ISOCHRONOUS RADIAL-SECTOR NON-SCALING FFAGS USING DIFFERENT F AND D FIELD PROFILES

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FFAG'14, Brookhaven National Laboratory, 22-26 September 2014

ISOCHRONISM

For an ion of rest mass m_0 and charge q, constant orbit frequency ω, independent of energy $\gamma m_0 c^2$ and average radius R (circumf./2π), implies that

$$\boxed{B = \frac{\gamma m_0 \omega}{q} = \gamma B_c\,, \qquad R = \frac{\beta c}{\omega} \equiv \beta R_c\,,} \tag{1}$$

where B denotes the average field around a closed orbit in the mid-plane, B_c the "central field" and R_c the "cyclotron radius".

The resultant positive field gradient (and average field index k) produces a defocusing contribution to the vertical betatron tune ν_z:

$$\boxed{\left(\Delta\nu_z^2\right)_{isoc.} = -k = -\frac{R}{B}\frac{dB}{dR} = -\frac{\beta}{\gamma}\frac{d\gamma}{d\beta} = -\beta^2\gamma^2\,.} \tag{2}$$

To compensate for this Thomas suggested an azimuthal variation:

$$\boxed{B_z(r,\theta) = B(r)\left(1 + f\cos N\theta\right),} \tag{3}$$

to produce a scalloped orbit and an edge focusing contribution:

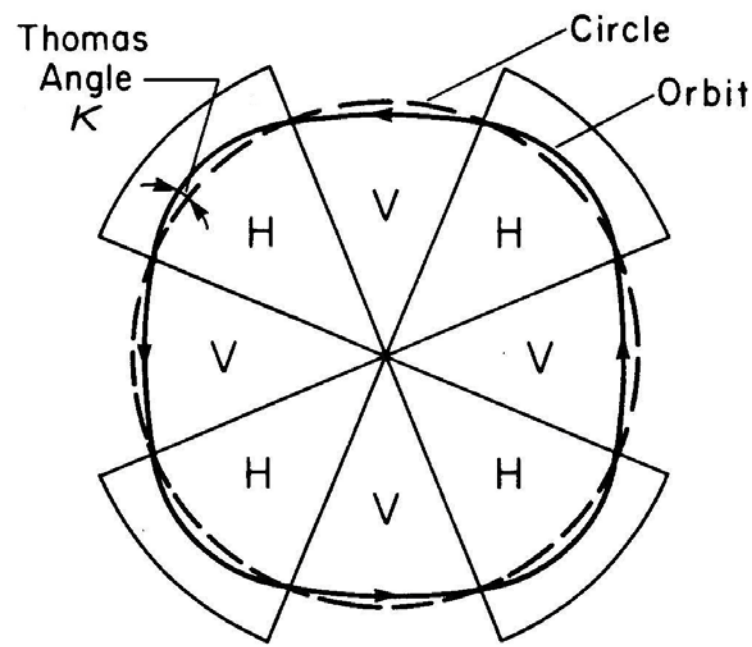
$$\boxed{\left(\Delta\nu_z^2\right)_{Thomas} = \tfrac{1}{2} f^2\,.}$$

SECTOR-FOCUSED CYCLOTRONS

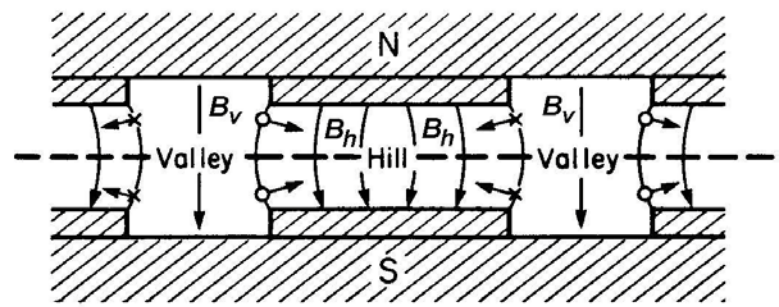
For a general variation $B_z(\theta)$ with N-fold symmetry the edge focusing is given by the "magnetic flutter" (i.e. the mean square variation in B_z:

$$F^2 \equiv \frac{\left\langle \left[B_z(\theta) - B\right]^2 \right\rangle}{B^2}.$$

But it's hard to achieve $F^2 > 0.1$ in single-yoke "compact" cyclotrons, limiting such radial-sector machines to ~50 MeV (p).



Kerst's introduction of alternating focusing by giving the hill edges a spiral angle ε further enhances the focusing:



$$\nu_z^2 \approx -\beta^2\gamma^2 + F^2\left(1 + \tan^2\varepsilon\right). \qquad (4)$$

With this powerful enhancement it has been possible to build pill-box cyclotrons to accelerate protons to 230 MeV ($\beta^2\gamma^2 = 0.55$).

SEPARATE-SECTOR CYCLOTRONS

Breaking the magnet into separate hill sectors with field-free valleys between them not only provides a more benign environment for rf and other equipment, but also increases the potential flutter. For hard-edge magnets occupying a fraction h of the orbit circumference,

$$\boxed{F^2 = \frac{1}{h} - 1.}$$

Thus the RIKEN SRC, with six 25°-wide radial sectors, is able to provide $F^2 \approx 1.4$. and accelerates light ions to 400 MeV/c ($\beta^2\gamma^2$ = 1.03). The PSI Ring Cyclotron, with eight 18°-wide spiral sectors, provides $F^2 = 1.5$ and accelerates protons to 590 MeV ($\beta^2\gamma^2$ = 1.65). Designs have been published for higher energies, ranging from 1-15 GeV.

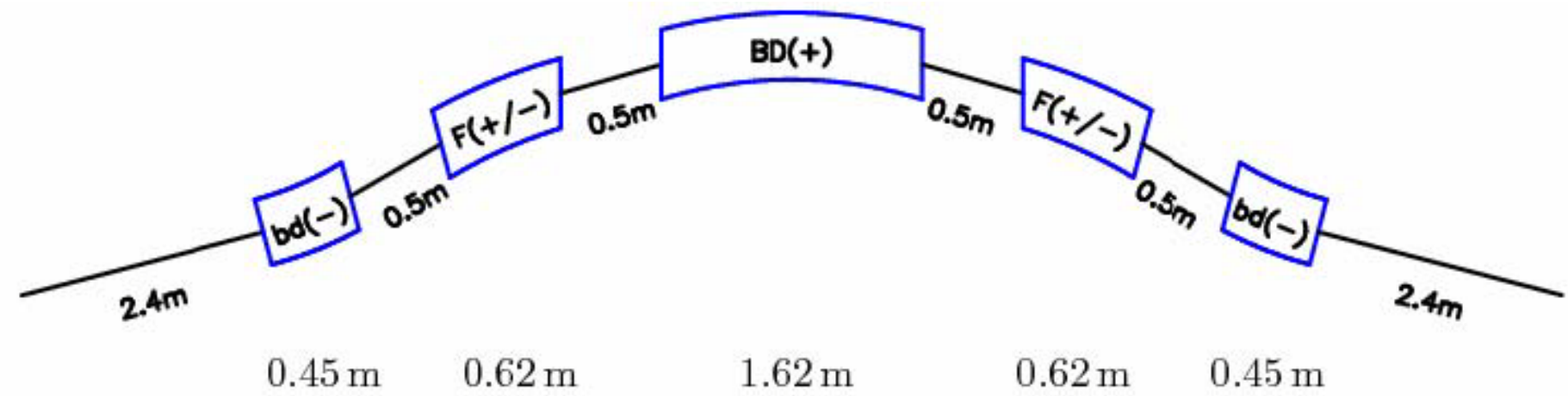
REVERSE-BEND CYCLOTRONS

- where $B_v = -B_h$, as in radial-sector FFAGs, can produce even higher flutter, and also significant AG focusing. A design with $h = 0.6$, $F^2 = 24$, (enough to counter $\beta^2\gamma^2$ at 3.7 GeV) was found to focus up to 5.9 GeV.

ISOCHRONOUS FFAGS I

But isochronous FFAGs are capable of much higher energies!

