Fragmentation From Space-Time Reciprocity

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arXiv:2003.02275 w/Felix Ringer, LBNL
arXiv:2010.02934

QCD Evolution 2021
May 13, 2021

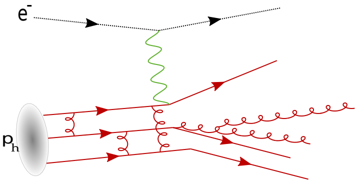
Fragmentation.

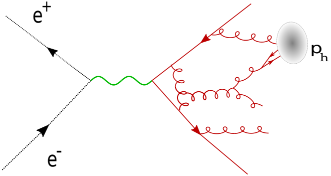
We wish to understand an object.

- Take the object, and randomly break it.
- What do the pieces look like?
- For QCD, the object is often the total momentum of a hadron or hard interaction.
- The pieces are partons or hadrons.

Thus we access the dynamical or static properties of QCD.

Fragmentation To understand QCD





The Fundamental Theorems:

$$PDF: \; F(x,Q^2) = \sum_i \int_x^1 \frac{dz}{z} f_{i/h}\Big(\frac{x}{z},\mu^2\Big) C_i^S\Big(z,\mu^2,Q^2\Big), \quad x = \frac{Q^2}{2P_h \cdot q}$$

$$FF: \; D(x,Q^2) = \sum_i \int_x^1 \frac{dz}{z} d_{h/i}\Big(\frac{x}{z},\mu^2\Big) C_i^T\Big(z,\mu^2,Q^2\Big), \quad x = \frac{2P_h \cdot q}{Q^2}$$

- f : randomly break a hadron into partons.
- d : randomly break a parton into hadrons.
- x is the momentum fraction.

Standard Approach to FF and PDFs

Usual procedure to tackle FF and PDFs.

- Regulate IR in **dimensional regularization.**
- Bare Non-perturbative functions *scaleless* and $\therefore = 1$.*
- Renormalized non-perturbative functions are the IR singularities.
- Integration of anomalous dimensions give IR singularities.

$$xD\Big(x,Q^2,\Lambda^2\Big)=\int_{c-i\infty}^{c+i\infty}\frac{dn}{2\pi i}x^{-n}\exp\Big(\int_0^{\alpha_s(\mu^2)}\frac{d\alpha}{\beta(\alpha,\epsilon)}\gamma^T(\alpha,n)\Big)\,\bar{C}^T\Big(n,Q^2,\mu^2\Big),$$

$$xF\Big(x,Q^2,\Lambda^2\Big)=\int_{c-i\infty}^{c+i\infty}\frac{dn}{2\pi i}x^{-n}\exp\Big(\int_0^{\alpha_s(\mu^2)}\frac{d\alpha}{\beta(\alpha,\epsilon)}\gamma^S(\alpha,n)\Big)\,\bar{C}^S\Big(n,Q^2,\mu^2\Big),$$

γ^S,γ^T are space-like & time-like DGLAP anomalous dimensions.

*Setting IR quantities to zero according to power counting

Standard Approach to FF and PDFs

Calculate $C^{S/T}$ and $\gamma^{S/T}$ at fixed order, discard singularities for some generic function for FF and PDFs, and fit.

Also known as the procedure that Ted hated on Tuesday.

Why Dimensional Regularization?

Mathematically, Dimensional regularization is nice!

- Minimal subtraction schemes.
- No power-law divergences.
- Mass independent renormalization group eqs.
- Gauge invariant, lorentz invariant.
- Ease of calculations: access high orders in perturbation theory.

PDFs: Dimensional regularization is particularly nice!

DIS: $x \sim O(1)$:

- Dispersion relations to Euclidean correlation functions.
- Dimensional Regularization cuts off Euclidean scale p^2 of each loop integral $\int d^d p$.
- Perform OPE onto twist-two operators to high orders in perturbation theory.

Issue with traditional approach

- Mathematical precision.
- Physical insight?
- Structural relationship between space-like and time-like fragmentation processes?

Our goals For the FF:

- Exploit a physically motivated regularization.
- Manifest structural relationship between Space-like and Time-like Processes in gauge theories.
- Maintain clear connection to Dim. Reg.*

*Still want to calculate!

