

## Near-Threshold Production of Heavy Quarkonium

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# Outline

- Introduction
- Gravitational form factors
- QCD interaction of a color dipole
- Photoproduction of  $J/\psi$
- Polarization and spin asymmetry
- Conclusions and outlook

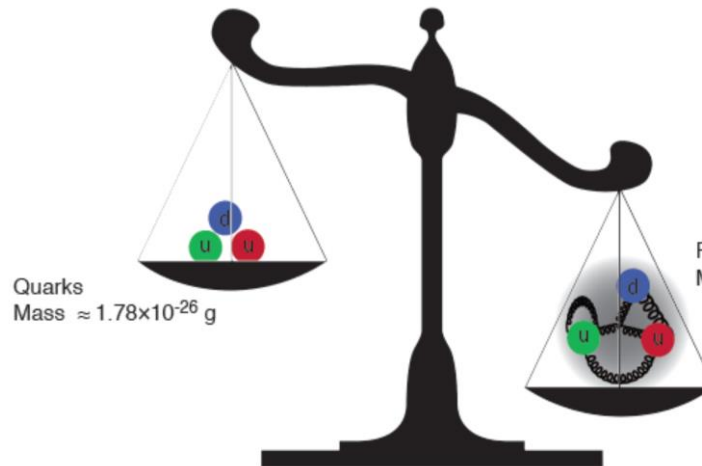
# Introduction to proton mass

-mass without mass

$$m_u \approx 2.2 \text{ MeV}$$

$$m_d \approx 4.7 \text{ MeV}$$

$$m_g = 0$$

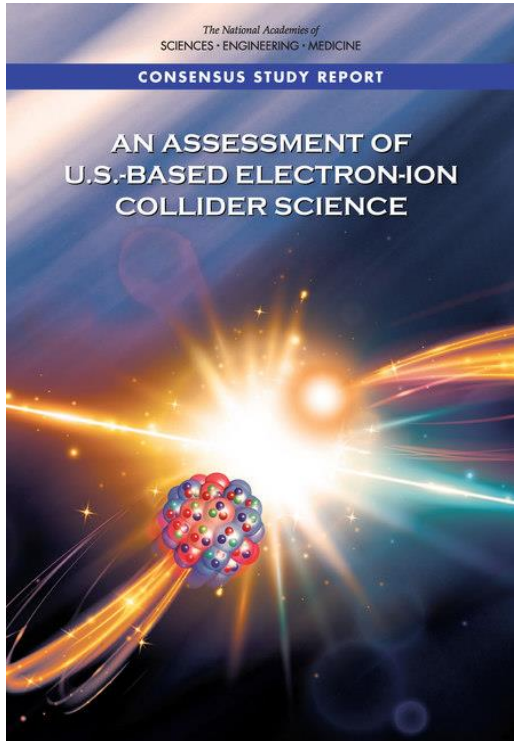


$$m_p \approx 938 \text{ MeV}$$

Most of the proton mass comes from dynamical energy!

# Introduction to proton mass

-importance scientific goal of EIC



## Finding 1:

An EIC can uniquely address three profound questions about nucleons--neutrons and protons--and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

# Introduction to proton mass

-previous works on heavy quarkonium production

- Vector dominance model

D. Kharzeev (1996),

D. Kharzeev et. al. (1999),

- Dispersive analysis

O. Gryniuk and M. Vanderhaeghen (2016)

O. Gryniuk et. al. (2020)

- Holographic QCD

Y. Hatta and D.-L. Yang (2018)

Y. Hatta, A. Rajan and D.-L. Yang (2019)

K. A. Mamo and I. Zahed (2020,2021)

- pQCD analysis at large momentum transfer

P. Sun, X.-B. Tong and F. Yuan (2021)

# Two important questions

- Quantum anomalous energy (QAE) X. Ji (1995),  
X. Ji, Y. Liu and A. Schafer (2021)

$$H_{\text{QCD}} = \int d^3\mathbf{x} T^{00}(0, \mathbf{x}) = H_q + H_m + H_g + H_a$$