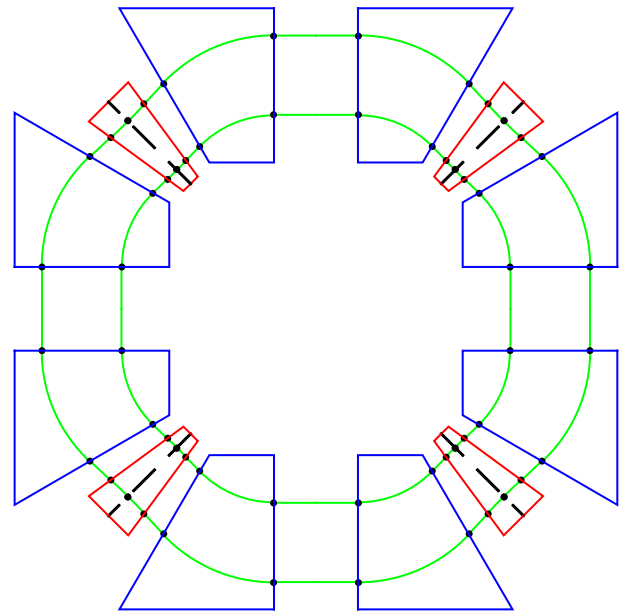
Grahame Rees has designed several FFAGs using novel 5-magnet "pumplet" cells, in which variations in field gradient and sign enable each magnet's function to vary with radius – providing great flexibility – even allowing well-matched insertions!



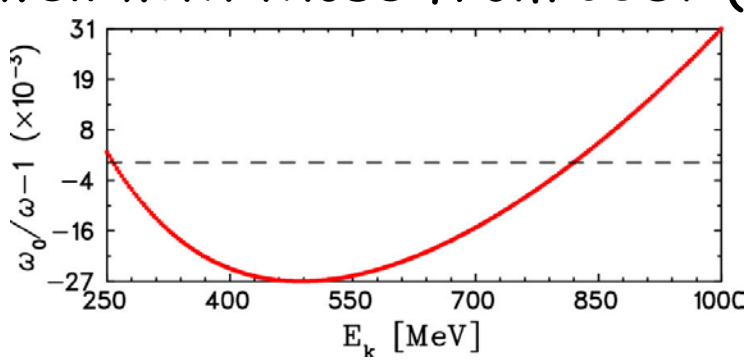
Among them was an isochronous "IFFAG" ($C = 1255$ m, $N = 123$, 16 turns) for 8-20 GeV muons – i.e. $5{,}900 < \beta^2\gamma^2 < 37{,}000$!

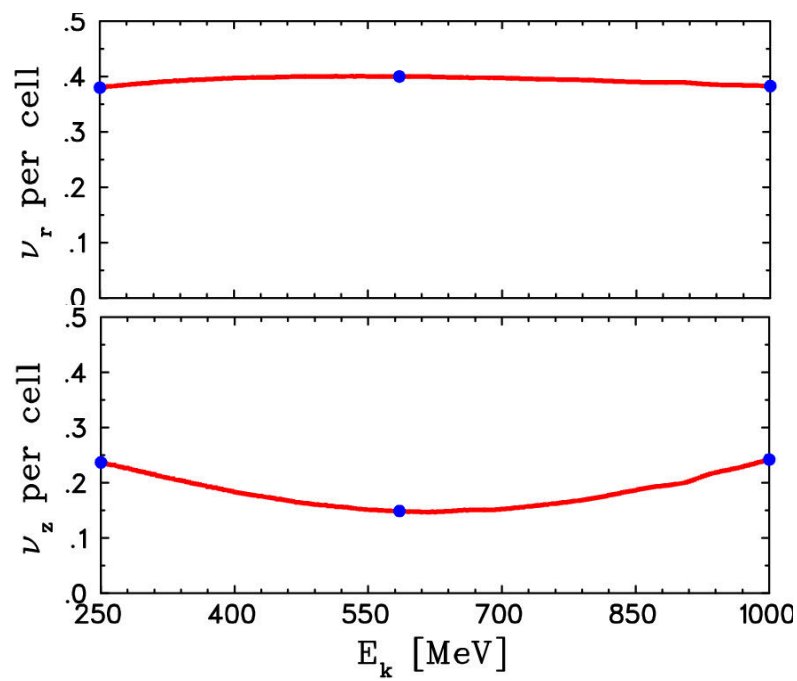
ISOCHRONOUS FFAGS II

Carol Johnstone has proposed a two-stage proton LNS-FFAG, operating at fixed frequency, for ADSR. Each stage uses a 4-cell FDF triplet lattice (right) with straight-sided (though not necessarily radial) edges and a specially determined $B(r)$ profile.



Tracking studies of the second stage (250-1000 MeV) with CYCLOPS have confirmed that the orbits are close to isochronous and the tunes (ν_z and ν_r) near constant. The CYCLOPS results (—) agree well with those from COSY (•).





DIFFERENT HILL AND VALLEY FIELD PROFILES I

Note that in all the cyclotron schemes described above the functional dependences of B_z on r and θ are assumed to be independent:

$$\boxed{B_Z(r,\theta) = R(r)\,\Theta(\theta),}$$

so that the radial dependence $R(r)$ is the same at all azimuths θ.

Thus in a radial-sector cyclotron there's only one free parameter – the flutter – determined by $\Theta(\theta)$ - available to control the vertical tune.

To provide more freedom of action and achieve positive vertical focusing at higher energies, we have explored a simpler possibility than in the FFAGs - allowing the radial field profiles in hills and valleys to differ – and assuming a polynomial variation with energy:

$$\boxed{B_h(\gamma) = H_0 + H_1\gamma + H_2\gamma^2 + H_3\gamma^3 + ...} \quad (5)$$

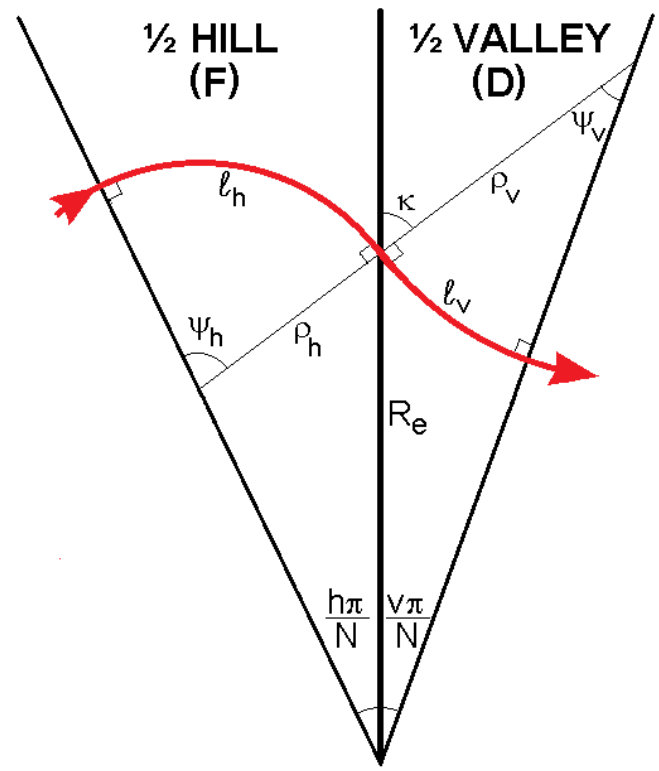
$$\boxed{B_v(\gamma) = V_0 + V_1\gamma + V_2\gamma^2 + V_3\gamma^3 + ...} \quad (6)$$

A COMPACT DESIGN WITH NEGATIVE VALLEY FIELDS

As a first step we considered a "compact" design with no drift spaces, negative valley fields, hard edges, and B_h and B_v each constant along equilibrium orbits. If $\ell_h = \rho_h \psi_h$ and $\ell_v = \rho_v \psi_v$ are the arc lengths within a half-cell, then to maintain isochronism,

$$\ell_h H_1 + \ell_v V_1 = \frac{\pi}{N} B_c R_c \beta \quad \text{and}$$

$$\ell_h H_n + \ell_v V_n = 0 \ (n \neq 1).$$



If the hill coefficients H_n are specified, the valley coefficients V_n must satisfy:

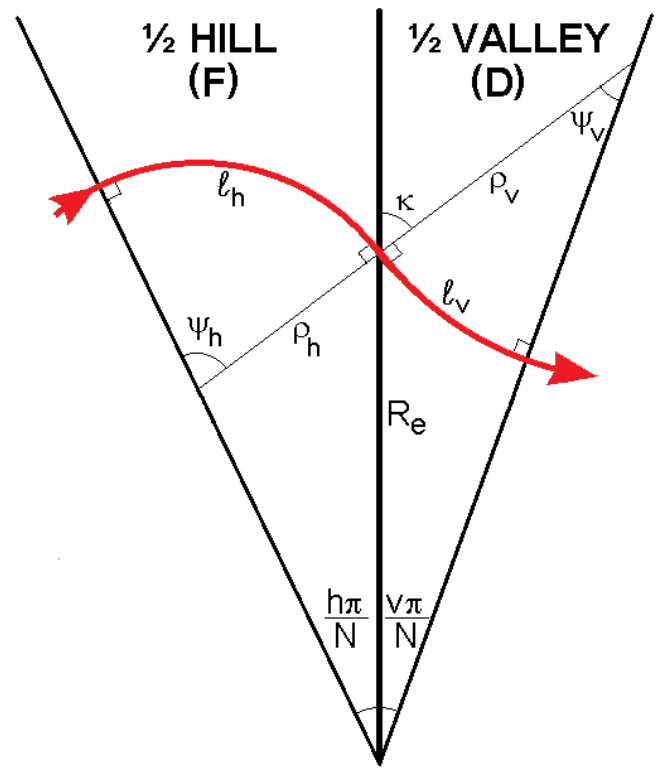
$$V_1 = \frac{\pi}{N} \frac{B_c R_c}{\ell_v} \beta - \frac{\ell_h}{\ell_v} H_1 \quad \text{and}$$

$$V_n = -\frac{\ell_h}{\ell_v} H_n \ (n \neq 1).$$

So to compute them we need the values of ℓ_h and ℓ_v.

