TMDs for spin-1 hadrons

(TMDs: Transverse-momentum-dependent parton distribution functions)

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QCD Evolution Workshop 2021
(Online) UCLA, Los Angeles, USA, May 10-14, 2021,
https://indico.bnl.gov/event/6803/timetable/#all.detailed

Recent related works on spin-1 hadrons in 2021,
http://research.kek.jp/people/kumanos/
(1) SK and Qin-Tao Song, Phys. Rev. D 103 (2021) 014025.
(2) A. Arbuzov *et al*. (NICA project),
Prog. Nucl. Part. Phys. 119 (2021) 103858 (arXiv:2011.15005).

May 14, 2021

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Note on my notations
Gluon transversity: $\Delta_T g$
Tensor-polarized gluon distribution: $\delta_T g$

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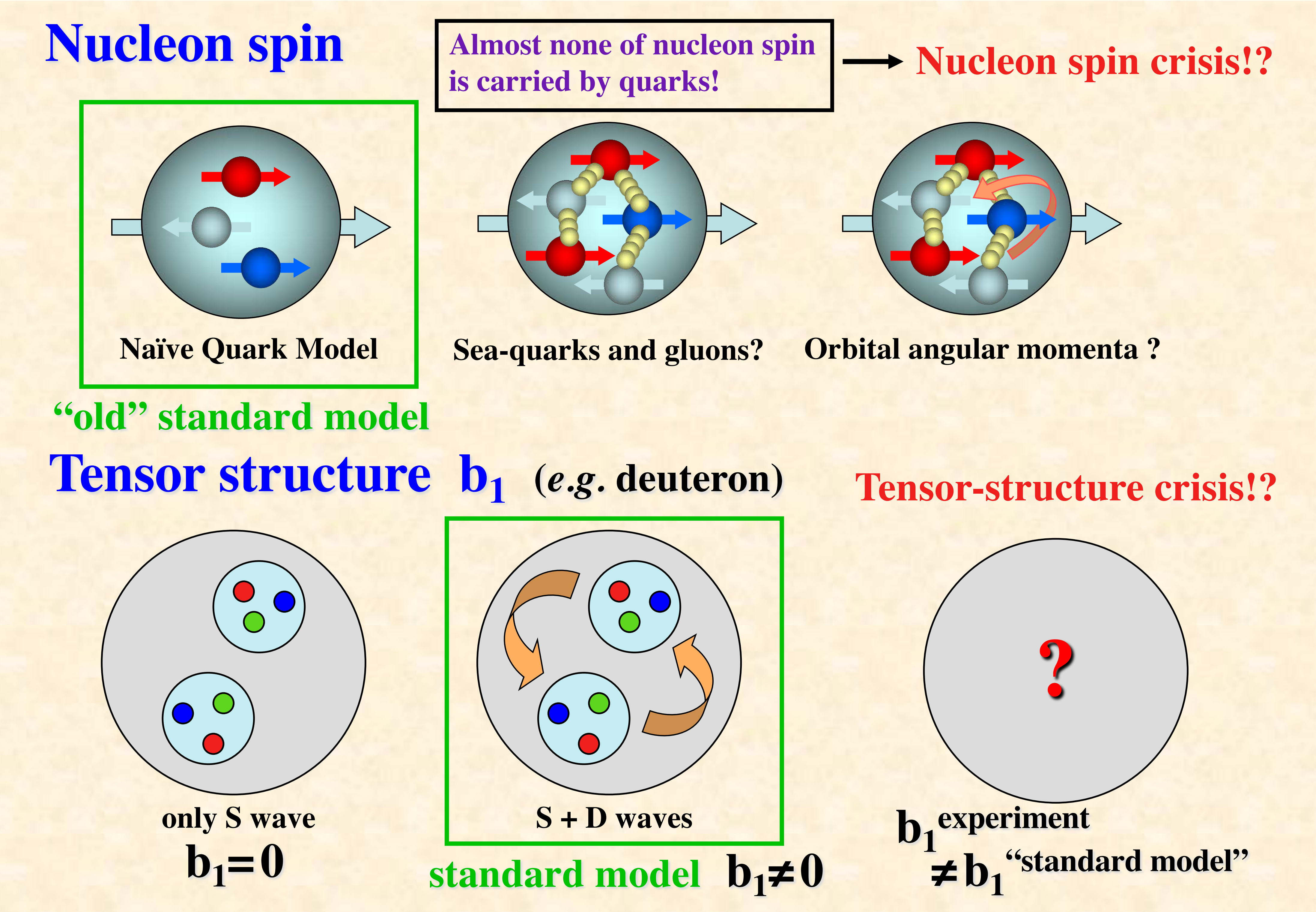
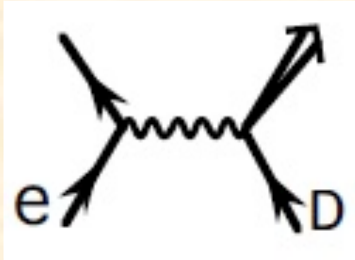
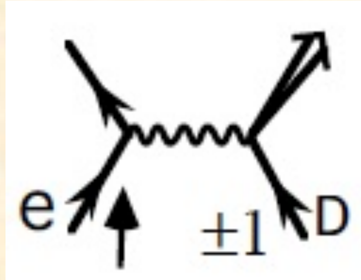

Nucleon spin
Almost none of nucleon spin is carried by quarks!
Nucleon spin crisis!?
Naïve Quark Model
Sea-quarks and gluons?
Orbital angular momenta ?
"old" standard model
Tensor structure b_1 (*e.g.* deuteron)
Tensor-structure crisis!?
only S wave
$b_1=0$
?
S + D waves
standard model $b_1 \neq 0$
$b_1^{\text{experiment}}$
$\neq b_1^{\text{"standard model"}}$

Structure Functions

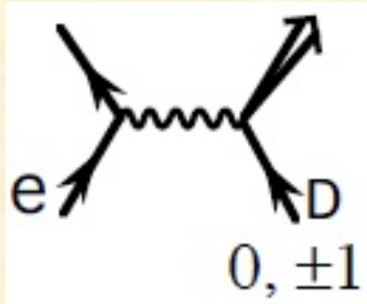
$$F_1 \propto \langle d\sigma \rangle$$



$$g_1 \propto d\sigma(\uparrow,+1) - d\sigma(\uparrow,-1)$$



$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$



note: $\sigma(0) - \frac{\sigma(+1)+\sigma(-1)}{2} = 3\langle\sigma\rangle - \frac{3}{2}\left[\sigma(+1)+\sigma(-1)\right]$

Parton Model

$$F_1 = \frac{1}{2}\sum_i e_i^2 \left(q_i + \bar{q}_i\right) \qquad q_i = \frac{1}{3}\left(q_i^{+1} + q_i^{0} + q_i^{-1}\right)$$

$$g_1 = \frac{1}{2}\sum_i e_i^2 \left(\Delta q_i + \Delta \bar{q}_i\right) \qquad \Delta q_i = q_{i\uparrow}^{+1} - q_{i\downarrow}^{+1}$$

$$\left[q_\uparrow^H\left(x,Q^2\right)\right]$$

$$b_1 = \frac{1}{2}\sum_i e_i^2 \left(\delta_T q_i + \delta_T \bar{q}_i\right) \qquad \delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

HERMES results on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.

27.6 GeV/c $\longrightarrow \bullet$ $\rightleftarrows$, 0

positron **deuteron**

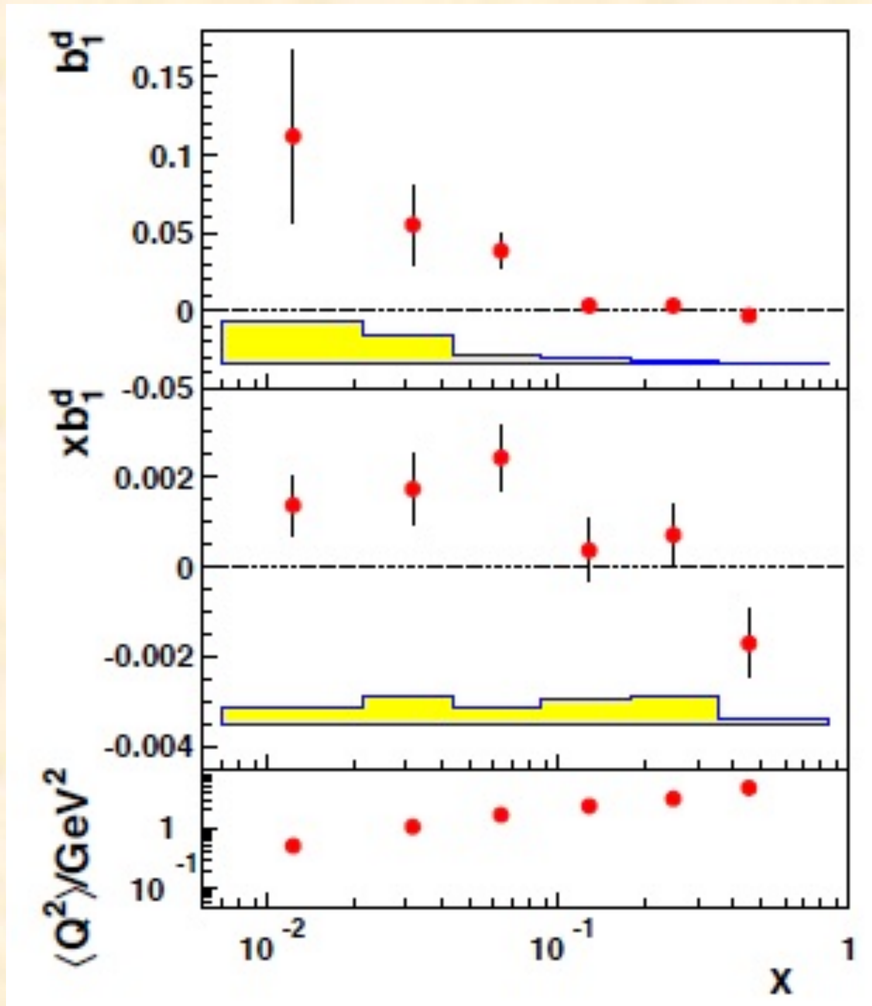
b_1 measurement in the kinematical region

$$0.01 < x < 0.45, \quad 0.5 \text{ GeV}^2 < Q^2 < 5 \text{ GeV}^2$$

b_1 sum in the restricted Q^2 range $Q^2 > 1$ GeV^2

$$\int_{0.02}^{0.85} dx\, b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5$ GeV^2



$$\int dx\, b_1^D(x) = \lim_{t\to 0} -\frac{5}{12}\frac{t}{M^2} F_Q(t) + \underline{\sum_i e_i^2 \int dx \delta_T \bar{q}_i(x)} = 0 \ ?$$

b_1 sum rule: F. E. Close and SK, PRD 42 (1990) 2377.