Space-Time Reciprocity in QCD

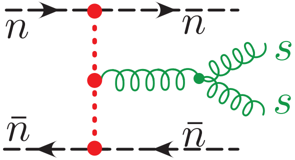
Intuition: All phenomena in forward scattering or initial state physics has an exact counterpart in jet physics, with perhaps calculable conformal anomalies. Examples:

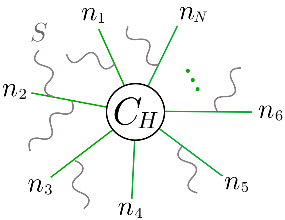
- Banfi-Marchesini-Smye Eq. and BK/B-JIMWLK Eqs. [**Hatta 0810.0889**]
- Collins-Soper Kernel and Threshold Anomalous Dimensions [**Li, Zhu 1604.01404, Vladimirov 1610.05791**]
- Reciprocity of Space-like and Time-like Anomalous Dimensions
  $\gamma^S(n + 2\gamma^T(n)) = \gamma^T(n)$
  $\gamma^T(n - 2\gamma^S(n)) = \gamma^S(n)$
  [**Basso, Korchemsky hep-th/0612247, Dokshitzer et al. hep-ph/0511302**]
- Realization of space-like anomalous dimensions in scalings of Light-Ray OPE. **(See Ian Moult's talk.)**

Soft processes in forward scattering

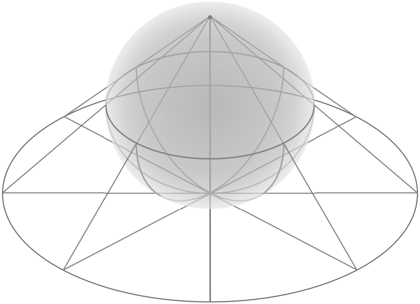
$\leftrightarrow$

Soft process in jet physics.





Geometrical picture of Reciprocity:
Stereographic projection of $S^2 \to \mathbb{R}^2$



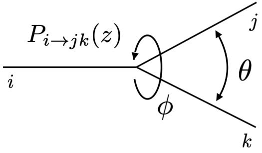
Eikonal lines of n and $\bar{n}$-collinear directions piercing the transverse plane of forward scattering,

$\leftrightarrow$

Eikonal lines emanating from a hard scattering vertex.

Intuition:

- PDF's accessed via a cutoff in transverse momentum/position in DIS as $x \to 0$.
- Dim Reg. respects: $\int d^d p = \int d\bar{n}\cdot p \int dn\cdot p \int d^{d-2}p_\perp$.
- Transverse positions maps to celestial angles.
- Access FF via cutoff in angles.



- Emissions are collinear when they are close in *angle* on celestial sphere.
- $\int d^d p = \int d\bar{n}\cdot p \int dn \cdot p \int d^{d-2}p_\perp$,
  Transverse momentum: $p_\perp \sim z\theta Q$, so angle or energy?
- But IR divergences in fragmentation from $\theta \to 0$

Angular Ordering Hypothesis

In fragmentation:

- Angular seperation *defines* collinearity on the celestial sphere
- Try factorization in angles, not transverse momenta.

This is old idea (1980$\sim$ 1985), *illuminated by reciprocity*: Mueller, Dokshitzer, Marchesini, ...

Angular Ordering Hypothesis Revisited

- Regulate and factorize fragmentation according to angles.
- Reciprocity: The evolution equations defined in such a factorization will be governed from kernel of *space-like branching*.
- Since space-like branching *enjoys* Dim. Reg., we keep this too, *to define the space-like branching kernels in perturbation theory.*

arXiv:2003.02275 D.N., Felix Ringer

Angular Regularization in fragmentation

- Introduce Fragmentation Cross-section with minimum branching angle R_{ir} to observed hadron:

$$D_{\triangleleft}\Big(x, R_{ir}^2, \frac{\mu^2}{Q^2}\Big)$$

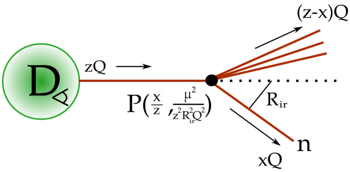
- x energy fraction of observed hadron.
- Always work in $d = 4 - 2\epsilon$ dimensions.
- μ mass scale of dimensional regularization.