ORBIT GEOMETRY FOR THE COMPACT DESIGN

Computing the valley field B_v requires a knowledge of ℓ_h and ℓ_v, and therefore of the bending angles ψ_h and ψ_v and the radii of curvature – of which ρ_v itself depends on B_v!



These parameters may nevertheless be evaluated by invoking their various geometrical relationships, which after some manipulation yield a transcendental equation for ψ_h, from which the other parameters follow:

$$\boxed{\psi_h + (\psi_h - \pi/N)\frac{\sin\psi_h \sin[(1-h)\pi/N]}{\sin(\psi_h - \pi/N)\sin(h\pi/N)} = \frac{B_c\beta\gamma}{B_h(\gamma)}.}$$

This must be solved numerically, but a good starting point is to make the approximation $R_e = \beta R_c$, giving:

$$\boxed{\psi_{h0} = \arcsin\left(\frac{B_h(\gamma)}{\gamma B_c}\sin\left(h\pi/N\right)\right).}$$

BETATRON TUNES

To calculate the tunes we take a lumped-element approach (validated by tracking with CYCLOPS in previous studies):

$$\boxed{\cos(2\pi\nu/N) = \tfrac{1}{2}\mathrm{Tr}(M_e M_v M_e M_h),}$$

where M_e is the standard 2×2 matrix for a thin lens, while for vertical motion, M_v and M_h are those for focusing and defocusing sector magnets respectively. For M_e we need the focal power g of the edge crossing, given by:

$$\boxed{g = \frac{B_h - B_v}{B_c R_c \beta\gamma} \tan\left(\psi_h - \frac{h\pi}{N}\right).}$$

For M_v and M_h we need the phase advances $\phi_{h,v} = \ell_{h,v}\sqrt{K_{h,v}}$, where:

$$\boxed{K_h = \frac{dB_h/dr}{B_c R_c \beta\gamma} = \frac{\gamma^2}{B_c R_c^2}\left(H_1 + 2H_2\gamma + 3H_3\gamma^2 + \ldots\right),}$$

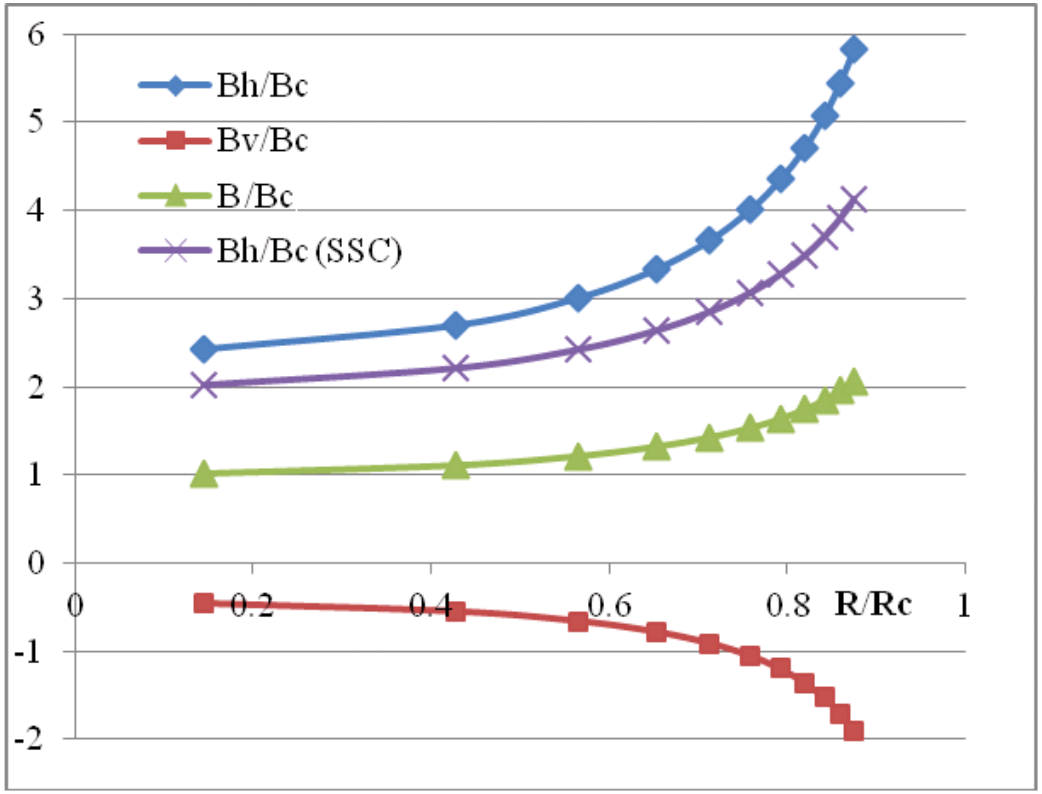
$$\boxed{K_v = \frac{dB_v/dr}{B_c R_c \beta\gamma} = \frac{\gamma^2}{B_c R_c^2}\left(V_1 + 2V_2\gamma + 3V_3\gamma^2 + \ldots\right).}$$

For horizontal motion $\phi^*_{h,v} = \ell_{h,v}\sqrt{K^*_{h,v}}$, where $K^*_{h,v} = (1/\rho_{h,v})^2 \pm K_{h,v}$.

CASES STUDIED

A number of cases were studied to investigate the effect on the tunes of simply adding a γ^2 component to the conventional γ-variation (i.e. $H_n = 0 = H_{n>2}$). Their dependence on the hill fraction h and the number of sectors N were also investigated, though most runs were made for $h = 0.5$ and $N = 8$.

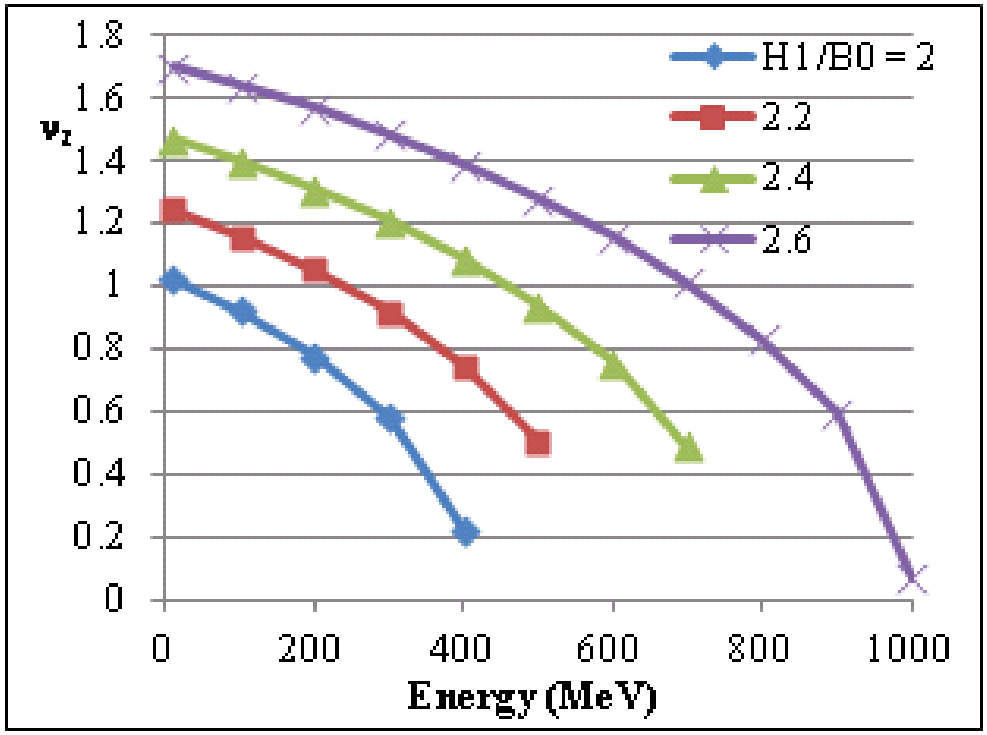
The figure displays an example of such field profiles for $H_1 = 2B_c$, $H_2 = 0.4B_c$ - one of the more promising cases studied, for which ν_z varies little from 10-1000 MeV.