$$\int \frac{dx}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3}\int dx\left[u_v - d_v\right] + \underline{\frac{2}{3}\int dx\left[\bar{u} - \bar{d}\right]} \neq 1/3$$

Drell-Yan experiments probe these antiquark distributions.

Standard model prediction for b_1 of deuteron

Convolution model: $A_{hH,hH}(x,Q^2) = \varepsilon_{h'}^{*\mu} W_{\mu\nu}^{H'H} \varepsilon_h^{\nu} = \int \frac{dy}{y} \sum_s f_s^H(y) \hat{A}_{hs,hs}(x/y,\, Q^2)$

$$b_1 = A_{+0,+0} - \frac{A_{++,++} + A_{+-,+-}}{2}, \quad \hat{A}_{+\uparrow,+\uparrow} = F_1 - g_1, \quad \hat{A}_{+\downarrow,+\downarrow} = F_1 + g_1$$

Nucleon momentum distribution: $f^H(y) \equiv f_\uparrow^H(y) + f_\downarrow^H(y) = \int d^3p\, y\, |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E-p_z}{M_N}\right)$

D-state admixture: $\phi^H(\vec{p}) = \phi_{\ell=0}^H(\vec{p}) + \phi_{\ell=2}^H(\vec{p})$

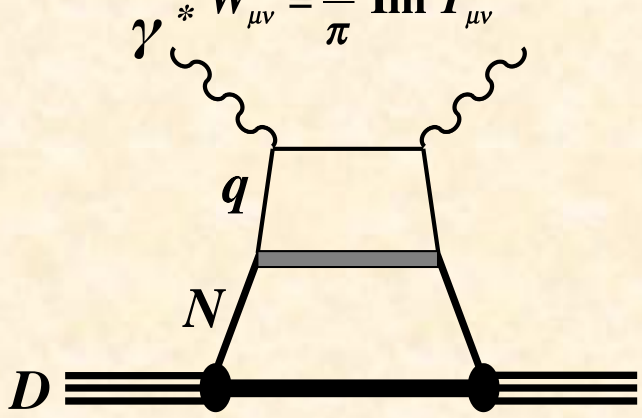
$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y,\, Q^2), \quad y = \frac{Mp\cdot q}{M_N P\cdot q} \simeq \frac{2p^-}{P^-}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y)+f^-(y)}{2}$$

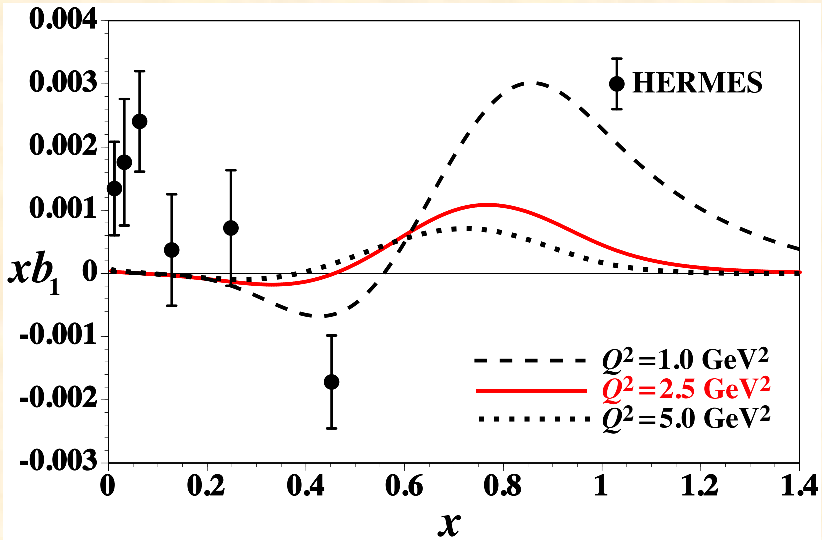
$$= \int d^3p\, y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p)\phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3\cos^2\theta - 1) \delta\left(y - \frac{p\cdot q}{M_N \nu}\right)$$

S-D term D-D term

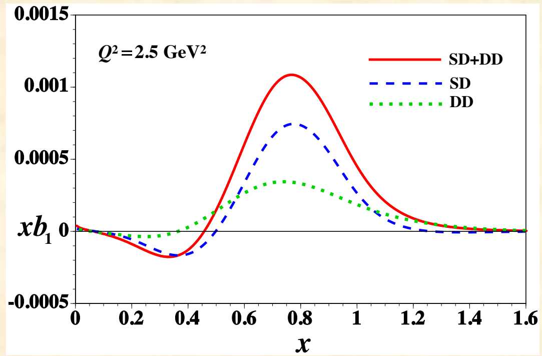
$$W_{\mu\nu} = \frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu}$$

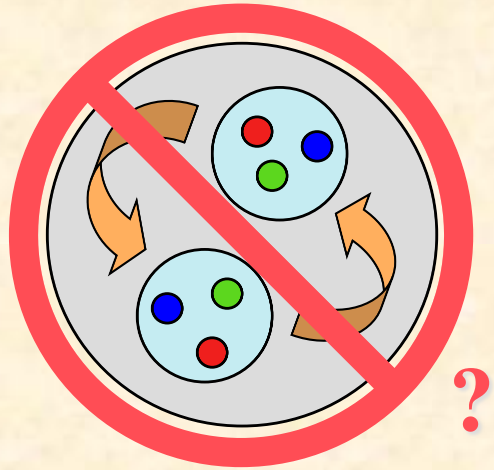


Standard model of the deuteron



W. Cosyn, Yu-Bing Dong, S. Kumano, M. Sargsian, Phys. Rev. D 95 (2017) 074036.





?!

S + D waves

$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$ at $x < 0.5$

Standard convolution model does not work for the deuteron tensor structure!?

Experimental possibility at Fermilab

Polarized fixed-target experiments at the Main Injector

Fermilab-E1039 (SpinQuest)

Drell-Yan experiment with a polarized proton target

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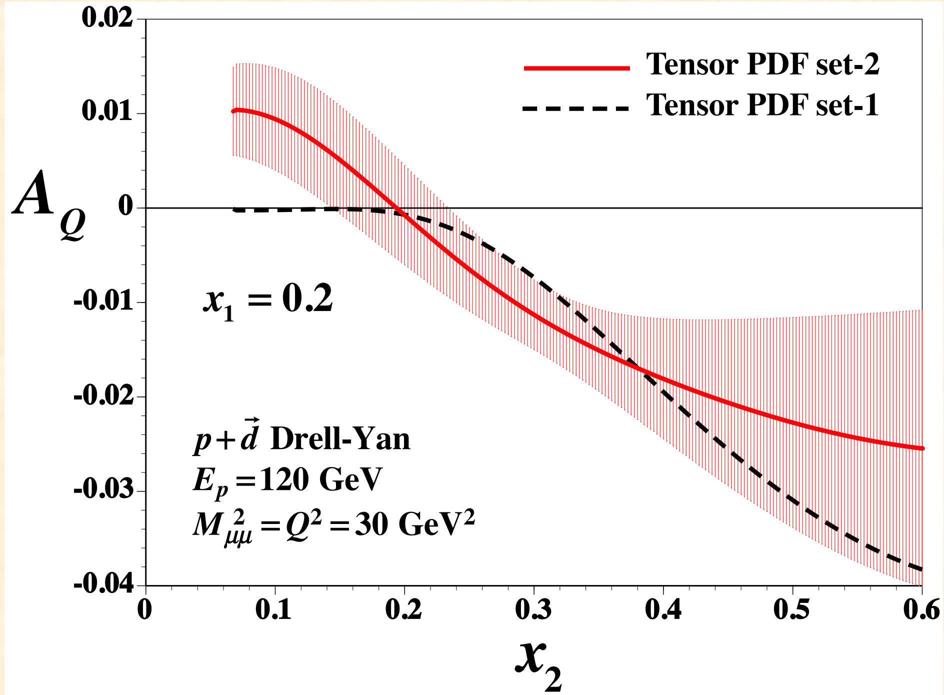
Tensor-polarized spin asymmetry

$$A_Q = \frac{\sum_a e_a^2 \left[q_a(x_A)\, \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A)\, \delta_T q_a(x_B) \right]}{\sum_a e_a^2 \left[q_a(x_A)\, \bar{q}_a(x_B) + \bar{q}_a(x_A)\, q_a(x_B) \right]}$$

$$\simeq \frac{\sum_a e_a^2 q_a(x_1)\, \delta_T \bar{q}_a(x_2)}{2\sum_a e_a^2 q_a(x_1)\, \bar{q}_a(x_2)} \quad \text{at large } x_F = x_1 - x_2$$



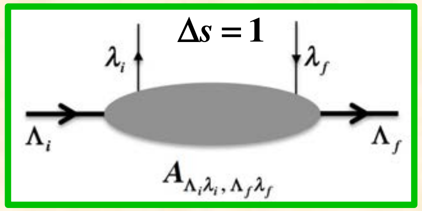
S. Kumano and Qin-Tao Song, Phys. Rev. D94 (2016) 054022.

