Mismatches between Perturbative and Non-perturbative Views of Hadronization in QCD

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Mismatches (in $e^+e^- \to$ hadrons):

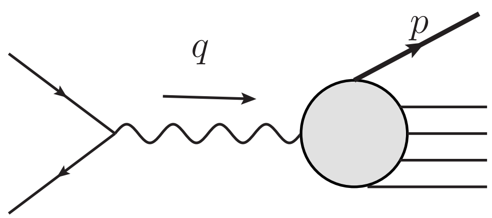
- View of factorization in pQCD v. QCD reality in hadronization of jets etc.
- Insight/intuition of what happens space-time v. Momentum-space calculations and experiments.
- Related: string model v. factorization, MCEG.

Implications

I'll start with the most elementary (and old and simple-minded) ideas, and build up.

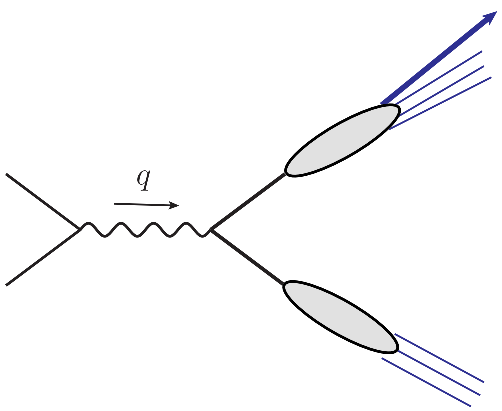
(See JCC & Ted Rogers, arXiv:1801.02704)

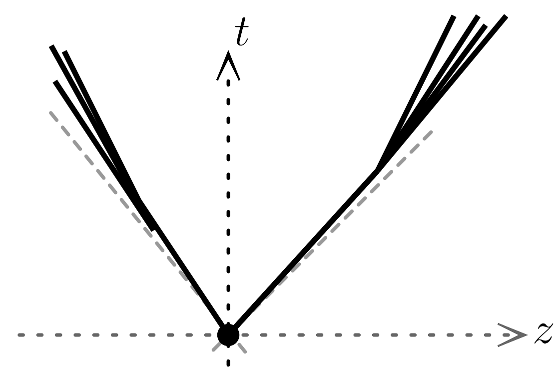
Initial (old) insights for $e^+e^- \to$ hadrons with identified hadrons

E.g.,  at, say, $Q = 100\,\mathrm{GeV}$.

Short-distance core (hence perturbative).
Space-time scale $\sim 1/Q = 10^{-2}\,\mathrm{GeV}^{-1} = 2 \times 10^{-3}\,\mathrm{fm}$.

A first idea/approximation: $q\bar{q}$ production, then hadronization:

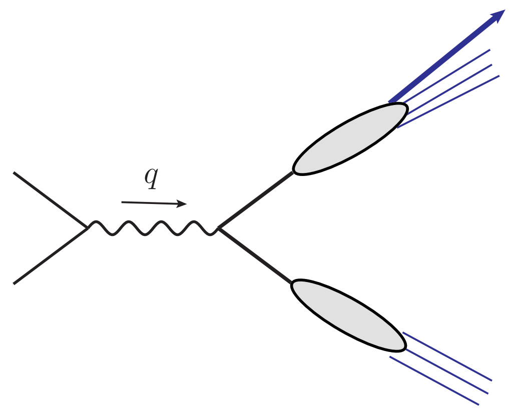


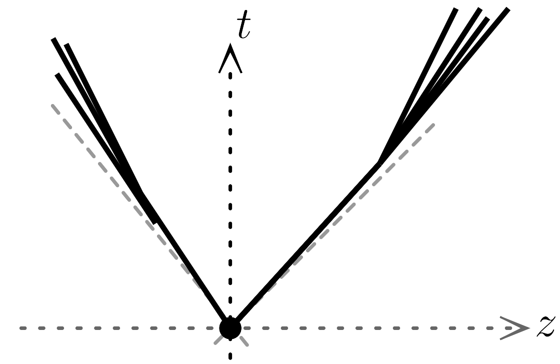


Quark virtuality $M^2 \implies$ (boosted) time-scale $\sim \dfrac{1}{M} \times \dfrac{Q/2}{M}$
$M = 1\,\mathrm{GeV}(??) \implies$ time/distance scale $10\,\mathrm{fm}$.

$\implies$ Big separation of scales. Space-like separation of hadronization regions.