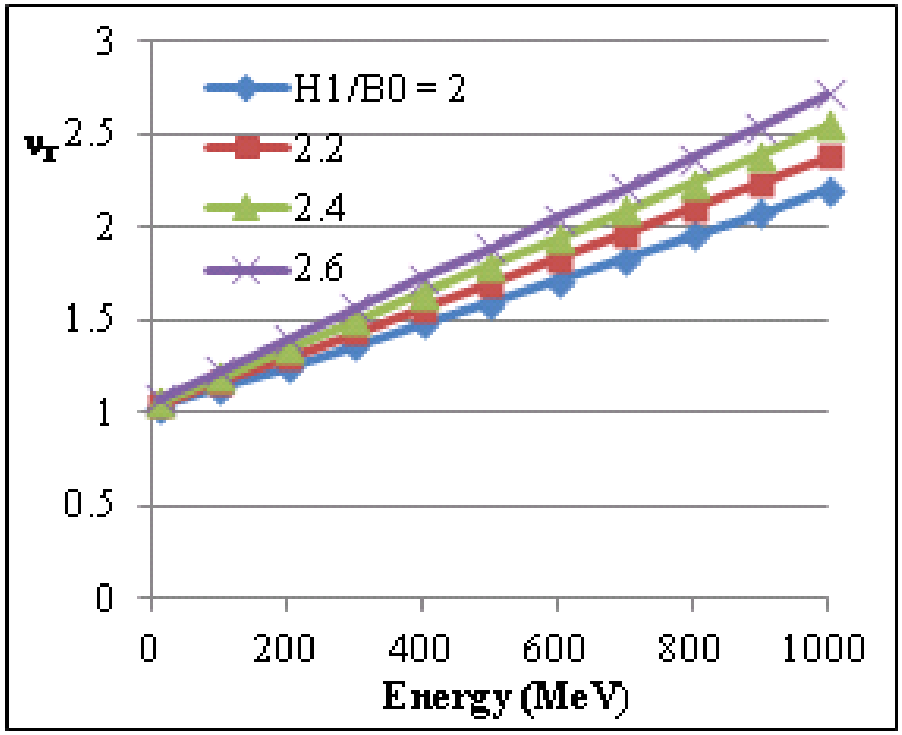


The comparison curve for a separate-sector cyclotron (SSC) with the same $h = 0.5$ shows that for $H_2 = 0.2H_1$ the B_h required would be 20-40% higher.

REFERENCE CASE: NO γ^2 COMPONENT (SEPARATE-SECTOR & NEGATIVE-BEND CYCLOTRONS)

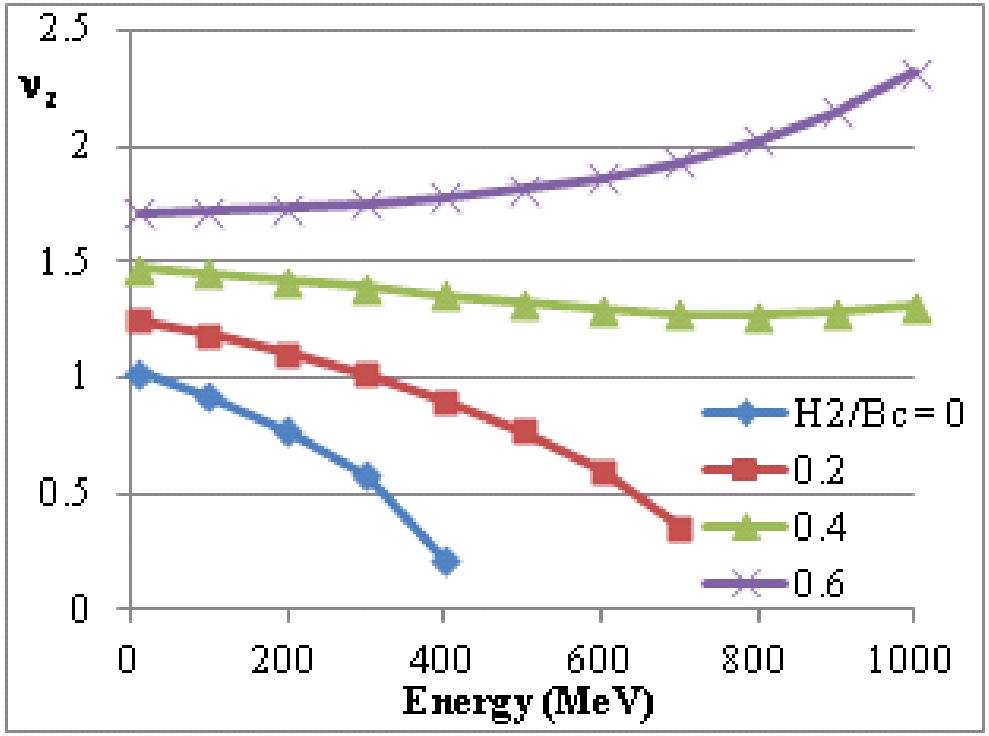
($H_2 = 0$, $h = 0.5$, $N = 8$)