Gluon transversity $\Delta_T g$

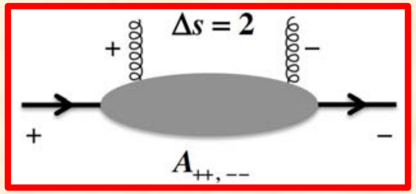
Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, **conservation** $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ +\frac{1}{2}+\frac{1}{2}\right) - A\left(+\frac{1}{2}-\frac{1}{2},\ +\frac{1}{2}-\frac{1}{2}\right)$

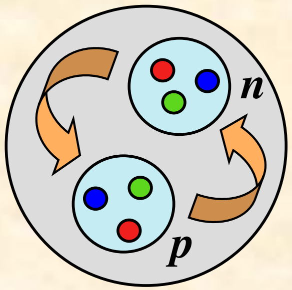
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \to \lambda_f = -\frac{1}{2}$ **quark spin flip** $(\Delta s = 1)$



Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1,\ -1-1)$, $\cancel{A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)}$ **not possible for nucleon**



Note: Gluon transversity does not exist for spin-1/2 nucleons.



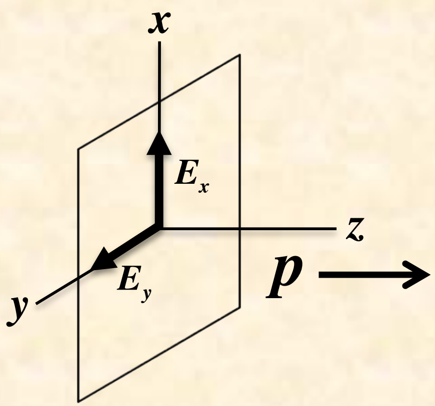
S + D waves

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \iff$ **still** $\Delta_T g = 0$

$\downarrow$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?

Gluon transversity distribution in deuteron



Linear-polarization difference: $d\sigma(E_x - E_y) \propto \Delta_T g$

$$\Delta_T g(x) = \int \frac{d\xi^-}{2\pi} x p^+ \, e^{ixp^+\xi^-} \left\langle pE_x \left| A^x(0)A^x(\xi) - A^y(0)A^y(\xi) \right| pE_x \right\rangle_{\xi^+=\vec{\xi}_T=0}$$

$$= g_{\hat{x}/\hat{x}} - g_{\hat{y}/\hat{x}}$$

$g_{\hat{y}/\hat{x}}$ = gluon distribution with the gluon linear polarization ε_y in the deuteron linear polarization E_x

Polarization vectors $\vec{E}_x = \vec{\varepsilon}_x = (1,\ 0,\ 0)$, $\vec{E}_y = \vec{\varepsilon}_y = (0,\ 1,\ 0)$

Confusing situation of gluon transversity

(no consensus even on its notation: publication # ≈ different notation #)

$$\Delta_2 G(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \qquad [13,\ 44],$$

$$a(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \qquad [23,\ 25],$$

$$\Delta_L g(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \qquad [19],$$

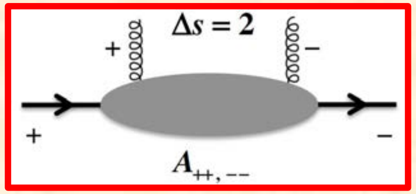
$$\delta G(x) = -g_{\hat{x}/\hat{x}}(x) + g_{\hat{y}/\hat{x}}(x) \qquad [26,\ 45],$$

$$h_{1TT,g}(x) = -g_{\hat{x}/\hat{x}}(x) + g_{\hat{y}/\hat{x}}(x) \qquad [36,\ 38,\ 46],$$

$$\Delta_T g(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x) \qquad [47],\ \text{this work},$$

→ One can imagine how premature this field is!

Gluon transversity $\Delta_T g$



Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

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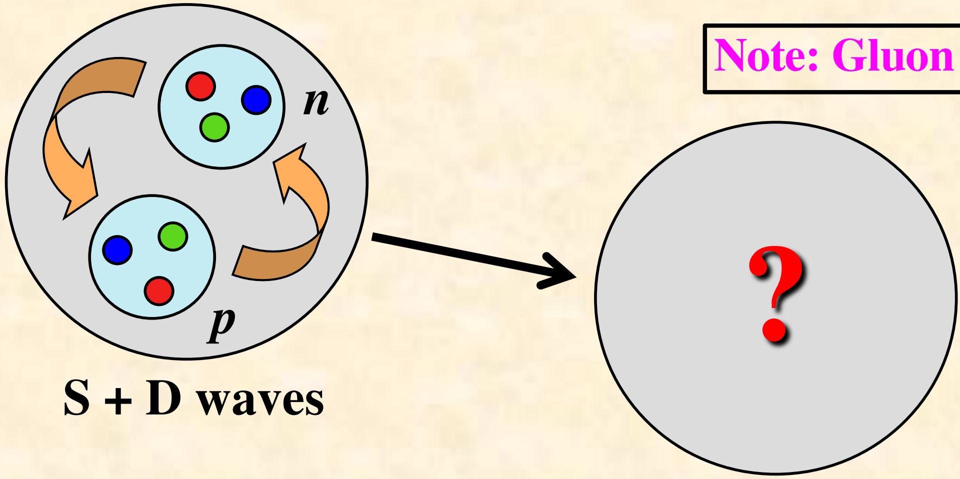
Note: Gluon transversity does not exist for spin-1/2 nucleons.



S + D waves

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \iff$ still $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?

Drell-Yan cross section

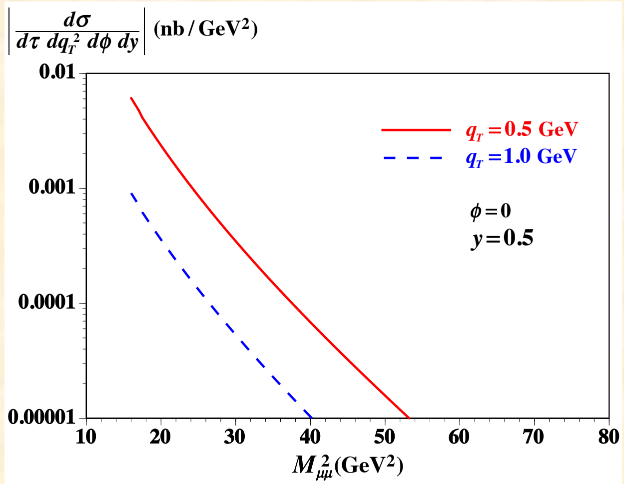
$$\frac{d\sigma_{pd\to\mu^+\mu^- X}(E_x - E_y)}{d\tau\, dq_T^2\, d\phi\, dy} = \frac{\alpha^2 \alpha_s C_F q_T^2}{6\pi s^3} \cos(2\phi) \int_{\min(x_a)}^{1} dx_a \frac{1}{(x_a x_b)^2 (x_a - x_1)(\tau - x_a x_2)^2}$$

$$\times \sum_q e_q^2 x_a \left[q_A(x_a) + \bar{q}_A(x_a) \right] x_b \Delta_T g_B(x_b)$$

$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad \min(x_a) = \frac{x_1 - \tau}{1 - x_2}, \quad x_b = \frac{x_a x_2 - \tau}{x_a - \tau}$$

= (unpolarized PDFs of proton) * (gluon transversity distribution in the deuteron)

S. Kumano and Qin-Tao Song, PRD101 (2020) 054011 & 094013.



New proposal at Fermilab-PAC in 2021 (D. Keller) !

TMDs for spin-1 hadrons

S. Kumano and Qin-Tao Song,
Phys. Rev. D 103 (2021) 014025.

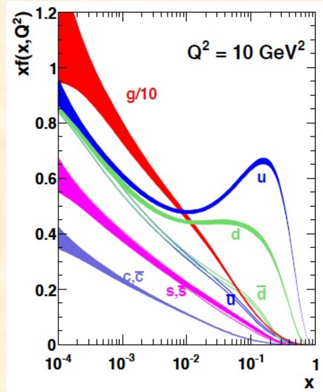
GTMD and Wigner distribution for various structure functions

Form factor

$\int dx\, d^2k_T$

PDF (Parton Distribution Function)

$\int d^2k_T,\ \Delta \to 0$



GTMD $W(x, \vec{k}_T, \Delta) \xrightarrow[\Delta^+ \to 0]{}$ **Wigner** $W(x, \vec{k}_T, \vec{r}_T)$

$\Delta = p' - p$

$\int d^2k_T$

3D world

GPD (Generalized Parton Distribution)

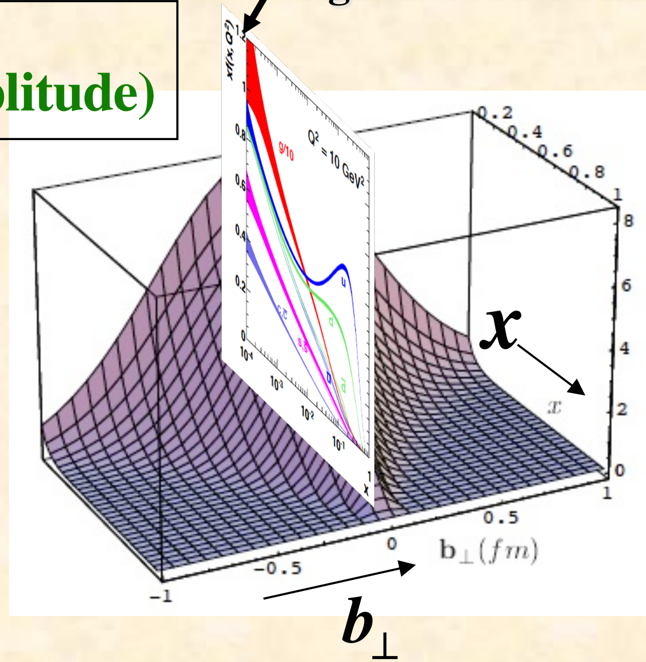
$\xrightarrow[s \leftrightarrow t]{}$ **GDA (Generalized Distribution Amplitude)**

By the two-photon process $\gamma^* \gamma \to h\bar{h}$.