What happens as $R_{ir} \to 0$?

Angular Evolution: Conjectured Equation [D.N., Felix Ringer]

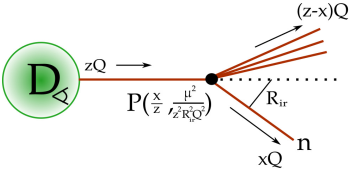
Changing cutoff R_{ir} (pure Yang-Mills):

$$R_{ir}^2 \frac{d}{dR_{ir}^2} x^{1+2\epsilon} D_{\sphericalangle}\Big(x, R_{ir}^2, \frac{\mu^2}{Q^2}\Big) = \rho\Big(\frac{\mu^2}{R_{ir}^2 Q^2}\Big) x^{1+2\epsilon} D_{\sphericalangle}\Big(x, R_{ir}^2, \frac{\mu^2}{Q^2}\Big)$$
$$+ \int_x^1 \frac{dz}{z} P\Big(\frac{x}{z}; \frac{\mu^2}{z^2 R_{ir}^2 Q^2}\Big) z^{1+2\epsilon} D_{\sphericalangle}\Big(z, R_{ir}^2, \frac{\mu^2}{Q^2}\Big).$$



- $zR_{ir}Q$ is transverse momentum of *parent parton* WRT fragmented hadron's direction.

Details of Kernel



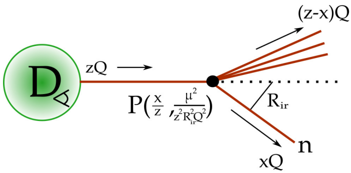
Kernels P, ρ have expansions:

$$P\left(\frac{x}{z};\frac{\mu^2}{z^2R_{ir}^2Q^2}\right)=\sum_{\ell=0}^{\infty}p^{(\ell)}\left(\frac{x}{z},\alpha_s,\epsilon\right)\times\left(\frac{\mu^2}{z^2R_{ir}^2Q^2}\right)^{(\ell+1)\epsilon}$$

$$\rho\left(\frac{\mu^2}{R_{ir}^2Q^2}\right)=-\sum_{\ell=0}^{\infty}\beta_i\left(\frac{C_A\alpha_s}{\pi}\right)^{\ell+1}\left(\frac{\mu^2}{R_{ir}^2Q^2}\right)^{(\ell+1)\epsilon}$$

β_i are the coefficients of the QCD beta function. $p^{(\ell)}$ has expansions in both ϵ and α_s.

Details of Kernel: Reciprocity



P is determined from the anomalous dimensions for twist-two local operators in $4-2\epsilon$-dimensions.

Manifesting the structural relationship between DIS and SIA.

Angular Evolution to Angular Factorization

Then define angular-order fragmentation function, evolved between R_{uv} and R_{ir}

$$D_{\triangleleft}\left(x, R_{ir}^2, \frac{\mu^2}{Q^2}\right) = \int_x^1 \frac{dz}{z} d_{\triangleleft}\left(\frac{x}{z}, R_{ir}^2, R_{uv}^2, \frac{\mu^2}{z^2 Q^2}\right) D_{\triangleleft}\left(z, R_{uv}^2, \frac{\mu^2}{Q^2}\right).$$

- $d_{\triangleleft}$ solves angular ordered evolution, and is a universal function.
- Simple boundary condition: $d_{\triangleleft}\left(x, R, R, \frac{\mu^2}{Q^2}\right) = \delta(1-x)$
- $D_{\triangleleft}\left(z, R_{uv}^2, \frac{\mu^2}{Q^2}\right)$ process dependent boundary condition.

What does this accomplish?

