QCD EVOLUTION 2021

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# TMDs in the Covariant Parton Model: A Generalized Systematic Approach

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Phys. Rev. D
103, 014024
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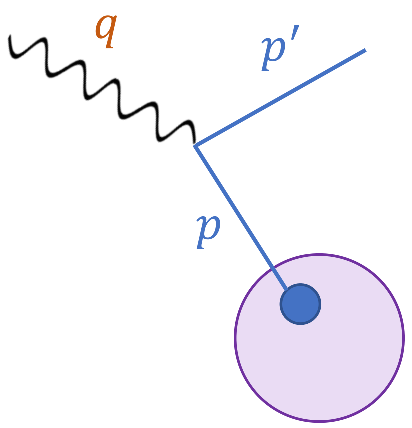
- Covariant Parton Model
- The Generalized CPM
- Numerical Results
- Consistency

## Basics of the CPM

P. Zavada, Phys. Rev. D **55**, 4290

- Partons are free on-shell particles with mass $m_q$
- Partonic structure of nucleon at rest is spherically symmetric
- No need to work in a specific frame including IMF
- It can be shown that $x = \frac{p^0+p^1}{M} \approx x_B$ when $p_T^2\, , m^2 \ll -q^2$
- Also $\mathrm{m}^2/\mathrm{M}^2 \leq x \leq 1$ and $-p_{max} \leq p^1 \leq p_{max} = \frac{M^2-m^2}{2M}$
- The distributions are related to covariant functions $\mathcal{G}(pP)$ and $\mathcal{H}(pP)$ :

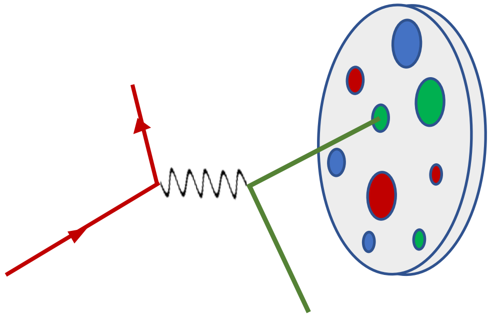
$$F(x) = \int \delta\left(\frac{p^0 + p^1}{M} - x\right) \mathcal{G}(p^0)\, d^3p$$




Nucleon with mass $M$

# Original formulation of covariant parton model based on DIS

Parton model assumption : $d\sigma(e^- p) = \int F(\xi) d\sigma(e^- q) d\xi$



$$P_\alpha P_\beta \frac{F_2}{Pq} - g_{\alpha\beta} F_1 + i\epsilon_{\alpha\beta\mu\nu} q^\mu \left[\frac{S^\nu}{Pq} g_1 + \frac{(Pq\, S^\nu - Sq\, P^\nu)}{(Pq)^2} g_2\right]$$

$$+ A(P_\alpha q_\beta + P_\beta q_\alpha) + B\, q_\alpha q_\beta = \frac{P_0}{M} \int \frac{\mathcal{G}(p^0)}{p^0} \left(2p_\alpha p_\beta - g_{\alpha\beta}\, pq\right) \delta\left((p+q)^2 - m^2\right) d^3p$$

$$+ \frac{P_0}{M} \int \frac{H(p^0)}{p^0} \, i\epsilon_{\alpha\beta\mu\nu} q^\mu\, m\, \omega^\nu \delta\left((p+q)^2 - m^2\right) d^3p$$

⇨ $f_1(\mathrm{x})$, $g_1(x)$, $g_2(x)$, and so $g_T(x)$ are obtained

$$\omega^\mu = -\frac{p \cdot S}{p \cdot P + mM} P^\mu + S^\mu - \frac{M}{m} \frac{p \cdot S}{p \cdot P + mM} p^\mu$$