Quark energy   Quark mass   Gluon energy   QAE

$$H_a = \int d^3\mathbf{x} \frac{9\alpha_s}{16\pi} (\mathbf{E}^2 - \mathbf{B}^2)$$

- Mass radius D. Kharzeev (2021), X. Ji (2021)

mass  $M = \int d^3\mathbf{x} \langle \mathbf{P} = 0 | T^{00}(0, \mathbf{x}) | \mathbf{P} = 0 \rangle$

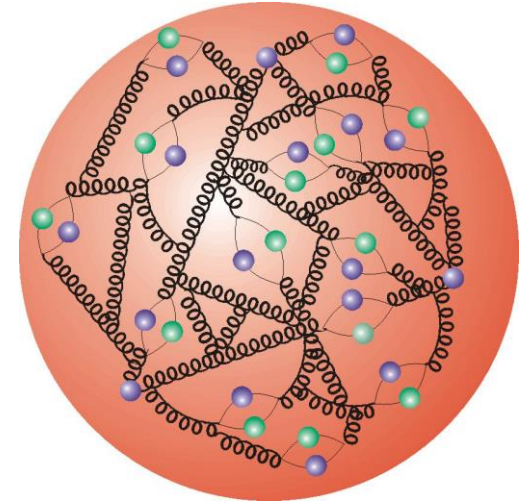
mass form factors  $G_m(Q^2) \langle P' | T^{00} | P \rangle = \bar{u}(P') G_m(-\Delta^2) u(P)$  (Breit frame)

mass radius  $\langle r^2 \rangle_m = -6 \frac{dG_m(Q^2)/M_N}{dQ^2} \Big|_{Q^2=0}$

# Gravitational form factors

Matrix element of EMT can be written into gravitation form factors. [X. Ji \(1996\)](#)

$$\begin{aligned} \langle P' | T_{q,g}^{\mu\nu} | P \rangle = & \bar{u}(P') \left[ A_{q,g}(t) \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g}(t) \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M_N} \right. \\ & \left. + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M_N} + \bar{C}_{q,g}(t) M_N g^{\mu\nu} \right] u(P) . \end{aligned}$$



Importance to mass structure

$$\text{QAE } H_a = \langle P | T_\mu^\mu | P \rangle = \frac{1}{4} (A_q(0) + A_g(0)) M_N$$

$$\text{Mass form factors } G_m(Q^2) = M_N A(Q^2) - B(Q^2) \frac{Q^2}{4M_N} + C(Q^2) \frac{Q^2}{M_N}$$

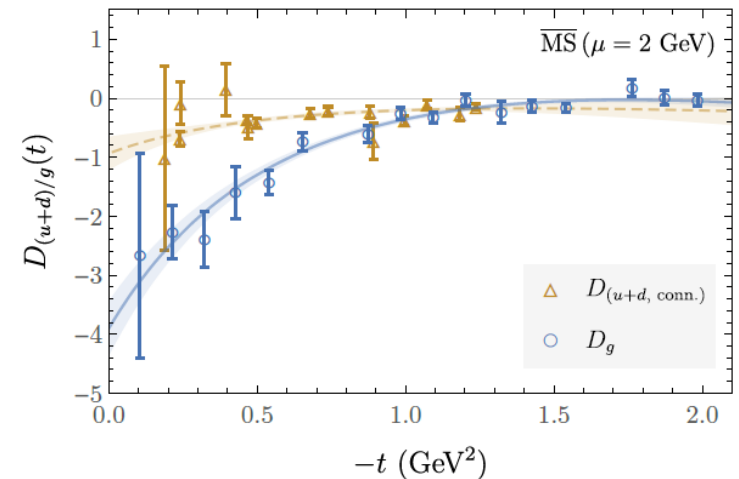
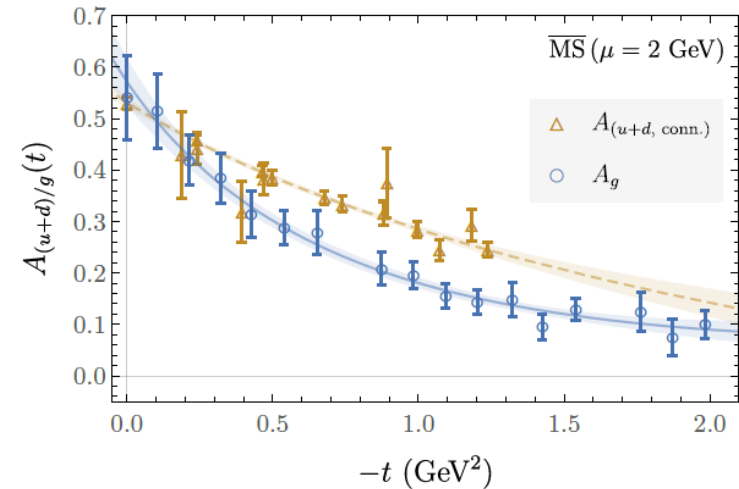
# Lattice calculation

