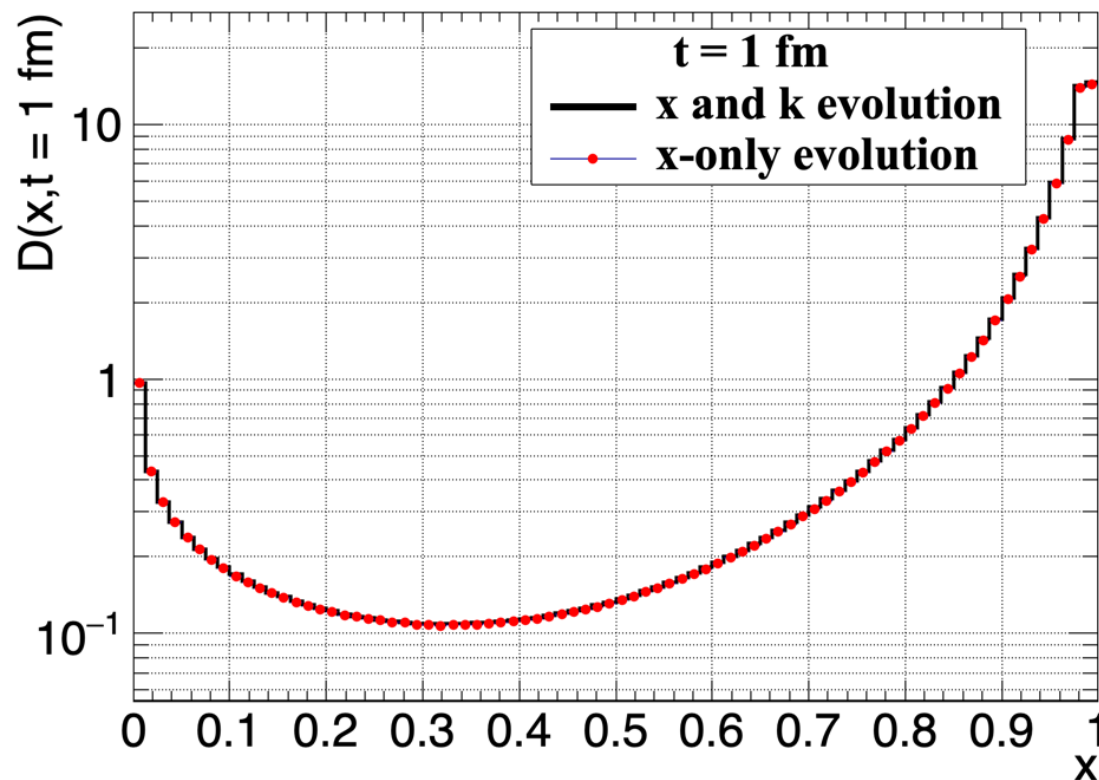
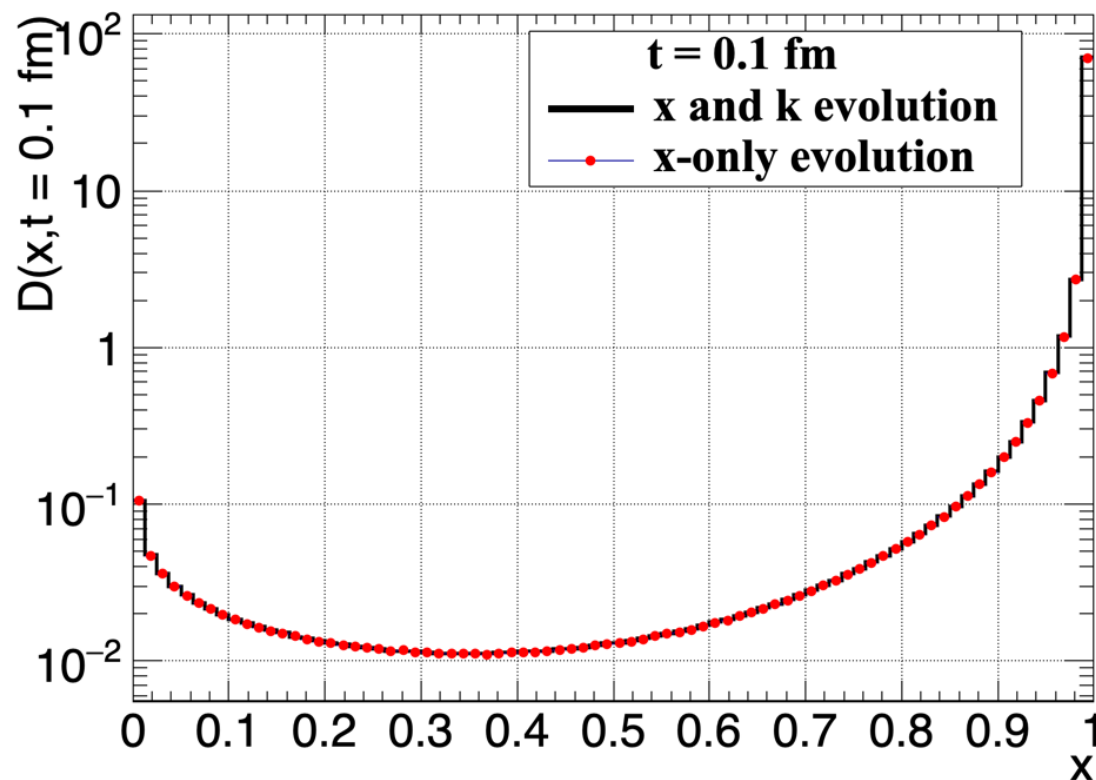
$$P(t) = \sum_{i=1}^N p_i(x, k_T, t)$$

