Extractions of Collinear and TMD Parton Distributions in the Pion

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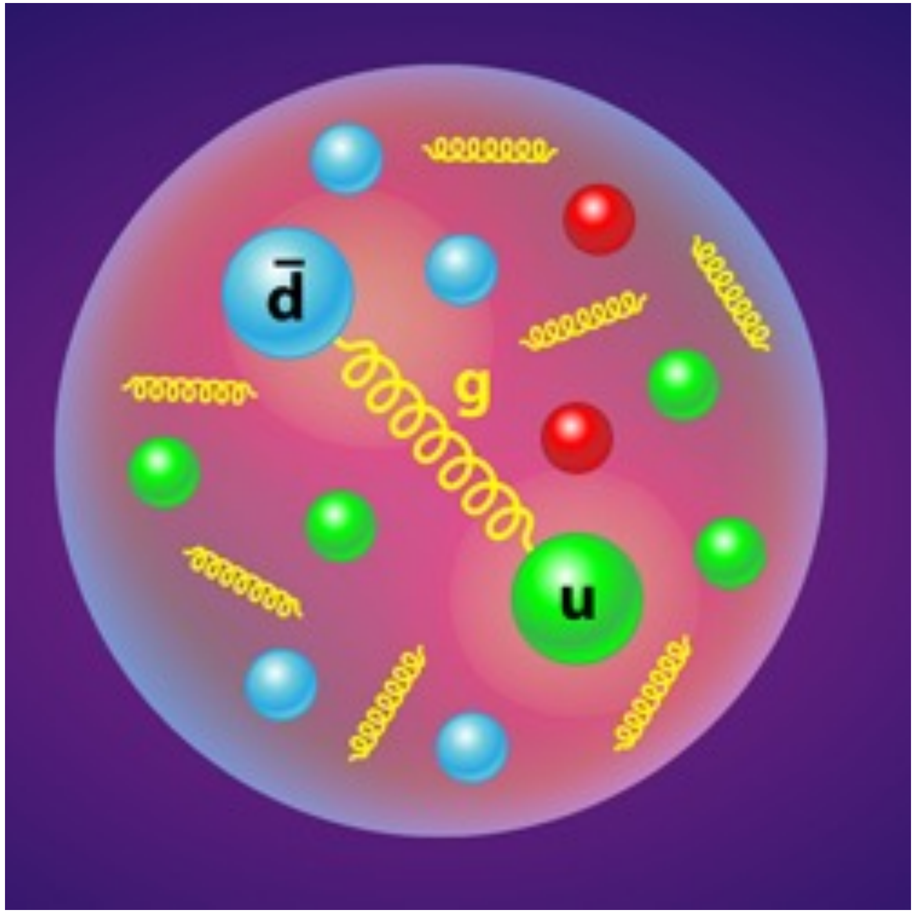
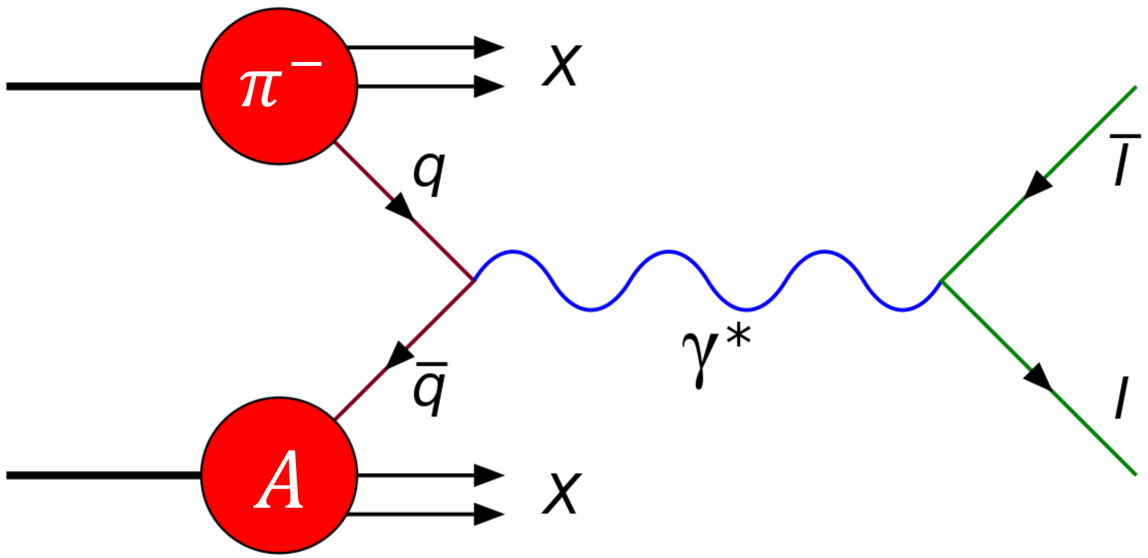
Pions

- Pion is the Goldstone boson associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry
- Lightest hadron as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies
- Simultaneously a pseudoscalar meson made up of q and $\bar{q}$ constituents



