HADRON PRODUCTION IN e^+e^- COLLISIONS WITHIN THE HELICITY FORMALISM AND THE Λ POLARIZING FF

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OUTLINE

- ❑ $e^{+}e^{-} \to h_1\ h_2 + X$: theory
  - ✓ TMD helicity formalism
  - ✓ TMD fragmentation functions for Spin-½ hadrons
  - ✓ Transverse Λ polarization
- ❑ Phenomenology
  - ✓ Fit of Belle data and extraction of the polarizing FF
  - ✓ Predictions for SIDIS@EIC
- ❑ Concluding remarks

$$e^+e^- \rightarrow \; h_1(P_1)\, h_2(P_2) + X$$

- Complete results for the azimuthal and polarization observables
  Boer, Jakob, Mulders 1997
- TMD factorization for small relative transverse momenta w.r.t. Q^2
  Collins 2011 - Echevarria, Idilbi Scimemi 2012

❑ $e^+e^- \rightarrow \; h_1(P_1)$ jet X

- ✓ Phenomenology: direct access to the intrinsic transverse momentum dependence
- ✓ Theory: recent developments on TMD factorization, role of soft factors, universality

  Kang, Shao, Zhao 2020 – Boglione,Simonelli 2021 – Gamberg, Kang, Shao, Terry, Zhao 2021
- ✓ Simplified model: check for consistency (see also *Anselmino, Kishore, Mukherjee 2019*)

$$e^+e^- \to h_1(P_1)\, h_2(P_2) + X$$

Master formula

Helicity density matrix

$$\rho^{h_1,S_1}_{\lambda_{h_1},\lambda'_{h_1}}\,\rho^{h_2,S_2}_{\lambda_{h_2},\lambda'_{h_2}}\,\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta\, dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

$$= \sum_{q_c,\bar{q}_d}\sum_{\{\lambda\}} \frac{1}{32\pi s}\,\frac{1}{4}\,\hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b}\,\hat{M}^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}\,\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1})\,\hat{D}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})$$

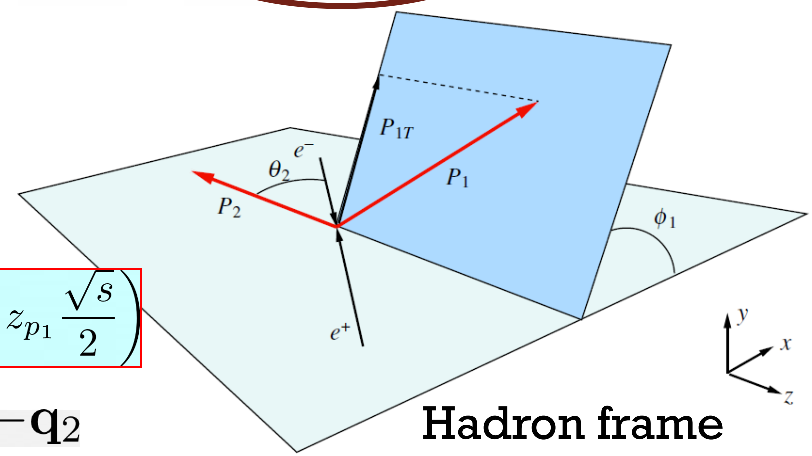
Fragm. Funct. (soft)

Helicity scatt. Amplitude (hard)

$$z_p = 2|\mathbf{P}_h|/\sqrt{s} \quad \simeq z\left(1-\frac{m_h^2}{z^2 s}\right) = z\,\xi$$

$$\boxed{P_2 \quad = \quad (E_2, 0, 0, -|\mathbf{P}_2|)} \qquad \boxed{\mathbf{P}_1 \simeq \left(P_{1T}\cos\phi_1, P_{1T}\sin\phi_1, z_{p_1}\frac{\sqrt{s}}{2}\right)}$$

$$q_2 \simeq \left(\frac{\sqrt{s}}{2}, -\frac{p_{\perp 2}}{z_{p_2}}\cos\varphi_2, -\frac{p_{\perp 2}}{z_{p_2}}\sin\varphi_2, -\frac{\sqrt{s}}{2}\right) \qquad \mathbf{q}_1 = -\mathbf{q}_2$$



Hadron frame

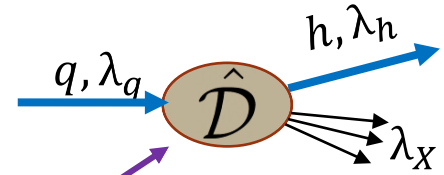
$$\frac{d\sigma^{e^+e^-\to h_1(S_1)h_2(S_2)\,X}}{d\cos\theta\, dz_1 dz_2 d^2\mathbf{P}_{1T}} = \int d^2\mathbf{p}_{\perp 2}\, d^2\mathbf{p}_{\perp 1}\, \delta^2(\mathbf{p}_{\perp 1} - \mathbf{P}_1 + \mathbf{p}_{\perp 2}\, z_{p_1}/z_{p_2})\, \frac{d\sigma^{e^+e^-\to h_1(S_1)h_2(S_2)\,X}}{d\cos\theta\, dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

QUARK TMD-FFS FOR SPIN-1/2 HADRONS

Hadron helicity density matrix

product of fragm. amplitudes

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q}\rho^q_{\lambda_q,\lambda'_q}\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$



$$\rho^{h,S_h}_{\lambda_h,\lambda'_h} = \frac{1}{2}\begin{pmatrix} 1+P^h_Z & P^h_X - iP^h_Y \\ P^h_X + iP^h_Y & 1-P^h_Z \end{pmatrix}$$

$$\sum\!\!\!\!\!\!\!\int_{X,\lambda_X}\hat{\mathcal{D}}_{\lambda_h,\lambda_X;\lambda_q}(z,\mathbf{p}_\perp)\hat{\mathcal{D}}^*_{\lambda'_h,\lambda_X;\lambda'_q}(z,\mathbf{p}_\perp)$$

polarization components
in the hadron helicity frame

$$D_{h/q}(z) = \frac{1}{2}\sum_{\lambda_q,\lambda_h}\int d^2\mathbf{p}_\perp D^{\lambda_h,\lambda_h}_{\lambda_q,\lambda_q}(z,\mathbf{p}_\perp)$$

Collinear unpolarized fragm. function

QUARK TMD-FFS FOR SPIN-1/2 HADRONS

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\,\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q} \rho^{q}_{\lambda_q,\lambda'_q}\,\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$

$$\boxed{\rho^{h,S_h}_{\lambda_h,\lambda'_h} = \frac{1}{2}\begin{pmatrix} 1+P^h_Z & P^h_X - iP^h_Y \\ P^h_X + iP^h_Y & 1-P^h_Z \end{pmatrix}}$$

Polarizations in the hadron helicity frame

$$\boxed{P^h_J\,\hat{D}_{h/q,s_q} = \hat{D}^{h/q}_{S_J/s_q} - \hat{D}^{h/q}_{-S_J/s_q} \equiv \Delta\hat{D}^{h/q}_{S_J/s_q}}$$

$$\begin{aligned}
(P^h_J\,\hat{D}_{h/q,s_T}) &= \Delta\hat{D}^{h/q}_{S_J/s_T} = \hat{D}^{h/q}_{S_J/s_T} - \hat{D}^{h/q}_{-S_J/s_T} \equiv \Delta\hat{D}^{h/q}_{S_J/s_T}(z,\mathbf{p}_\perp)\\
(P^h_J\,\hat{D}_{h/q,s_z}) &= \Delta\hat{D}^{h/q}_{S_J/s_z} = \hat{D}^{h/q}_{S_J/+} - \hat{D}^{h/q}_{-S_J/+} \equiv \Delta\hat{D}^{h/q}_{S_J/+}(z,\mathbf{p}_\perp)\\
\hat{D}_{h/q,s_T} &= \hat{D}_{h/q}(z,p_\perp) + \frac{1}{2}\,\Delta\hat{D}_{h/s_T}(z,\mathbf{p}_\perp)
\end{aligned}$$

8 TMD-FFs

QUARK TMD-FFS FOR SPIN-1/2 HADRONS

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\,\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q} \rho^{q}_{\lambda_q,\lambda'_q}\,\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$

$$\rho^{q}_{\lambda_q,\lambda'_q} = \frac{1}{2}\begin{pmatrix} 1+P^q_L & P^q_T e^{-i\phi_{s_q}} \\ P^q_T e^{i\phi_{s_q}} & 1-P^q_L \end{pmatrix}$$

Quark helicity density matrix

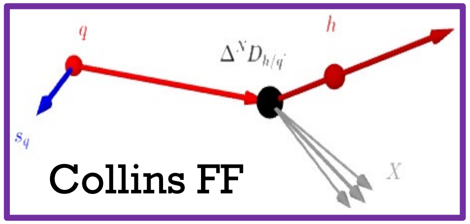
D^{++}_{++}, D^{++}_{--}, D^{++}_{+-}, D^{+-}_{++}, D^{+-}_{+-}, D^{+-}_{-+}

real complex real (q) / imm (g)