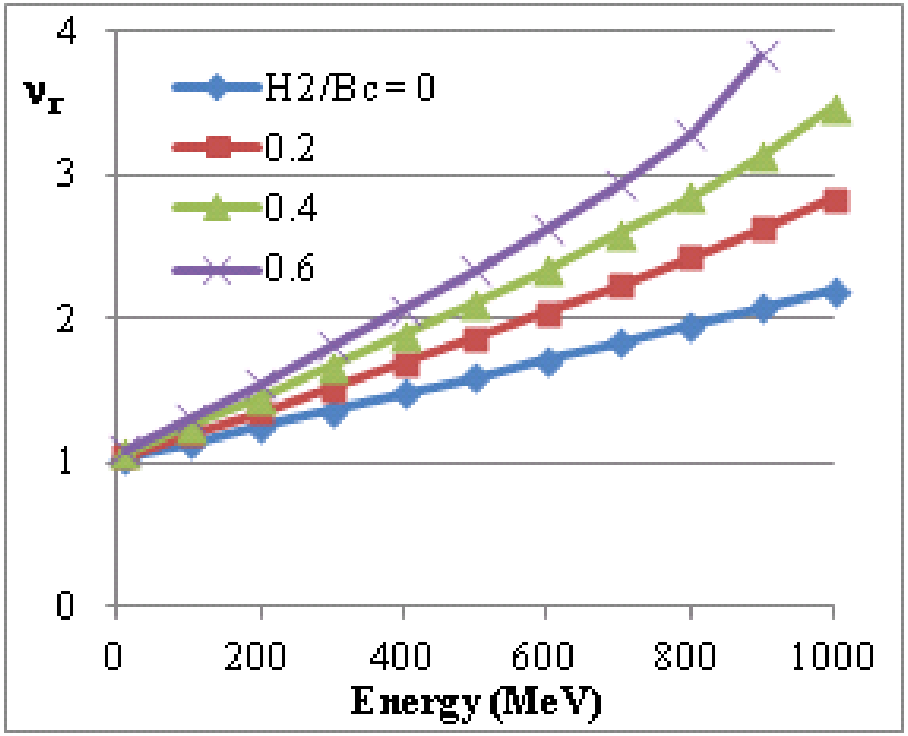




EFFECT OF ADDING A γ^2 COMPONENT

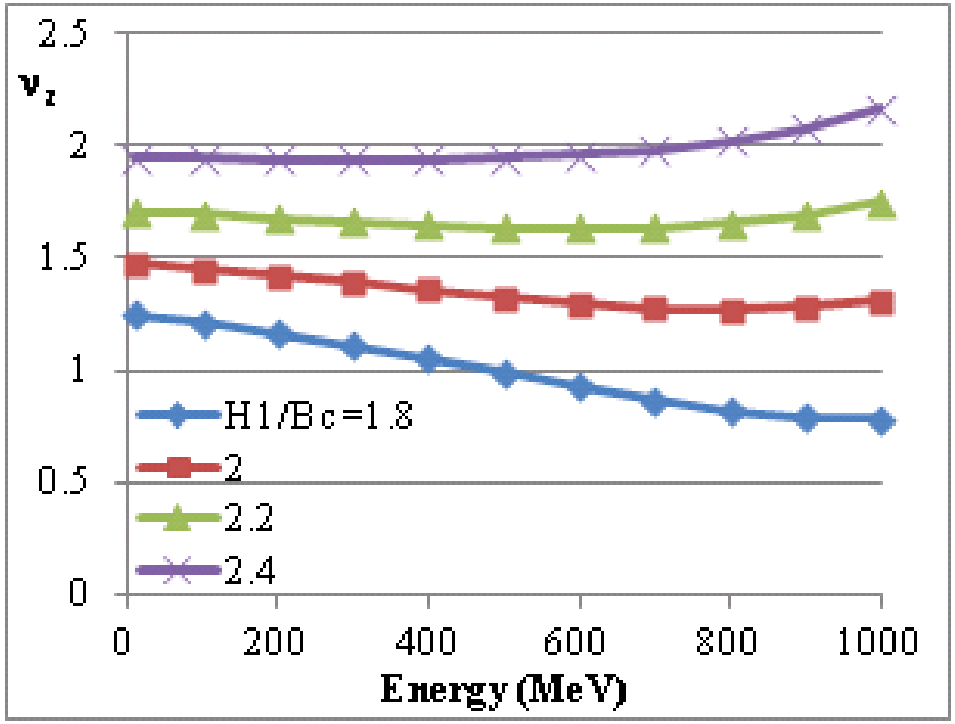
($H_1/B_c = 2$, $h = 0.5$, $N = 8$)

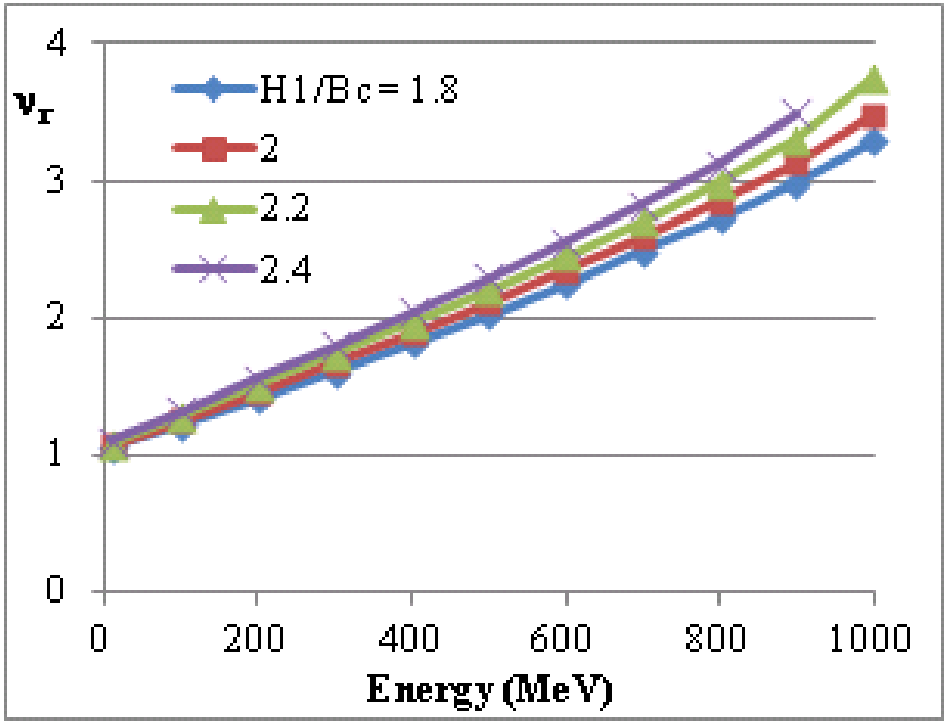




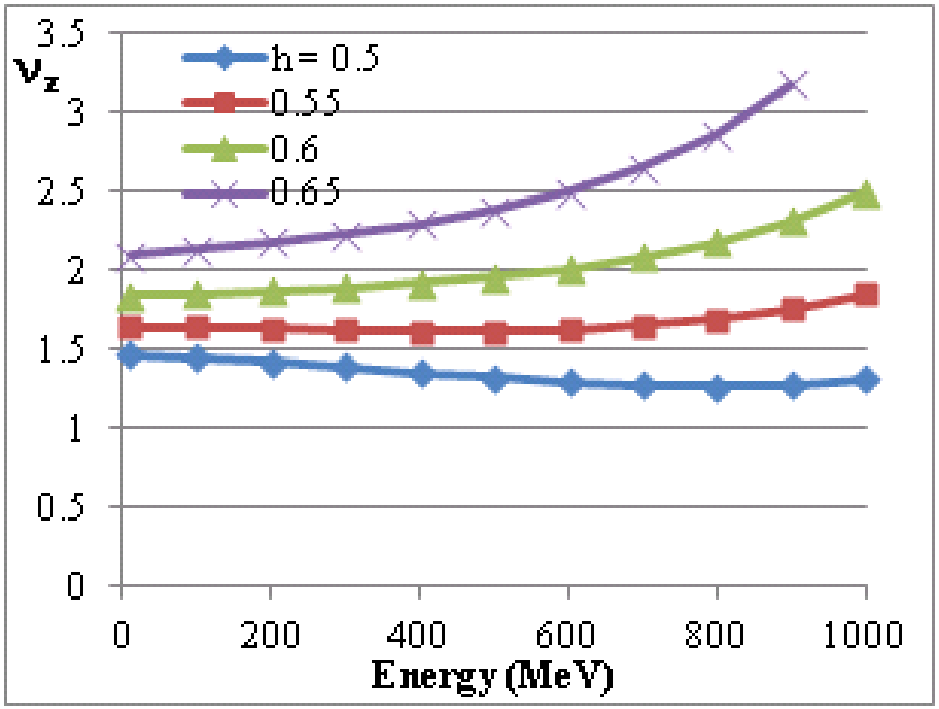
EFFECT OF VARYING H_1 FOR FIXED H_2

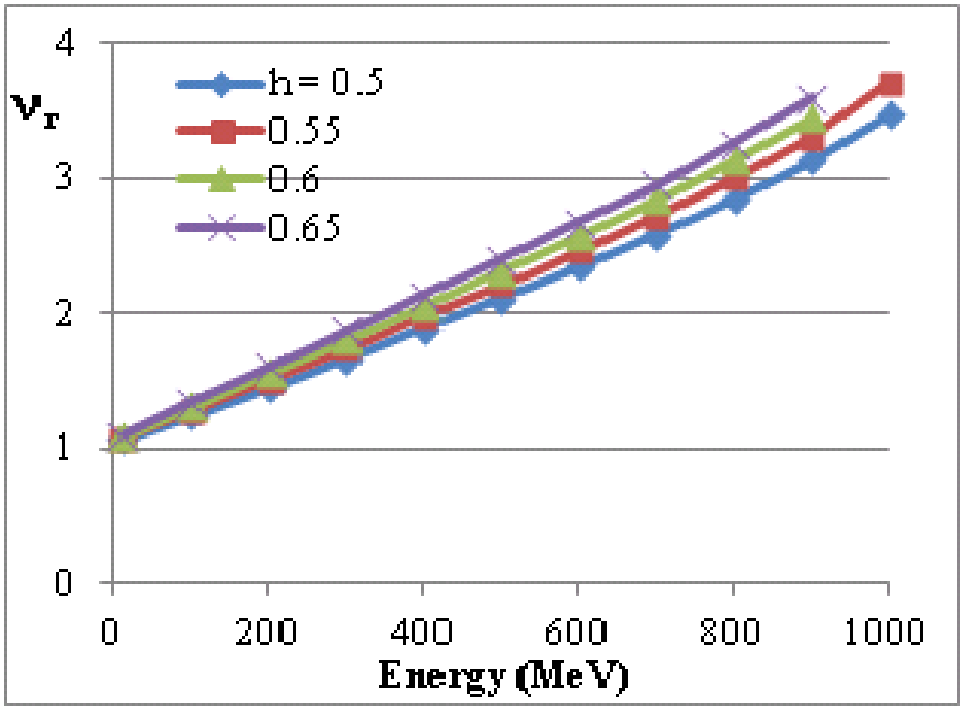
(H_2 = 0.4, h = 0.5, N = 8)





