Energy-Momentum Tensor Form Factors in QED

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Based on Metz, Pasquini, Rodini, 2104.04207

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Outline

Basic definitions of Energy-Momentum Tensor (EMT) and form factors

The D(t) form factor, why to bother?

Results in QED

‘Pressure’ and ‘shear’ distributions

Basic definitions

Symmetric and gauge-invariant EMT for massless-photon QED

$$
\begin{aligned}
T^{\mu\nu}_{\mathrm{QED}} &= T^{\mu\nu}_{e} + T'^{\mu\nu}_{\gamma} \\
T^{\mu\nu}_{e} &= Z_2\,\bar{\psi}\,\frac{i}{4}\,\gamma^{\{\mu}\overleftrightarrow{\partial}^{\nu\}}\psi - Z_2\,\mu^{2\,\varepsilon_{\mathrm{UV}}}\,e\,\bar{\psi}\,\gamma^{\{\mu}A^{\nu\}}\psi \\
T'^{\mu\nu}_{\gamma} &= -Z_3\,F^{\mu\alpha}F^{\nu}{}_{\alpha} + Z_3\,\frac{g^{\mu\nu}}{4}F^{\alpha\beta}F_{\alpha\beta}
\end{aligned}
$$

How can the EMT for QED incorporate a massive photon?

$$
T^{\mu\nu} = T^{\mu\nu}_{\mathrm{QED}} + m_\gamma^2\left(A^\mu A^\nu - \frac{g^{\mu\nu}}{2}A^2\right) + T^{\mu\nu}_{\mathrm{extra}}
$$

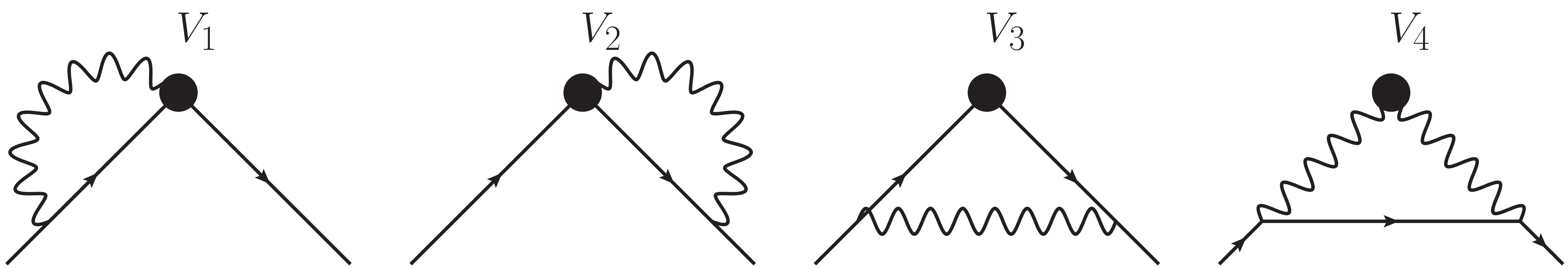
Stueckelberg Higgs mechanism ...

We used the Higgs model for a massive photon in the R_1 gauge

Metz et al., 2104.04207

EMT matrix elements

$$\langle e(p',s')|\, T_i^{\mu\nu}\, |e(p,s)\rangle = \bar{u}(p',s')\bigg(A_i(t)\, \frac{P^\mu P^\nu}{m_e} + J_i(t)\, \frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2m_e} + D_i(t)\, \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4m_e} + m_e\, \bar{C}_i(t)\, g^{\mu\nu} \bigg) u(p,s)$$



$$\langle e(p',s')|\, \mathrm{T}\left[T_i^{\mu\nu}(0)\exp\left(-ie\int d^4x\ \bar{\psi}\not{A}\psi\right)\right]|e(p,s)\rangle$$

Metz et al., 2104.04207 | Rodini et al., JHEP 09 (2020) 067 | Berends, Gastmans, Annals Phys. 98 (1976) 225 | Milton, PRD 15 (1977) 538

The D(t) form factor, why to bother?