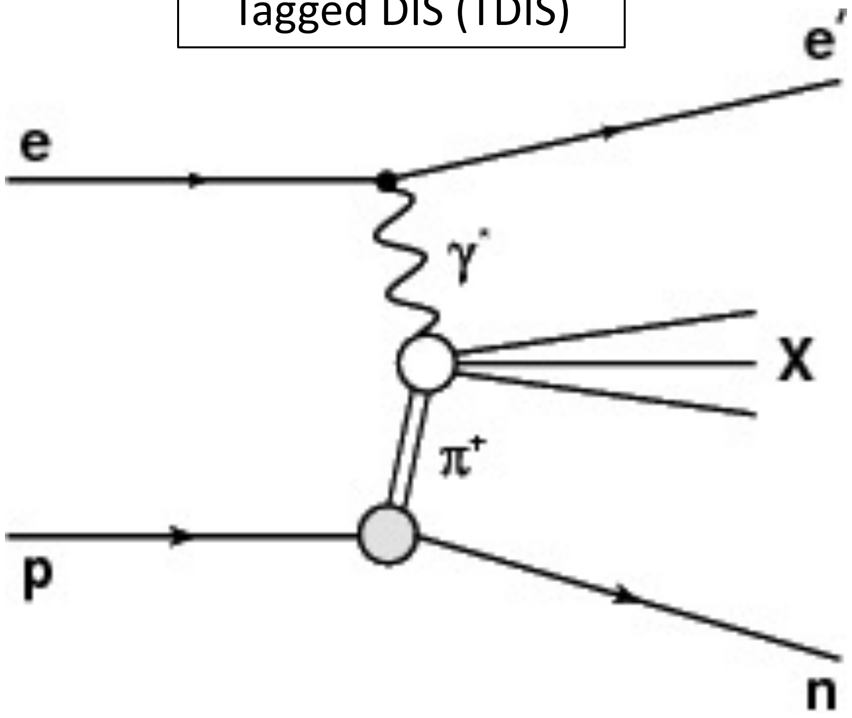
Experiments to Probe Pion Structure

- Drell-Yan (DY)



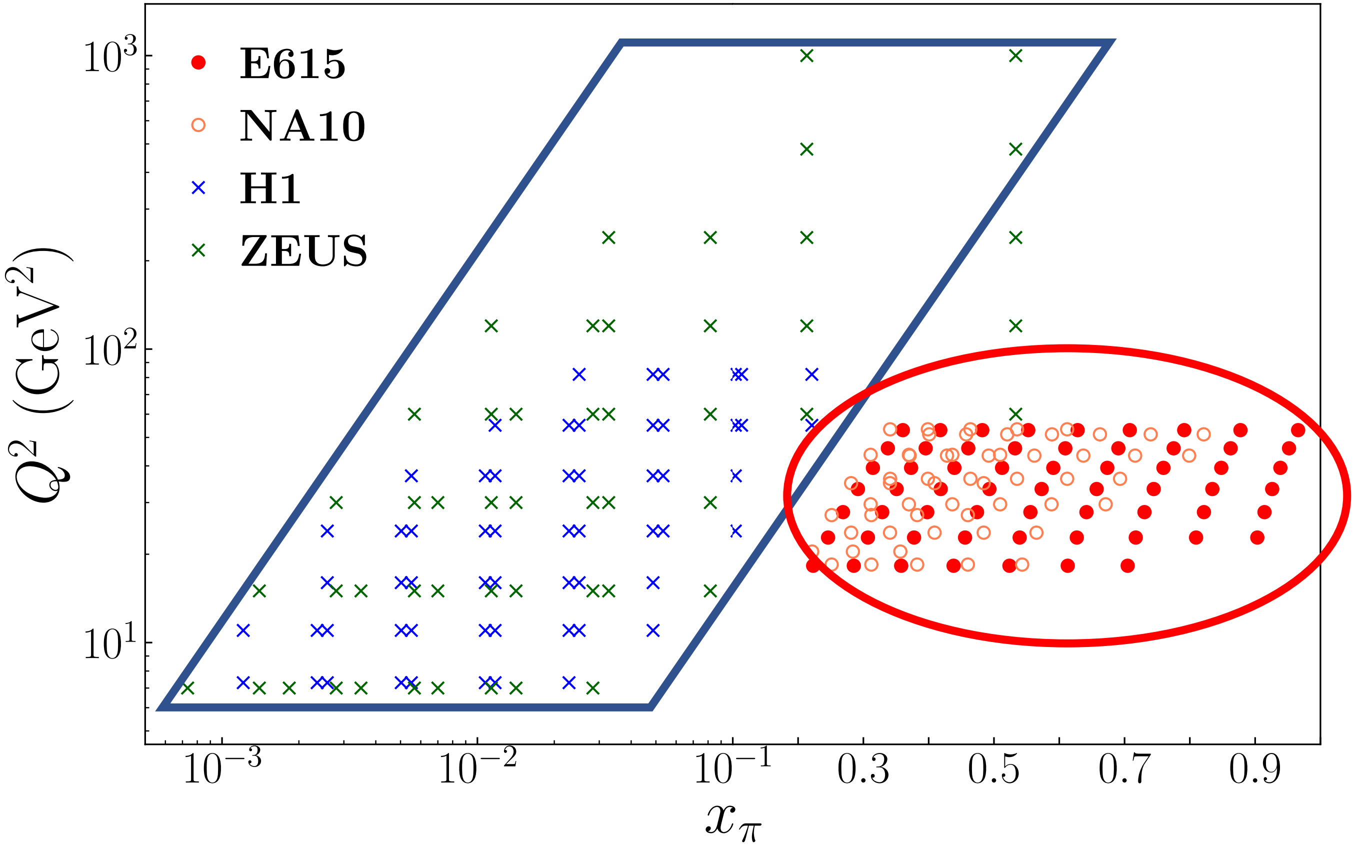
- Leading Neutron (LN)

Tagged DIS (TDIS)