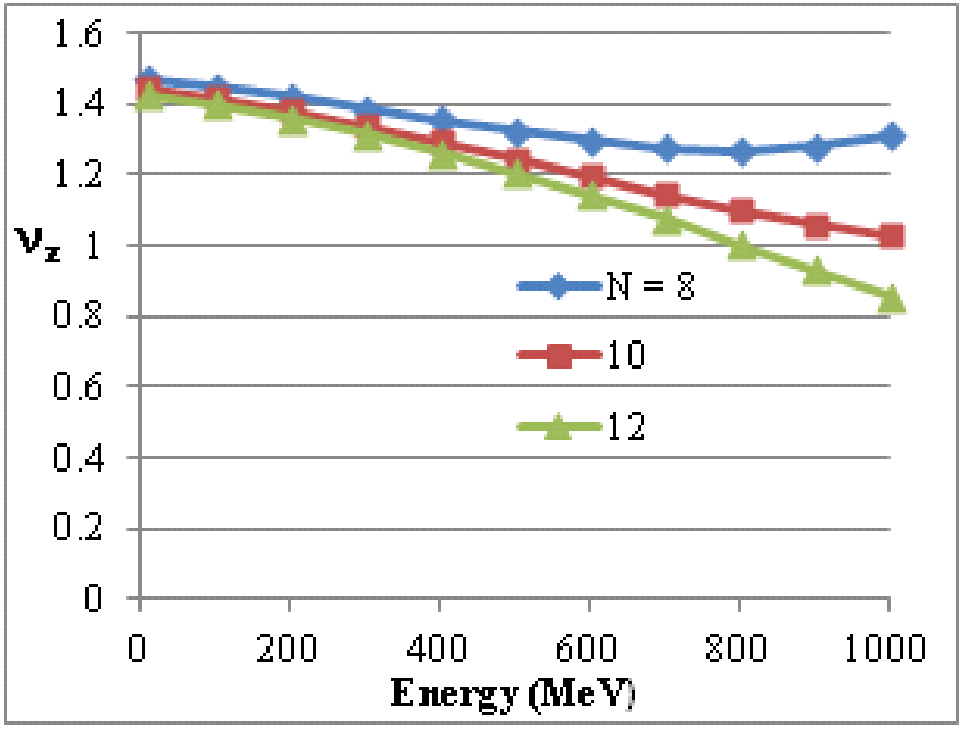
EFFECT OF VARYING h FOR FIXED H_1, H_2
($H_1/B_c = 2$, $H_2/B_c = 0.4$, $N = 8$)

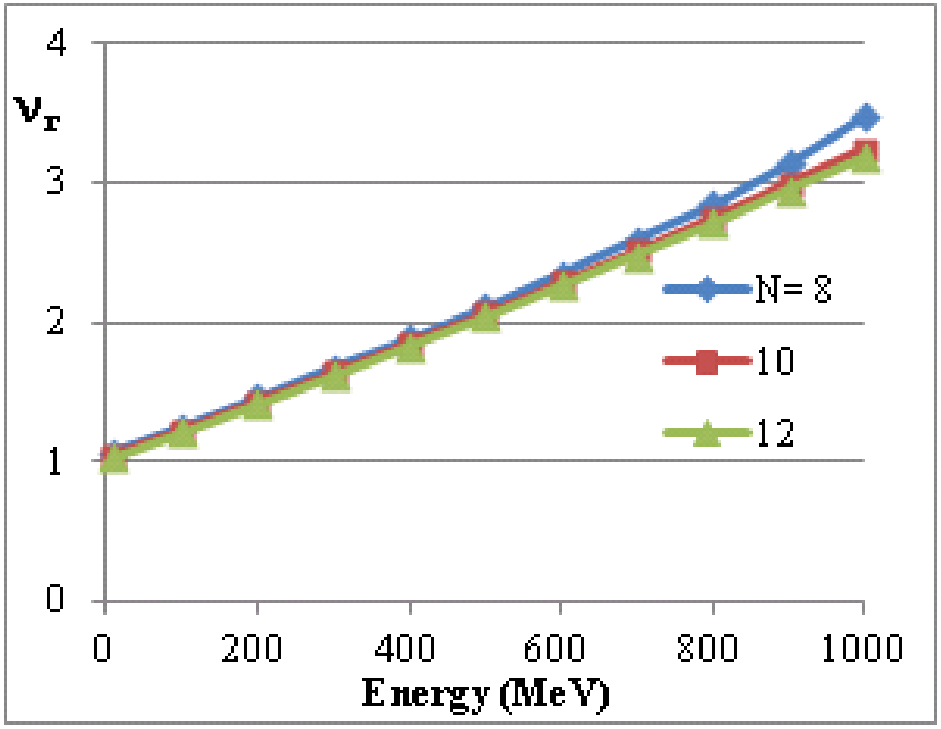




EFFECT OF VARYING N FOR FIXED H_1, H_2

($H_1/B_c = 2$, $H_2/B_c = 0.4$, $h = 0.5$)



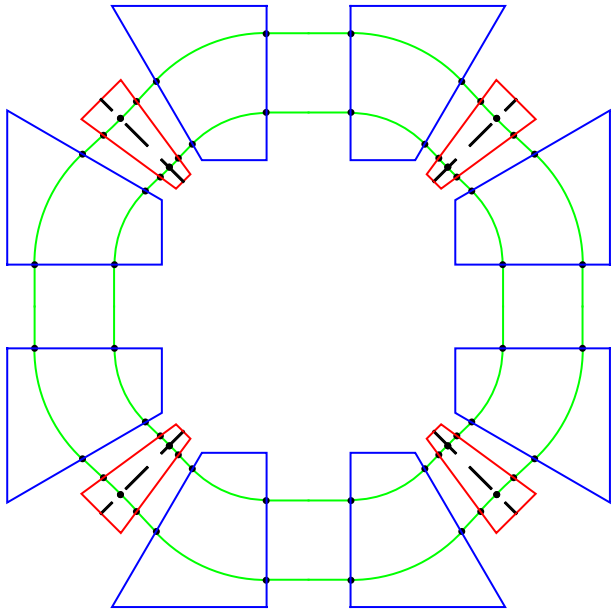


CONCLUSIONS ON THE "COMPACT" DESIGN

Adding a γ^2 component to the hill field - and subtracting an offsetting γ^2 component from the valley fields to maintain isochronism

- increases the flutter and gives stronger alternating gradients
- appears to be a possible way of providing radial-sector machines with sufficient vertical focusing to reach at least 1 GeV.
- though the radial tune still grows with energy: $\nu_r \propto \gamma$
- and the practicality of a design lacking field-free regions has not been taken into account, particularly with regard to finding suitable locations for the rf accelerating cavities and injection and extraction systems.

Our next step is therefore to consider a design with field-free drift spaces, e.g. with a FDF triplet design like Carol Johnstone's (though with radial sectors).

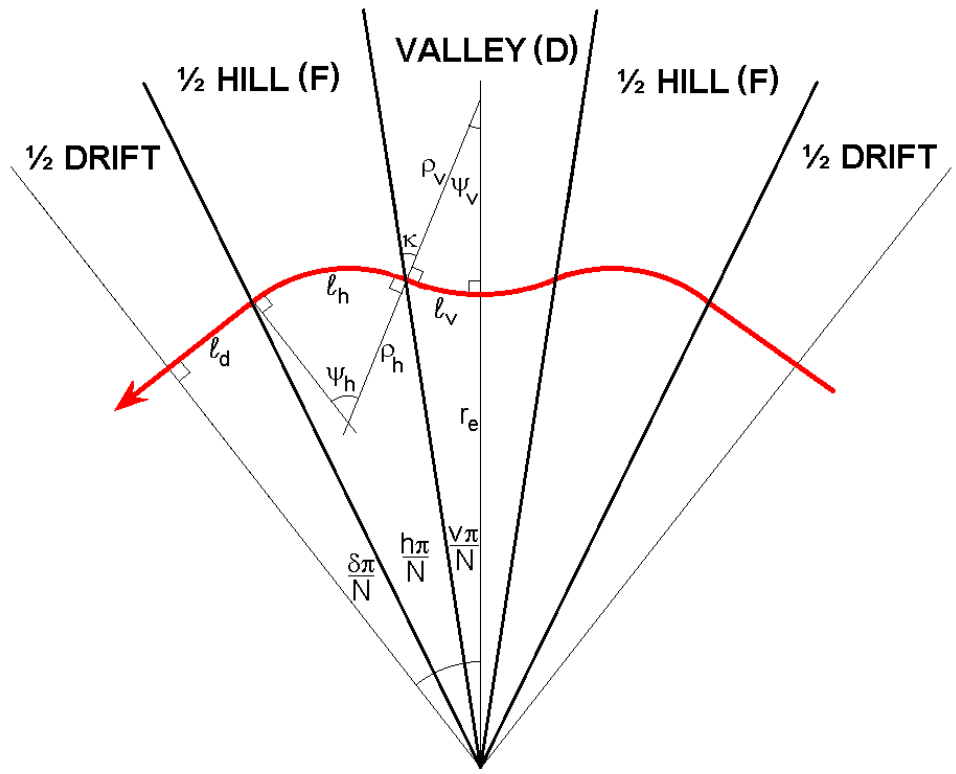


NEXT STEP – A SEPARATED TRIPLET DESIGN

The figure shows the sector and orbit geometry, using the same notation as before. The ½-drift sector angle is $\delta\pi/N$ where