8 real quantities
by parity

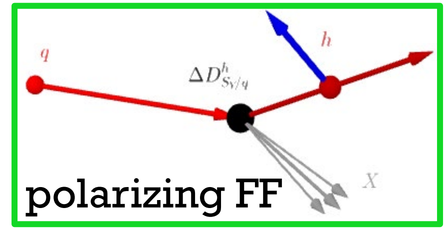
$$\begin{aligned}
\hat{D}_{h/q,s_q} &= (D^{++}_{++}+D^{++}_{--}) + 2P^q_T \mathrm{Im}D^{++}_{+-}\sin(\phi_{s_q}-\phi_h) \\
P^h_Z\hat{D}_{h/q,s_q} &= P^q_L(D^{++}_{++}-D^{++}_{--}) + 2P^q_T \mathrm{Re}D^{++}_{+-}\cos(\phi_{s_q}-\phi_h) \\
P^h_X\hat{D}_{h/q,s_q} &= 2P^q_L \mathrm{Re}D^{+-}_{++} + P^q_T(D^{+-}_{+-}+D^{+-}_{-+})\cos(\phi_{s_q}-\phi_h) \\
P^h_Y\hat{D}_{h/q,s_q} &= -2\mathrm{Im}D^{+-}_{++} + P^q_T(D^{+-}_{+-}-D^{+-}_{-+})\sin(\phi_{s_q}-\phi_h)
\end{aligned}$$

8 independent TMD Fragmentation Functions



T-odd & chiral-odd

UNIVERSAL



T-odd & chiral-even

		Hadron		
	Pol. States	**U**	**L**	**T**
Quark	**U**	$\hat{D}_{h/q}$		$\Delta \hat{D}_{h^\uparrow/q}$
	L		$\Delta \hat{D}^{h/q}_{S_Z/s_L}$	$\Delta \hat{D}^{h/q}_{S_X/s_L}$
	T	$\Delta \hat{D}_{h/q^\uparrow}$	$\Delta \hat{D}^{h/q}_{S_Z/s_T}$	$\Delta \hat{D}^{h/q}_{S_X/s_T}$ / $\Delta^- \hat{D}^{h/q}_{S_Y/s_T}$

Surviving in the collinear limit

OBSERVABLES (FEW EXAMPLES)

Unpol. Cross sect

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{2s}\sum_q e_q^2\Big\{(1+\cos^2\theta)D_{h_1/q}(z_1,p_{\perp 1})D_{h_2/\bar q}(z_2,p_{\perp 2}) + \frac{1}{4}\sin^2\theta\,\Delta^N D_{h_1/q^\uparrow}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_{h_1})$$

Single polariz. cross sect.

$$P_X^{h_1}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{4s}\sum_q e_q^2\sin^2\theta\,\Delta D^{h_1/q}_{S_X/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\sin(2\varphi_2+\phi_{h_1})$$

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{2s}\sum_q e_q^2\Big\{(1+\cos^2\theta)\Delta D^{h_1}_{S_Y/q}(z_1,p_{\perp 1})D_{h_2/\bar q}(z_2,p_{\perp 2}) + \frac{1}{2}\sin^2\theta\,\Delta^- D^{h_1}_{S_Y/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\cos(2\varphi_2+\phi_{h_1})\Big\}$$

$$P_Z^{h_1}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{4s}\sum_q e_q^2\sin^2\theta\,\Delta D^{h_1/q}_{S_Z/s_T}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar q^\uparrow}(z_2,p_{\perp 2})\sin(2\varphi_2+\phi_{h_1})$$

$$\hat M_{+-,+-} \simeq e^2 e_q\sqrt{3}\,(1+\cos\theta)\,e^{i\varphi_2}$$
$$\hat M_{-+,+-} \simeq e^2 e_q\sqrt{3}\,(1-\cos\theta)\,e^{-i\varphi_2}$$

$$\cos\phi_{h_1} \simeq \frac{P_{1T}}{p_{\perp 1}}\cos(\phi_1-\varphi_2) - \frac{z_{p_1}}{z_{p_2}}\frac{p_{\perp 2}}{p_{\perp 1}}$$

$$\sin\phi_{h_1} = \frac{P_{1T}}{p_{\perp 1}}\sin(\phi_1-\varphi_2)$$

TENSORIAL ANALYSIS

$$T^i = \frac{1}{P_{1T}} \int d^2\mathbf{p}_{\perp 2}\, p^i_{\perp 2}\, \Delta D^{h_1}(z_1, p_{\perp 1})\, \Delta D^{h_2}(z_2, p_{\perp 2})$$

$$T^{ij} = \frac{1}{P^2_{1T}} \int d^2\mathbf{p}_{\perp 2}\, p^i_{\perp 2} p^j_{\perp 2}\, \Delta D^{h_1}(z_1, p_{\perp 1})\, \Delta D^{h_2}(z_2, p_{\perp 2})$$

Euclidean tensors rank 1,2,…

depending only on P_{1T}

$$T^i = \frac{P^i_{1T}}{P_{1T}}\, S_1(P_{1T})$$

$$T^{ij} = \frac{P^i_{1T} P^j_{1T}}{P^2_{1T}}\, S_2(P_{1T}) + \delta^{ij}\, S_3(P_{1T})$$

Scalar functions,…

$$S_1(P_{1T}) = \frac{1}{P_{1T}} \int d^2\mathbf{p}_{\perp 2}\, (\mathbf{p}_{\perp 2} \cdot \hat{\mathbf{P}}_{1T})\, \Delta D^{h_1}\, \Delta D^{h_2}$$

$$S_2(P_{1T}) = \frac{1}{P^2_{1T}} \int d^2\mathbf{p}_{\perp 2}\, [2(\mathbf{p}_{\perp 2} \cdot \hat{\mathbf{P}}_{1T})^2 - p^2_{\perp 2}]\, \Delta D^{h_1}\, \Delta D^{h_2}$$

$$S_3(P_{1T}) = \frac{1}{P^2_{1T}} \int d^2\mathbf{p}_{\perp 2}\, [p^2_{\perp 2} - (\mathbf{p}_{\perp 2} \cdot \hat{\mathbf{P}}_{1T})^2]\, \Delta D^{h_1}\, \Delta D^{h_2}$$

$$\int d^2\mathbf{p}_{\perp 2}\, \cos\varphi_2\, \Delta D^{h_1}\, \Delta D^{h_2} = \cos\phi_1 \int d^2\mathbf{p}_{\perp 2}\, (\hat{\mathbf{p}}_{\perp 2} \cdot \hat{\mathbf{P}}_{1T})\, \Delta D^{h_1}\, \Delta D^{h_2}$$

$$\int d^2\mathbf{p}_{\perp 2}\, \sin\varphi_2\, \Delta D^{h_1}\, \Delta D^{h_2} = \sin\phi_1 \int d^2\mathbf{p}_{\perp 2}\, (\hat{\mathbf{p}}_{\perp 2} \cdot \hat{\mathbf{P}}_{1T})\, \Delta D^{h_1}\, \Delta D^{h_2}$$

$$\int d^2\mathbf{p}_{\perp 2}\, \cos^2\varphi_2\, \Delta D^{h_1}\, \Delta D^{h_2} = \frac{1}{2} \int d^2\mathbf{p}_{\perp 2} \left\{1 + \cos 2\phi_1\, [2(\hat{\mathbf{p}}_{\perp 2} \cdot \hat{\mathbf{P}}_{1T})^2 - 1]\right\} \Delta D^{h_1}\, \Delta D^{h_2}$$

$$\int d^2\mathbf{p}_{\perp 2}\, \sin^2\varphi_2\, \Delta D^{h_1}\, \Delta D^{h_2} = \frac{1}{2} \int d^2\mathbf{p}_{\perp 2} \left\{1 - \cos 2\phi_1\, [2(\hat{\mathbf{p}}_{\perp 2} \cdot \hat{\mathbf{P}}_{1T})^2 + 1]\right\} \Delta D^{h_1}\, \Delta D^{h_2}$$

as developed for SIDIS Anselmino *et al.* 2011

CONVOLUTIONS

$$\mathcal{C}[wD\bar{D}] = \sum_q e_q^2 \int d^2\mathbf{p}_{\perp 1} d^2\mathbf{p}_{\perp 2}\, \delta^{(2)}\big(\mathbf{p}_{\perp 1} - \mathbf{P}_{1T} + \mathbf{p}_{\perp 2}\, z_{p_1}/z_{p_2}\big)\, w(\mathbf{p}_{\perp 2}, \mathbf{P}_{1T}) D(z_1, p_{\perp 1}) \bar{D}(z_2, p_{\perp 2})$$