# PDFs in the covariant parton model (CPM)

$$f_1(x) = \int \frac{d^3p}{p_0}\ \mathcal{G}(p_0)\delta\left(x - \frac{p_0 + p_1}{M}\right)(p_0 + p_1)$$

$$g_1(x) = \int \frac{d^3p}{p_0}\ \mathcal{H}(p_0)\delta\left(x - \frac{p_0 + p_1}{M}\right)\left(p_0 + p_1 - \frac{p_T^2}{p_0 + m}\right)$$

$\mathcal{G}(p^0)$ and $\mathcal{H}(p^0)$ can be calculated using $f_1$ and $g_1$ extractions

$g_2$ and $g_T$ are also obtained… by introducing an auxiliary polarized process

$$h_1(x) = \int \frac{d^3p}{p_0}\ \mathcal{H}(p_0)\ \delta\left(x - \frac{p_0 + p_1}{M}\right)\left(p_0 + p_1 - \frac{p_T^2}{2(p_0 + m)}\right)$$

Efremov, Teryaev, and P. Závada
Phys. Rev. D **70**, 054018

- The Callan-Gross relation is exact $\quad F_2(x) = 2x\, F_1(x)$
- WW-approximation is not an approximation $\quad g_T^a(x) = \int_x^1 \frac{dy}{y} g_1^a(y) + \mathcal{O}(m/M)$

# TMDs in the covariant parton model

Efremov, Schweitzer, Teryaev, and Zavada Phys. Rev. D **80**, 014021

- Considering unintegrated hadronic/quark tensor with $\int d^3p \;\rightarrow\; \int dp_1$
- Finding the write projection and decomposition of the quark tensor
- Using the generic decomposition of the quark polarization vector

$$\omega^{\mu} = -\frac{p\cdot S}{p\cdot P + mM}\,P^{\mu} + S^{\mu} - \frac{M}{m}\frac{p\cdot S}{p\cdot P + mM}\,p^{\mu}$$

- Comparison to correlator projections on the Dirac structures and find TMDs
- T-even twist-2 TMDs and $g_T(x, p_T)$ are obtained

Consistent with colinear results?

A consistent and rather systematic approach could be explored

## Our new formulation

The quark correlation function is described by

$$\Phi^{q}_{ij}(p,P,S) = 2P_0\,\Theta(p_0)\,\delta(p^2-m^2)\,\bar{u}_j(p)\,u_i(p)\;\times\;\begin{cases}\mathcal{G}^q(p\cdot P) & \text{unpolarized case}\\ \mathcal{H}^q(p\cdot P) & \text{polarized case}\end{cases}$$

$$\Phi^{q\,[\Gamma]} = \frac{1}{2}\,\mathrm{Tr}[\Phi^q\,\Gamma] = P_0\,\Theta(p_0)\,\delta(p^2-m^2)\,\mathrm{Tr}\left[(\not{p}+m)\big(\mathcal{G}^q(p\cdot P)+\mathcal{H}^q(p\cdot P)\,\gamma_5\,\not{\omega}(p)\big)\,\Gamma\right]$$

Quark polarization vector

$$\Gamma\;:\;\mathbb{1}\,,\;\gamma^\mu,\;\gamma^\mu\gamma^5,\;\gamma^5\,,\;\sigma^{jk}\gamma^5$$

$$\bar{u}(p)\,\Gamma\,u(p) = \mathrm{Tr}\left[\frac{1}{2}(\not{p}+m)\big(1+\gamma_5\,\not{\omega}(p)\big)\,\Gamma\right]$$

One can obtain TMDs by comparing the generated terms with the corresponding decompositions of the quark correlator on Dirac structures

From standard QPM

**Twist-2**

$$\phi^{q[\gamma^+]} = f_1^q - \frac{\varepsilon^{jk} p_T^j S_T^k}{M} f_{1T}^{\perp q}$$

$$\phi^{q[\gamma^+\gamma_5]} = S_L\, g_1^q + \frac{\vec{p}_T \vec{S}_T}{M} g_{1T}^{\perp q}$$

$$\phi^{q[i\sigma^{j+}\gamma_5]} = S_T^j\, h_1^q + S_L \frac{p_T^j}{M} h_{1L}^{\perp q} + \frac{\kappa^{jk} S_T^k}{M^2} h_{1T}^{\perp q} + \frac{\varepsilon^{jk} p_T^k}{M} h_1^{\perp q}$$