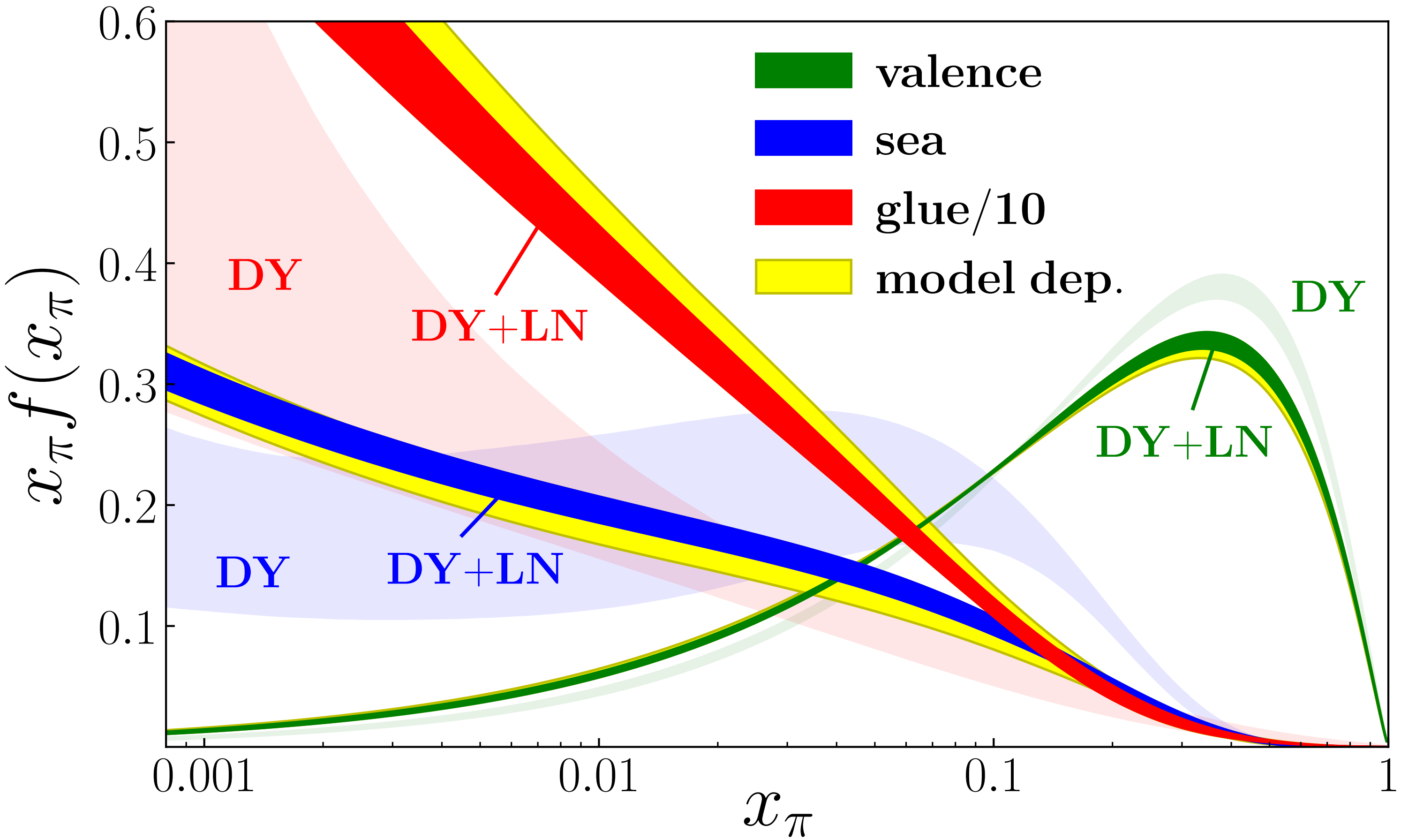
Kinematic Coverage

- Drell-Yan – large x_π
- Leading Neutron – small x_π
- Little kinematic overlap



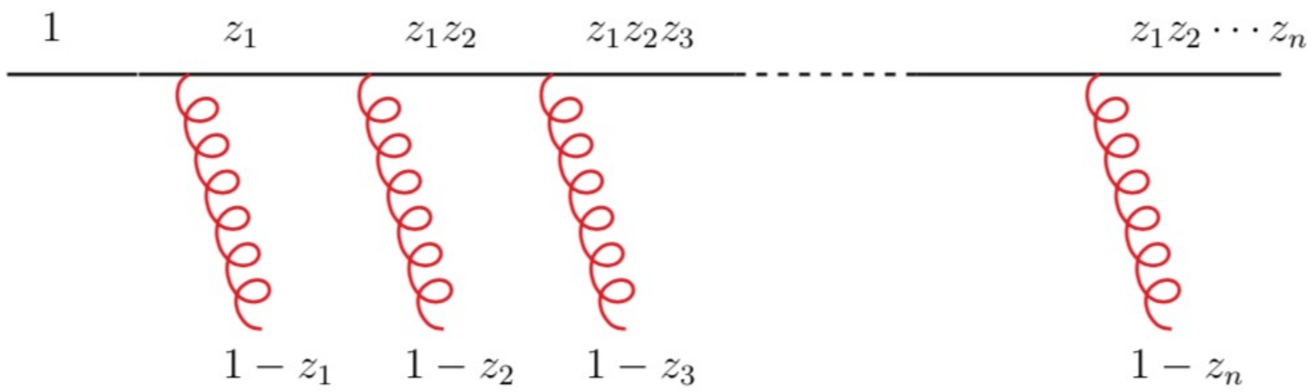
JAM18 Pion PDFs

- Lightly shaded bands – only Drell-Yan data
- Solid bands – fit to both Drell-Yan and LN data



PCB, N. Sato, W. Melnitchouk and Chueng-Ryong Ji, Phys. Rev. Lett. **121**, 152001 (2018).