Space-time view II

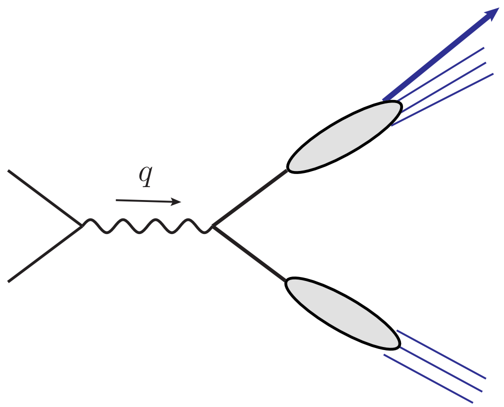


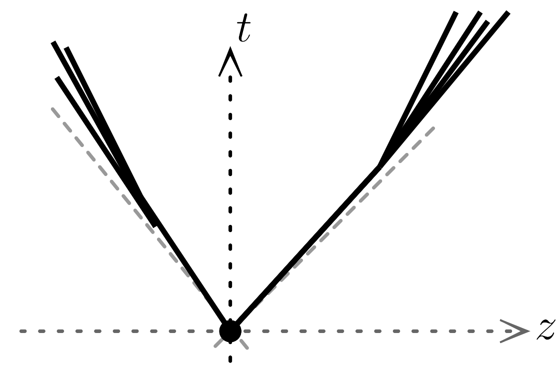


- Space-like separation of jet hadronization.
- Hence independent fragmentation of jets. (Modulo initial entangled spin state.)

$\Longrightarrow$ idea of fragmentation function . . .

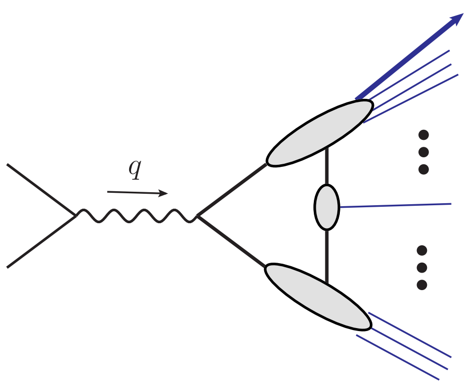
What's wrong with these pictures





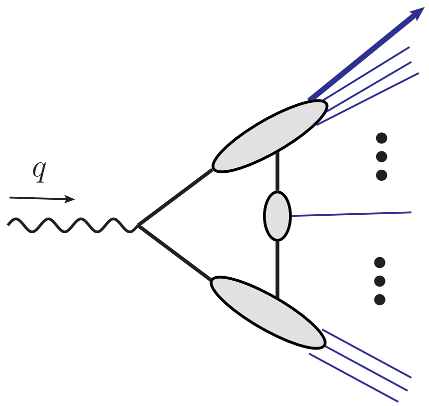
- Jets have quark quantum numbers
- Big rapidity gap between jets
- There's a gluon field between the q and $\bar{q}$

Minimum model scheme:

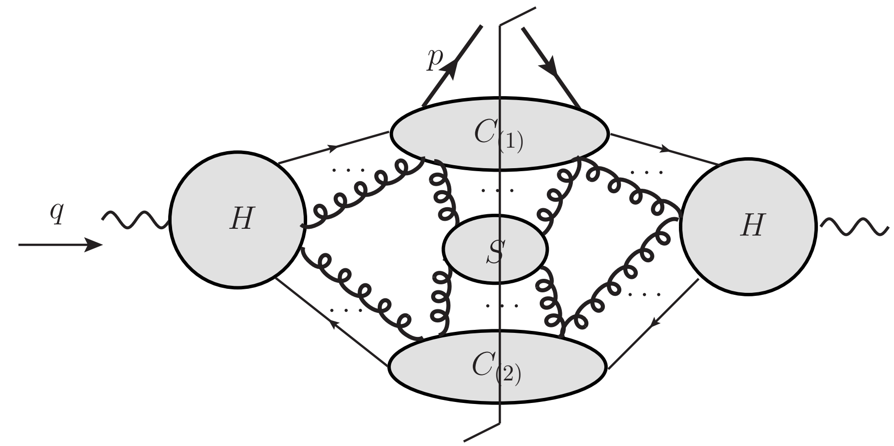


(Complications — see later)

Clash #1: Minimum acceptable model v. pQCD region analysis



× herm. conj.



Exchange of spin J (across big rapidity interval) $\Longrightarrow$ factor Q^{2J-2} w.r.t. LO.

So power Q^0 for gluon exchange, but $1/Q$ for quark exchange.

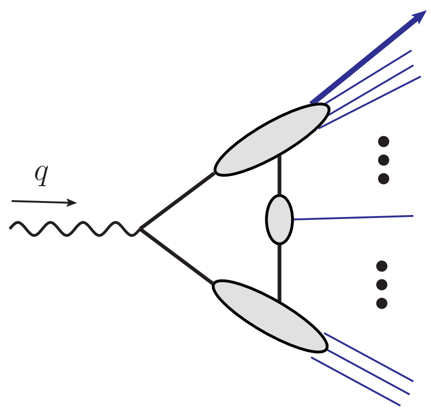
As $Q \to \infty$:

- Quark-exchange graph power-suppressed.
- pQCD leading region graph (gluon exchange) has big rapidity gap(s). (This is starting point of derivation of factorization.)
- At fixed perturbative order of graph, always have big rapidity gaps.
- *Recall: Use of Ward identities on gluons crossing big rapidity ranges was critical to getting factorization. Gives fragmentation functions etc with Wilson lines.*

Clash #2: Cannot exchange "sum" over graphs and $Q \to \infty$

Standard perturbative analysis (factorization *and* OPE) *assumes* it is sufficient to examine $Q \to \infty$ graph-by-graph.