**Twist-3**

$$\phi^{q[\mathbb{1}]} = \frac{M}{P^+}\left[ e^q - \frac{\varepsilon^{jk} p_T^j S_T^k}{M} e_T^{\perp q} \right]$$

$$\phi^{q[\gamma^j\gamma^5]} = \frac{M}{P^+}\left[ S_T^j\, g_T^q + S_L \frac{p_T^j}{M} g_L^{\perp q} + \frac{\kappa^{jk} S_T^k}{M^2} g_T^{\perp q} + \frac{\varepsilon^{jk} p_T^k}{M} g^{\perp q} \right]$$

$$\phi^{q[i\sigma^{+-}\gamma^5]} = \frac{M}{P^+}\left[ S_L\, h_L^q + \frac{\vec{p}_T \cdot \vec{S}_T}{M} h_T^q \right] \qquad \dots \dots$$

$$\Phi_{ij}^q(x, p_T, S) = \iint dp^-\, dp^+\ \Phi_{ij}^q(p, P, S)\ \delta(p^+ - x\, P^+)$$

$$\iint dp^-\, dp^+ \equiv \iint dp_0\, dp_1$$

Compare with the CPM correlator projection, TMDs will show up!

No gluons, no gauge links and so NO access to naïve T-odd TMDs!

## Twist-2 TMDs in CPM

$$f_1^q = \int \{dp_{\mathcal{G}}^1\}\,[Mx(p^0+m)]$$

$$g_1^q = \int \{dp_{\mathcal{H}}^1\}\,[Mx(p^0+m) - p_T^2]$$

$$g_{1T}^{\perp q} = \int \{dp_{\mathcal{H}}^1\}\,[M(Mx+m)]$$

$$h_1^q = \int \{dp_{\mathcal{H}}^1\}\,[m(p^0+m) - p_T^2/2]$$

$$h_{1L}^{\perp q} = \int \{dp_{\mathcal{H}}^1\}\,[-M(Mx+m)]$$

$$h_{1T}^{\perp q} = \int \{dp_{\mathcal{H}}^1\}\,[-M^2]$$

$$\{dp_{\mathcal{G}}^1\} = \frac{dp^1}{p^0}\frac{\mathcal{G}^q(p^0)}{p^0+m}\,\delta\left(x - \frac{p^0+p^1}{M}\right)$$

$$\{dp_{\mathcal{H}}^1\} = \frac{dp^1}{p^0}\frac{\mathcal{H}^q(p^0)}{p^0+m}\,\delta\left(x - \frac{p^0+p^1}{M}\right)$$