Soft Gluon Resummation



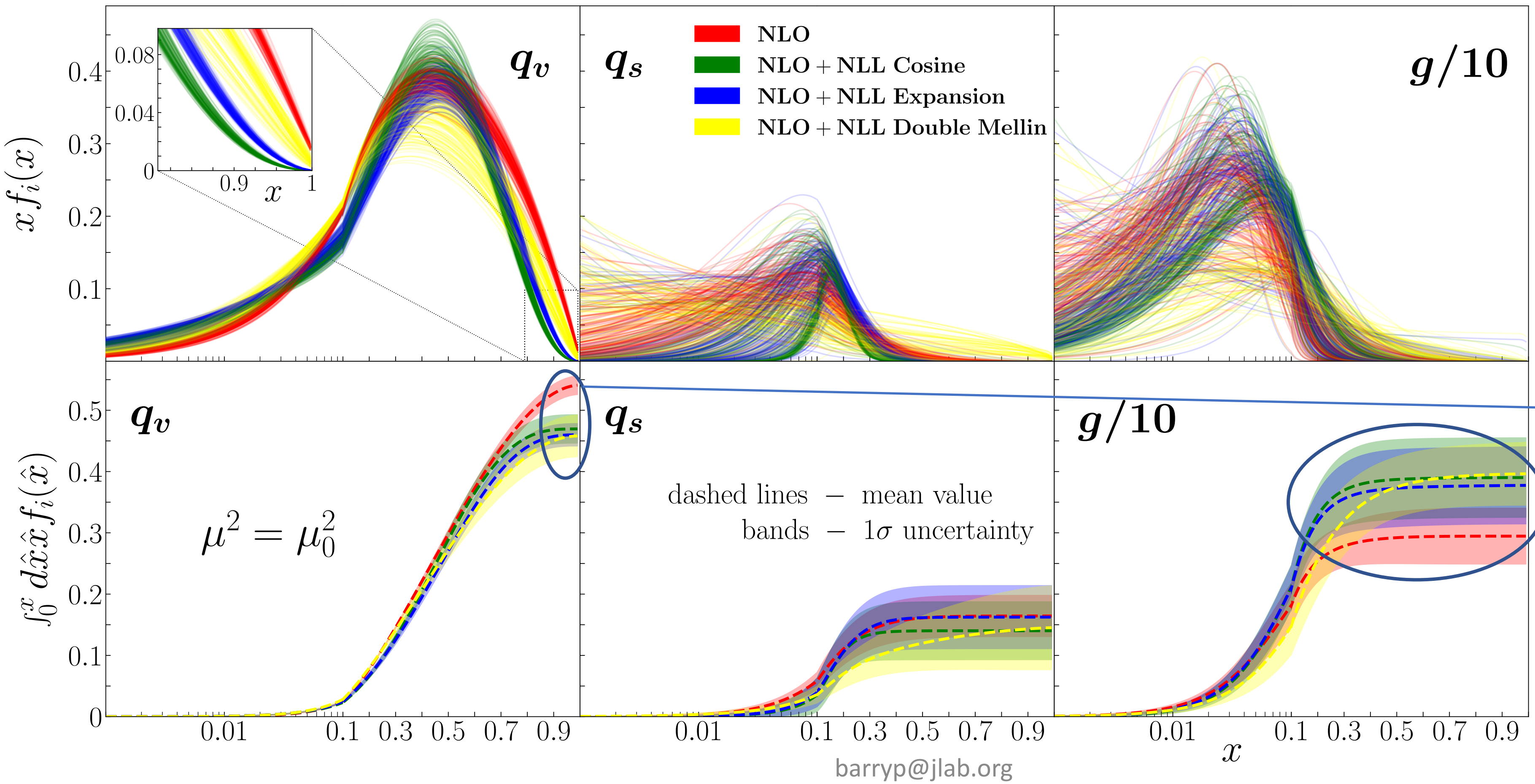
- Fixed-target Drell-Yan notoriously has large-x_F contamination of higher orders
- Large logarithms may spoil perturbation
- Focus on corrections to the most important $q\bar{q}$ channel
- Resum contributions to all orders of α_s

Methods of Resummation

- Calculation performed in conjugate space
- Data are reported as $\frac{d\sigma}{dQ^2 dY}$ -- need two transforms
- We can perform a Mellin-Fourier transform to account for the rapidity
  - A cosine appears while doing Fourier transform; options:
    1) Take first order expansion, cosine ≈ 1
    2) Keep cosine intact
- Can additionally perform a Double Mellin transform
- Explore the different methods and analyze effects