Hagler et. al. (2008), Shanahan et. al. (2018)

Gravitational form factors  
calculated with lattice QCD.

$$\langle r^2 \rangle_m \sim (0.74 \text{ fm})^2$$

Dominated by gluon contribution





# Color dipole as a probe

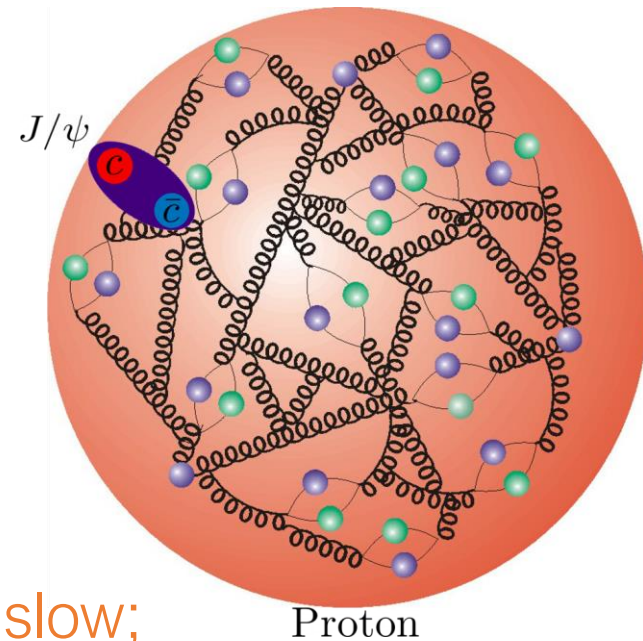
D. Kharzeev (1996),  
M.B. Voloshin (1979)

$$\mathcal{H}_{\text{int}} = -\xi^a \mathbf{r} \cdot \mathbf{E}^a \quad \text{with} \quad \xi^a = t_1^a - t_2^a$$

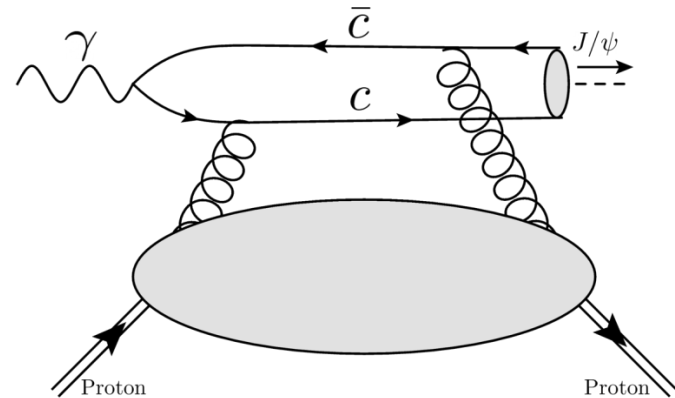
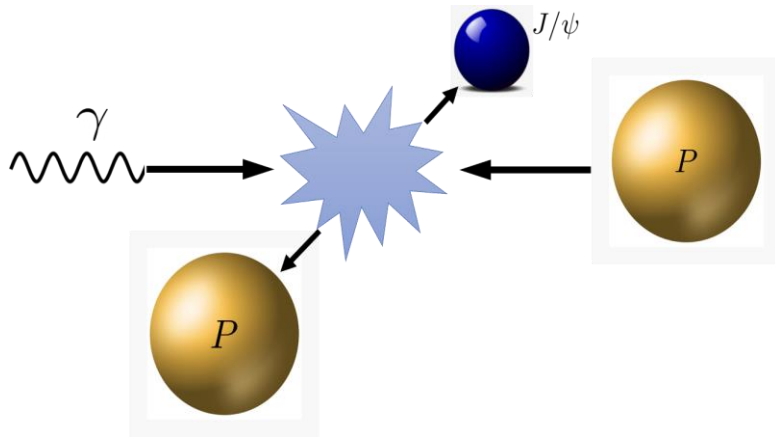
Quadratic Stark effect in QCD –  
measures the color electric field  $\mathbf{E}^2$