Unpolarized

Polarized

One can readily see relations between the TMDs

## Twist-3 TMDs in CPM

$$f^{\perp q} = \int \{dp_{\mathcal{G}}^1\}\,[M(p^0+m)]$$

$$e^q = \int \{dp_{\mathcal{G}}^1\}\,[m(p^0+m)]$$

$$g_T^q = \int \{dp_{\mathcal{H}}^1\}\,[m(p^0+m) + p_T^2/2]$$

$$g_L^{\perp q} = \int \{dp_{\mathcal{H}}^1\}\,[M(Mx - p^0)]$$

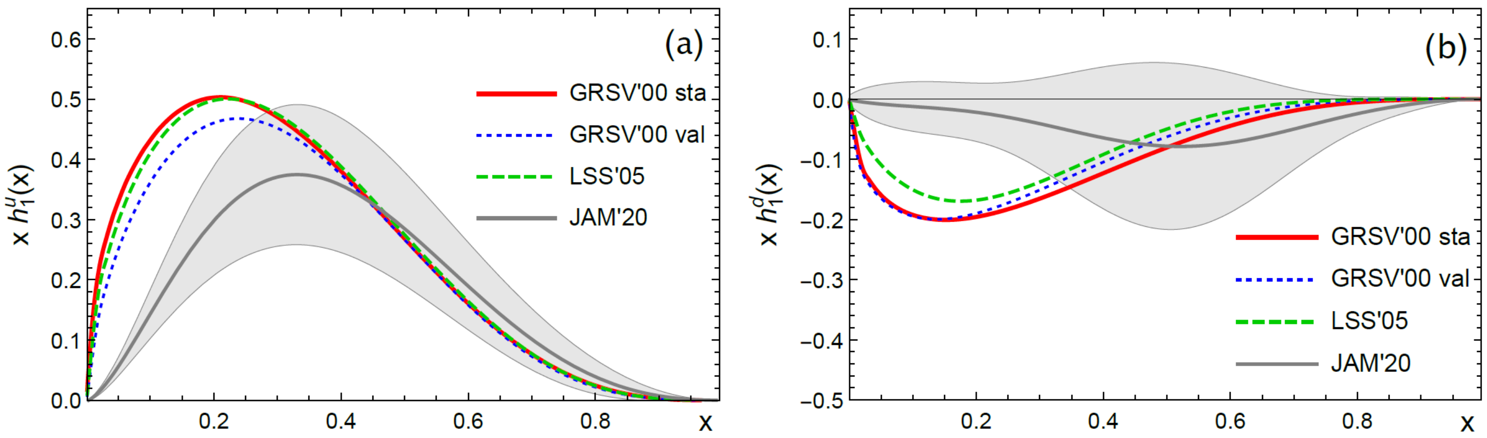
$$g_T^{\perp q} = \int \{dp_{\mathcal{H}}^1\}\,[M^2]$$

$$h_L^q = \int \{dp_{\mathcal{H}}^1\}\,[m(p^0+m) + p_T^2]$$

$$h_T^q = \int \{dp_{\mathcal{H}}^1\}\,[M(p^0 - Mx)]$$

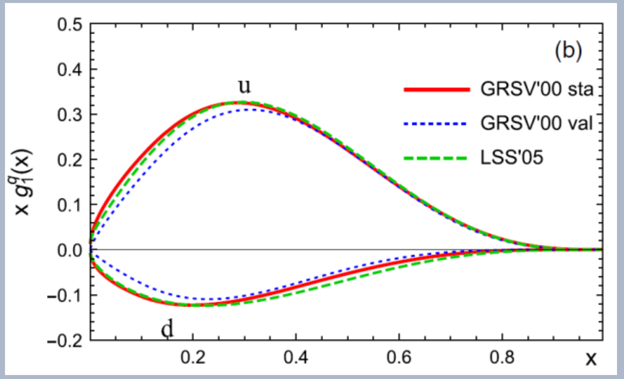
$$h_T^{\perp q} = \int \{dp_{\mathcal{H}}^1\}\,[M(p^0+m)]$$