PDF Results

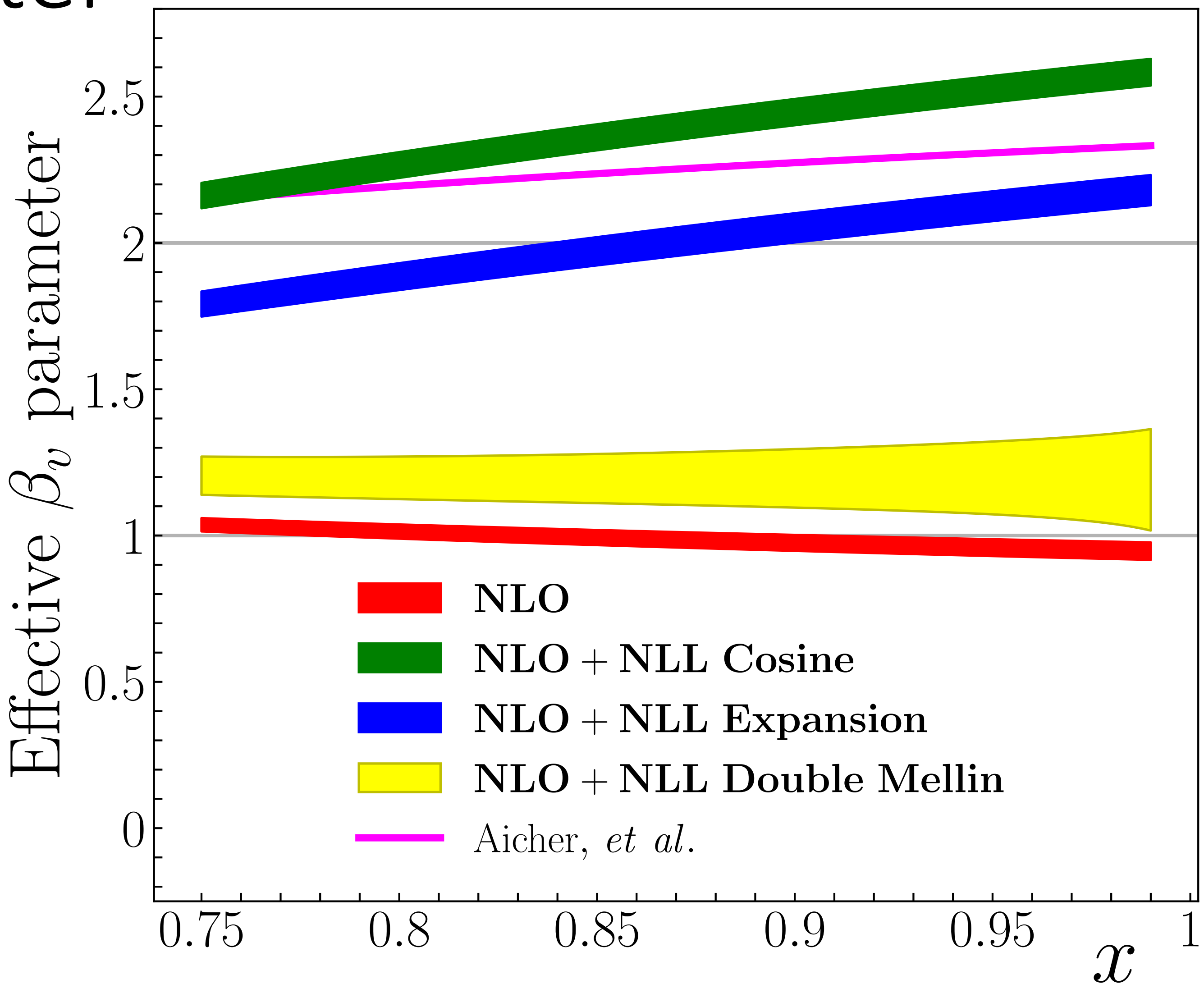
- Large x behavior in valence depends on prescription



Large x momentum fraction in the valence shifts to low x gluon momentum fraction

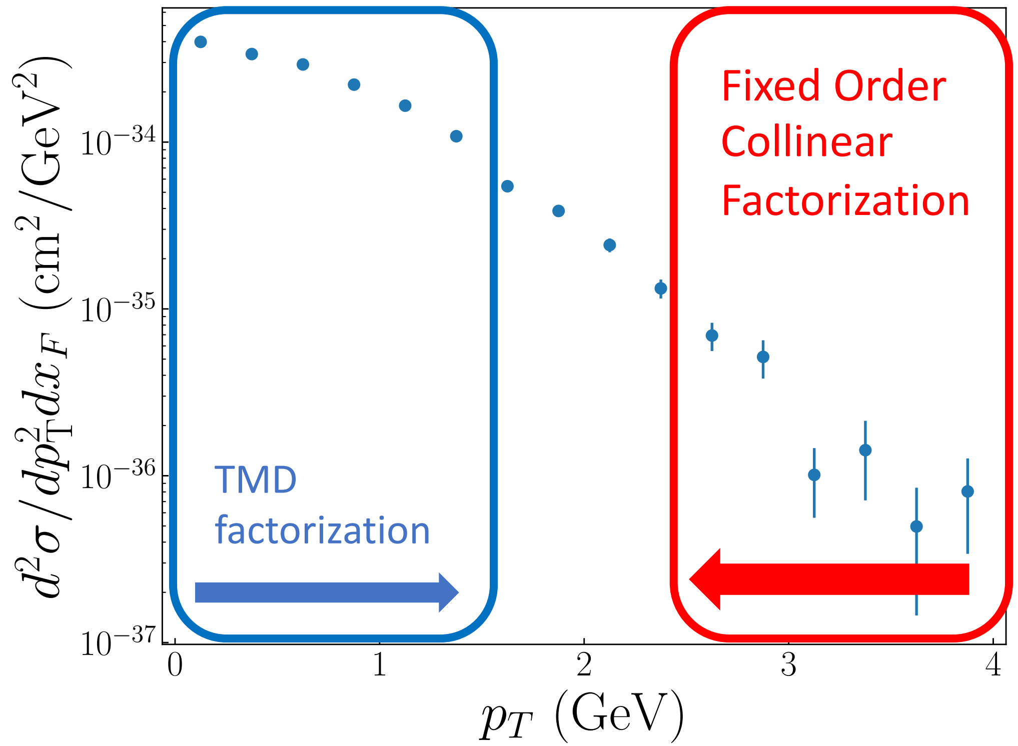
Effective β_v parameter

- $q_v(x) \sim (1-x)^{\beta_v}$ as $x \to 1$
- Threshold resummation does not give universal behavior of β_v
- NLO and double Mellin give $\beta_v \approx 1$
- Cosine and Expansion give $\beta_v > 2$