But to avoid large rapidity gaps in any perturbative model, need graphs of minimal order proportional to #(hadrons): $n \propto \ln(Q/m)$, as in



$\times$ herm. conj.

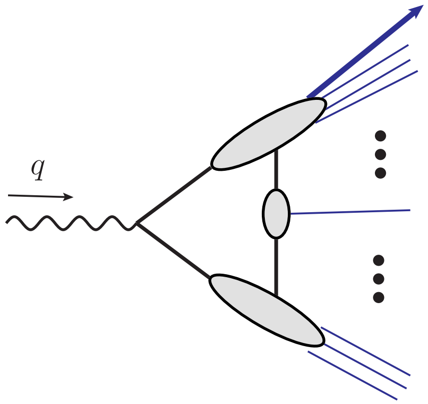
Rough estimate of size w.r.t. LO:

$$\alpha_s^{cn} = \exp[-C\ln(Q/m)\ln(1/\alpha_s)] = \left(\frac{m}{Q}\right)^{C\ln(1/\alpha_s)}$$

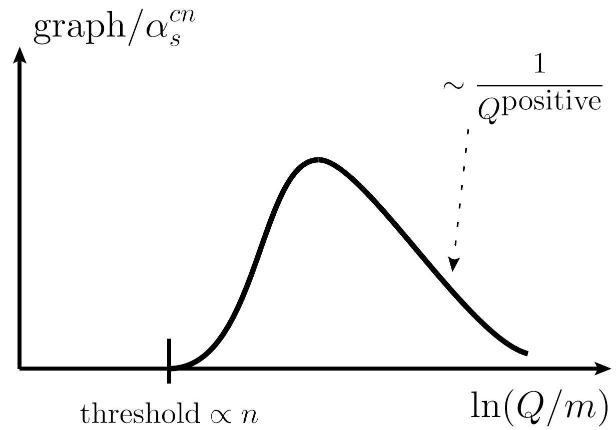
Suppressed if effective coupling weak in IR (e.g., QED). But not in QCD.

pQCD region analysis misses this. (Illegal exchange of sum$_n$ and $\lim_{Q\to\infty}$.)

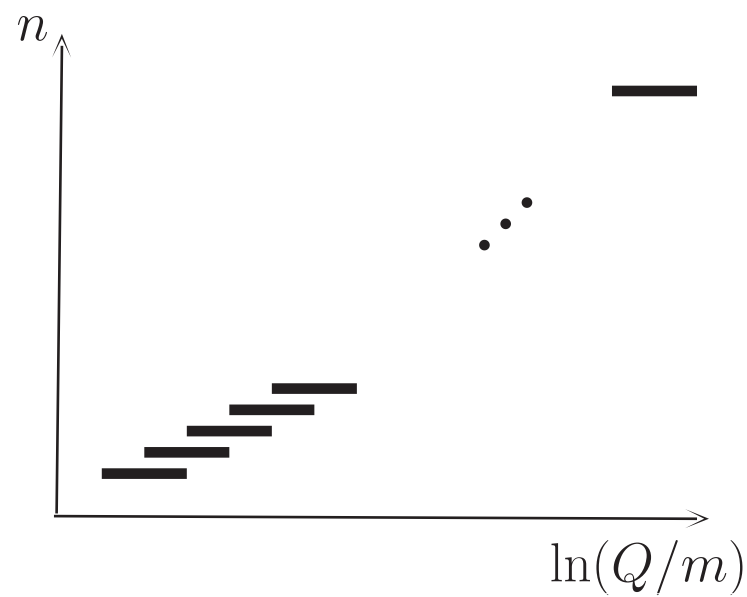
How order of important graphs changes with Q



$\times$ herm. conj. $\Longrightarrow$



Given n: dominant range of Q:

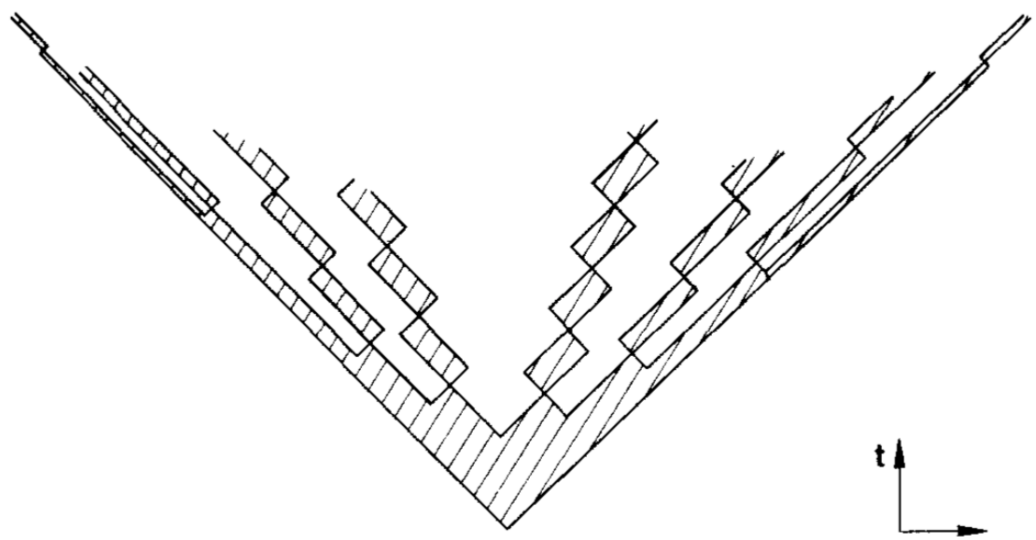


Strong effective coupling in IR: Each graph vanishes as $Q \to \infty$, but sum doesn't.

String model etc of jet hadronization (from 1970s)

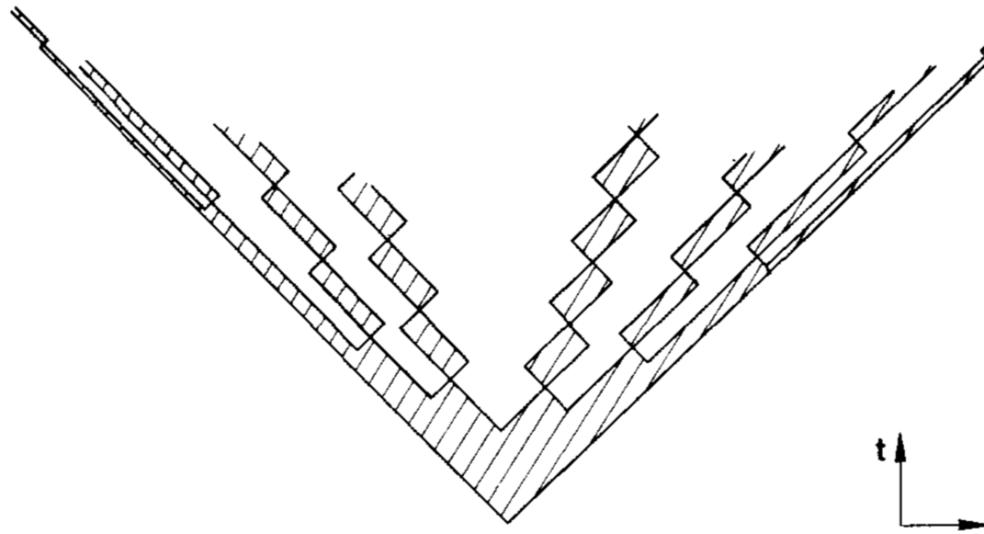
Started with

- Casher, Kogut, Susskind: perturbative view (1973).
- Artru & Mennessier (1974. . .), Lund (. . .): string, yoyo model.
- Feynman & Field (1978)



Basics: – Gluon field collapses to flux tube, constant energy/length
– Quark-pair production at constant rate per unit space-time volume.

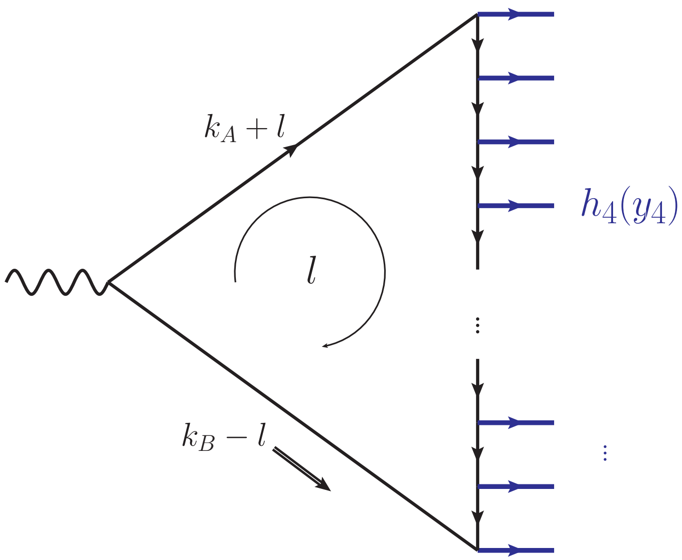
Space-time structure with string hadronization



Qualitative implications for non-perturbative hadronization:

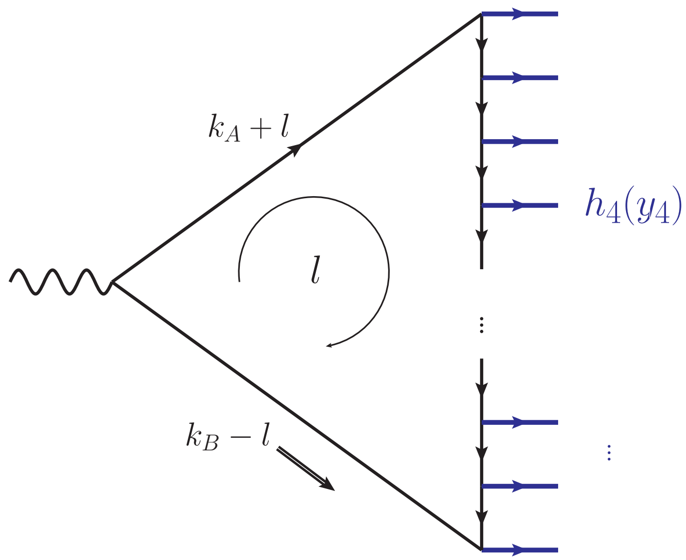
- It is *near* a space-like hyperbola $t^2 - z^2 \sim 1/m^2$.
- It gives hadronization of leading hadrons on the time-dilated time scale Q/m^2.
- Domain of perturbative physics is outside the hyperbola $t^2 - z^2 \lesssim 1/m^2$.
- "Final-state interactions" after perturbative part of process cannot be delimited by a single choice of time coordinate.

Most elementary quark-exchange graphs



- Basic quark momenta $[(+,-,T)]$: $k_A = \left(Q/\sqrt{2}, 0, \mathbf{0}_T\right)$, $k_B = \left(0, Q/\sqrt{2}, \mathbf{0}_T\right)$.
- Final-state rapidity range such that e^y is between about Q/m and m/Q.
- Hadron j of rapidity y has $p_j \sim m\left(e^y, e^{-y}, 1\right)$.
- *If* all lines have virtuality $O(m^2)$
  - Except near extreme rapidity, most of p_j^+ from top, p_j^- from bottom.
  - Hence positive $+$ momentum goes down vertical quark line. $(l^+ > 0)$
  - And positive $-$ momentum goes *up* quark line. $(l^- < 0)$

Elementary quark-exchange graphs in space-time I

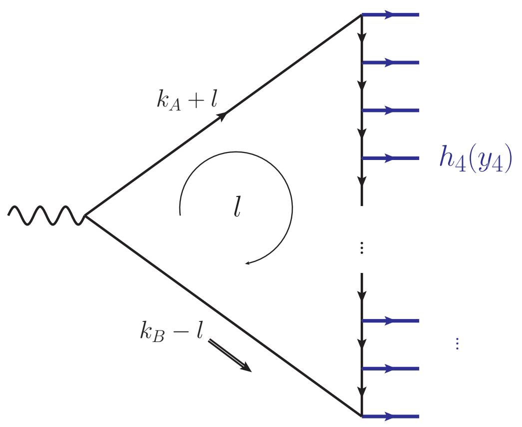


- Flow of $+$ momentum goes down vertical quark line.
- And positive $-$ momentum comes *up* quark line.
- Neglect l^- compared with large k_B^- etc at bottom.
- Similarly for l^+.

Generic denominator for l^- (always with positive k^+ when relevant)

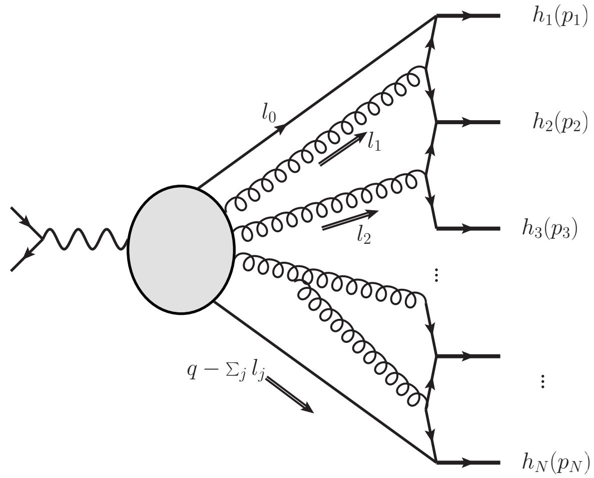
$$\frac{1}{2k^+l^- + \ldots + i\epsilon}$$

Elementary quark-exchange graphs in space-time II



- For each denominator, l^- is neglected or in $\dfrac{1}{2k^+l^- + \ldots + i\epsilon}$ with $k^+ > 0$
- Deform l^- from $O(m^2/Q)$ to $O(m)$.
- Then initial quark lines have virtuality mQ.
- Hence time scale for hadronization is $\dfrac{1}{\sqrt{mQ}} \times \dfrac{Q}{\sqrt{mQ}} = \dfrac{1}{m}$
- *Does not match expectation from string model.*
- *Unless* the vertical line has *effective* propagators and vertices in gluon field, without momentum conservation.

Need gluons (of course)



Main longitudinal momentum components for hadrons are from the horizontal gluons.