$$\delta + h + v = 1.$$

If the values of N, h, δ, γ and the hill coefficients H_n are specified, B_h and ρ_h are set, and the bending angle ψ_h can again be obtained by solving a transcendental equation:



$$\frac{B_c\beta\gamma}{B_h(\gamma)} = \psi_h + \sin\delta\pi/N\left[\cos\delta\pi/N - \cos\left(\psi_h - \delta\pi/N\right)\right] + \frac{\left[\sin\left(\psi_h - \delta\pi/N\right) + \sin\delta\pi/N\right]}{\sin h\pi/N}\left[\cos h\pi/N \sin\delta\pi/N + \frac{\left(\psi_h - \pi/N\right)\sin\left[\left(1-h-\delta\right)\pi/N\right]}{\sin\left(\psi_h - \pi/N\right)}\right].$$

- and from this ψ_v, ρ_v, ℓ_v, ℓ_h, ℓ_d, and r_e determined.

BETATRON TUNES

The valley field coefficients V_n satisfy the same equations as before (slide 8), so we now have all the quantities needed to evaluate the transfer matrices for the hill and valley half-sectors M_{sh}, M_{sv}, the edges M_{ed}, M_{ev}, and the drift M_d. If the left half-cell matrix is written

$$M_L = M_d M_{ed} M_{sh} M_{ev} M_{sv} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

then that for right half-cell matrix is:

$$M_R = M_{sv} M_{ev} M_{sh} M_{ed} M_d = \begin{pmatrix} d & b \\ c & a \end{pmatrix},$$

and the betatron tune ν is given by:

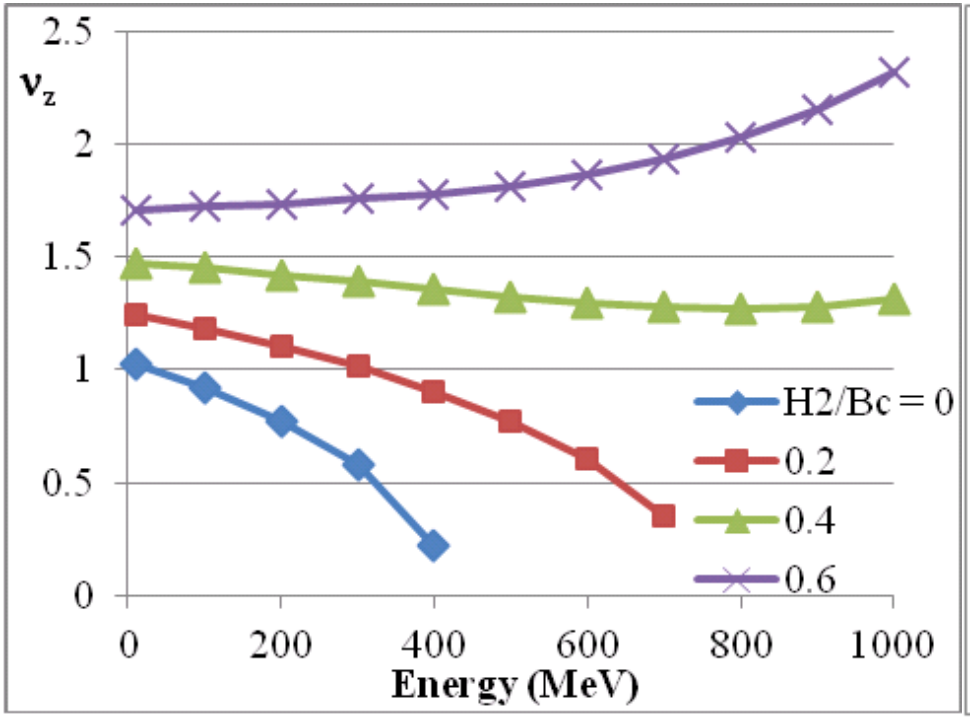
$$\boxed{\cos(2\pi\nu / N) = \tfrac{1}{2}\mathrm{Tr}(M_L M_R) = ad + bc.}$$

Some results are shown in the following slides, all computed for $N = 8$ and $H_1 = 2B_c$.

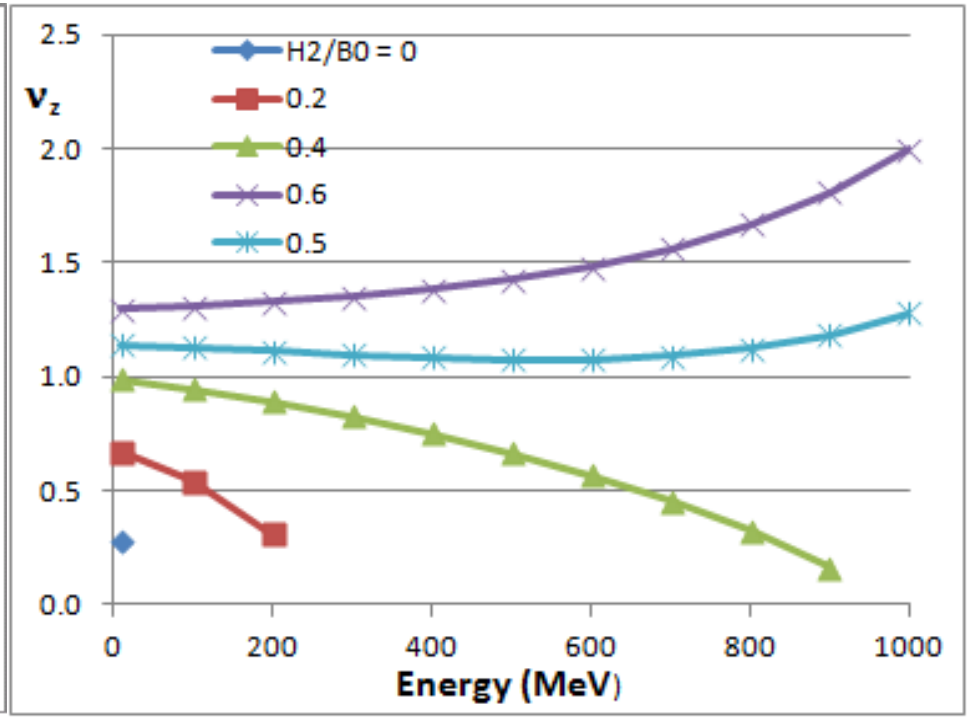
EFFECT ON ν_z OF ADDING AN $H_2\gamma^2$ FIELD COMPONENT

($H_1/B_c = 2$, $h = 0.5$, $N = 8$)