It is the least known of the global properties of particles

For non-vanishing momentum transfer
D(t) is related to the pressure and shear distributions

D(0) can be used to define an energy radius
and a mechanical radius of the system

D(0) is linked to the stability of the system

Polyakov, Schweitzer, IJMPA 33 (2018) 1830025

Metz et al., 2104.04207

Varma, Schweitzer, PRD 102 (2020) 014047

Lorcé et al., EPJC 79 (2019) 89

D(t) for an electron in QED

For the D(t) form factor at one loop
we do not need to worry about renormalization

$$D_i(\tau^2, \lambda^2) = \int_0^1 dx \int_0^{1-x} dy \frac{f_i(x,y)}{\tau^2 + a_i(x,y,\lambda^2)}$$

$$\tau^2 = \frac{-t}{m_e^2} > 0 \qquad \lambda = \frac{m_\gamma}{m_e}$$

$$f_e(x,y) = \frac{\alpha}{\pi} \frac{(x-2)(1-x-2y)^2}{y(1-x-y)} \qquad a_e(x,y,\lambda^2) = \frac{(1-x)^2 + x\lambda^2}{y(1-x-y)}$$

$$f_\gamma(x,y) = \frac{\alpha}{\pi} \frac{1-x-(1+x)(1-x-2y)^2}{y(1-x-y)} \qquad a_\gamma(x,y,\lambda^2) = \frac{x^2 + (1-x)\lambda^2}{y(1-x-y)}$$

Polyakov, Schweitzer, IJMPA 33 (2018) 1830025

Metz et al., 2104.04207

Varma, Schweitzer, PRD 102 (2020) 014047

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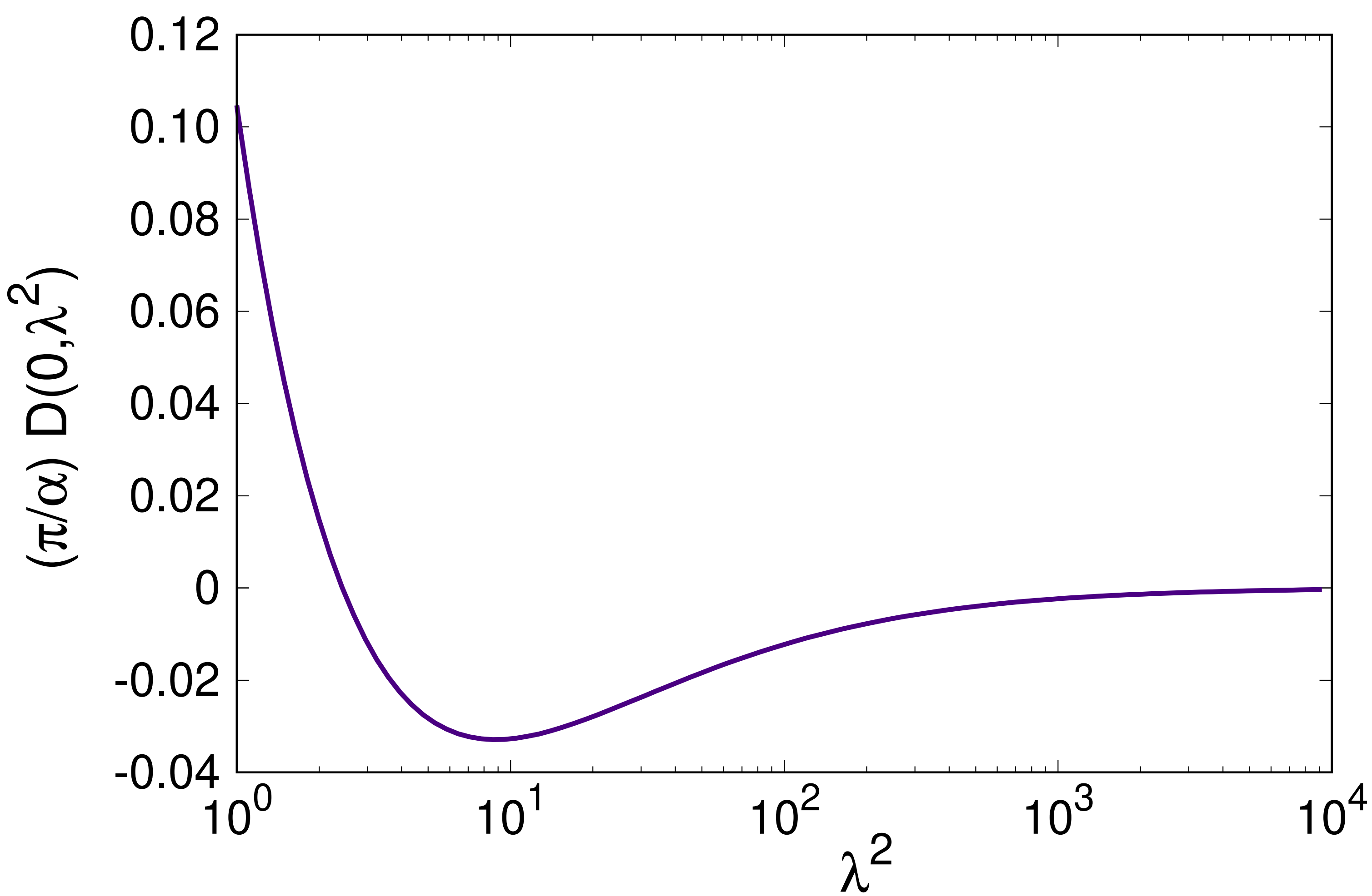
Limits