$\Delta \to 0$

TMD (Transverse Momentum Dependent) parton distribution

e.g. HERA studies

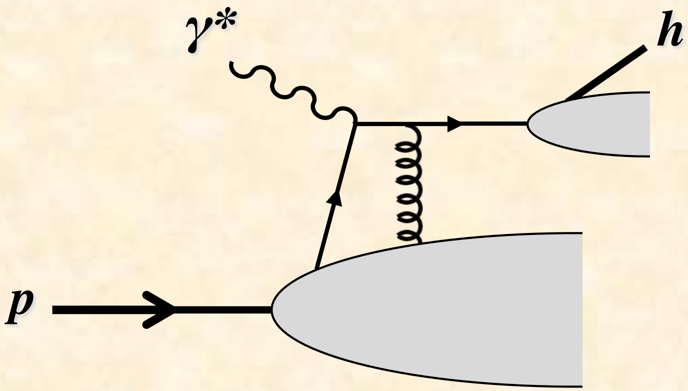


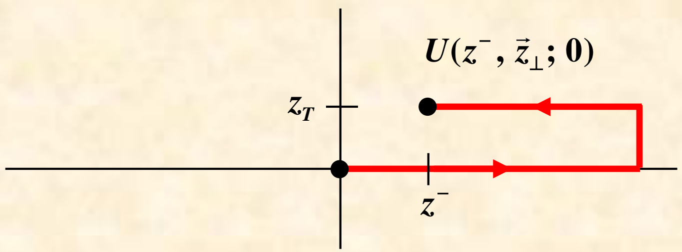
Importance of color flow (gauge link) in semi-inclusive DIS and Drell-Yan processes

$$q(x,k_\perp)=\int\frac{dz^- d^2z_\perp}{2(2\pi)^3}\,e^{-ixp^+z^-+i\vec{k}_\perp\cdot\vec{z}_\perp}\left\langle p\left|\overline{\psi}(z^-,\vec{z}_\perp)\,\gamma^+U(z^-,\vec{z}_\perp;0)\psi(0)\right|p\right\rangle\Big|_{z^+=0}$$

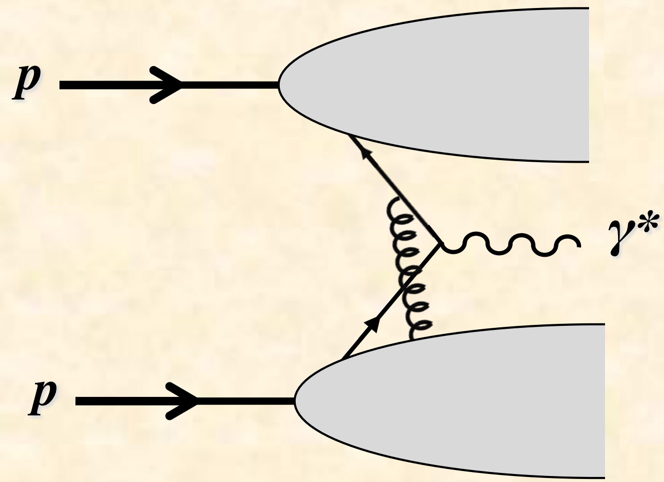
Semi-inclusive DIS (deep inelastic scattering):

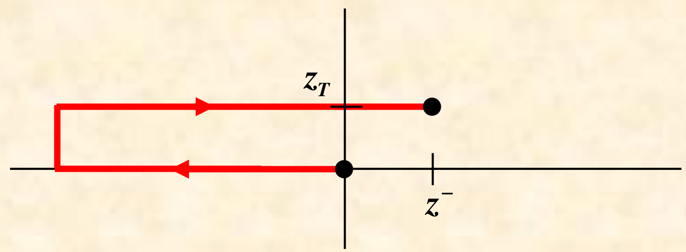
$e+p\rightarrow e'+h+X$





Drell-Yan process: $p+p\rightarrow\mu^+\mu^-+X$





TMDs for spin-1 hadrons

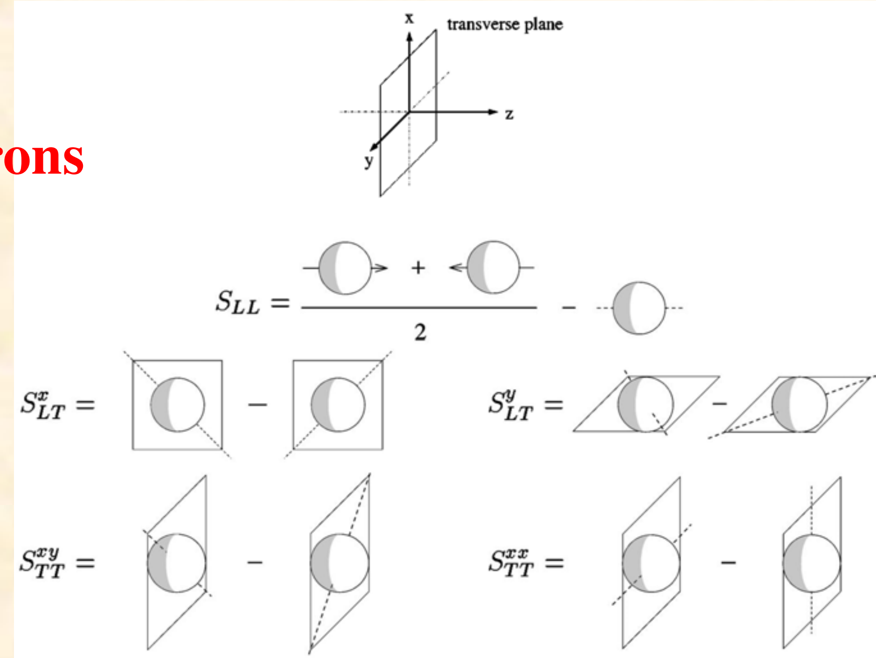
Twist-2 TMDs

Bacchetta-Mulders, PRD 62 (2000) 114004.

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5$ / σ^{i+}) T-even	T ($i\sigma^{i+}\gamma_5$ / σ^{i+}) T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

Spin-1/2 nucleon

Spin-1 hadrons



Twist-2 collinear PDFs

$[\cdots]$= chiral odd

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5$ / σ^{i+}) T-even	T ($i\sigma^{i+}\gamma_5$ / σ^{i+}) T-odd
U	f_1					
L			$g_{1L}\,(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}\,(b_1)$					
LT						*1
TT						

*1 Because of the time-reversal invariance, the collinear PDF $h_{1LT}(x)$ vanishes. However, since the time-reversal invariance cannot be imposed in the fragmentation functions, we should note that the corresponding fragmentation function $H_{1LT}(z)$ should exist as a collinear fragmentation function. (see our PRD paper for the details)

TMD correlation functions for spin-1 hadrons

$$\Phi_{ij}(k,P,T)=\int\frac{d^4\xi}{(2\pi)^4}e^{ik\cdot\xi}\left\langle P,T\right|\bar\psi_j(0)W(0,\xi)\psi_i(\xi)\left|P,T\right\rangle$$

$$W(0,\xi)=P\exp\left[-ig\int_0^\xi d\xi\cdot A(\xi)\right]$$

Spin vector: $S^\mu=S_L\frac{P^+}{M}\bar n^\mu-S_L\frac{M}{2P^+}n^\mu+S_T^\mu$

Tensor: $T^{\mu\nu}=\frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar n^\mu\bar n^\nu+\frac{P^+}{M}\bar n^{\{\mu}S_{LT}^{\nu\}}-\frac{2}{3}S_{LL}\left(\bar n^{\{\mu}n^{\nu\}}-g_T^{\mu\nu}\right)+S_{TT}^{\mu\nu}-\frac{M}{2P^+}n^{\{\mu}S_{LT}^{\nu\}}+\frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^\mu n^\nu\right]$

Tensor part (twist-2): Bacchetta, Mulders, PRD 62 (2000) 114004

$$\Phi(k,P,T)=\left(\frac{A_{13}}{M}I+\frac{A_{14}}{M^2}\not P+\frac{A_{15}}{M^2}\not k+\frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu}+\left[A_{17}\gamma_\nu+\left(\frac{A_{18}}{M}P^\rho+\frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho}+\frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}$$

Tensor part (twist-2, 3, 4): n^μ dependent terms are added for up to twist 4.

[For the spin-1/2 nucleon: Goeke, Metzand, Schlegel, PLB 618 (2005) ,90; Metz, Schweitzer, Teckentrup, PLB 680 (2009) 141.]