Solve evolution equation,
Evolve to $R_{ir} = 0$,
Get Dim. Reg. fragmentation cross-section:

$$D(x,Q^2) = \lim_{R_{ir}\to 0}\int_x^1 \frac{dz}{z} d_{\triangleleft}\Big(\frac{x}{z}, R_{ir}^2, R_{uv}^2, \frac{\mu^2}{z^2Q^2}\Big) D_{\triangleleft}\Big(z, R_{uv}^2, \frac{\mu^2}{Q^2}\Big)$$

$$= \int_{c-i\infty}^{c+i\infty} \frac{dn}{2\pi i} x^{-n} \exp\Big(\int_0^{\alpha_s(\mu^2)} \frac{d\alpha}{\beta(\alpha,\epsilon)} \gamma^T(\alpha,n)\Big) \bar{C}^T\Big(n, Q^2, \mu^2\Big)$$

- Calculates: γ^T from γ^S, as $\gamma^S(n + 2\gamma^T(n)) = \gamma^T(n)$
- Resums Soft Region: $d_{\triangleleft}$ contains **all** logarithmically enhanced terms as $x \to 0$ in both γ^T and C^T of Standard Approach.

Checks: Does it work?

Predicts resummed terms $\ln x$ to N^2LL in C^T, N^3LL in γ^T, reproducing perturbative expansion up-to and at 3 loops.

What does this accomplish?

Angular factorization *combined* with Dim. Reg. resums soft region of fragmentation and fully connects the fragmentation cross-section to the initial state physics of DIS and PDFs.

- How far can we go?

New object: Angular Ordered Fragmentation Function

$$D_{\triangleleft}\left(x, R_{ir}^2, \frac{\mu^2}{Q^2}\right) = \int_x^1 \frac{dz}{z} d_{\triangleleft}\left(\frac{x}{z}, R_{ir}^2, R_{uv}^2, \frac{\mu^2}{z^2 Q^2}\right) D_{\triangleleft}\left(z, R_{uv}^2, \frac{\mu^2}{Q^2}\right).$$

$d_{\triangleleft}$ and its evolution equation are the critical new objects.

- $R_{ir} \to 0$ calculates and resums the perturbative expansion for coefficient function, IR divergences, and cross-section.
- $R_{uv} \to \infty$?

Why $R_{uv} \to \infty$?

$$d_{\triangleleft}\left(x, R_{ir}^2, R_{uv}^2, \frac{\mu^2}{Q^2}\right).$$

Effective Field Theory Intuition:

- To isolate UV physics, take IR cutoff 0.
- To isolate IR physics, take UV cutoff to ∞.

$$d_{\triangleleft}\left(x, 0, 1, \frac{\mu^2}{Q^2}\right) : \text{ Coefficient Function and its IR divergences}$$

$$d_{\triangleleft}\left(x, 1, \infty, \frac{\mu^2}{Q^2}\right) : \text{ Fragmentation Function and its UV divergences}$$

An Enticing Relation

$$d_{\lessdot}\left(x, 0, \infty, \frac{\mu^2}{Q^2}\right) = \delta(1-x)$$

Why?

$d_{\lessdot}$ is defined via angular ordered evolution equation *while* maintaining Dim. Reg.:

- Solving $\rightarrow$ iterated integrals of power-laws in Dim. Reg.
- Taking limits to 0 and ∞ gives scaleless integrals.
- Perturbative expansion set to zero.

Implication (mellin space):

$$1 = \bar{d}_{\triangleleft}\Big(n, 0, \infty, \frac{\mu^2}{Q^2}\Big) = \bar{d}_{\triangleleft}\Big(n, 0, 1, \frac{\mu^2}{Q^2}\Big)\bar{d}_{\triangleleft}\Big(n, 1, \infty, \frac{\mu^2}{Q^2}\Big)$$

So:

$$\bar{d}_{\triangleleft}\Big(n, 0, 1, \frac{\mu^2}{Q^2}\Big) = \left(\bar{d}_{\triangleleft}\Big(n, 1, \infty, \frac{\mu^2}{Q^2}\Big)\right)^{-1}$$

The FF is the reciprocal of the universal parts of the coefficient function!

Recall $\bar{d}_{\triangleleft}$ is defined via a differential equation, and we are just using the semi-group property of solutions to diff. eq.'s. and behavior of scaleless integrals in Dim. Reg.

Coefficient Function $\leftrightarrow$ Fragmentation Function

The FF is the reciprocal of the universal parts of the coefficient function!

This is non-sense?

Fragmentation Function to *jets* is perturbative.

Coefficient Function is perturbative, even when resummed.

A Question.

What is the boundary between jets and hadrons when it comes to hard scattering processes?