# Leading twist results

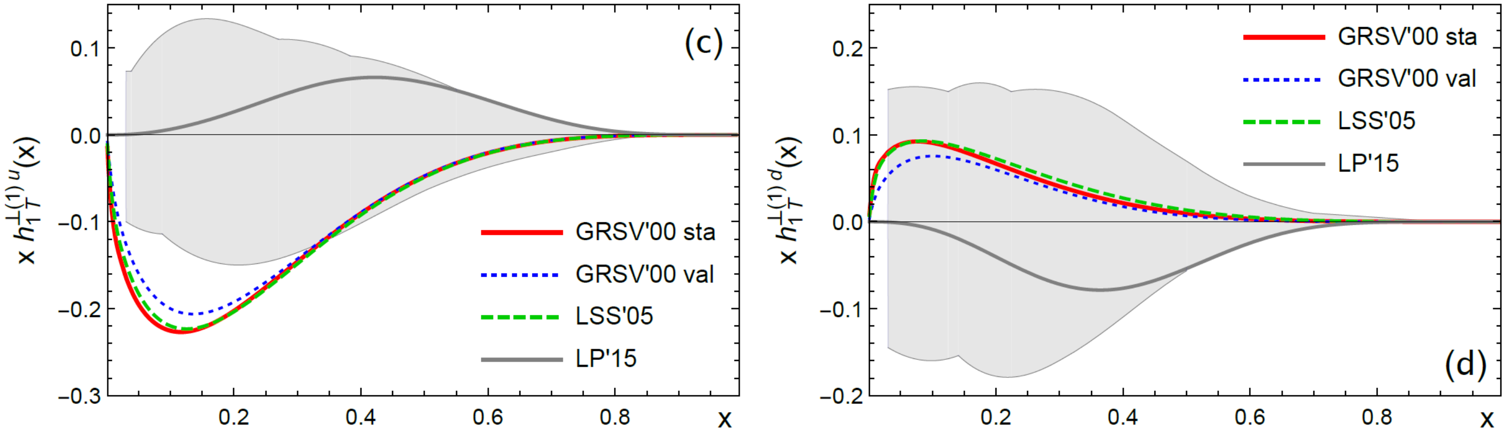



Cammarota *et al*, PRD 102, 054002 (2020)

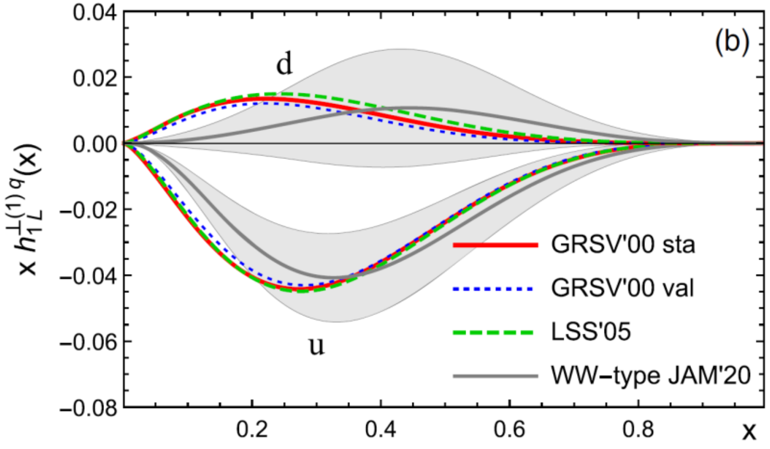
Input $g_1$ which inverts $\mathcal{H}(p_0)$