Kumano-Song-2020, for the details see KEK-TH-2258

$$\Phi(k,P,T\,|\,n)=\boxed{\left(\frac{A_{13}}{M}I+\frac{A_{14}}{M^2}\not P+\frac{A_{15}}{M^2}\not k+\frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu}+\left[A_{17}\gamma_\nu+\left(\frac{A_{18}}{M}P^\rho+\frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho}+\frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}}$$

Bacchetta -Mulders

$$+\left(\frac{B_{21}M}{P\cdot n}k_\mu+\frac{B_{22}M^3}{(P\cdot n)^2}n_\mu\right)n_\nu T^{\mu\nu}+i\gamma_5\varepsilon_{\mu\tau\rho\sigma}P^\rho\left(\frac{B_{23}}{(P\cdot n)M}k^\tau n^\sigma k_\nu+\frac{B_{24}M}{(P\cdot n)^2}k^\tau n^\sigma n_\nu\right)T^{\mu\nu}$$

$$+\left[\frac{B_{25}}{P\cdot n}\not n k_\mu k_\nu+\left(\frac{B_{26}M^2}{(P\cdot n)^2}\not n+\frac{B_{28}}{P\cdot n}\not P+\frac{B_{30}}{P\cdot n}\not k\right)k_\mu n_\nu+\left(\frac{B_{27}M^4}{(P\cdot n)^3}\not n+\frac{B_{29}M^2}{(P\cdot n)^2}\not P+\frac{B_{31}M^2}{(P\cdot n)^2}\not k\right)n_\mu n_\nu+\frac{B_{32}M^2}{P\cdot n}\gamma_\mu n_\nu\right]T^{\mu\nu}$$

$$-\left[\varepsilon_{\mu\tau\rho\sigma}\gamma^\tau P^\rho\left(\frac{B_{34}}{P\cdot n}n^\sigma k_\nu+\frac{B_{33}}{P\cdot n}k^\sigma n_\nu+\frac{B_{35}M^2}{(P\cdot n)^2}n^\sigma n_\nu\right)+\varepsilon_{\lambda\tau\rho\sigma}k^\lambda\gamma^\tau P^\rho n^\sigma\left(\frac{B_{36}}{P\cdot nM^2}k_\mu k_\nu+\frac{B_{37}}{(P\cdot n)^2}k_\mu n_\nu+\frac{B_{38}M^2}{(P\cdot n)^3}n_\mu n_\nu\right)\right]\gamma_5 T^{\mu\nu}$$

$$+\varepsilon_{\mu\tau\rho\sigma}k^\tau P^\rho n^\sigma\left(\frac{B_{39}}{(P\cdot n)^2}k_\nu+\frac{B_{40}M^2}{(P\cdot n)^3}n_\nu\right)\not n\gamma_5 T^{\mu\nu}$$

$$+\sigma_{\rho\sigma}\left[P^\rho k^\sigma\left(\frac{B_{41}}{(P\cdot n)M}k_\mu n_\nu+\frac{B_{42}M}{(P\cdot n)^2}n_\mu n_\nu\right)+P^\rho n^\sigma\left(\frac{B_{43}}{(P\cdot n)M}k_\mu k_\nu+\frac{B_{44}M}{(P\cdot n)^2}k_\mu n_\nu+\frac{B_{45}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu}$$

$$+\sigma_{\rho\sigma}\left[k^\rho n^\sigma\left(\frac{B_{46}}{(P\cdot n)M}k_\mu k_\nu+\frac{B_{47}M}{(P\cdot n)^2}k_\mu n_\nu+\frac{B_{48}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu}+\sigma_{\mu\sigma}\left[n^\sigma\left(\frac{B_{49}M}{P\cdot n}k_\nu+\frac{B_{50}M^3}{(P\cdot n)^2}n_\nu\right)+\left(\frac{B_{51}M}{P\cdot n}P^\sigma+\frac{B_{52}M}{P\cdot n}k^\sigma\right)n_\nu\right]T^{\mu\nu}$$

New terms in our paper

From this correlation function, new tensor-polarized TMDs are defined in twist-3 and 4 in addition to twist-2 ones.

Terms associated with $n=\frac{1}{\sqrt2}(1,\,0,\,0,\,-1)$

Twist-2 TMDs for spin-1 hadrons

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2}\mathrm{Tr}\left[\Phi^{[\Gamma]}(x, k_T, T)\,\Gamma\right] = \frac{1}{2}\mathrm{Tr}\left[\int dk^- \Phi(k, P, T \mid n)\,\Gamma\right]$$

$$\Phi^{[\gamma^+]}(x, k_T, T) = f_{1LL}(x, k_T^2) S_{LL} - f_{1LT}(x, k_T^2)\frac{k_T \cdot S_{LT}}{M} + f_{1TT}(x, k_T^2)\frac{k_T \cdot S_{TT} \cdot k_T}{M^2}$$

$$\Phi^{[\gamma^+\gamma_5]}(x, k_T, T) = g_{1LT}(x, k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu} k_{T\nu}}{M} + g_{1TT}(x, k_T^2)\frac{S_{TT\mu\rho}k_T^{\rho}\varepsilon_T^{\mu\nu} k_{T\nu}}{M^2}$$

$$\Phi^{[\sigma^{i+}]}(x, k_T, T) = h_{1LL}^{\perp}(x, k_T^2)\frac{S_{LL}k_T^i}{M} + h'_{1LT}(x, k_T^2) S_{LT}^i - h_{1LT}^{\perp}(x, k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2}$$
$$- h'_{1TT}(x, k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M} + h_{1TT}^{\perp}(x, k_T^2)\frac{k_T^i k_T \cdot S_{TT}\cdot k_T}{M^3}$$

$$F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2}F^{\perp}(x, k_T^2)$$

Tensor: $T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar{n}^{\mu}\bar{n}^{\nu} + \frac{P^+}{M}\bar{n}^{\{\mu}S_{LT}^{\nu\}} - \frac{2}{3}S_{LL}\left(\bar{n}^{\{\mu}n^{\nu\}} - g_T^{\mu\nu}\right) + S_{TT}^{\mu\nu} - \frac{M}{2P^+}n^{\{\mu}S_{LT}^{\nu\}} + \frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^{\mu}n^{\nu}\right]$

Correlation function:
$$\Phi(k, P, T \mid n) = \left(\frac{A_{13}}{M}I + \frac{A_{14}}{M^2}\not{P} + \frac{A_{15}}{M^2}\not{k} + \frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^{\rho}k^{\sigma}\right)k_{\mu}k_{\nu}T^{\mu\nu} + \left[A_{17}\gamma_{\nu} + \left(\frac{A_{18}}{M}P^{\rho} + \frac{A_{19}}{M}k^{\rho}\right)\sigma_{\nu\rho} + \frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^{\rho}k^{\sigma}\gamma^{\tau}\gamma_5\right]k_{\mu}T^{\mu\nu}$$
$$+\left(\frac{B_{21}M}{P\cdot n}k_{\mu} + \frac{B_{22}M^3}{(P\cdot n)^2}n_{\mu}\right)n_{\nu}T^{\mu\nu} + i\gamma_5\varepsilon_{\mu\tau\rho\sigma}P^{\rho}\left(\frac{B_{23}}{(P\cdot n)M}k^{\tau}n^{\sigma}k_{\nu} + \frac{B_{24}M}{(P\cdot n)^2}k^{\tau}n^{\sigma}n_{\nu}\right)T^{\mu\nu}$$
$$+\left[\frac{B_{25}}{P\cdot n}\not{n}k_{\mu}k_{\nu} + \left(\frac{B_{26}M^2}{(P\cdot n)^2}\not{n} + \frac{B_{28}}{P\cdot n}\not{P} + \frac{B_{30}}{P\cdot n}\not{k}\right)k_{\mu}n_{\nu} + \left(\frac{B_{27}M^4}{(P\cdot n)^3}\not{n} + \frac{B_{29}M^2}{(P\cdot n)^2}\not{P} + \frac{B_{31}M^2}{(P\cdot n)^2}\not{k}\right)n_{\mu}n_{\nu} + \frac{B_{32}M^2}{P\cdot n}\gamma_{\mu}n_{\nu}\right]T^{\mu\nu}$$
$$-\left[\varepsilon_{\mu\tau\rho\sigma}\gamma^{\tau}P^{\rho}\left(\frac{B_{34}}{P\cdot n}n^{\sigma}k_{\nu} + \frac{B_{33}}{P\cdot n}k^{\sigma}n_{\nu} + \frac{B_{35}M^2}{(P\cdot n)^2}n^{\sigma}n_{\nu}\right) + \varepsilon_{\lambda\tau\rho\sigma}k^{\lambda}\gamma^{\tau}P^{\rho}n^{\sigma}\left(\frac{B_{36}}{P\cdot nM^2}k_{\mu}k_{\nu} + \frac{B_{37}}{(P\cdot n)^2}k_{\mu}n_{\nu} + \frac{B_{38}M^2}{(P\cdot n)^3}n_{\mu}n_{\nu}\right)\right]\gamma_5 T^{\mu\nu}$$
$$+\varepsilon_{\mu\tau\rho\sigma}k^{\tau}P^{\rho}n^{\sigma}\left(\frac{B_{39}}{(P\cdot n)^2}k_{\nu} + \frac{B_{40}M^2}{(P\cdot n)^3}n_{\nu}\right)\not{n}\gamma_5 T^{\mu\nu}$$
$$+\sigma_{\rho\sigma}\left[P^{\rho}k^{\sigma}\left(\frac{B_{41}}{(P\cdot n)M}k_{\mu}n_{\nu} + \frac{B_{42}M}{(P\cdot n)^2}n_{\mu}n_{\nu}\right) + P^{\rho}n^{\sigma}\left(\frac{B_{43}}{(P\cdot n)M}k_{\mu}k_{\nu} + \frac{B_{44}M}{(P\cdot n)^2}k_{\mu}n_{\nu} + \frac{B_{45}M^3}{(P\cdot n)^3}n_{\mu}n_{\nu}\right)\right]T^{\mu\nu}$$
$$+\sigma_{\rho\sigma}\left[k^{\rho}n^{\sigma}\left(\frac{B_{46}}{(P\cdot n)M}k_{\mu}k_{\nu} + \frac{B_{47}M}{(P\cdot n)^2}k_{\mu}n_{\nu} + \frac{B_{48}M^3}{(P\cdot n)^3}n_{\mu}n_{\nu}\right)\right]T^{\mu\nu} + \sigma_{\mu\sigma}\left[n^{\sigma}\left(\frac{B_{49}M}{P\cdot n}k_{\nu} + \frac{B_{50}M^3}{(P\cdot n)^2}n_{\nu}\right) + \left(\frac{B_{51}M}{P\cdot n}P^{\sigma} + \frac{B_{52}M}{P\cdot n}k^{\sigma}\right)n_{\nu}\right]T^{\mu\nu}$$