The photon mass regulates the soft divergence for $\tau^2 \to 0$



$$D_\gamma(\tau^2 \ll 1, \lambda^2 = 0) \simeq \frac{\alpha\pi}{4\sqrt{\tau^2}}$$

$$D_\gamma(\tau^2 = 0, \lambda^2 \ll 1) \simeq \frac{\alpha}{3\lambda}$$

$$D_e(\tau^2 = 0, \lambda^2 = 0) = -\frac{5\alpha}{18\pi}$$

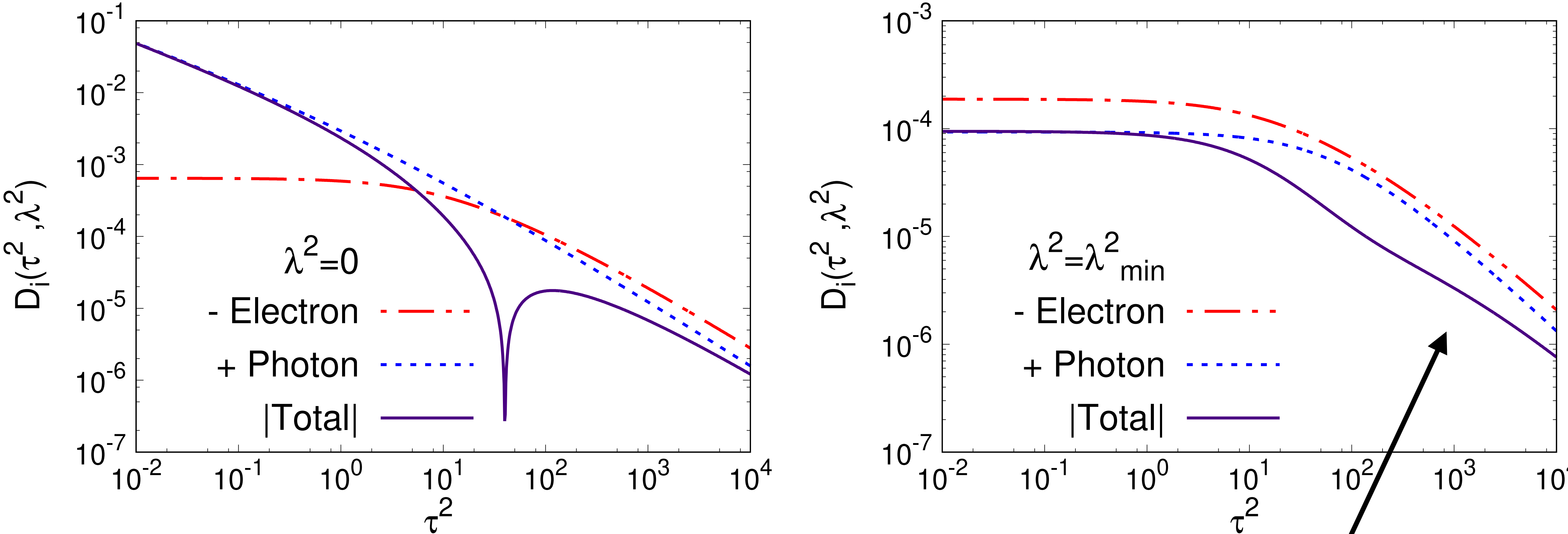
$$D_e(\tau^2 \gg 1, \lambda^2 = 0) = \frac{\alpha}{\pi}\frac{10 - 5\log\left(\tau^2\right)}{3\tau^2}$$