Unpol. cross section

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 dz_2 d^2\mathbf{P}_{1T}} = \frac{3\pi\alpha^2}{2s}\left\{(1+\cos^2\theta) F_{UU} + \sin^2\theta\cos(2\phi_1) F_{UU}^{\cos(2\phi_1)}\right\}$$

$$F_{UU} = \sum_q e_q^2 \int d^2\mathbf{p}_{\perp 2}\, D_{h_1/q}(z_1, p_{\perp 1}) D_{h_2/\bar{q}}(z_2, p_{\perp 2}) = \mathcal{C}[D_1 \bar{D}_1]$$

Tensorial analysis

$$\begin{aligned}\cos(2\phi_1)\, F_{UU}^{\cos(2\phi_1)} &= \sum_q e_q^2 \int d^2\mathbf{p}_{\perp 2} \frac{1}{4}\left[\frac{P_{1T}}{p_{\perp 1}}\cos(\phi_1 + \varphi_2) - \frac{z_{p_1}}{z_{p_2}}\frac{p_{\perp 2}}{p_{\perp 1}}\cos(2\varphi_2)\right]\Delta^N D_{h_1/q^\uparrow}\Delta^N D_{h_2/\bar{q}^\uparrow} \\ &= \cos(2\phi_1)\, \mathcal{C}\left[\left\{\frac{\mathbf{p}_{\perp 2}\cdot\mathbf{P}_{1T}}{z_1 z_2} - \frac{\xi_1}{\xi_2}\left[2\left(\frac{\mathbf{p}_{\perp 2}\cdot\hat{\mathbf{P}}_{1T}}{z_2}\right)^2 - \frac{p_{\perp 2}^2}{z_2^2}\right]\right\}\frac{H_1^\perp \bar{H}_1^\perp}{M_{h_1} M_{h_2}}\right]\end{aligned}$$

in agreement with *Boer, Jakob, Mulders 1997*

TRANSVERSE POLARIZATION

$$\mathbf{P}_T^{h_1} = P_X^{h_1}\,\hat{\mathbf{X}}_{h_1} + P_Y^{h_1}\,\hat{\mathbf{Y}}_{h_1} = P_{x_L}^{h_1}\,\hat{\mathbf{x}}_L + P_{y_L}^{h_1}\,\hat{\mathbf{y}}_L = P_T^{h_1}(\cos\phi_{S_1}^L\,\hat{\mathbf{x}}_L + \sin\phi_{S_1}^L\,\hat{\mathbf{y}}_L)$$

From the helicity to the lab frame

$$P_T^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 dz_2 d^2\mathbf{P}_{1T}} = \frac{3\pi\alpha^2}{4s}\Big\{\Big(1+\cos^2\theta\Big)\sin(\phi_1-\phi_{S_1}^L)\boxed{F_{TU}^{\sin(\phi_1-\phi_{S_1}^L)}}$$
$$+\ \sin^2\theta\Big(\sin(\phi_1+\phi_{S_1}^L)\boxed{F_{TU}^{\sin(\phi_1+\phi_{S_1}^L)}} + \sin(3\phi_1-\phi_{S_1}^L)\boxed{F_{TU}^{\sin(3\phi_1-\phi_{S_1}^L)}}\Big)\Big\}$$

$$\boxed{F_{TU}^{\sin(\phi_1-\phi_{S_1}^L)}} = \mathcal{C}\Big[\Big(\frac{\xi_1}{\xi_2}\frac{\mathbf{p}_{\perp 2}\cdot\hat{\mathbf{P}}_{1T}}{z_2} - \frac{P_{1T}}{z_1}\Big)\boxed{\frac{D_{1T}^{\perp}\bar{D}_1}{M_{h_1}}}\Big]$$

Polarizing FF

$$\boxed{F_{TU}^{\sin(\phi_1+\phi_{S_1}^L)}} = \mathcal{C}\Big[\Big(\frac{\mathbf{p}_{\perp 2}\cdot\hat{\mathbf{P}}_{1T}}{z_2}\Big)\boxed{\frac{H_1\bar{H}_1^{\perp}}{M_{h_2}}}\Big]$$

$$\boxed{F_{TU}^{\sin(3\phi_1-\phi_{S_1}^L)}} = \mathcal{C}\Big[\Big\{\frac{\xi_1^2}{\xi_2^2}\Big[4\Big(\frac{\mathbf{p}_{\perp 2}\cdot\hat{\mathbf{P}}_{1T}}{z_2}\Big)^3 - 3\,\frac{p_{\perp 2}^2}{z_2^2}\Big(\frac{\mathbf{p}_{\perp 2}\cdot\hat{\mathbf{P}}_{1T}}{z_2}\Big)\Big]$$
$$+\ \Big(\frac{\mathbf{p}_{\perp 2}\cdot\mathbf{P}_{1T}}{z_1z_2}\Big)\frac{P_{1T}}{z_1} - 2\,\frac{\xi_1}{\xi_2}\Big[2\Big(\frac{\mathbf{p}_{\perp 2}\cdot\hat{\mathbf{P}}_{1T}}{z_2}\Big)^2 - \frac{p_{\perp 2}^2}{z_2^2}\Big]\frac{P_{1T}}{z_1}\Big\}\boxed{\frac{H_{1T}^{\perp}\bar{H}_1^{\perp}}{2M_{h_1}^2M_{h_2}}}\Big].$$

TRANSVERSE POLARIZATION

Projection along the normal to the hadron plane [lab frame] (phenom.)

$$\hat{\mathbf{n}} = \frac{-\mathbf{P}_2 \times \mathbf{P}_1}{|\mathbf{P}_2 \times \mathbf{P}_1|} = -\sin\phi_1 \hat{\mathbf{x}}_L + \cos\phi_1 \hat{\mathbf{y}}_L \qquad P_n^{h_1} \equiv \mathbf{P}^{h_1} \cdot \hat{\mathbf{n}} = \mathbf{P}_T^{h_1} \cdot \hat{\mathbf{n}}$$

$$P_n^{h_1} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{d\cos\theta dz_1 dz_2 d^2\mathbf{P}_{1T}} = \frac{3\pi\alpha^2}{4s}\Big\{\Big(1+\cos^2\theta\Big)\Big[-\boxed{F_{TU}^{\sin(\phi_1-\phi_{S_1}^L)}}\Big] + \sin^2\theta\Big(F_{TU}^{\sin(\phi_1+\phi_{S_1}^L)} - F_{TU}^{\sin(3\phi_1-\phi_{S_1}^L)}\Big)\cos(2\phi_1)\Big\}$$

By integrating over P_{1T}, the $\cos(2\phi_1)$ term is washed out, leaving

$$\boxed{F_{TU}^{\sin(\phi_1-\phi_{S_1}^L)}} = \mathcal{C}\left[\left(\frac{\xi_1}{\xi_2}\frac{\mathbf{p}_{\perp 2}\cdot\hat{\mathbf{P}}_{1T}}{z_2} - \frac{P_{1T}}{z_1}\right)\frac{D_{1T}^{\perp}\bar{D}_1}{M_{h_1}}\right]$$

$$= \mathcal{C}\left[\left(\frac{z_{p_1}}{z_{p_2}}\frac{\mathbf{p}_{\perp 2}\cdot\hat{\mathbf{P}}_{1T}}{p_{\perp 1}} - \frac{P_{1T}}{p_{\perp 1}}\right)\Delta D_{h_1^\uparrow/q} D_{h_2/\bar{q}}\right]$$

Access to the polarizing FF

PHENOMENOLOGY

- TMD polarizing fragmentation function *Boer, Jakob, Mulders 1997*
- First attempt to describe $P_T(\Lambda)$ data in unpolarized hadron-hadron collisions within a phenomenological TMD model *Anselmino, Boer, UD, Murgia 2001*
- Twist-three approach: *Kanazawa, Koike 2001*

2019

- Belle data for $P_T(\Lambda)$ in $e^+e^- \to \Lambda^\uparrow h + X$ and $e^+e^- \to \Lambda^\uparrow(\text{jet}) + X$