Hadron \ Quark	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^{\perp}]$
L			g_{1L}		$[h_{1L}^{\perp}]$	
T		$f_{1T}^{\perp}$	g_{1T}		$[h_1], [h_{1T}^{\perp}]$	
LL	f_{1LL}					$[h_{1LL}^{\perp}]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^{\perp}]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^{\perp}]$

Spin-1 hadron TMDs

Hadron \ Quark	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			$g_{1L}\,(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}\,(b_1)$					
LT						*1
TT						

Collinear PDFs

Twist-3 TMDs for spin-1 hadrons

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2}\mathrm{Tr}\left[\Phi^{[\Gamma]}(x, k_T, T)\,\Gamma\right] = \frac{1}{2}\mathrm{Tr}\left[\int dk^- \Phi(k, P, T\,|\,n)\,\Gamma\right], \quad F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2} F^{\perp}(x, k_T^2)$$

$$\Phi^{[\gamma^i]}(x, k_T, T) = \frac{M}{P^+}\left[f_{LL}^{\perp}(x, k_T^2)\frac{S_{LL}k_T^i}{M} + f'_{LT}(x, k_T^2)S_{LT}^i - f_{LT}^{\perp}(x, k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2} - f'_{TT}(x, k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M} + f_{TT}^{\perp}(x, k_T^2)\frac{k_T^i k_T\cdot S_{TT}\cdot k_T}{M^3} \right]$$

$$\Phi^{[1]}(x, k_T, T) = \frac{M}{P^+}\left[e_{LL}(x, k_T^2)S_{LL} - e_{LT}^{\perp}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + e_{TT}^{\perp}(x, k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2} \right]$$

$$\Phi^{[i\gamma_5]}(x, k_T, T) = \frac{M}{P^+}\left[e_{LT}(x, k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu}k_{T\nu}}{M} - e_{TT}(x, k_T^2)\frac{S_{TT\mu\rho}k_T^{\rho}\varepsilon_T^{\mu\nu}k_{T\nu}}{M^2} \right]$$

$$\Phi^{[\gamma^i\gamma_5]}(x, k_T, T) = \frac{M}{P^+}\left[-g_{LL}^{\perp}(x, k_T^2)\frac{S_{LL}\varepsilon_T^{ij}k_{Tj}}{M} - g'_{LT}(x, k_T^2)\varepsilon_T^{ij}S_{LTj} + g_{LT}^{\perp}(x, k_T^2)\frac{\varepsilon_T^{ij}k_{Tj}S_{LT}\cdot k_T}{M^2} + g'_{TT}(x, k_T^2)\frac{\varepsilon_T^{ij}S_{TTjl}k_T^l}{M} - g_{TT}^{\perp}(x, k_T^2)\frac{\varepsilon_T^{ij}k_{Tj}k_T\cdot S_{TT}\cdot k_T}{M^3} \right]$$

$$\Phi^{[\sigma^{-+}]}(x, k_T, T) = \frac{M}{P^+}\left[h_{LL}(x, k_T^2)S_{LL} - h_{LT}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + h_{TT}(x, k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2} \right]$$

$$\Phi^{[\sigma^{ij}]}(x, k_T, T) = \frac{M}{P^+}\left[h_{LT}^{\perp}(x, k_T^2)\frac{S_{LT}^i k_T^j - S_{LT}^j k_T^i}{M} - h_{TT}^{\perp}(x, k_T^2)\frac{S_{TT}^{il}k_{Tl}k_T^j - S_{TT}^{jl}k_{Tl}k_T^i}{M^2} \right]$$

*2, *3 Because of the time-reversal invariance, the collinear PDFs $g_{LT}(x)$ and $h_{LL}(x)$ do not exist. However, the corresponding new collinear fragmentation functions $G_{LT}(z)$ and $H_{LL}(z)$ should exist. (see our PRD paper for the details)

Hadron \ Quark	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^{\perp}$ $[e]$			$g^{\perp}$		$[h]$
L		$f_L^{\perp}$ $[e_L]$	$g_L^{\perp}$		$[h_L]$	
T		$f_T, f_T^{\perp}$ $[e_T, e_T^{\perp}]$	$g_T, g_T^{\perp}$		$[h_T], [h_T^{\perp}]$	
LL	$f_{LL}^{\perp}$ $[e_{LL}]$			$g_{LL}^{\perp}$		$[h_{LL}]$
LT	$f_{LT}, f_{LT}^{\perp}$ $[e_{LT}, e_{LT}^{\perp}]$			$g_{LT}, g_{LT}^{\perp}$		$[h_{LT}], [h_{LT}^{\perp}]$
TT	$f_{TT}, f_{TT}^{\perp}$ $[e_{TT}, e_{TT}^{\perp}]$			$g_{TT}, g_{TT}^{\perp}$		$[h_{TT}], [h_{TT}^{\perp}]$

New TMDs

$[\cdots]$= chiral odd

Hadron \ Quark	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT	f_{LT}			*2		
TT						

New collinear PDFs

Twist-4 TMDs for spin-1 hadrons

$$\Phi^{[\Gamma]}(x,k_T,T) \equiv \frac{1}{2}\mathrm{Tr}\left[\Phi^{[\Gamma]}(x,k_T,T)\,\Gamma\right] = \frac{1}{2}\mathrm{Tr}\left[\int dk^- \Phi(k,P,T\,|\,n)\,\Gamma\right], \quad F(x,k_T^2) \equiv F'(x,k_T^2) - \frac{k_T^2}{2M^2}F^\perp(x,k_T^2)$$

$$\Phi^{[\gamma^-]}(x,k_T,T) = \frac{M^2}{P^{+2}}\left[f_{3LL}(x,k_T^2)S_{LL} - f_{3LT}(x,k_T^2)\frac{S_{LT}\cdot k_T}{M} + f_{3TT}(x,k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\right]$$

$$\Phi^{[\gamma^-\gamma_5]}(x,k_T,T) = \frac{M^2}{P^{+2}}\left[g_{3LT}(x,k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu}k_{T\nu}}{M} + g_{3TT}(x,k_T^2)\frac{S_{TT\mu\rho}k_T^\rho\varepsilon_T^{\mu\nu}k_{T\nu}}{M^2}\right]$$

$$\Phi^{[\sigma^{i-}]}(x,k_T,T) = \frac{M^2}{P^{+2}}\left[h_{3LL}^\perp(x,k_T^2)\frac{S_{LL}k_T^i}{M} + h'_{3LT}(x,k_T^2)S_{LT}^i - h_{3LT}^\perp(x,k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2} - h'_{3TT}(x,k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M} + h_{3TT}^\perp(x,k_T^2)\frac{k_T^i k_T\cdot S_{TT}\cdot k_T}{M^3}\right]$$

*4 Because of the time-reversal invariance, $h_{3LT}(x)$ does not exist; however, the corresponding new collinear fragmentation function $H_{3LT}(z)$ should exist because the time-reversal invariance does not have to be imposed.

Quark / Hadron	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}]$, $[h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}]$, $[h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}]$, $[h_{3TT}^\perp]$

New TMDs

$[\cdot\cdot\cdot]$= chiral odd

Quark / Hadron	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						*4
TT						

New collinear PDFs

Sum rules for TMDs of spin-1 hadrons

Twist-2 TMDs

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5$ / σ^{i+})	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1]$, $[h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}]$, $[h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}]$, $[h_{1TT}^\perp]$

Time-reversal invariance in colliear corrlation functions (PDFs)

$$\int d^2k_T \Phi_{\text{T-odd}}(x,k_T^2) = 0$$

Sum rules for the TMDs of spin-1 hadrons

$$\int d^2k_T\, h_{1LT}(x,k_T^2) = 0, \qquad \int d^2k_T\, g_{LT}(x,k_T^2) = 0,$$

$$\int d^2k_T\, h_{LL}(x,k_T^2) = 0, \qquad \int d^2k_T\, h_{3LT}(x,k_T^2) = 0$$

For example, in the twist-4

$$\int d^2k_T\, h_{3LT}(x,k_T^2) \equiv \int d^2k_T \left[h'_{3LT}(x,k_T^2) - \frac{k_T^2}{2M^2} h_{3LT}(x,k_T^2) \right] = 0$$

$$\Phi^{[\sigma^{i-}]} = \frac{M^2}{P^{+2}} \left[h_{3LL}^\perp(x,k_T^2) S_{LL} \frac{k_T^i}{M} + h'_{3LT}(x,k_T^2) S_{LT}^i - h_{3LT}^\perp(x,k_T^2) \frac{k_T^i S_{LT}\cdot k_T}{M^2} - h'_{3TT}(x,k_T^2) \frac{S_{TT}^{ij} k_{T\,j}}{M} + h_{3TT}^\perp(x,k_T^2) \frac{k_T\cdot S_{TT}\cdot k_T}{M^2} \frac{k_T^i}{M} \right]$$