$$D_\gamma(\tau^2 \gg 1, \lambda^2 = 0) = \frac{\alpha}{\pi}\frac{2 + 2\log\left(\tau^2\right)}{3\tau^2}$$

Metz et al., 2104.04207

Milton, PRD 15 (1977) 538

Berends, Gastmans, Annals Phys. 98 (1976) 225



Mechanical radius

$$\langle r^2(\lambda^2)\rangle_{\rm mech} = \frac{6\,D(0,\lambda^2)}{m_e^2\int_0^\infty d\tau^2 D(\tau^2,\lambda^2)}$$

Undefined for $\lambda^2 = 0$

Due to the slow fall-off at large moment transfer one has

$$\langle r^2(\lambda^2\neq 0)\rangle_{\rm mech} = 0$$

Position space

$\Delta^0 = 0$

$$\hat{D}_i(\rho,\lambda^2) = \int \frac{d^3\boldsymbol{\tau}}{(2\pi)^3} e^{-i\boldsymbol{\tau}\cdot\boldsymbol{\rho}} D_i(\tau^2,\lambda^2) = \frac{1}{2\pi^2\rho}\,\mathrm{FST}\left(\tau D_i(\tau^2,\lambda^2);\tau,\rho\right)$$

$$\hat{\bar{C}}_i(\rho,\lambda^2) = \int \frac{d^3\boldsymbol{\tau}}{(2\pi)^3} e^{-i\boldsymbol{\tau}\cdot\boldsymbol{\rho}} \bar{C}_i(\tau^2,\lambda^2)$$

$$\hat{D}_i(\rho,\lambda^2) = \int_0^1 dx \int_0^{1-x} dy\, \frac{f_i(x,y)}{4\pi\rho}\, e^{-\rho\sqrt{a_i(x,y,\lambda^2)}}$$

$$\hat{p}_i(\rho,\lambda^2) = \frac{p_i(\rho,\lambda^2)}{m_e^4} = \frac{1}{6\rho^2}\frac{d}{d\rho}\rho^2\frac{d}{d\rho}\hat{D}_i(\rho,\lambda^2) - \hat{\bar{C}}_i(\rho,\lambda^2)$$

$$\hat{s}_i(\rho,\lambda^2) = \frac{s_i(\rho,\lambda^2)}{m_e^4} = -\frac{\rho}{4}\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}\hat{D}_i(\rho,\lambda^2)$$

Metz et al., 2104.04207 | Polyakov, Schweitzer, IJMPA 33 (2018) 1830025 | Lorcé et al., EPJC 79 (2019) 89

Position space

$$\hat{\bar{C}}_i(\rho, \lambda^2) = \phi_i(\lambda^2)\,\frac{\delta'(\rho)}{\rho} \qquad \bar{C}_i(\tau^2, \lambda^2) = \text{constant}$$

Von Laue condition (which directly follows from EMT conservation)

$$\int_0^\infty d\rho\rho^2 \hat{p}(\rho, \lambda^2) = \frac{1}{12\pi}\int_0^\infty d\tau\tau^3 D(\tau^2, \lambda^2)\delta'(\tau) = 0$$

$$\bar{C}_e(\tau^2, \lambda^2) + \bar{C}_\gamma(\tau^2, \lambda^2) = 0 \quad \forall \tau^2, \lambda^2$$

$$\int_0^\infty d\rho\rho^2 \hat{p}_{i,D}(\rho, \lambda^2) = \frac{1}{12\pi}\int_0^\infty d\tau\tau^3 D_i(\tau^2, \lambda^2)\delta'(\tau) = 0$$