- Can we take our resummed FF to jets to its extreme limit?
- Local Parton-Hadron Duality (Y. I. Azimov, Y. L. Dokshitzer, V. A. Khoze, and S. Troyan)

Model for Fragmentation Functions from Perturbation Theory

Take:

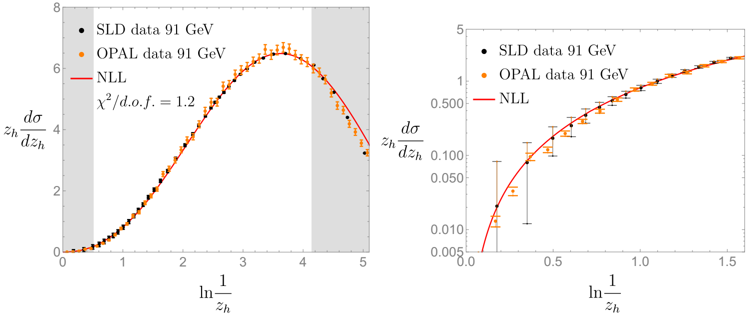
$$\bar{d}_{\triangleleft}\left(n, 1, \infty, \frac{\mu^2}{Q^2}\right)$$

as fragmentation function in Standard Approach.

- Run down to $\mu \sim \Lambda_{QCD}$, fit for normalization Λ_{QCD}.
- Use $x \to 0$ resummed Coef., FF, γ^T. (Derived from angular-ordered evolution).

Model for FF from Perturbation Theory

Works almost everywhere at Z-pole:



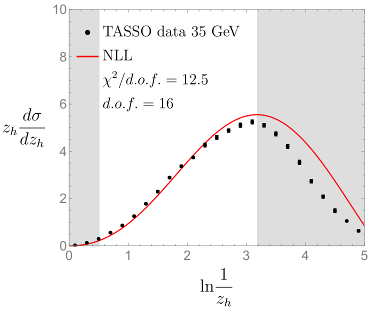
$e^+e^- \to h^\pm + X$

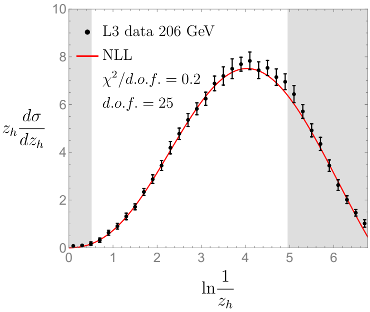
Excluded Gray Regions: hadron mass corrections to energy fraction, threshold region.

Red curve is a global fit to data from SLD@91, L3@206, ALEPH@189, TASSO@35&44, and TOPAZ@58 GeV, but $\chi^2/d.o.f$ given in plot is goodness-of-fit for the data set pictured only, to show how well the pictured data set is described. Total $\chi^2/d.o.f$ for red curve is 4.6 over all experiments.

Model for FF from Perturbation Theory

Evolution:





$$e^+e^- \to h^\pm + X$$

Excluded Gray Regions: hadron mass corrections to energy fraction, threshold region.

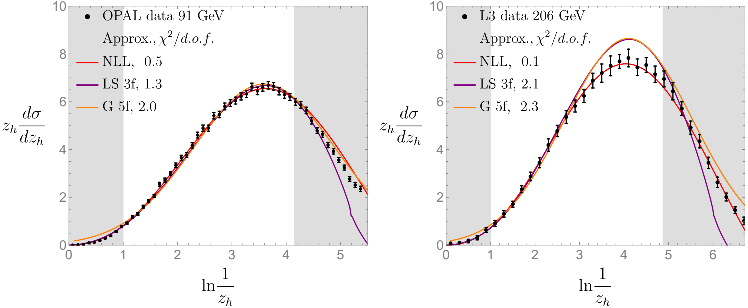
Lower Q data, hadron mass corrections?

Conclusions

- Jet physics and forward scattering physics: two branches, same tree.
- Reciprocity determines *fragmentation* spectrum from the scaling laws governing *PDF*.
- BFKL
- How far can we go?