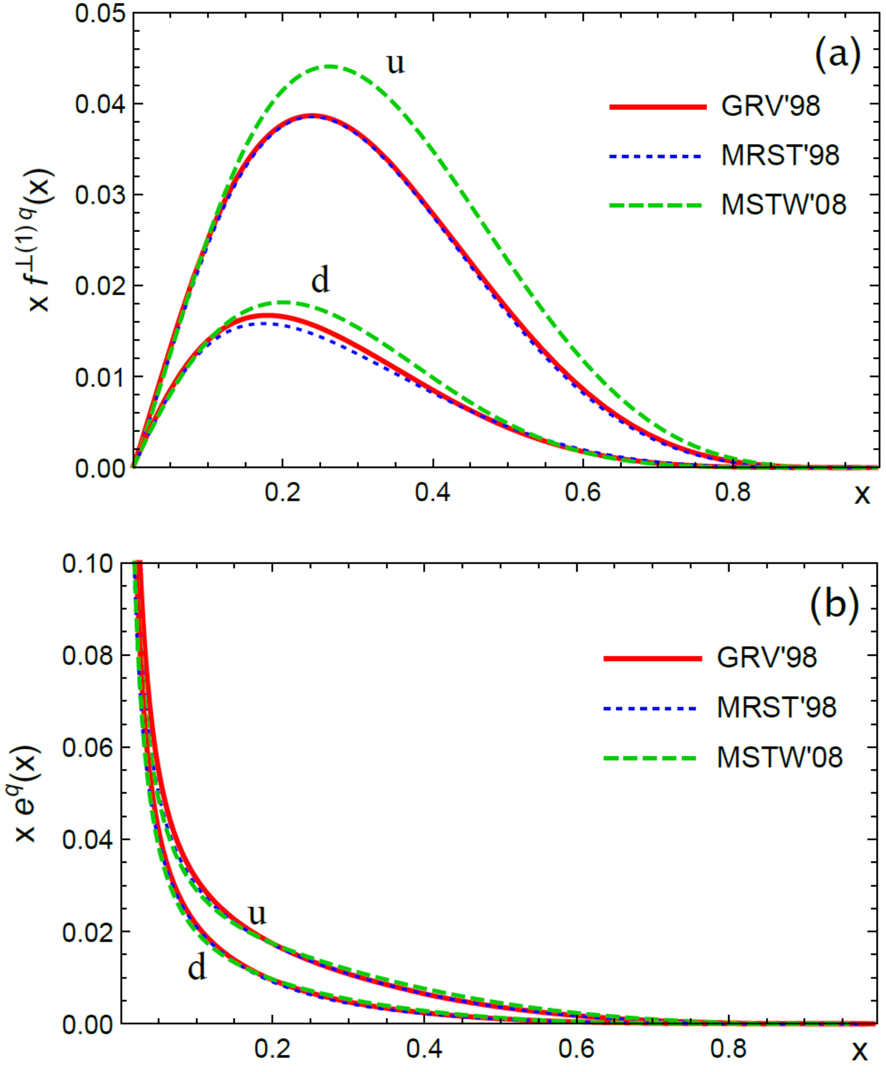

PRD 63, 094005 (2001)
PRD 73, 034023 (2006)




Lefky and Prokudin, PRD 91, 034010 (2015)