Ignoring the $\bar{C}_i$ form factor,
the separate electron and photon contributions are conserved

Metz et al., 2104.04207 | Polyakov, Schweitzer, IJMPA 33 (2018) 1830025 | Lorcé et al., EPJC 79 (2019) 89

Position space

$$\hat{p}_{i,\mathrm{fin.}}(\rho,\lambda^2) = \int_0^1 dx \int_0^{1-x} dy\ e^{-\rho\sqrt{a_i(x,y,\lambda^2)}} f_i(x,y)\,\frac{a_i(x,y,\lambda^2)}{24\pi\rho}$$

$$\hat{p}_{i,D,\mathrm{sing.}}(\rho) = \frac{\delta'(\rho)}{12\pi\rho}\int_0^1 dx \int_0^{1-x} dy\ f_i(x,y)$$

$$\hat{s}_{i,\mathrm{fin.}}(\rho,\lambda^2) = -\int_0^1 dx \int_0^{1-x} dy\ e^{-\rho\sqrt{a_i(x,y,\lambda^2)}} f_i(x,y)\,\frac{3+3\sqrt{a_i(x,y,\lambda^2)}\rho + a_i(x,y,\lambda^2)\rho^2}{16\pi\rho^3}$$

$$\hat{s}_{i,\mathrm{sing.}}(\rho) = \frac{3\delta(\rho)-\rho\delta'(\rho)}{8\pi\rho^2}\int_0^1 dx \int_0^{1-x} dy f_i(x,y)$$

In the large-distance limit we find

$$\hat{p}(\rho\to\infty,\lambda^2=0) \simeq \hat{p}_\gamma(\rho\to\infty,\lambda^2=0) = \frac{\alpha}{24\pi\rho^4} + \ldots$$

$$\hat{s}(\rho\to\infty,\lambda^2=0) \simeq \hat{s}_\gamma(\rho\to\infty,\lambda^2=0) = -\frac{\alpha}{4\pi\rho^4} + \ldots$$