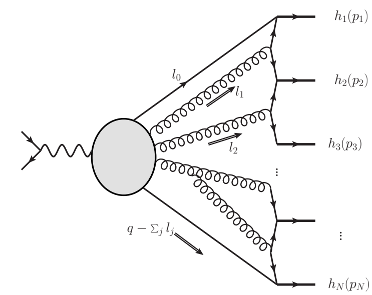
Now space-time structure matches string model, but in a Feynman-graph formulation that covers perturbative domain and models non-perturbative physics.

Clash #3: Space-time localization v. precise beam energies

Experiments have precise beam energies, $\Delta p / p = \text{few} \times 10^{-3}$ or smaller.

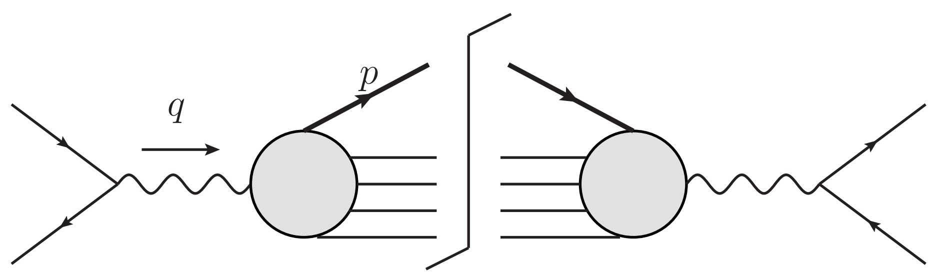
Size of wave packet $\sim \dfrac{1}{\Delta p} = \dfrac{1}{Q} \times \dfrac{p}{\Delta p}$.

Amplitude with wave function $f(z)$ is $\int \mathrm{d}^4 z \, f(z)$



Interference between different positions and times, to get momentum conservation.

$\int \int \mathrm{d}^4 z \, f(z) \times$



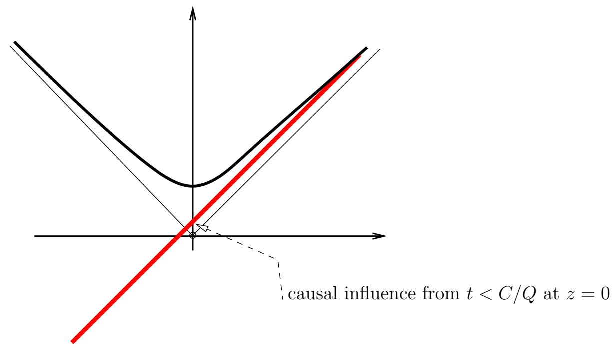
$\times f(z')^* \, \mathrm{d}^4 z'$

Positions of hard scatter on left and right differ by up to wave packet size.

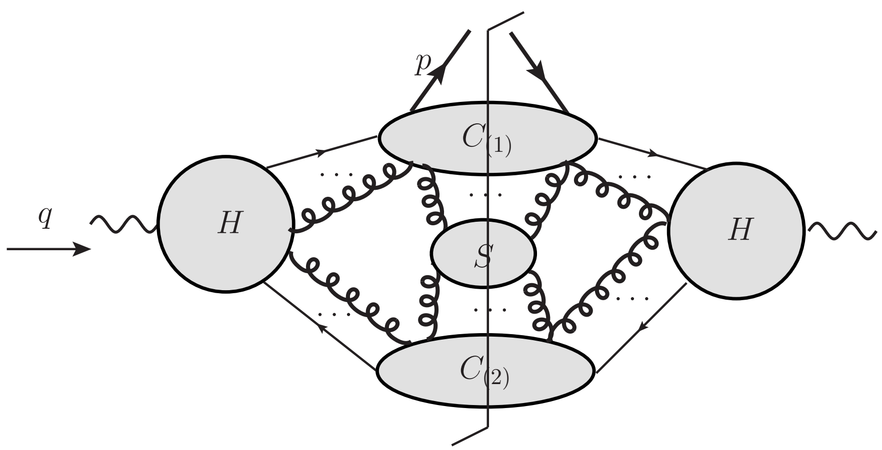
$\Longrightarrow$??

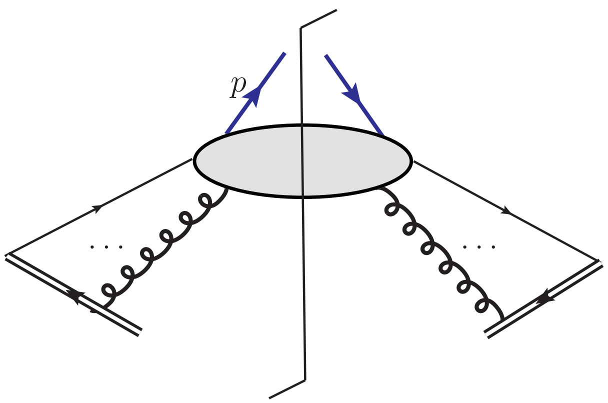
How to get pQCD factorization to match correct hadronization?