p_{T}-dependent spectrum for pion data

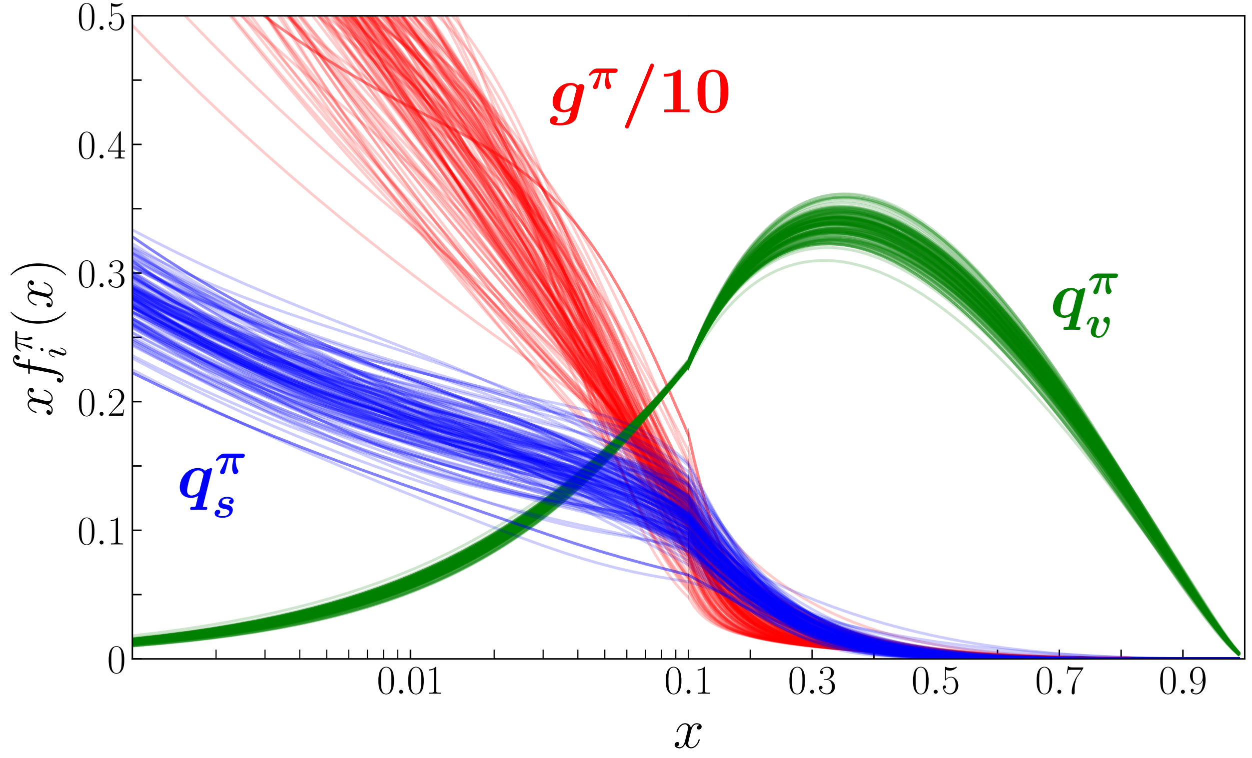
- Various factorization theorems break down in certain regions of p_{T}
- Errors are related with $\mathcal{O}(p_{\mathrm{T}}/Q)$ (low-p_{T}) or $\mathcal{O}(m/p_{\mathrm{T}})$ (large-p_{T})



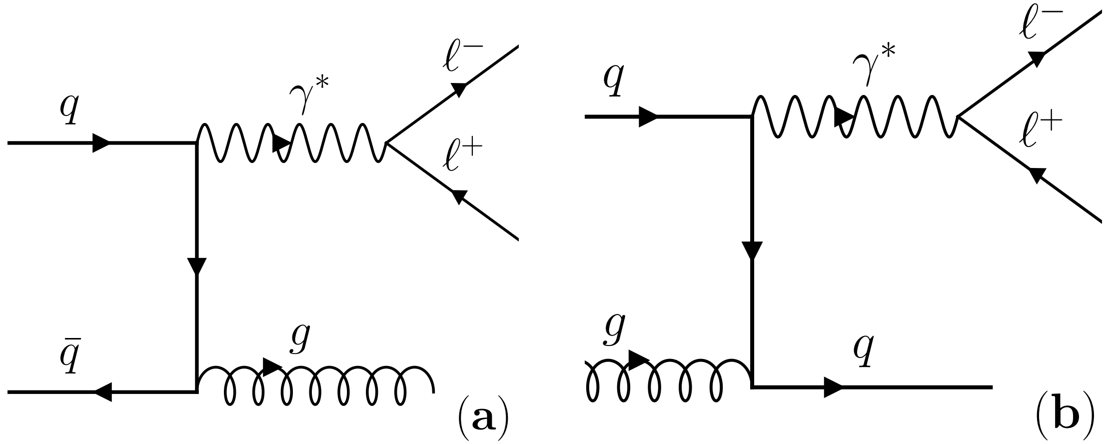
E615 πW Drell-Yan

Phys. Rev. D **39**, 92 (1989).