In agreement with the known asymptotic limits

In agreement with the photon dominance in the low-t region

Metz et al., 2104.04207 | Donoghue et al., PLB 529 (2002) 132

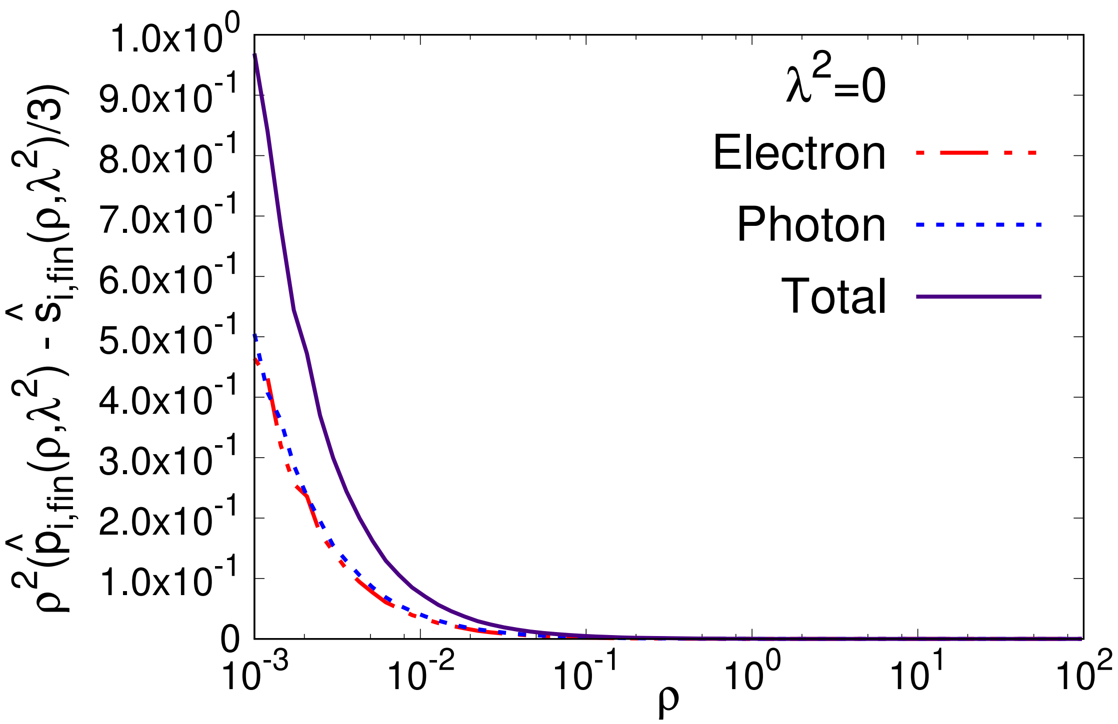
Force distributions

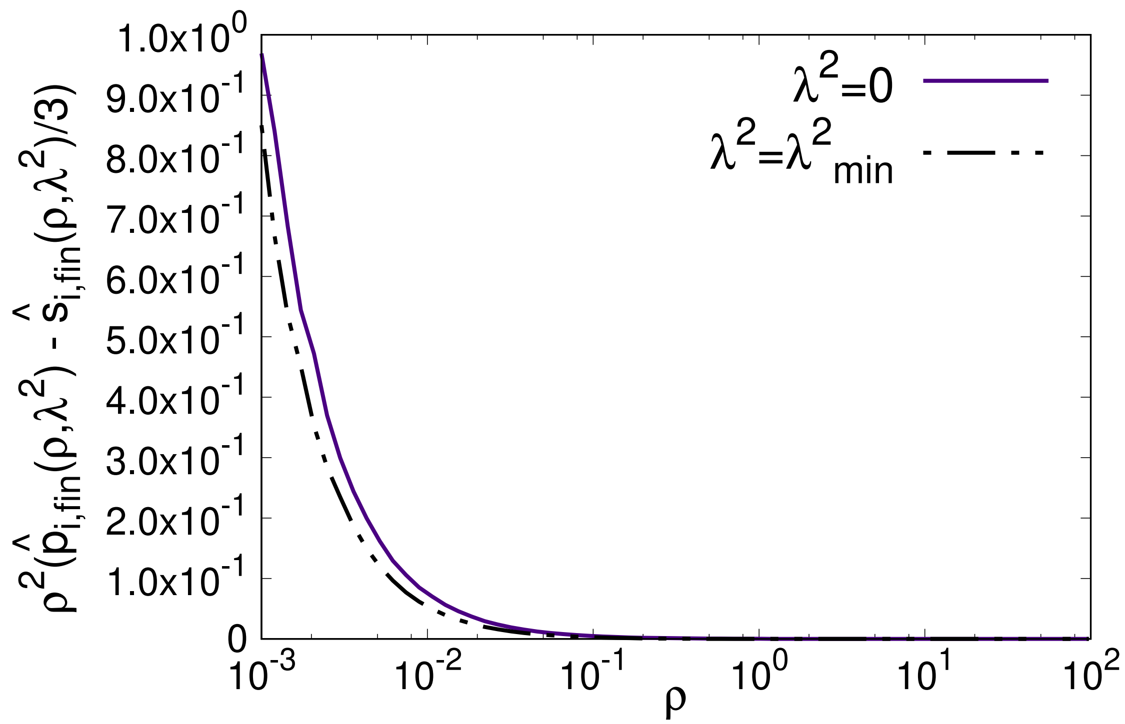
Forces per unit area
experienced by a spherical shell of radius ρ

$$\frac{dF_n}{dS_n} = \frac{2}{3}\hat{s}(\rho,\lambda^2) + \hat{p}(\rho,\lambda^2)$$