Space-like separation of jet hadronization:



Regions v. operator definition of fragmentation function

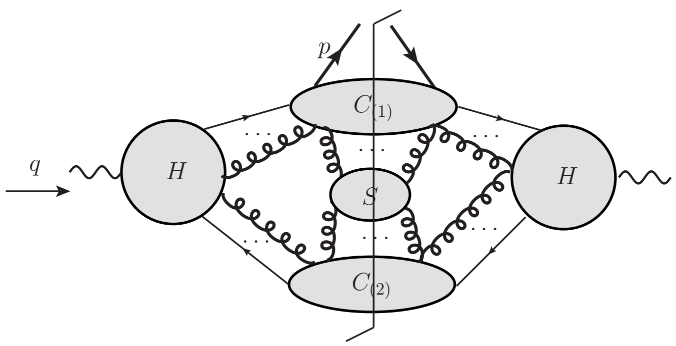




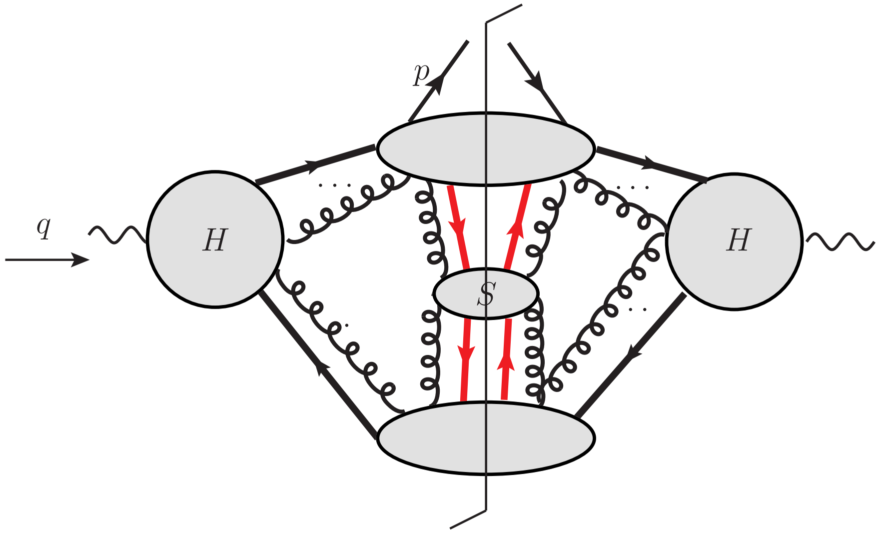
Possible approach: Only effect on right-moving jet from left-moving jet is early history of left-moving object with (anti)-quark parton numbers. Same effects with Wilson line instead of true antiquark.

Corrected region graph

1. Insert quark exchange in standard region graph:



to



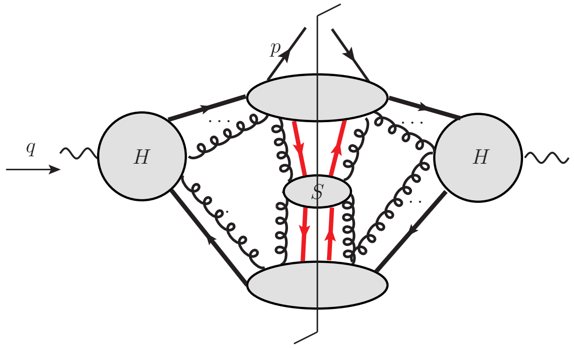
Recall: non-suppression of quark exchange when we sum over number of final-state particles.

2. And in part closest to final-state (hadronization region) remove requirement of big rapidity difference between collinear blobs and "soft" blob.

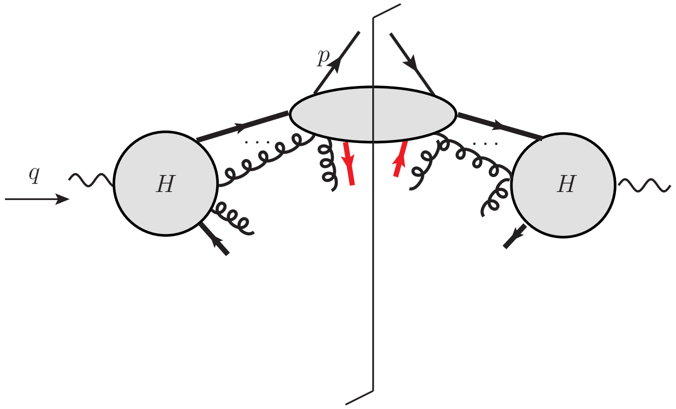
3. But large rapidity difference remains at earlier times, closer to hard scattering, as in standard region analysis.