  Guan et al. (Belle Coll.) 2019

FACTORIZED GAUSSIAN MODEL

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle} \qquad \langle p_\perp^2\rangle_{\rm pol} = \frac{M_{\rm pol}^2}{M_{\rm pol}^2+\langle p_\perp^2\rangle}\,\langle p_\perp^2\rangle$$

$$\Delta D_{h^\uparrow/q}(z,p_\perp) = \Delta D_{h^\uparrow/q}(z)\,\frac{\sqrt{2e}\,p_\perp}{M_{\rm pol}}\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{\rm pol}}}{\pi\langle p_\perp^2\rangle}$$

$$\mathcal{P}_n(z_1,z_2) = \sqrt{\frac{e\pi}{2}}\,\frac{1}{M_{\rm pol}}\,\frac{\langle p_\perp^2\rangle_{\rm pol}^2}{\langle p_{\perp 1}^2\rangle}\,\frac{z_2}{\left\{[z_1(1-m_{h_1}^2/(z_1^2 s))]^2\langle p_{\perp 2}^2\rangle + z_2^2\langle p_\perp^2\rangle_{\rm pol}\right\}^{1/2}} \times \frac{\sum_q e_q^2\,\Delta D_{h_1^\uparrow/q}(z_1)D_{h_2/\bar q}(z_2)}{\sum_q e_q^2\,D_{h_1/q}(z_1)D_{h_2/\bar q}(z_2)} \qquad P^{h_1}\cdot\hat n$$

- $e^+e^- \to h_1(P_1)$ jet X within a simplified TMD scheme

$$\mathcal{P}_T(z_1,p_{\perp 1}) = \frac{\sum_q e_q^2\,\Delta D_{h_1^\uparrow/q}(z_1,p_{\perp 1})}{\sum_q e_q^2\,D_{h_1/q}(z_1,p_{\perp 1})}$$

orthogonal to the thrust plane

FIT OF BELLE DATA

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \;\; - 0.9]$ excluded → 96/128 data points
- Λ(jet) : $z_{\Lambda} = [0.5 \;\; - 0.9]$ excluded → 24/32 data points

- **Unpol. FFs**: DSS07 for π/K , AKK08 for Λ

$$\langle p_\perp^2 \rangle = 0.2 \;\; \mathrm{GeV}^2$$

$$\boxed{D_{\Lambda/\bar{q}}(z_p) = (1 - z_p) D_{\Lambda/q}(z_p)}$$

flavour separation

- **Polarizing FF**

$$\Delta D_{\Lambda^\uparrow/q}(z) = N_q z^{a_q} (1-z)^{b_q} \frac{(a_q + b_q)^{(a_q+b_q)}}{a_q^{a_q} b_q^{b_q}} D_{\Lambda/q}(z)$$

$|N_q| \leq 1$ Positivity bound

✓ No TMD evolution (fixed-scale analysis)

FIT OF BELLE DATA

$$N_u,\ N_d,\ N_s,\ N_{\rm sea},\ a_s,\ b_u,\ b_{\rm sea} \qquad \langle p_\perp^2\rangle_{\rm pol}$$

$\Lambda + \pi/K$ data fit : $\chi^2_{\rm dof} = 1.26$

N_u	$=$	$0.67^{+0.33}_{-0.32}$	N_d	$=$	$-0.65^{+0.32}_{-0.35}$
N_s	$=$	$-1.00^{+0.02}_{-0.00}$	$N_{\rm sea}$	$=$	$-0.40^{+0.18}_{-0.28}$
a_s	$=$	$2.17^{+0.76}_{-0.71}$			
b_u	$=$	$3.36^{+3.15}_{-2.31}$	$b_{\rm sea}$	$=$	$2.14^{+3.06}_{-1.92}$
$\langle p_\perp^2\rangle_{\rm pol}$	$=$	$0.056^{+0.026}_{-0.019}$ GeV^2			

2σ uncertainty (250K sets)

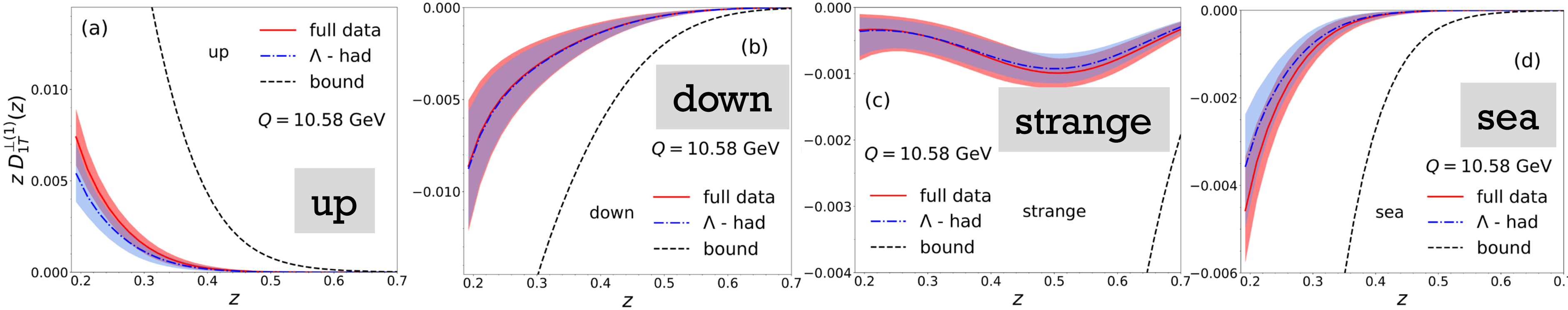
$$\Delta\chi^2 = 15.79$$

Similar analysis in *Callos, Kang, Terry 2020*

Full-data fit : $\chi^2_{\rm dof} = 1.94$

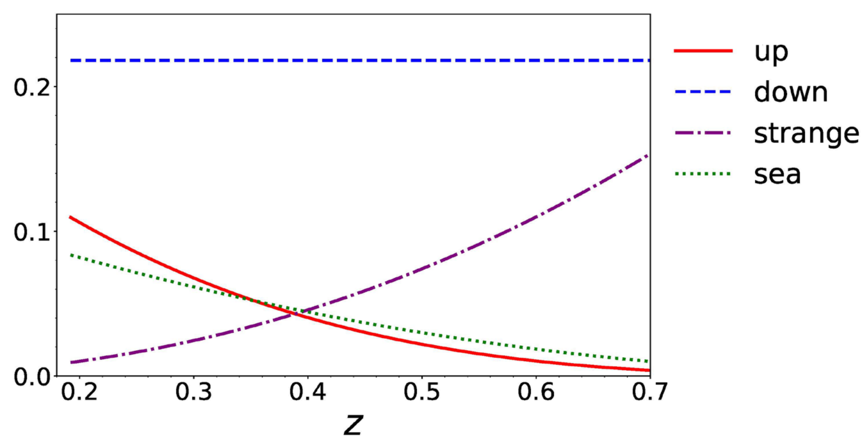
First moments of the polarizing FF

$$D_{1T}^{\perp(1)}(z) = \int d^2\mathbf{p}_\perp \frac{p_\perp}{2zm_h} \Delta D_{h^\uparrow/q}(z, p_\perp)$$



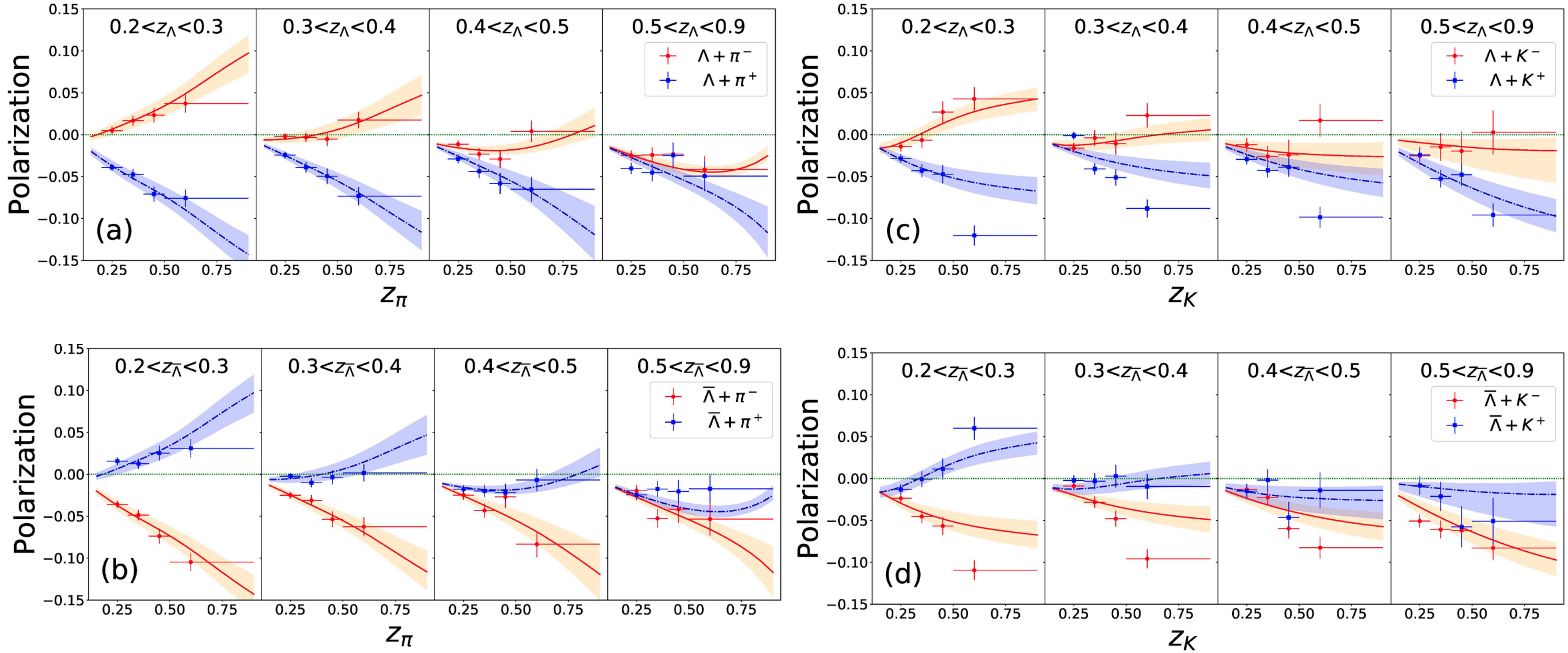
Full-data (including Λ (jet)) and Λ*-had* extractions
Consistency: central lines and (overlapping) uncertainty bands!!!