The quarkonium needs to be small;  
The relative motion in the quarkonium is slow;  
Perturbatively calculable;

Heavy quarkonium is needed!



# Heavy quarkonium production



**Near threshold**,  $J/\psi$  slowly goes through the proton and measures the  $E^2$  distribution inside.

$$M_P = 938 \text{ MeV},$$

$$M_{J/\psi} = 3097 \text{ MeV},$$

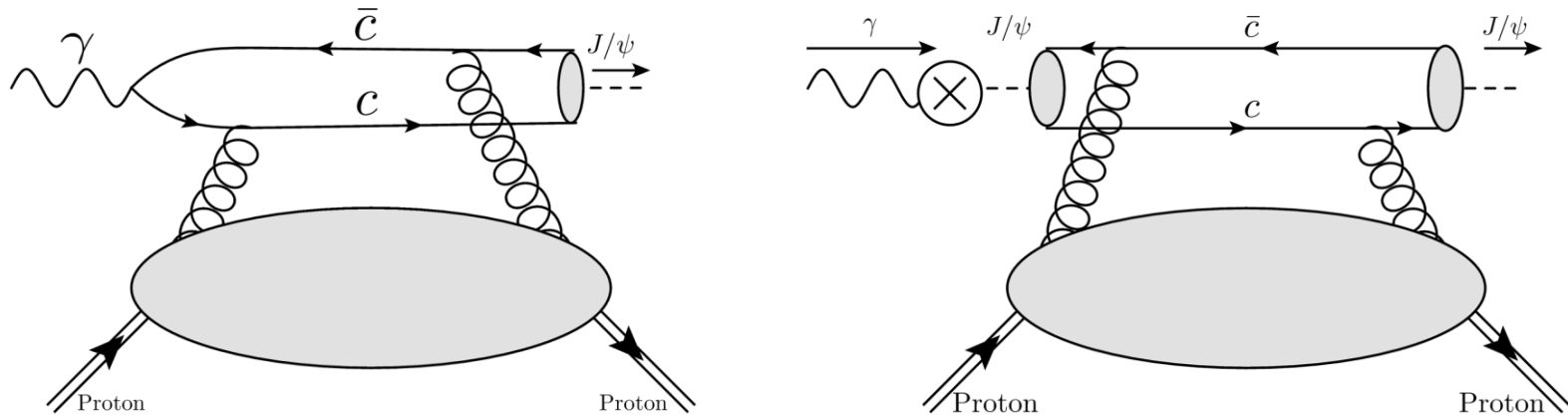
$$W_{th} = 4.035 \text{ GeV} \rightarrow 4.8 \text{ GeV (JLab)}$$

$$E_{th}^\gamma = 8.210 \text{ GeV} \rightarrow 10.8 \text{ GeV (JLab)}$$

# Vector dominance model

V. Barger et. al. (1975),  
D. Kharzeev et. al. (1999)

In VDM, the photoproduction  $\sigma$  is extrapolated to  $t=0$ , and matched to  $J/\psi$ -proton forward scattering  $\sigma$ .