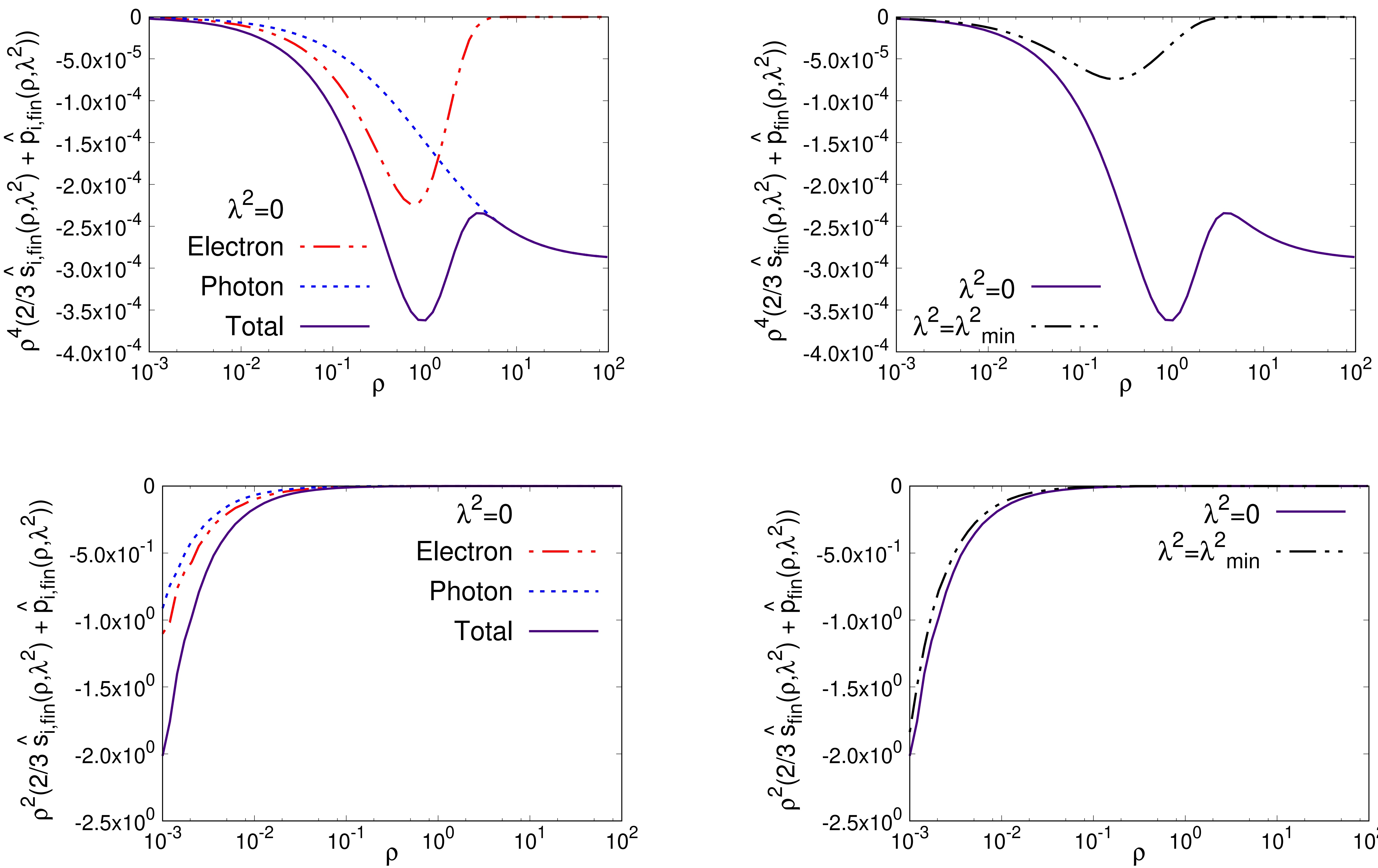
$$\frac{dF_t}{dS_t} = \hat{p}(\rho,\lambda^2) - \frac{1}{3}\hat{s}(\rho,\lambda^2)$$

Normal force





Tangential force



Conclusions

The D-term is as a fundamental quantity
as the mass, the spin, etc…

QED corrections to the D(t) form factor
are in principle present also in QCD calculations

One-loop QCD calculations for a quark state are identical to QED,
provide with the appropriate color factor

From the D(t) form factor one can derive:

The mechanical radius The energy radius

The pressure and shear distributions