[B. Chen, Y. Zhu, J. Hu and J. C. Principe, System Parameter Identification. Information Criteria and Algorithms. Elsevier, 2013, <https://doi.org/10.1016/C2012-0-01233-1>] (Delta-Entropy)



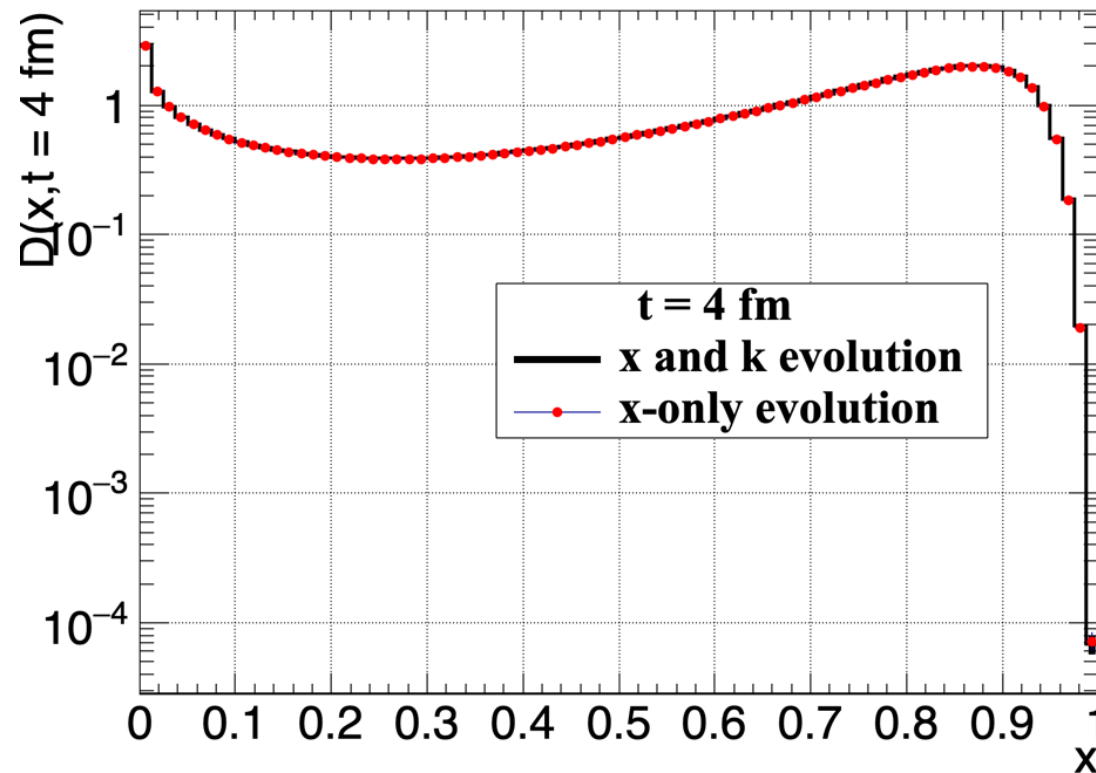