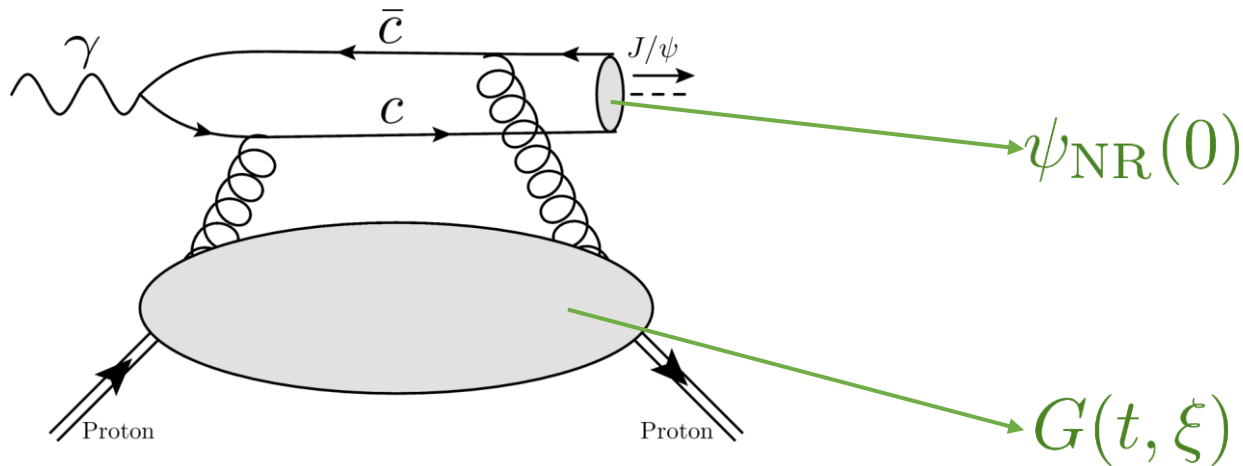


$$\frac{d\sigma_{\gamma N \rightarrow \psi N}}{dt}(s, t=0) = \frac{3\Gamma(\psi \rightarrow e^+e^-)}{\alpha m_\psi} \left( \frac{k_{\psi N}}{k_{\gamma N}} \right)^2 \frac{d\sigma_{\psi N \rightarrow \psi N}}{dt}(s, t=0)$$

$E^2$  Distribution (OPE)

# QCD factorization YG, X. Ji and Y. Liu (2021)

The cross section can be analyzed using QCD factorization in the heavy quark limit near the threshold.

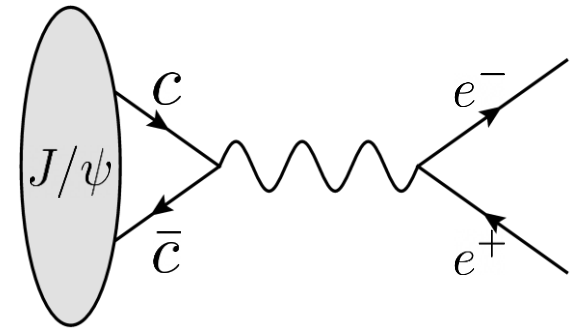


$$\begin{aligned} \frac{d\sigma}{dt} &= \frac{e^2 e_Q^2}{16\pi (W^2 - M_N^2)^2} \frac{1}{2} \sum_{\text{polarization}} |\mathcal{M}(\varepsilon_V, \varepsilon)|^2, \\ &= \frac{\alpha_{\text{EM}} e_Q^2}{4 (W^2 - M_N^2)^2} \frac{(16\pi \alpha_S)^2}{3M_V^3} |\psi_{\text{NR}}(0)|^2 |G(t, \xi)|^2, \end{aligned}$$

# Distribution amplitude

$\psi_{\text{NR}}(0)$ : Non-relativistic distribution amplitude of  $J/\psi$

To the leading order of  $\alpha_s$ ,  $J/\psi$  can be estimated as hydrogen like system.