|Ratios| of the first moments w.r.t. their bounds



Associated production

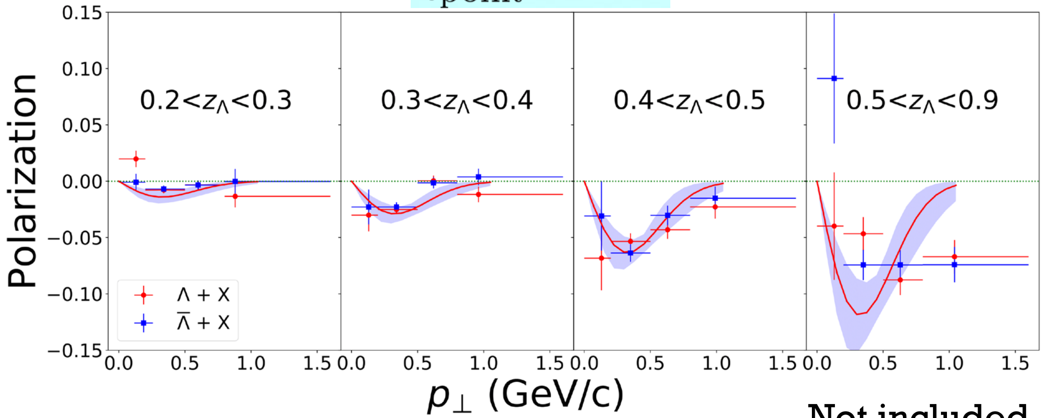
$\chi^2_{\text{point}} = 0.8$ $\chi^2_{\text{point}} = 1.5$



- simpler fits with only two pFFs (*u*=*d*, *s* and no sea) ➡ higher χ^2
- pion well described (also including the highest *z* bin)
- kaon, some tension (data?, unpol FFs?, parametrization of polFF?)

Inclusive Λ production [full-data analysis]

$\chi^2_{\rm point} = 2.75$



Not included

- Less good description…but not bad!
- Theory
  - ✓ Polarization = 0 at $p_\perp = 0$
  - ✓ $P(\Lambda) = P(\bar{\Lambda})$

- not *clearly* visible in the data

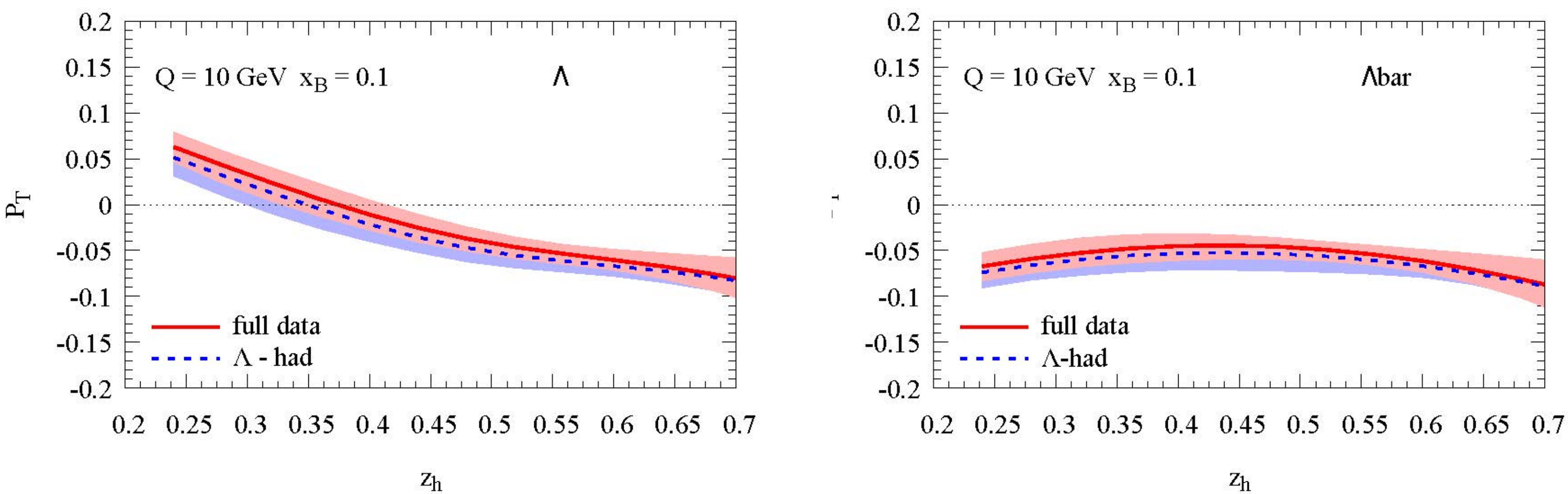
TRANSVERSE Λ POLARIZATION IN SIDIS

$$P_T(x_B, z_h) = \frac{\sqrt{2e\pi}}{2M_p}\frac{\langle p_\perp^2\rangle_{\rm pol}^2}{\langle p_\perp^2\rangle}\frac{1}{\sqrt{\langle p_\perp^2\rangle_{\rm pol} + \xi_p^2\langle k_\perp^2\rangle}} \times \frac{\sum_q e_q^2 f_{q/p}(x_B)\Delta D_{\Lambda^\uparrow/q}(z_h)}{\sum_q e_q^2 f_{q/p}(x_B) D_{\Lambda/q}(z_h)}$$

Transverse w.r.t.
the target-Λ plane

$$\xi_p = z_h\left(1 - \frac{m_h^2}{z_h^2 Q^2}\frac{x_B}{1-x_B}\right)$$

EIC kinematics



CONCLUDING REMARKS

- General helicity formalism for $e^+e^- \rightarrow h_1 \ h_2 + X$
- Classification of all leading-twist quark (and gluon) TMD-FFs for spin-1/2 hadrons
- Fit of Belle e^+e^- data on transverse Λ polarization and extraction of the polarizing FF
- Polarizing FF
  - ✓ three different valence polFFs (up, down and strange) + sea
  - ✓ relative sign between up and down polFFs
  - ✓ extraction of the $p_\perp$ dependence within a Gaussian ansatz
- SIDIS measurements @EIC important to complement these results