## Subleading unpolarized TMDs

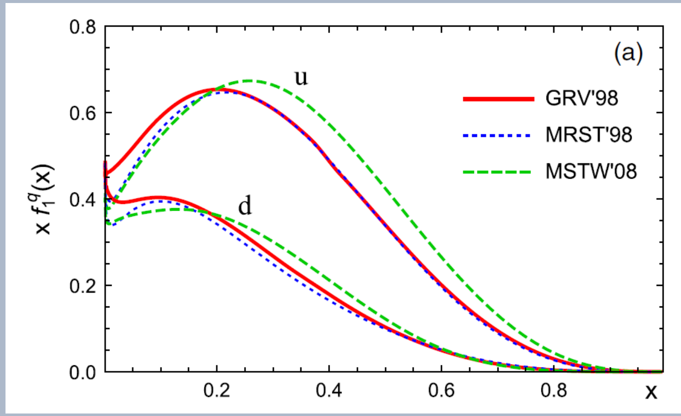



$$x\, f^{\perp q} = f_1$$

$$x\, e^q = \frac{m}{M} f_1$$

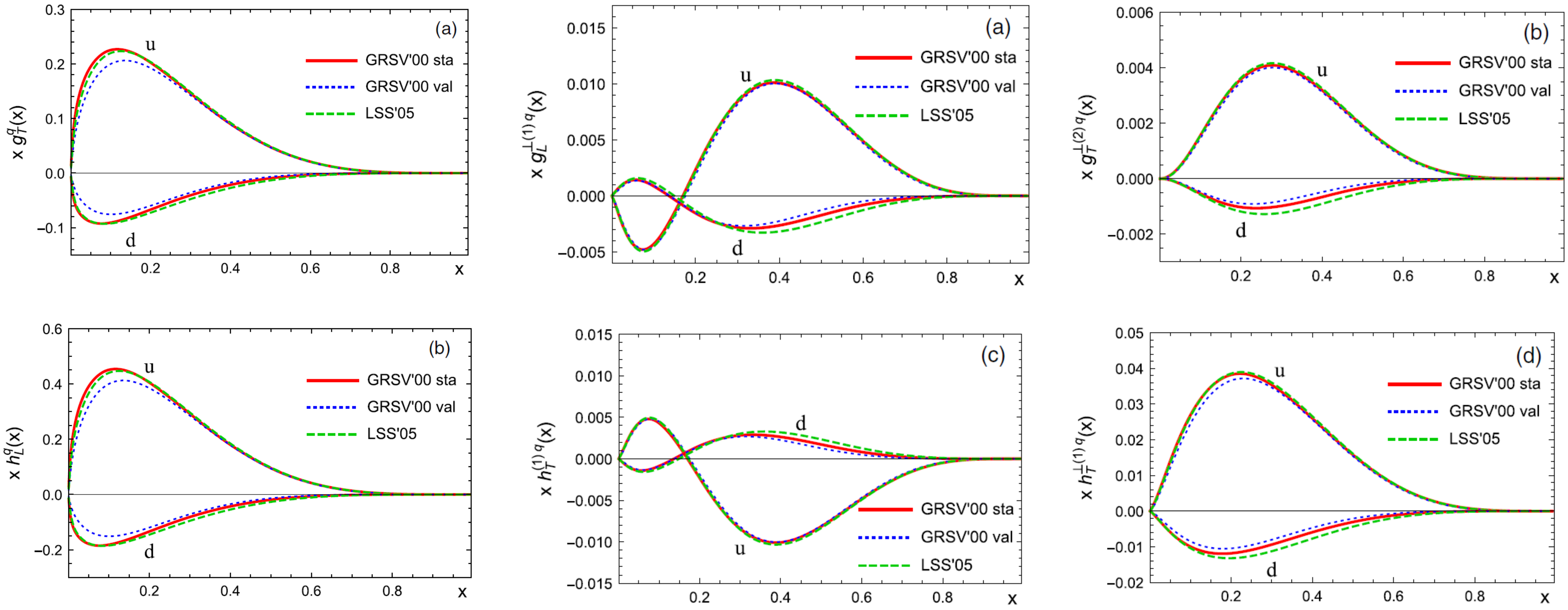
We used $m = 5\ MeV$. Becomes important at extremely small x

Input $f_1$ which inverts $\mathcal{G}(p_0)$




Eur. Phys. J. C 5, 461 (1998)
Eur. Phys. J. C 4, 463 (1998)
Eur. Phys. J. C 63, 189 (2009)

## Subleading polarized TMDs



## New perspective: Lorentz Invariant Amplitudes

$$\Phi^q(P,p,S) = MA_1^q + \not{P}A_2^q + \not{p}A_3^q + \frac{i}{2M}[\not{P},\not{p}]A_4^q + i(p\cdot S)\gamma_5 A_5^q + M\not{S}\gamma_5 A_6^q + \frac{p\cdot S}{M}\not{P}\gamma_5 A_7^q + \frac{p\cdot S}{M}\not{p}\gamma_5 A_8^q + \frac{[\not{P},\not{S}]}{2}\gamma_5 A_9^q + \frac{[\not{p},\not{S}]}{2}\gamma_5 A_{10}^q + \frac{p\cdot S}{2M^2}[\not{P},\not{p}]\gamma_5 A_{11}^q + \frac{1}{M}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho S_\sigma A_{12}^q + \mathcal{O}(B_i)$$

$B_i$ amplitudes are absent because Wilson lines in the CPM

$$\Phi^{q\,[\Gamma]} = \frac{1}{2}\,\mathrm{Tr}[\Phi^q\,\Gamma] = \mathrm{P}_0\,\Theta(p_0)\,\delta(p^2-m^2)\,\mathrm{Tr}\left[(\not{p}+m)\left(\mathcal{G}^q(p\cdot P) + \mathcal{H}^q(p\cdot P)\,\gamma_5\,\not{a}(p)\right)\Gamma\right]$$