$$\Gamma(V \rightarrow e^+e^-) = \frac{16\pi\alpha_{\text{EM}}^2 e_Q^2 |\psi_{\text{NR}}(0)|^2}{M_V^2} \left(1 - \frac{16\alpha_s}{3\pi}\right)$$

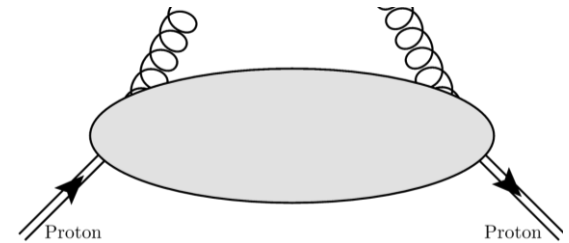
Higher order terms can be analyzed with NRQCD.

R. Van Royen and V. F. Weisskopf(1967)  
B. Gittelman et. al. (1975)  
G. T. Bodwin et. al. (2006)

# Hadronic matrix element

$G(t, \xi)$  : Hadronic matrix element

$$G(t, \xi) = \frac{1}{2\xi} \int_{-1}^1 dx \mathcal{A}(x, \xi) F_g(x, \xi, t) .$$



Hard Kernel:  $\mathcal{A}(x, \xi) \equiv \frac{1}{x + \xi - i0} - \frac{1}{x - \xi + i0} .$

Gluon GPD:  $F_g(x, \xi, t) \equiv \frac{1}{(\bar{P}^+)^2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P' \left| \text{Tr} \left\{ F^{+i} \left( -\frac{\lambda n}{2} \right) F_i^+ \left( \frac{\lambda n}{2} \right) \right\} \right| P \right\rangle ,$

Consistent with leading twist deeply virtual meson production result

[J. Koempel et. al. \(2011\)](#)

# Kinematics near threshold

$t$  and  $\xi$  are large near threshold.

$$\mathcal{A}(x, \xi) \equiv \frac{1}{x + \xi - i0} - \frac{1}{x - \xi + i0}.$$

$$\downarrow$$

$$\frac{1}{2\xi}$$

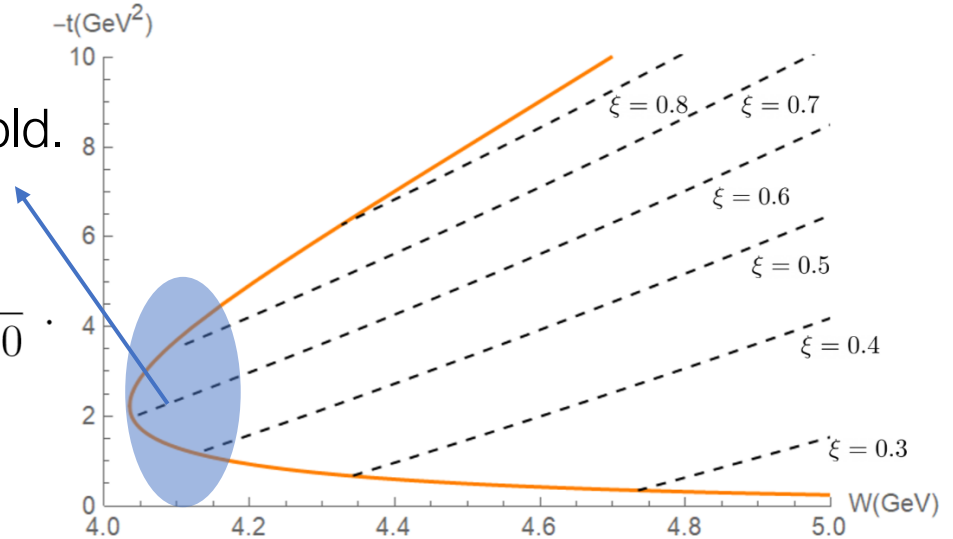


FIG. 2:  $\xi$  on the  $(W, -t)$  plane in the kinematically allowed region with  $M_{I/\gamma b} = 3.097$  GeV.