THANKS for the ATTENTION

BACK-UP SLIDES

CORRESPONDENCE WITH AMSTERDAM NOTATION

Projection of the correlator $\Delta(z, \mathbf{k}_T)$

$(-z\,\mathbf{k}_T) \quad = \quad \mathbf{p}_\perp$

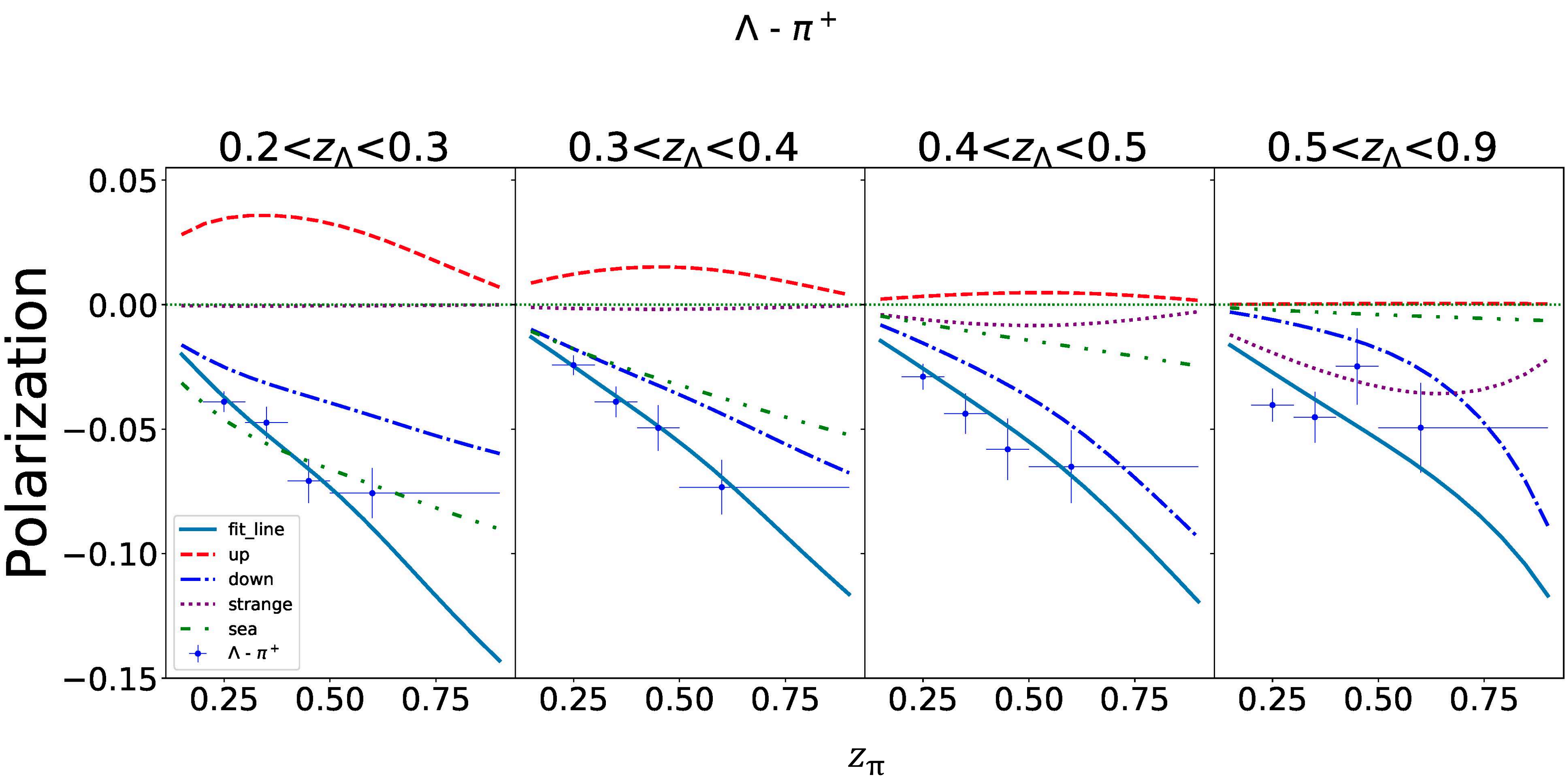
$$
\begin{aligned}
D_1(z,p_\perp) &= D^{++}_{++} + D^{++}_{--} = D_{h/q} \\
\frac{p_\perp}{zM_h} D^\perp_{1T}(z,p_\perp) &= -2\,\mathrm{Im}D^{+-}_{++} = \Delta D^h_{S_Y/q} = \Delta D_{h^\uparrow/q} \\
G_{1L}(z,p_\perp) &= D^{++}_{++} - D^{++}_{--} = \Delta D^{h/q}_{S_Z/s_L} \\
\frac{p_\perp}{zM_h} G_{1T}(z,p_\perp) &= 2\,\mathrm{Re}D^{++}_{+-} = \Delta D^{h/q}_{S_X/s_L} \\
\frac{p_\perp}{zM_h} H^\perp_{1L}(z,p_\perp) &= 2\,\mathrm{Re}D^{+-}_{++} = \Delta D^{h/q}_{S_Z/s_T} \\
\frac{2p_\perp}{zM_h} H^\perp_1(z,p_\perp) &= 4\,\mathrm{Im}D^{++}_{+-} = \Delta D_{h/q^\uparrow} \\
H_1(z,p_\perp) &= D^{+-}_{+-} \\
\frac{p_\perp^2}{z^2M_h^2} H^\perp_{1T}(z,p_\perp) &= 2\,D^{+-}_{-+}
\end{aligned}
$$