Twist-3 TMDs

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^\perp$ $[e]$			$g^\perp$		$[h]$
L		$f_L^\perp$ $[e_L]$	$g_L^\perp$		$[h_L]$	
T		f_T, $f_T^\perp$ $[e_T, e_T^\perp]$	g_T, $g_T^\perp$		$[h_T]$, $[h_T^\perp]$	
LL	$f_{LL}^\perp$ $[e_{LL}]$			$g_{LL}^\perp$		$[h_{LL}]$
LT	f_{LT}, $f_{LT}^\perp$ $[e_{LT}, e_{LT}^\perp]$			g_{LT}, $g_{LT}^\perp$		$[h_{LT}]$, $[h_{LT}^\perp]$
TT	f_{TT}, $f_{TT}^\perp$ $[e_{TT}, e_{TT}^\perp]$			g_{TT}, $g_{TT}^\perp$		$[h_{TT}]$, $[h_{TT}^\perp]$

Twist-4 TMDs

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}]$, $[h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}]$, $[h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}]$, $[h_{3TT}^\perp]$

New fragmentations for spin-1 hadrons

Corresponding fragmentation functions exist for the spin-1 haddrons

simply by changing function names and kinematical variables.

TMD distribution functions: $f,\ g,\ h,\ e\ ;\quad x,\ k_T,\ S,\ T,\ M,\ n,\ \gamma^+,\ \sigma^{i+}$

$$\Downarrow$$

TMD fragmentation functions: $D,\ G,\ H,\ E\ ;\quad z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

Fragmentation functions:
X. Ji, Phys. Rev. D 49, 114 (1994).

Summary on our recent spin-1 TMD studies

TMDs of spin-1 hadrons

- **TMDs: interdisciplinary field of physics**
  (Color Aharonov-Bohm effect, Color entanglement)
- **We proposed new 30 TMDs and 3 PDFs in twist 3 and 4.**
- **New sum rules for TMDs.**
- **New TMD fragmentation functions.**

Twist-3 TMD: $f_{LL}^{\perp}, e_{LL}, f_{LT}, f_{LT}^{\perp}, e_{1T}, e_{1T}^{\perp}, f_{TT}, f_{TT}^{\perp}, e_{TT}, e_{TT}^{\perp},$
$g_{LL}^{\perp}, g_{LT}, g_{LT}^{\perp}, g_{TT}, g_{TT}^{\perp}, h_{1L}, h_{LT}, h_{LT}^{\perp}, h_{TT}, h_{TT}^{\perp}$

Twist-4 TMD: $f_{3LL}, f_{3LT}, f_{3TT}, g_{3LT}, f_{3TT}, h_{3LL}^{\perp}, h_{3LT}, h_{3LT}^{\perp}, h_{3TT}, h_{3TT}^{\perp}$

Twist-3 PDF: e_{LL}, f_{LT}

Twist-4 PDF: f_{3LL}

Sum rules: $\int d^2k_T\, g_{LT}(x, k_T^2) = \int d^2k_T\, h_{LL}(x, k_T^2) = \int d^2k_T\, h_{3LL}(x, k_T^2) = 0$

TMD distribution functions: $f,\ g,\ h,\ e\ ;\quad x,\ k_T,\ S,\ T,\ M,\ n,\ \gamma^+,\ \sigma^{i+}$

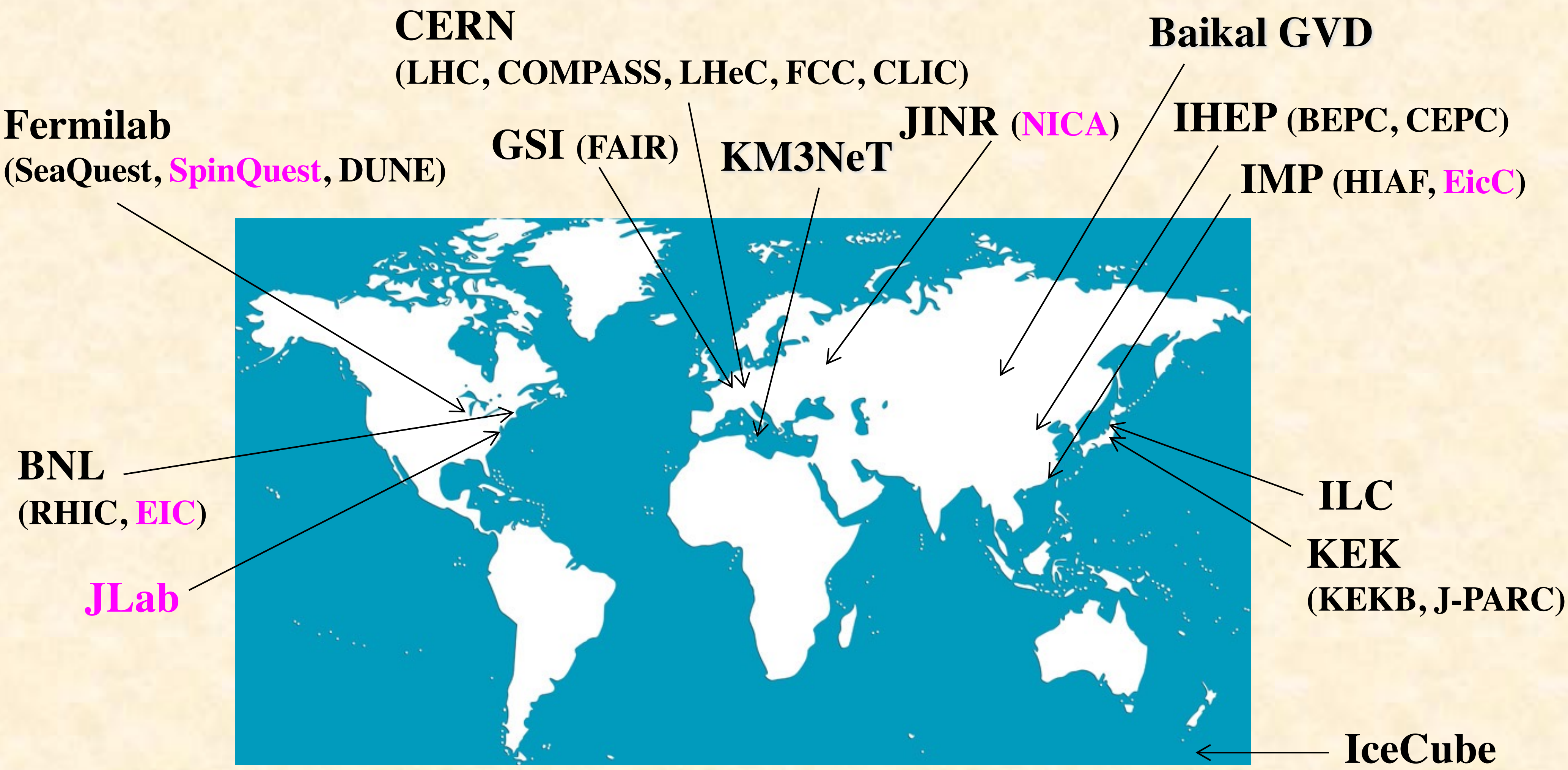
$$\Downarrow$$

TMD fragmentation functions: $D,\ G,\ H,\ E\ ;\quad z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

Experimental prospects on structure functions of spin-1 hadrons

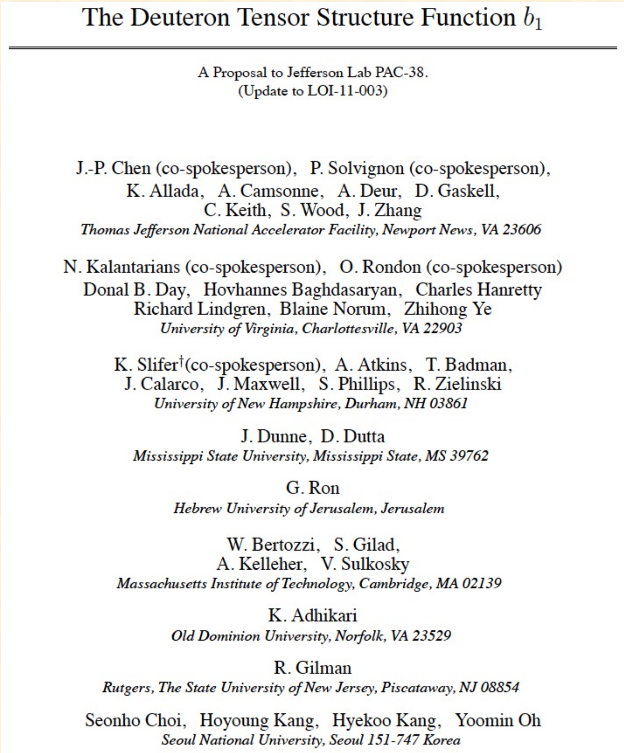
(no TMD experiment yet)

High-energy hadron physics experiments



Facilities on spin-1 hadron structure functions including future possibilities.