- $A_4$, $A_5$ and $A_{12}$ are zero (naïve T-odd)
- All other $A_i$ amplitudes are obtained
- We find only two independent amplitudes (by choice):

$$A_1 = \frac{m}{M}A_3\,,\ A_6 = \frac{m}{M}A_{10}\,,\ A_7 = \frac{m}{p^0+m}A_3\,, \ldots$$

$$A_3 = \mathrm{P}_0\,\Theta(p_0)\,\delta(p^2-m^2)\,\mathcal{G}^q(pP)$$

$$A_{10} = -\mathrm{P}_0\,\Theta(p_0)\,\delta(p^2-m^2)\,\mathcal{H}^q(pP)$$

D’Alesio, Leader, and Murgia, Phys. Rev. D 81, 036010 found three independent amplitudes. We are currently investigating how our approach compares to theirs.

# Consistency

✓ EOM relation are all satisfied

✓ Lorentz invariant relations (LIRs) hold true

$$g_T^q(x) \overset{\rm LIR}{=} g_1^q(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{\perp(1)q}(x)\,,$$
$$h_L^q(x) \overset{\rm LIR}{=} h_1^q(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)q}(x)\,,$$
$$h_T^q(x) \overset{\rm LIR}{=} -\frac{\mathrm{d}}{\mathrm{d}x} h_{1T}^{\perp(1)q}(x)\,,$$
$$g_L^{\perp q}(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_T^{\perp(1)q}(x) \overset{\rm LIR}{=} 0\,,$$
$$h_T^q(x,p_T) - h_T^{\perp q}(x,p_T) \overset{\rm LIR}{=} h_{1L}^{\perp q}(x,p_T)\,,$$

$$xe = x\tilde{e} + \frac{m}{M} f_1,$$
$$xf^\perp = x\tilde{f}^\perp + f_1,$$
$$xg_T' = x\tilde{g}_T' + \frac{m}{M} h_{1T},$$
$$xg_T^\perp = x\tilde{g}_T^\perp + g_{1T} + \frac{m}{M} h_{1T}^\perp,$$
$$xg_T = x\tilde{g}_T - \frac{p_T^2}{2M^2} g_{1T} + \frac{m}{M} h_1,$$
$$xg_L^\perp = x\tilde{g}_L^\perp + g_{1L} + \frac{m}{M} h_{1L}^\perp,$$
$$xh_L = x\tilde{h}_L + \frac{p_T^2}{M^2} h_{1L}^\perp + \frac{m}{M} g_{1L},$$
$$xh_T = x\tilde{h}_T - h_1 + \frac{p_T^2}{2M^2} h_{1T}^\perp + \frac{m}{M} g_{1T},$$
$$xh_T^\perp = x\tilde{h}_T^\perp + h_1 + \frac{p_T^2}{2M^2} h_{1T}^\perp.$$

✓ WW-type relation are exact

$$g_T^a(x) = \int_x^1 \frac{dy}{y} g_1^a(y) + \tilde{g}'^a_T(x) + \mathcal{O}(m/M)$$

$$h_L^a(x) = 2x \int_x^1 \frac{dy}{y^2} h_1^a(y) + \tilde{h}'^a_L(x) + \mathcal{O}(m/M)$$

✓ linear and non-linear model relations, supported in other model studies

✓ The positivity constraints were already verified in

Efremov, Teryaev, and P. Závada
Phys. Rev. D **70**, 054018

## The CPM features in the new formulation

- All polarized and unpolarized T-even TMDs are systematically obtained,
- all can be expressed in terms of $f_1(x, p_T)$ and $g_1(x, p_T)$ which in turn, they are obtained by the collinear PDFs $f_1(x)$ and $g_1(x)$
- T-odd TMDs are absent
- TMD relations expected in QCD or supported by other quark models are satisfied

## And possibly:

- Exploring the anti-quark distributions
- Generalization to include off-shell-ness effects
- Wish to access T-odd TMDs

THANK YOU!