High p_T – JAM20 Pion PDFs



N.Cao, PCB, N. Sato, and W. Melnitchouk
arXiv:2103.02159 [hep-ph]



- Successfully describe data with a scale $\mu = p_\mathrm{T}/2$

Low p_{T} – TMD factorization

CSS

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H_{j\bar{j}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}$$

$$\times\, e^{-\boxed{g_{j/A}(x_A,b_{\mathrm{T}};b_{\max})}} \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}_{j/j_A}^{\mathrm{PDF}}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times\, e^{-\boxed{g_{\bar{j}/B}(x_B,b_{\mathrm{T}};b_{\max})}} \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}_{\bar{j}/j_B}^{\mathrm{PDF}}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times \exp\left\{-\boxed{g_K(b_{\mathrm{T}};b_{\max})}\ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Non-perturbative TMDs to extract

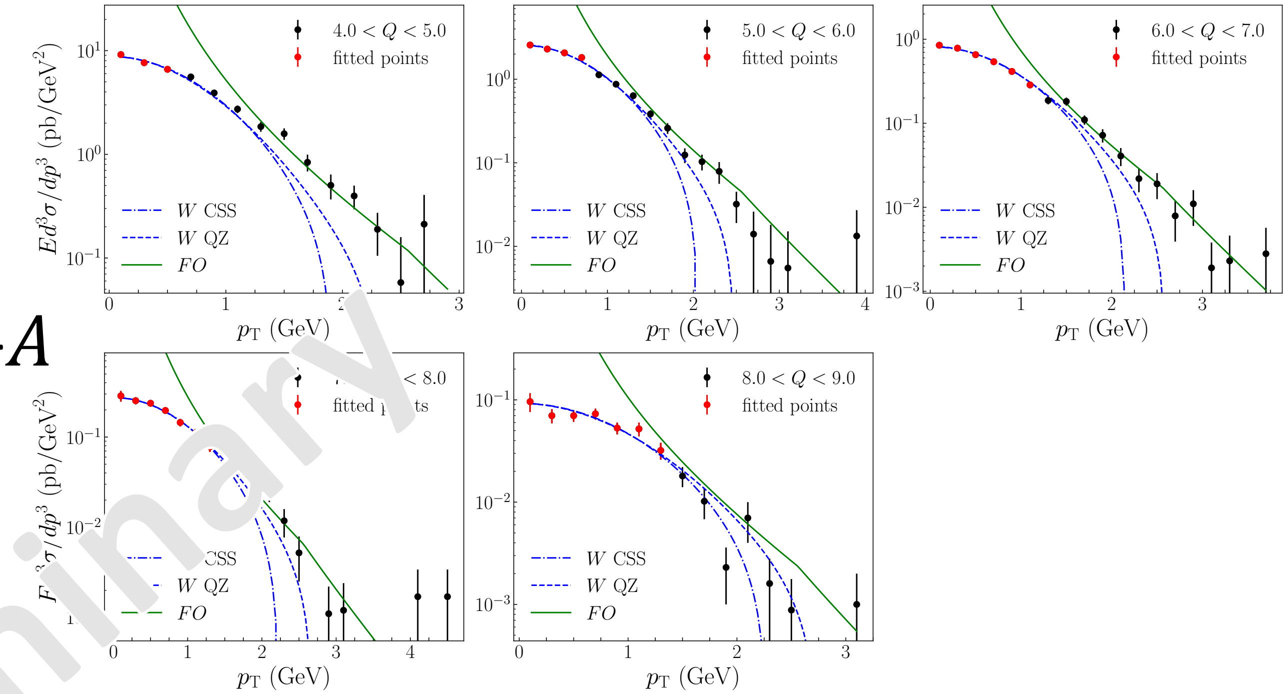
Qiu-Zhang

$$\tilde{W}^{QZ}(b,Q,x_A,x_B) = \begin{cases} \tilde{W}(b,Q,x_A,x_B), & b \leq b_{max}, \\ \tilde{W}(b_{max},Q,x_A,x_B)\boxed{F_{QZ}^{NP}(b,Q,x_A,x_B;b_{max})}, & b > b_{max}, \end{cases}$$

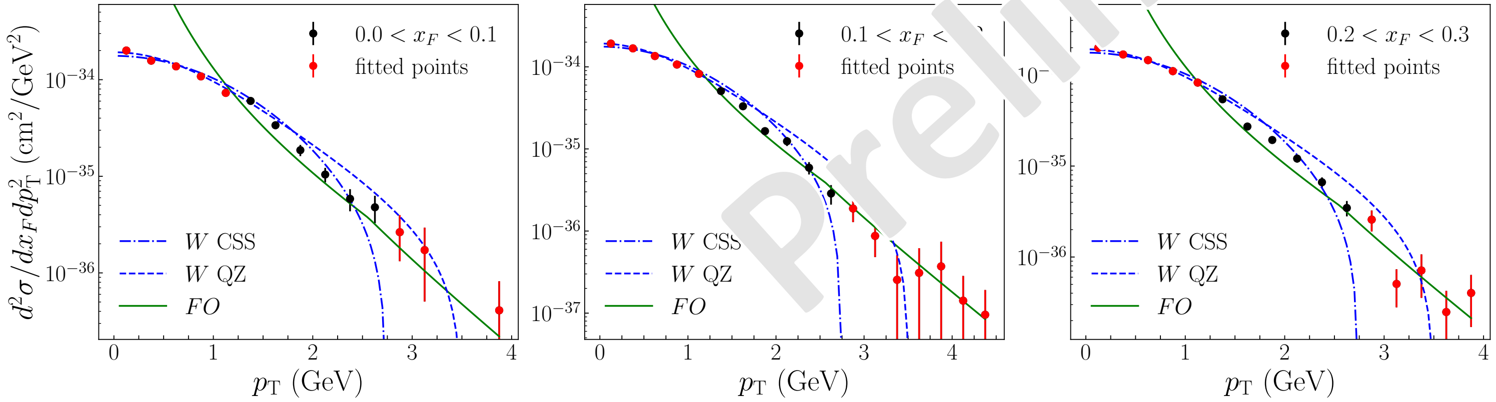
Fit Results

Success in fitting *simultaneously* proton and pion Drell-Yan data

- E288 p-A



- E615 π-A



Future Work

- Look to lattice data to discriminate the large-x_π behavior of valence quark PDF
- Explore matching procedure between W and FO terms in p_T spectrum
- Include more Drell-Yan datasets in TMD extractions