JLab PAC-38 (Aug. 22-26, 2011) proposal, PR12-11-110


The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

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Seonho Choi, Hoyoung Kang, Hyekoo Kang, Yoomin Oh
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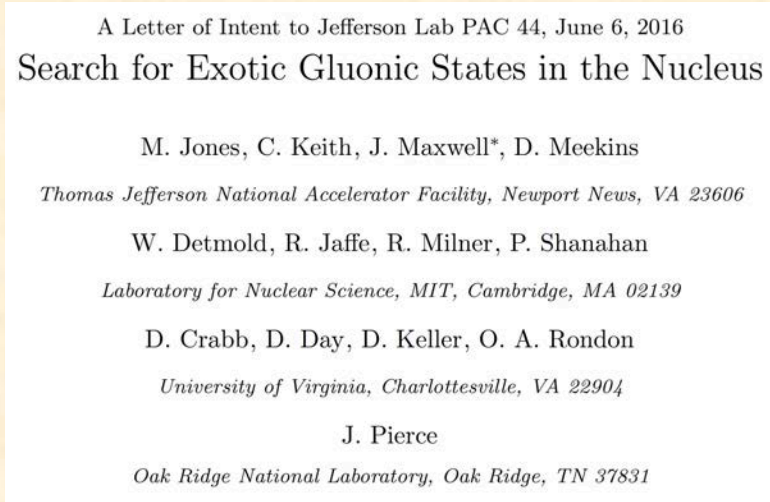
LoI, arXiv:1803.11206


A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

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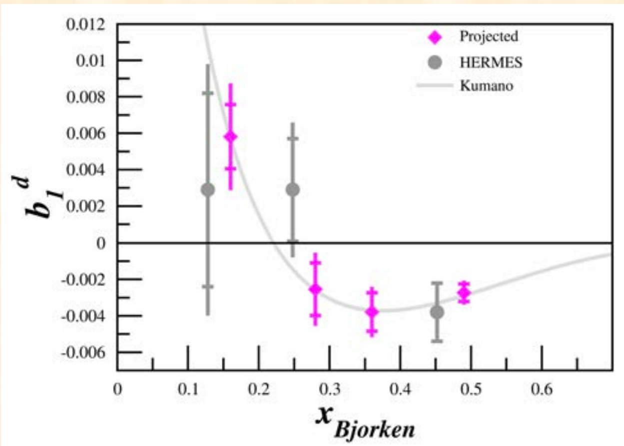
J. Pierce
Oak Ridge National Laboratory, Oak Ridge, TN 37831

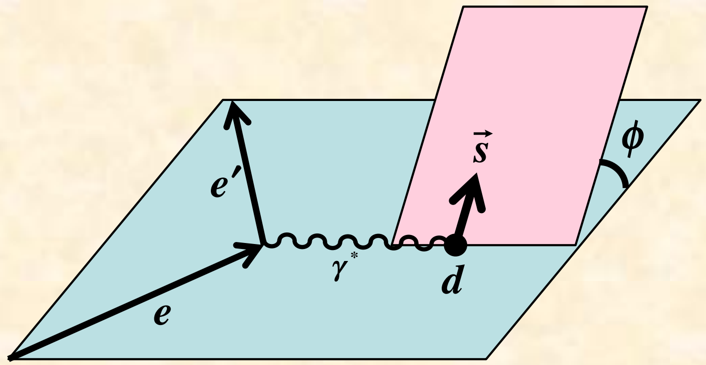
$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$



$$\left.\frac{d\sigma}{dx\,dy\,d\phi}\right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4}\left[xy^2 F_1(x,Q^2) + (1-y)F_2(x,Q^2) - \frac{1}{2}x(1-y)\Delta(x,Q^2)\cos(2\phi)\right]$$

$$\Delta(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3}\Delta_T g(y,Q^2)$$

By looking at the deuteron-polarization angle ϕ,
the quark transversty $\Delta_T g$ can be measured.



Experimental possibility at Fermilab in 2020's

Polarized fixed-target experiments at the Main Injector, Proton beam = 120 GeV

J-PARC?

Fermilab-E1039

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

List of Collaborators:

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Fermilab experimentalists are interested in the gluon transversity by replacing the E1039 proton target for the deuteron one. (Spokesperson of E1039: D. Keller) However, there was no theoretical formalism until our work.

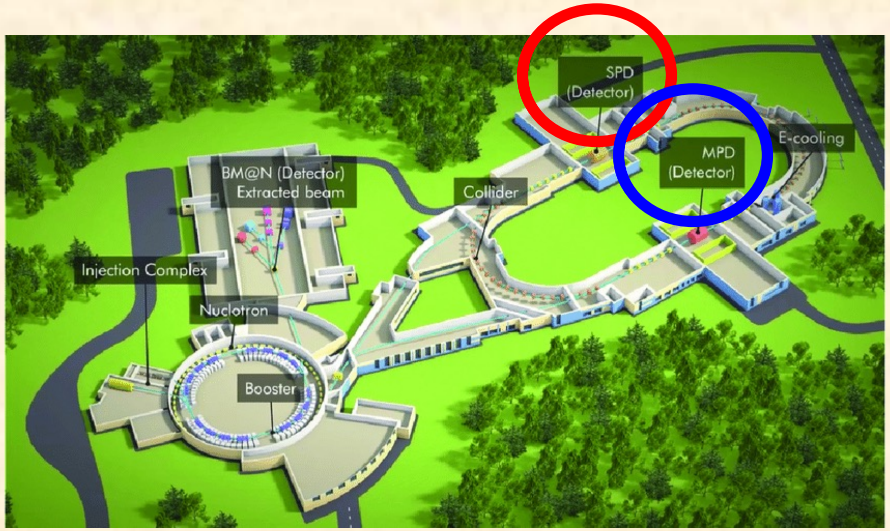
The Transverse Structure of the Deuteron with Drell-Yan

D. Keller[1]

[1] *University of Virginia, Charlottesville, VA 22904*

New proposal is being written for a Fermilab-PAC in 2021.

Nuclotron-based Ion Collider fAcility (NICA)



SPD (Spin Physics Detector for physics with polarized beams)
MPD (MultiPurpose Detector for heavy ion physics)

$\vec{p}+\vec{p}: \quad \sqrt{s_{pp}} = 12 \sim 27$ GeV

$\vec{d}+\vec{d}: \quad \sqrt{s_{NN}} = 4 \sim 14$ GeV

$\vec{p}+\vec{d}$ is also possilbe.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al.* (NICA project), Prog. Nucl. Part. Phys. 119 (2021) 103858 (arXiv:2011.15005).

Unique opportunity in high-energy spin physics, especially on the deuteron spin physics.
→ Theoretical formalisms need to be developed.

It is a timely project in 2020's in competition with JLab, Fermilab, and EIC (possibly also J-PARC, GSI-FAIR, EicC).

Personal studies on tensor structure of the deuteron

- Sum rule for b_1
  F. E. Close and SK, Phys. Rev. D42 (1990) 2377.
- Polarized proton-deuteron Drell-Yan: General formalism
  M. Hino and SK, Phys. Rev. D59 (1999) 094026.
- Polarized proton-deuteron Drell-Yan: Parton model
  M. Hino and SK, Phys. Rev. D60 (1999) 054018.
- Extraction of $\Delta\bar{u}/\Delta\bar{d}$ and $\Delta_T\bar{u}/\Delta_T\bar{d}$ from polarized pd Drell-Yan
  SK and M. Miyama, Phys. Lett. B497 (2000) 149.
- Projections to $b_1, \ldots, b_4$ from $W_{\mu\nu}$
  T.-Y. Kimura and SK, Phys. Rev. D 78 (2008) 117505.
- Tensor-polarized distributions from HERMES data
  SK, Phys. Rev. D82 (2010) 017501.
- Tensor-polarization asymmetry in pd Drell-Yan
  SK and Qin-Tao Song, Phys. Rev. D94 (2016) 054022.
- Convolution calculation for b_1
  W. Cosyn, Yu-Bing Dong, SK, M. Sargsian, Phys. Rev. D 95 (2017) 074036.
- Gluon transversity by proton-deuteron Drell-Yan
  SK and Qin-Tao Song, Phys. Rev. D 101 (2020) 054011 & 094013.
- TMDs up to twist 4 for spin-1 hadrons
  SK and Qin-Tao Song, Phys. Rev. D 103 (2021) 014025.
- Gluon distributions at NICA SPD
  A. Arbuzov *et al*., Prog. Nucl. Part. Phys. 119 (2021) 103858.

Future experiments: JLab, Fermilab, NICA, EIC, EicC, · · ·

Motived by the following works.

Hoodbhoy-Jaffe-Manohar (1989)

↓

Polarized deuteron acceleration at RHIC:
E. D. Courant, Report BNL-65606 (1998)

↓

HERMES measurement on b_1 (2005)

↓

Future possibilities
at JLab, Fermilab, NICA, · · ·

↓

JLab PAC-38 proposal, PR12-11-110,
J.-P. Chen *et al.* (2011) → approved!
Fermilab-E1039, under consideration.

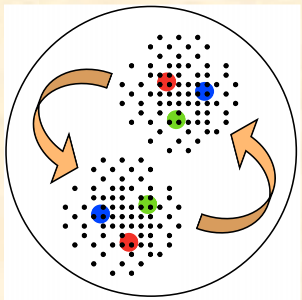
Summary

Spin-1 structure functions of the deuteron (new spin structure)

- tensor structure in quark-gluon degrees of freedom
- gluon transversity
- new signature beyond “standard” hadron physics?
- experiments: JLab (approved), Fermilab (to be proposed), ... ,
  NICA (in progress), COMPASS?, EIC, EicC, ...
- TMDs: interdisciplinary field of physics
  e.g. Color Aharonov-Bohm effect, Color entanglement

We proposed new TMDs and PDFs in twist 3 and 4.

standard model



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The End

The End