Polarizing FF

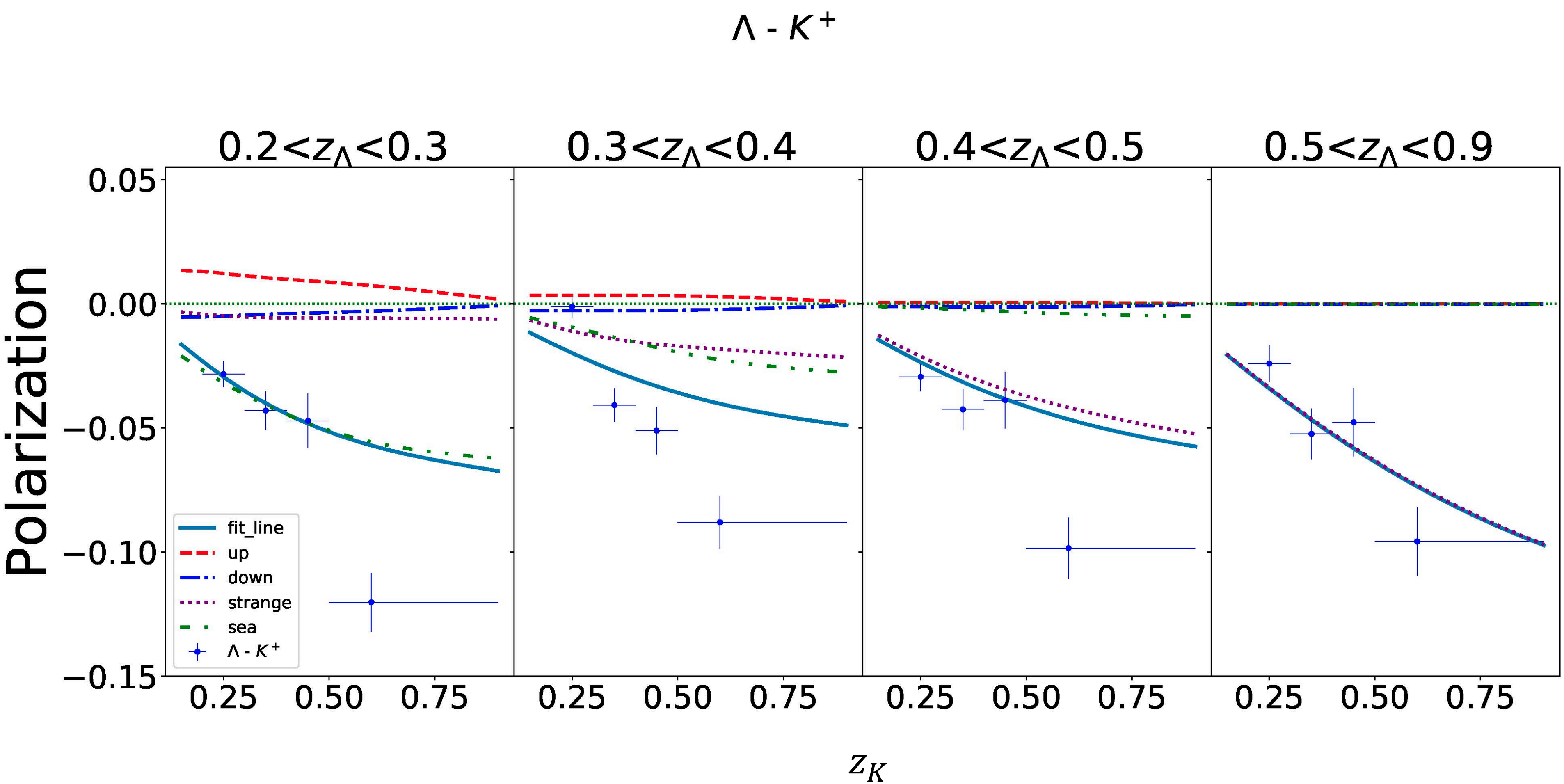
Collins FF

Extension of the Trento Convention results

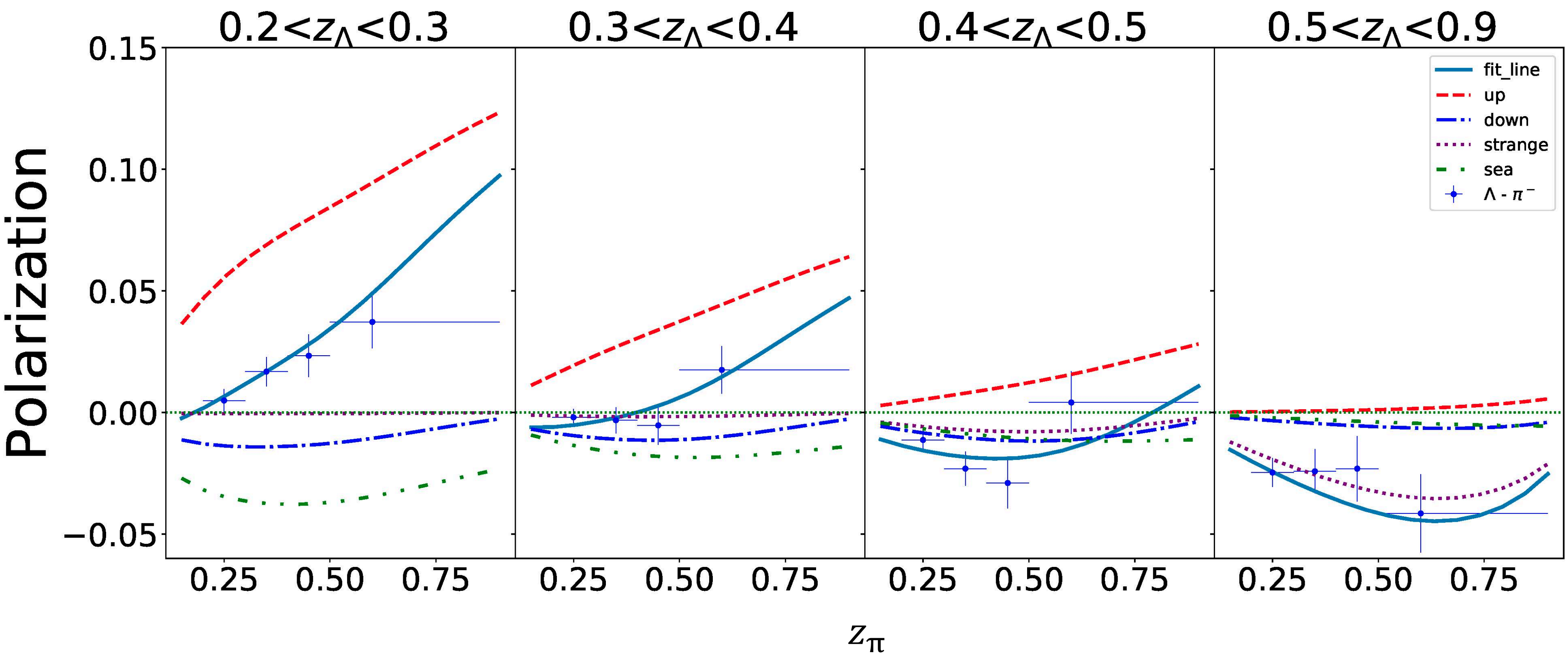
Bacchetta, UD, Diehl, Miller 2004



Λ - π+
0.2<zΛ<0.3
0.3<zΛ<0.4
0.4<zΛ<0.5
0.5<zΛ<0.9
Polarization
0.05
0.00
−0.05
−0.10
−0.15
0.25
0.50
0.75
zπ
fit_line
up
down
strange
sea
Λ - π+

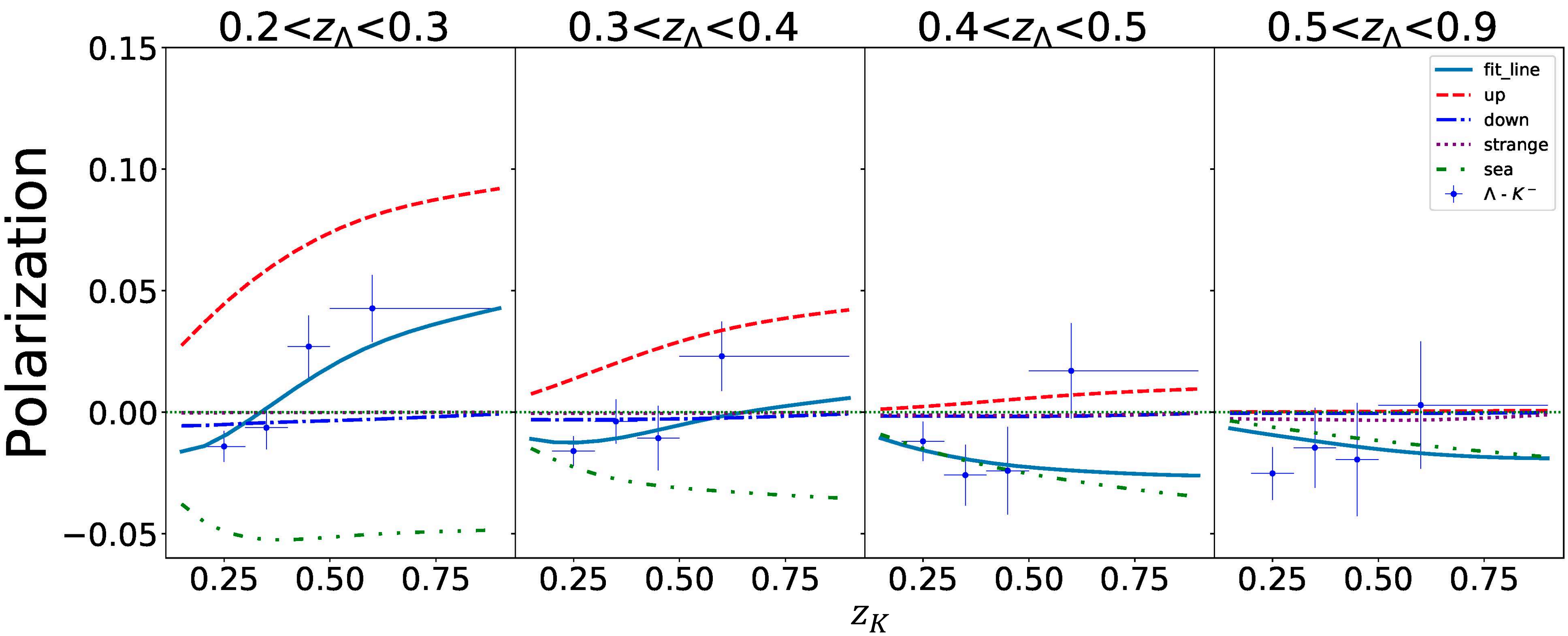

Λ - K^+
0.2<z_Λ<0.3
0.3<z_Λ<0.4
0.4<z_Λ<0.5
0.5<z_Λ<0.9
Polarization
0.05
0.00
−0.05
−0.10
−0.15
0.25
0.50
0.75
z_K
fit_line
up
down
strange
sea
Λ - K^+

Λ - π^-



0.2<z_Λ<0.3
0.3<z_Λ<0.4
0.4<z_Λ<0.5
0.5<z_Λ<0.9
Polarization
0.15
0.10
0.05
0.00
−0.05
0.25
0.50
0.75
fit_line
up
down
strange
sea
Λ - π^-
z_π

Λ - K^-



0.2<z_Λ<0.3
0.3<z_Λ<0.4
0.4<z_Λ<0.5
0.5<z_Λ<0.9
Polarization
0.15
0.10
0.05
0.00
−0.05
0.25
0.50
0.75
z_K
fit_line
up
down
strange
sea
Λ - K^-

FIT OF BELLE DATA

$$N_u,\ N_d,\ N_s,\ N_{\rm sea},\ a_s,\ b_u,\ b_{\rm sea} \qquad \langle p_\perp^2\rangle_{\rm pol}$$

Full data fit : $\chi^2_{\rm dof} = 1.94$

$N_u = 0.47^{+0.32}_{-0.20}$	$N_d = -0.32^{+0.13}_{-0.13}$
$N_s = -0.57^{+0.29}_{-0.43}$	$N_{\rm sea} = -0.27^{+0.12}_{-0.20}$
$a_s = 2.30^{+1.08}_{-0.91}$	
$b_u = 3.50^{+2.33}_{-1.82}$	$b_{\rm sea} = 2.60^{+2.60}_{-1.74}$
$\langle p_\perp^2\rangle_{\rm pol} = 0.10^{+0.02}_{-0.02}$ GeV^2	