Taking the leading moment of  $x$ ,

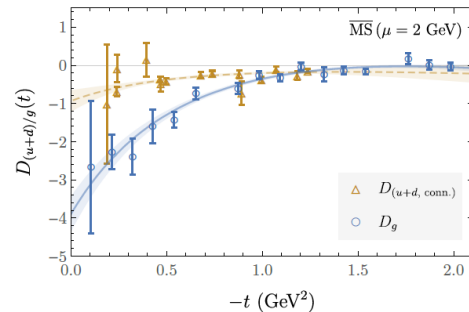
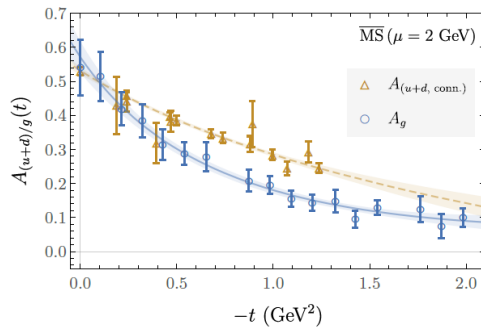
$$G(t, \xi) = \frac{1}{2\xi^2(\bar{P}^+)^2} \langle P' | T_g^{++} | P \rangle ,$$

Measurement of the gravitational form factors.

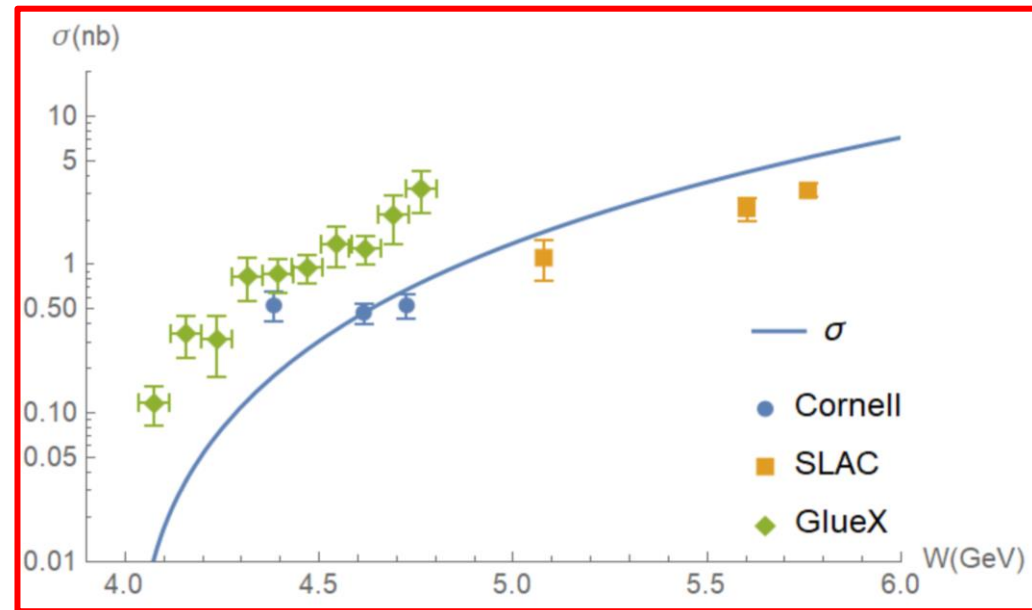
# Connect lattice results with exp.

$$\langle P' | T_g^{++} | P \rangle \sim G(t, \xi)$$

Lattice calculated  
form factors



cross section  $\frac{d\sigma}{dt}$



QCD works quite well!