Causality

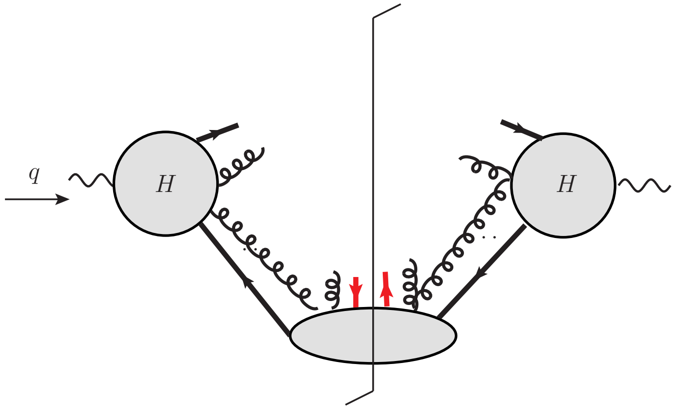
Enhanced region graph:



Causal influences on top jet:



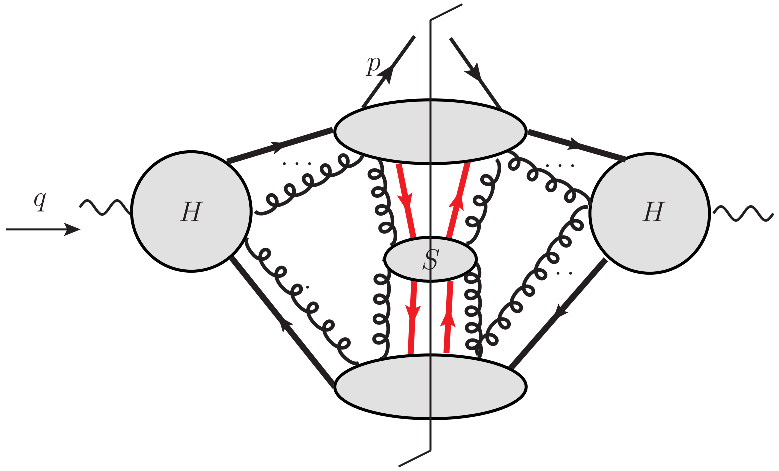
Causal influences on bottom jet:



(I drew graphs so vertex positions indicate typical vertex separations in (t, z).)

Put together

- Central/soft part has no causal influence on jet hadronization, so censor it



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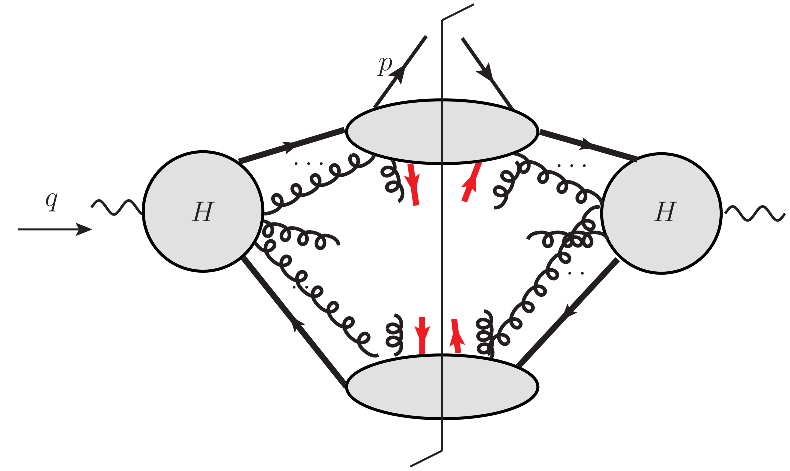

- But earlier parts (nearer hard scattering) are still dominated by gluons across big rapidity ranges. (Perturbative!)
- So we can hope to recover the critical parts of use of Ward identity. (Future work!)
- Technical: How to implement censoring technically in factorization derivation?
- Clash #3 again: It endangers causality argument/intuition/insight:
  - Vertex positions on left and separated in the expected way. (On right, similarly.)
  - But hard vertices integrated over large wave packet region.
  - QM interference between emissions from different space-time positions of Hs

So I've proposed two complementary ideas

1. Same space-time structure applies to operator definition of fragmentation function
   - No causal effect on fast hadron of whether opposite side is Wilson line or a genuine opposite jet.
2. Treat amplitudes/graphs for one-particle inclusive hadron production partly as being in space-time.
   - Draw region of causal influence on formation of fast hadron.
   - Censor anything outside. Need a new kind of unitarity argument.)
   - See structures that match standard region analysis.

Easy to say/propose these ideas. Harder to get correct, valid implementation.

Upshot

- Missing physics in standard Feynman graph approach to obtaining high Q asymptotes in pQCD.
- But it's known missing physics.
- Dangers in taking literally what's written.

- Space-time view is critical to understanding.
- Space-time localization in Feynman graphs needs to be better codified.
- Need technical implementation of "censoring" of causally irrelevant regions of graph/space-time.

- Standard QM has evolution of state in time.
  - QFT converts to Heisenberg picture.
  - Ideal: to find and formalize a more local approach to state evolution in space-time.
  - How to deal with non-localization of beams of precise momenta?
- We examined the real physical regions, not the extra ultra soft ones with perturbative massless gluons. (Cf. SCET-1 v. SCET-2.)