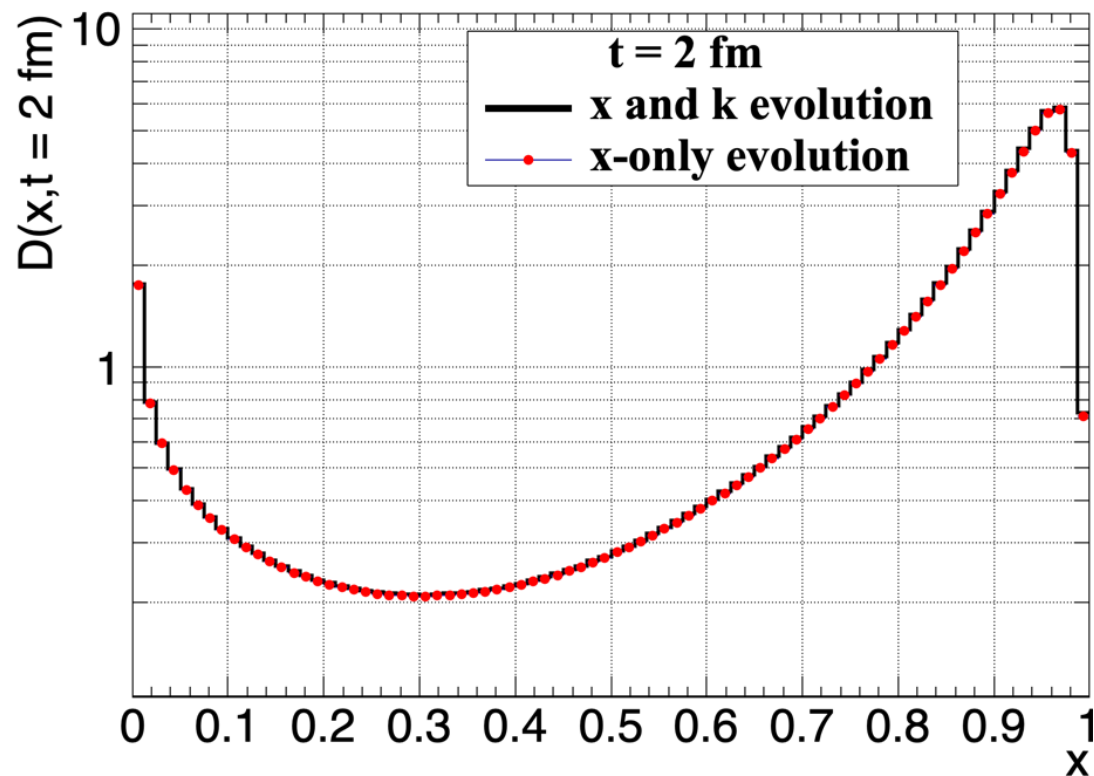
[Blanco, Kutak, Płaczek, MR, Straka, arXiv:2009.03876]

Turbulent behavior (1/2)



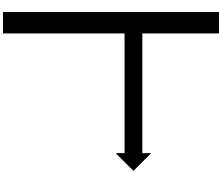
[Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