Backup

Bayesian Inference

- Minimize the χ^2 for each replica

Normalization parameter

$$\chi^2(\boldsymbol{a}, \text{data}) = \sum_e \left(\sum_i \left[\frac{d_i^e - \sum_k r_k^e \beta_{k,i}^e - t_i^e(\boldsymbol{a})/n_e}{\alpha_i^e} \right]^2 + \left(\frac{1 - n_e}{\delta n_e} \right)^2 + \sum_k \left(r_k^e \right)^2 \right)$$

- Perform N total χ^2 minimizations and compute statistical quantities

Expectation value $$\mathrm{E}[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(\boldsymbol{a}_k),$$

Variance $$\mathrm{V}[\mathcal{O}] = \frac{1}{N} \sum_k \left[\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}] \right]^2,$$

TMD Non-Perturbative Parametrization

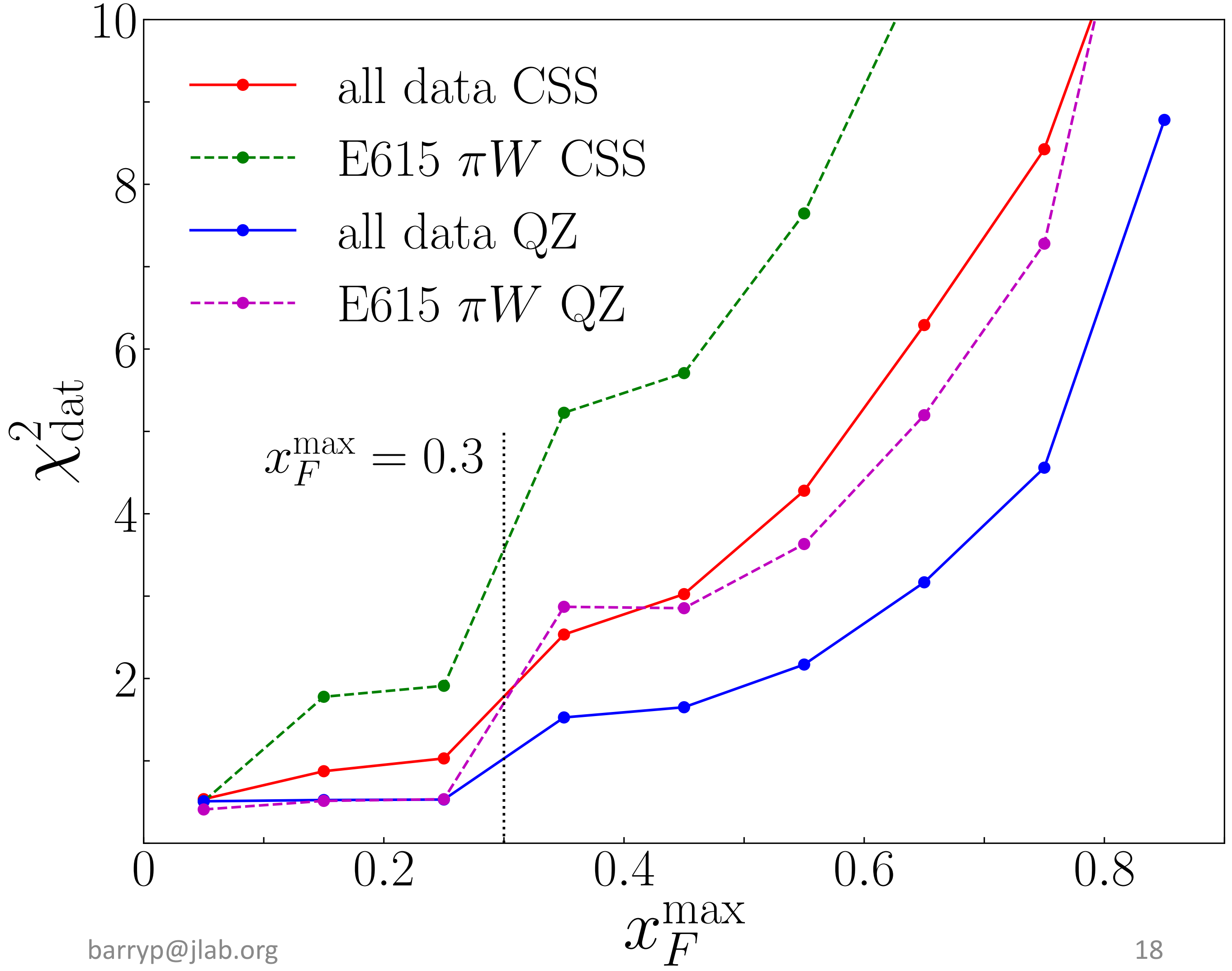
- Parametrize g_j for each hadron and g_K as in Slide 21

$$g_{j/h}(x, b_T^2) = -b_T^2 g_1 (\frac{1}{2} + g_3 \log(10x))$$

$$g_K(b_T) = -b_T^2 \frac{g_2}{2}$$

χ^2 as a function of $x_F^{\mathbf{max}}$

- Phenomenologically determine where TMD factorization breaks down in large x_F data



E288 pp

- Calculate large-p_{T} FO term with $\mu = Q$
- Not a good description