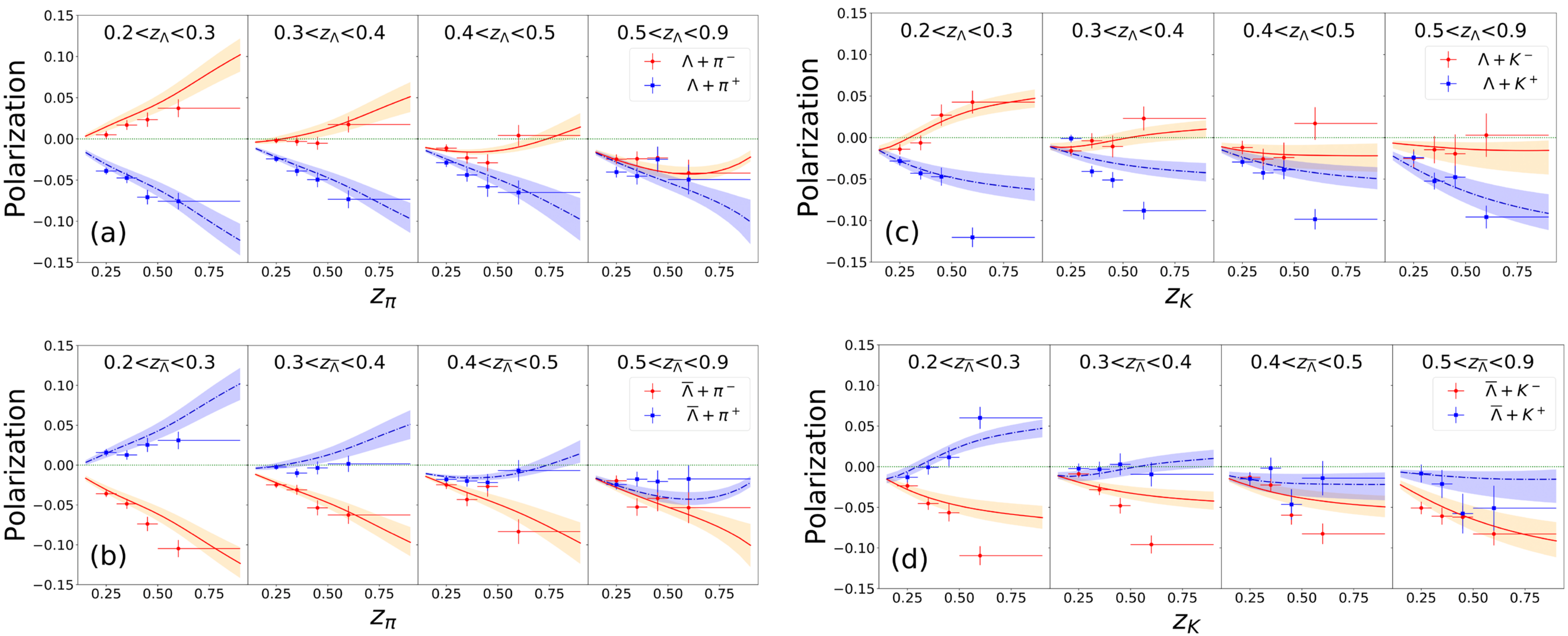
Errors correspond to 2σ uncertainty (250K sets)

$\Delta\chi^2 = 15.79$

$\Lambda + \pi/K$ data fit : $\chi^2_{\rm dof} = 1.26$

Similar analysis (associated production) Callos, Kang, Terry 2020

Associated production [full-data analysis]



$\chi^2_{\rm point} = 1.55/0.8$

$\chi^2_{\rm point} = 1.61/1.5$

- simpler fits with only two pFFs (u=d and s and no sea) ➡ higher χ^2
- pion well described (also including the highest z bin)
- kaon, some tension (data, FFs?)

Uncertainty band

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - F(x_i; \boldsymbol{a})}{\sigma_i} \right)^2$$

- N measurements y_i at known points x_i, with variance σ_i^2.
- $F(x_i; \boldsymbol{a})$ depends *non-linearly* on M unknown parameters a_i.
- Best fit: $\chi^2_{\rm min} \to \boldsymbol{a}_0$

Error band: all sets of parameters such that $\chi^2(\boldsymbol{a}_j) \leq \chi^2_{\rm min} + \Delta\chi^2$

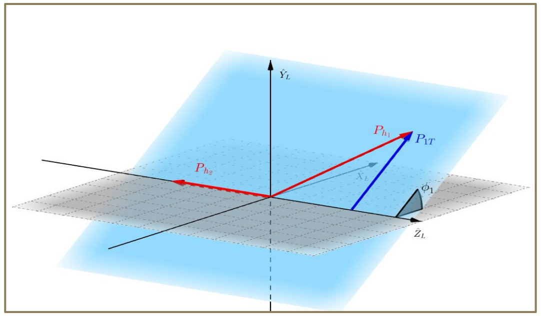
- $\Delta\chi^2 = 1 \leftrightarrow 1$-$\sigma$: small errors, uncorrelated parameters, linearity, χ^2 parabolic
- $\Delta\chi^2$: fixed according to the coverage probability

$$P = \int_0^{\Delta\chi^2} \frac{1}{2\Gamma(M/2)} \left(\frac{\chi^2}{2}\right)^{(M/2)-1} \exp\left(-\frac{\chi^2}{2}\right) \mathrm{d}\chi^2 .$$

P= probability that true set of parameters falls inside the hypervolume

$[P = 0.68 \leftrightarrow 1$-$\sigma$, $P = 0.95 \leftrightarrow 2$-$\sigma]$

TRANSVERSE Λ POLARIZATION



$$\mathcal{P}_T = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\text{unp}}},$$

- Measured along $\hat{\boldsymbol{n}} = -\boldsymbol{P}_{h_2} \times \boldsymbol{P}_{h_1}$ (hadron frame)
- Proper projection from the hadron helicity frame

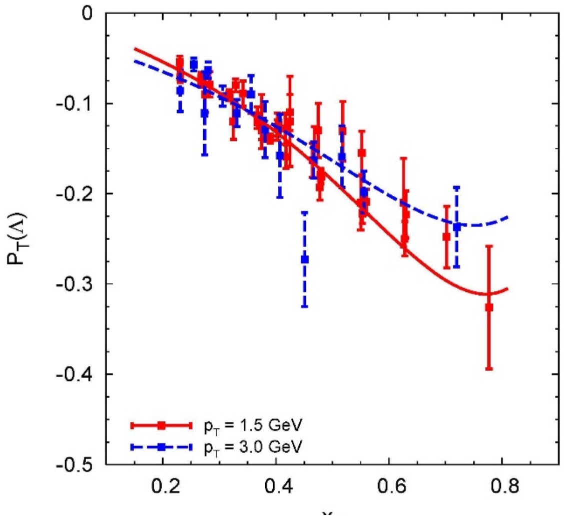
Polarizing FF

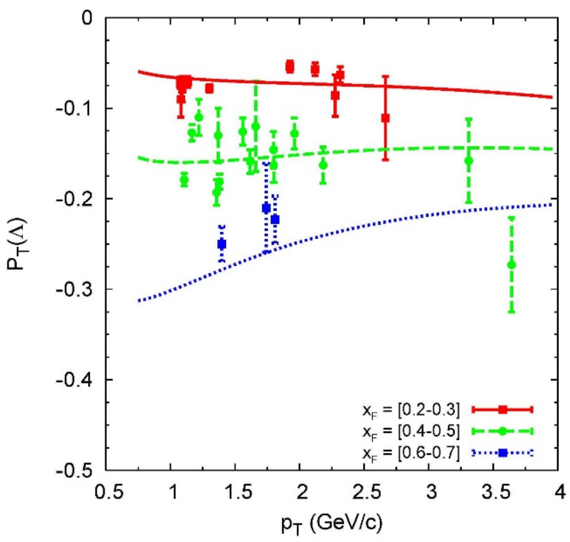
$$P_Y^{h_1} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta d\mathrm{PS}_{12}} = \frac{3\pi\alpha^2}{2s} \sum_q e_q^2 \Big\{ (1+\cos^2\theta) \Delta D^{h_1}_{S_Y/q}(z_1, p_{\perp 1}) D_{h_2/\bar{q}}(z_2, p_{\perp 2}) + \frac{1}{2} \sin^2\theta \Delta^- D^{h_1}_{S_Y/s_T}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \cos(2\varphi_2 + \phi_{h_1}) \Big\}$$

$$d\mathrm{PS}_{12} = dz_1 d^2\boldsymbol{p}_{\perp 1} dz_2 d^2\boldsymbol{p}_{\perp 2}$$

Collins FF

- By integrating over Φ_1 $\qquad d^2\mathbf{p}_{\perp 1} \to d^2\mathbf{P}_{T1}$
  - ✓ the transverse component along X is washed out
  - ✓ the contribution from the Collins FF is washed out





A LOOK BACK…

$$P_T(\Lambda) = \frac{d\sigma^{AB\to\Lambda^\uparrow X} - d\sigma^{AB\to\Lambda^\downarrow X}}{d\sigma^{AB\to\Lambda^\uparrow X} + d\sigma^{AB\to\Lambda^\downarrow X}}$$

- pA collisions (late 70's)

 - fixed target exp.s - $E \backsim 400$ GeV
- Λ self-analyzing: a powerful tool
- Transverse Λ polarization w.r.t. the production plane
  - large and negative
  - increasing up to $p_T \sim 1$ GeV then a plateau
  - increasing with x_F in the plateau regime
  - $\overline{\Lambda}$ no polarization
  - Almost zero in collinear pQCD at leading twist