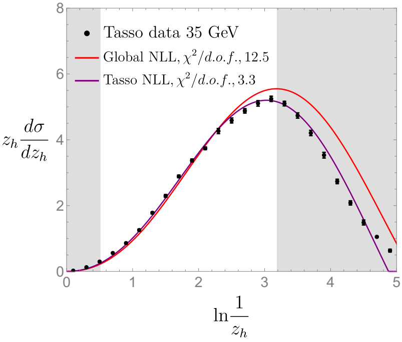
MLLA

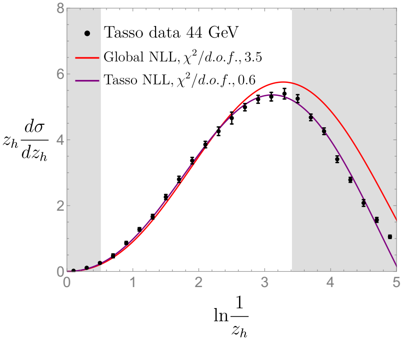
Angular ordering is old, used to derive "mixed-leading log" approximation (LS and G curves below), with unclear systematics.



NLL is fragmentation from space-time duality, clear systematics.

Vary Λ_{QCD}





- Red $\Lambda_{QCD} = 1.0$ GeV
- Purple $\Lambda_{QCD} = 2.0$ GeV

Mellin Space Angular Evolution

$$R_{ir}^2\frac{d}{dR_{ir}^2}\bar{d}_{\sphericalangle}(n,R_{ir}^2,R_{uv}^2,\mu^2,Q^2)=\sum_{\ell=1}^{\infty}\rho^{(\ell-1)}(\alpha_s;\epsilon)\Big(\frac{\mu^2}{R_{ir}^2Q^2}\Big)^{\ell\epsilon}\bar{d}_{\sphericalangle}\Big(n,R_{ir}^2,R_{uv}^2,\mu^2,Q^2\Big)$$

$$+\sum_{\ell=1}^{\infty}\bar{P}^{(\ell-1)}\Big(n-2\epsilon;\alpha_s;\epsilon\Big)\Big(\frac{\mu^2}{R_{ir}^2Q^2}\Big)^{\ell\epsilon}\bar{d}_{\sphericalangle}\Big(n-2\ell\epsilon,R_{ir}^2,\mu^2,Q^2\Big)\,.$$

Mellin Space Angular Evolution II

Setting $\mu = Q$, and writing:

$$\bar{d}_{\triangleleft}(n, R_{ir}^2, R_{uv}^2, \mu^2, \mu^2) = \bar{d}_{\triangleleft}(n, R_{ir}^2, R_{uv}^2)\,,$$

$$I\Big(\ell_1; n; R_{ir}, R_{uv}\Big) = 2\int_{R_{ir}}^{R_{uv}} \frac{d\theta_1}{\theta_1^{1+2\ell_1\epsilon}}\bar{P}^{(\ell_1-1)}(n-2\epsilon),$$

$$I\Big(\ell_1, ..., \ell_k; n; R_{ir}, R_{uv}\Big) = 2^k \int_{R_{ir}}^{R_{uv}} \frac{d\theta_1}{\theta_1^{1+2\ell_1\epsilon}}\bar{P}^{(\ell_1-1)}(n-2\epsilon)$$
$$\times \int_{R_{ir}}^{R_{uv}} \prod_{i=2}^{k} \frac{d\theta_i}{\theta_i^{1+2\ell_i\epsilon}}\Theta(\theta_i - \theta_{i-1})\bar{P}^{(\ell_i-1)}\Big(n - 2\epsilon(1+\sum_{j=1}^{i-1}\ell_j)\Big)\,, \text{if } k > 1\,.$$

Then the iterative expansion is obtained to be:

$$\bar{d}_{\triangleleft}\Big(n, R_{ir}^2, R_{uv}^2\Big) = \exp\Big(-\int_{R_{ir}^2}^{R_{uv}^2}\frac{d\theta^2}{\theta^2}\rho(\theta^{-2})\Big)\Big(1 + \sum_{\ell_1} I(\ell_1; n; R_{ir}, R_{uv})$$
$$+ \sum_{\ell_1,\ell_2} I(\ell_1, \ell_2; n; R_{ir}, R_{uv}) + \sum_{\ell_1,\ell_2,\ell_3} I(\ell_1, \ell_2, \ell_3; n; R_{ir}, R_{uv}) + ...\Big)\,.$$