# Fitting gravitational form factors

$$A_g(t) = \frac{A_g(0)}{\left(1 - \frac{t}{m_A^2}\right)^2},$$

$$C_g(t) = \frac{C_g(0)}{\left(1 - \frac{t}{m_C^2}\right)^2}.$$

$$A_g(0) = 0.58$$

$$m_C = 0.48 \text{ GeV}$$

Fixed

$$C_g(0) = -0.84 \pm 0.82$$

$$m_A = 1.64 \pm 0.11 \text{ GeV}$$

Fit

$$\langle r^2 \rangle_m \approx (0.68 \text{ fm})^2$$

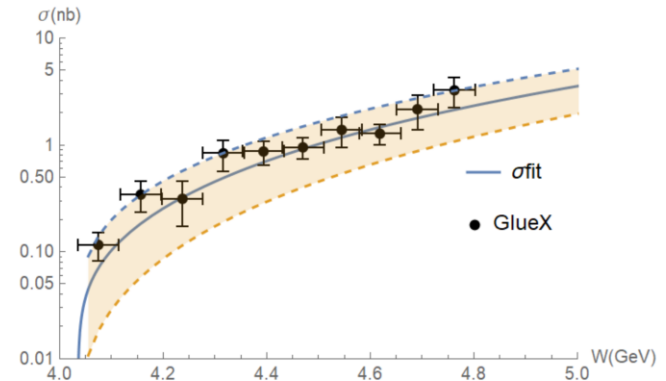


FIG. 6: Fit total cross section for  $J/\psi$  production compared with the total cross section measured at GlueX [36]. The 95% confidence band is shown as the shaded region hereafter.

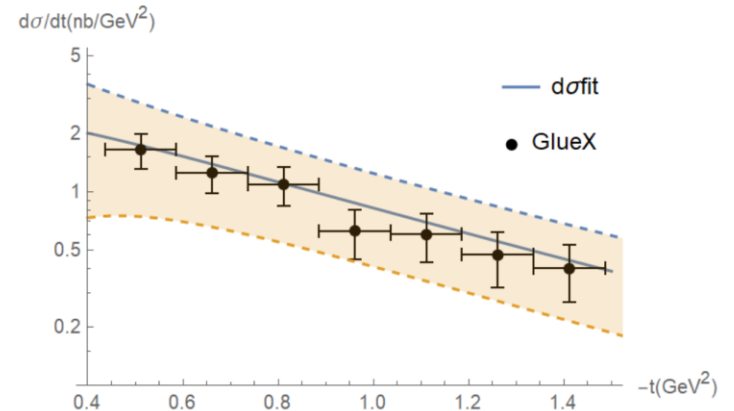


FIG. 7: Fit differential cross section for  $J/\psi$  production compared with the differential cross section at  $W = 4.58$  GeV measured at GlueX [36].

# More $J/\psi$ data at JLab

Z.-E. Meziani

	GlueX HALL D	HMS+SHMS HALL C	CLAS 12 upgrade I HALL B	SoLID HALL A
$J/\psi$ counts (photo-prod.)	~469 published, 10k phase I+II	4k	14k	804k
$J/\psi$ Rate (electro-prod.)			1k	21k
Photon-proton luminosity				
Acceptance	$\sim 2\pi$	$< 3 \times 10^{-4}$	$< 2\pi$	$\sim 2\pi$
When?	Finished	Finished	Proposed	~8 years?

More data at JLab will allow much more precise measurements of gravitational form factors.

# Polarization measurement

The leading amplitude before summing over polarizations reads

$$\mathcal{M}(\varepsilon_V, \varepsilon) = \frac{8\sqrt{2}\pi\alpha_S(M_V)}{M_V^2} \phi^*(0) G(t, \xi) (\varepsilon_V^* \cdot \varepsilon) .$$

- Photon polarization  $\varepsilon^\mu$  and vector meson polarization  $\varepsilon_V^\mu$  are factored out.
- The proton polarizations are contained in the hadronic matrix element  $G(t, \xi)$  -- polarization measurements provide more information on the gravitational form factors.

# Conclusions

- Factorization formula to the leading order in the heavy quark limit.
- Measurements of the gluonic gravitational form factors .
- Proton polarization measurements can be helpful.

# Outlook

- Higher order correction in  $\mathcal{O}\left(\frac{M_N}{M_V}\right)$  and  $\mathcal{O}(\alpha_S)$
- Higher twist effects including three-gluon exchange
- Factorization theorem to higher/all order