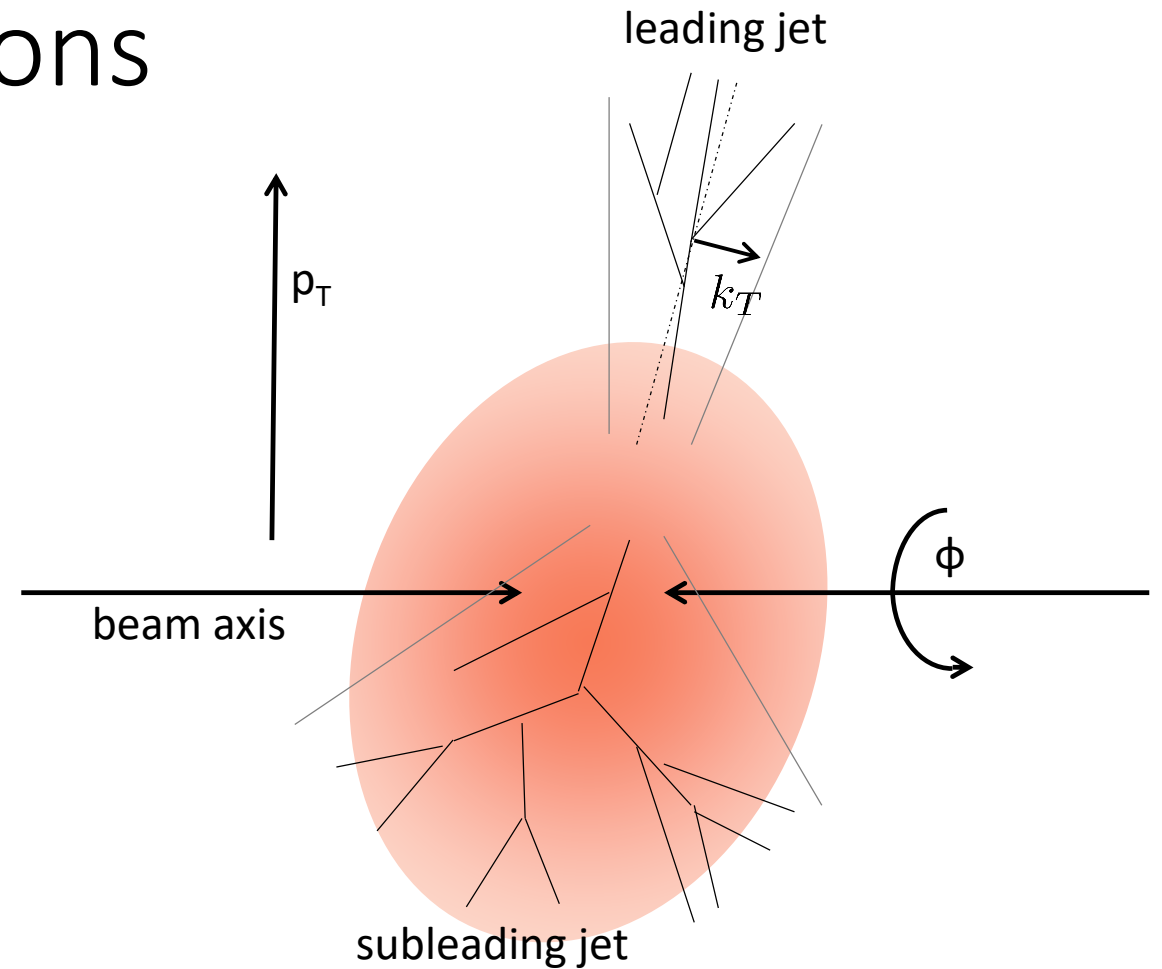
Turbulent behavior (2/2)



[Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Jets in Heavy Ion collisions

Jets interact with medium  Jet Quenching!
probe of the medium



Medium Model for Di-jet studies

Bjorken Model:

$$T(t) = T_0 \left(\frac{t_0}{t} \right)^{\frac{1}{3}}$$

T ...temperature at time t

T_0 ...temperature at time t_0

fixed		free		resulting	
c_q	3.7	t_0	0.6 fm/c	$\langle \hat{q} \rangle$	0.54 GeV ² /fm
c_n	5.228	t_L	5 fm/c	$\langle n \rangle$	0.154 GeV ³
		T_0	0.4 GeV	$\langle m_D \rangle$	0.684 GeV

From Phenomenology (the JET-Collaboration):

$$\hat{q}(T) = c_q T^3$$

[JET Collaboration, Burke et al.: Phys. Rev.C90(2014) 014909]

From HTL:

$$m_D^2 = \left(\frac{N_C}{3} + \frac{N_F}{6} \right) g^2 T^2$$

cf. [Laine, Vuorinen: Lect. Notes Phys.925(2016)pp.1–281, 1701.01554]

Bose-Einstein/Fermi-Dirac Distribution



Taylor expansion, lowest order:

$$n(T) = n_q + n_{\bar{q}} + n_g = c_n T^3$$

cf.: [K.C.Zapp, PhD-Thesis, Heidelberg U., 2008]

Diffusion approximation (1/2)

$$\begin{aligned} \frac{\partial}{\partial t} D(x, \mathbf{k}, t) = & \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] \\ & + \int \frac{d^2 \mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t), \end{aligned}$$



$$\begin{aligned} \frac{\partial}{\partial t} D(x, \mathbf{k}, t) = & \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] \\ & + \frac{1}{4} \hat{q} \nabla_k^2 \left[D(x, \mathbf{k}, t) \right]. \end{aligned}$$

Diffusion approximation (2/2)