$\delta = 0$



$\delta = 0.3$

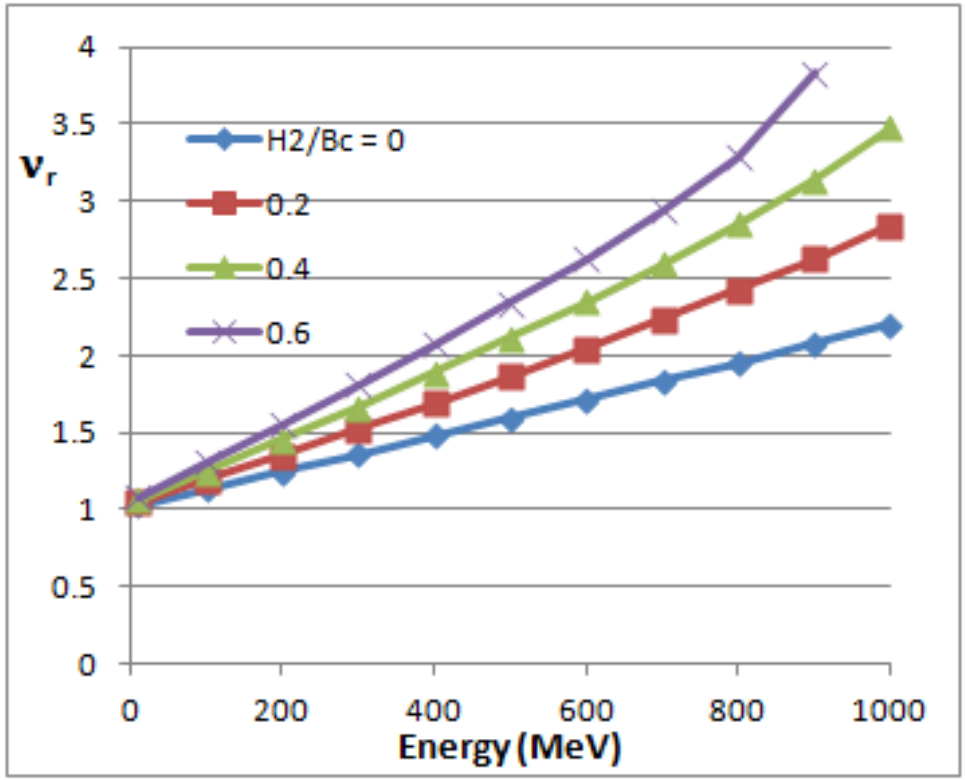


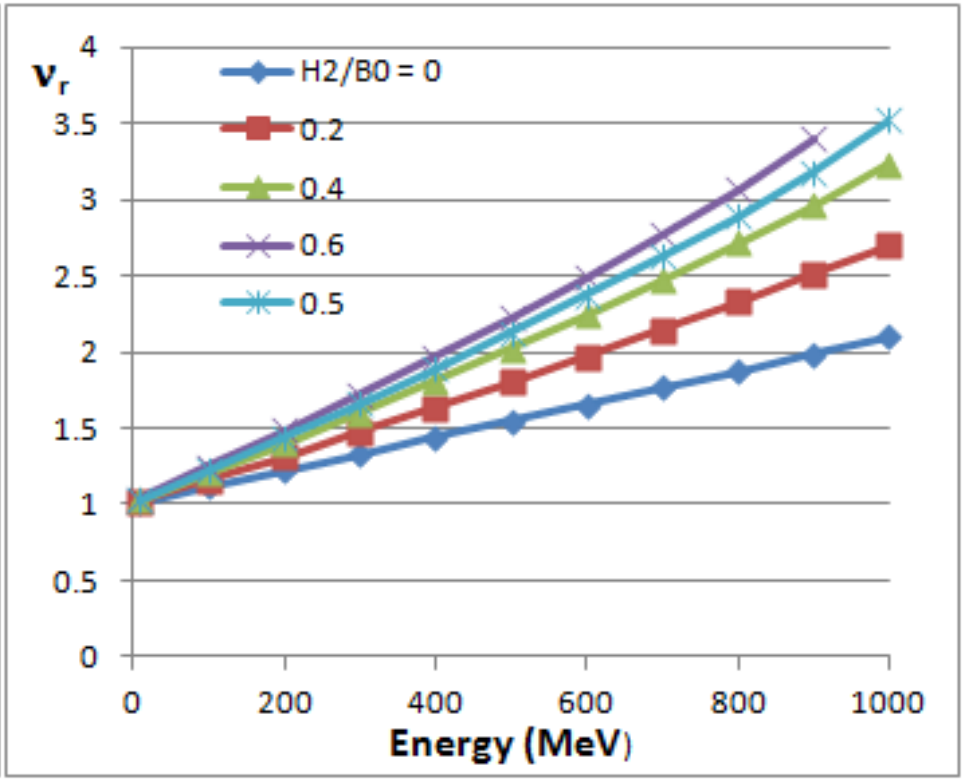
- In both cases, increasing H_2 increases the vertical tune ν_z
- Inserting a drift lowers the vertical tune ν_z
- The energy variation of the vertical tune ν_z can be minimized by a suitable choice of H_2.

EFFECT ON ν_r OF ADDING AN $H_2\gamma^2$ FIELD COMPONENT

($H_1/B_c = 2$, $h = 0.5$, $N = 8$)

$\delta = 0$ **$\delta = 0.3$**

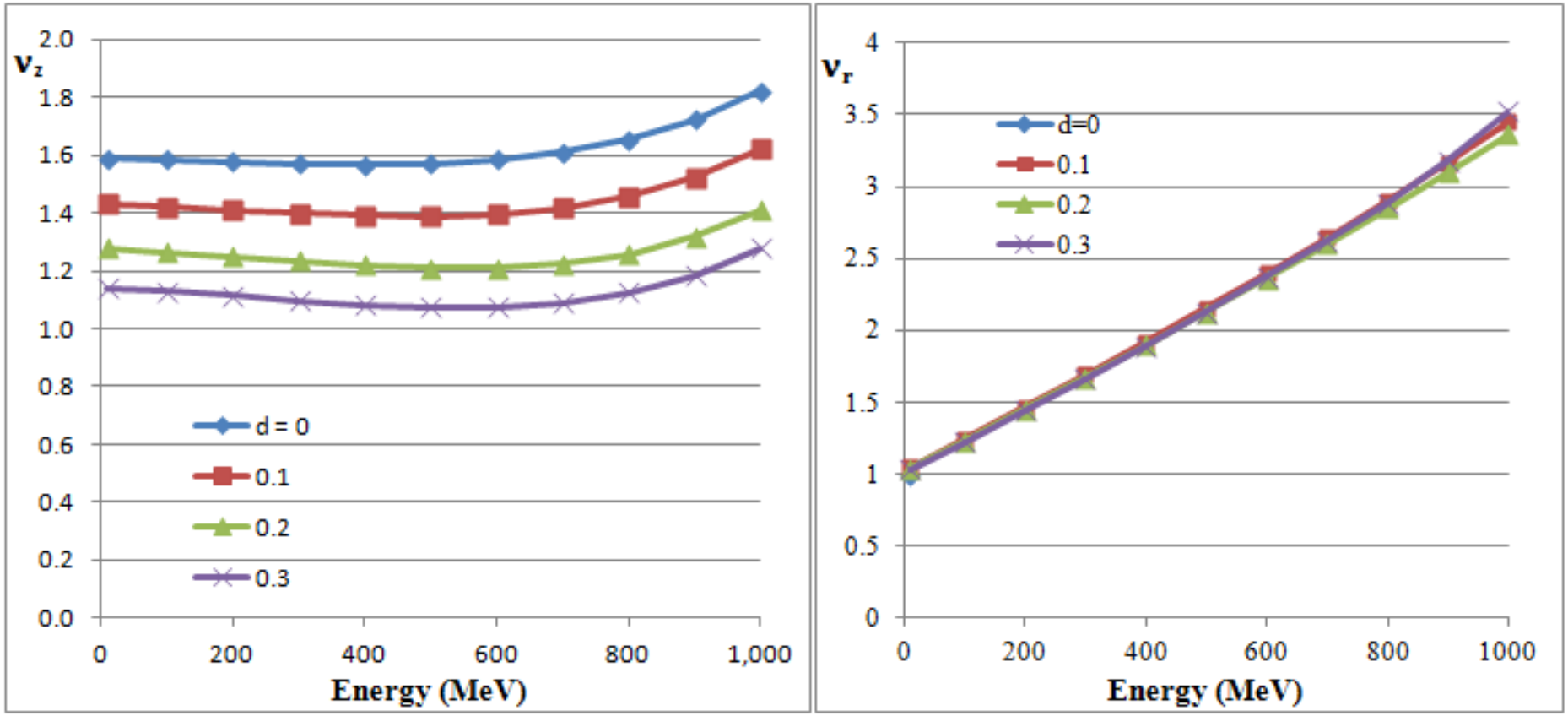




- In both cases increasing H_2 increases the growth rate of the radial tune ν_r with energy;
- Inserting a drift slightly reduces the growth rate.

EFFECT ON ν_z, ν_r OF VARYING THE DRIFT WIDTH δ

($H_1/B_c = 2$, $H_2/B_c = 0.5$, $\underline{h = 0.5}$, $N = 8$)

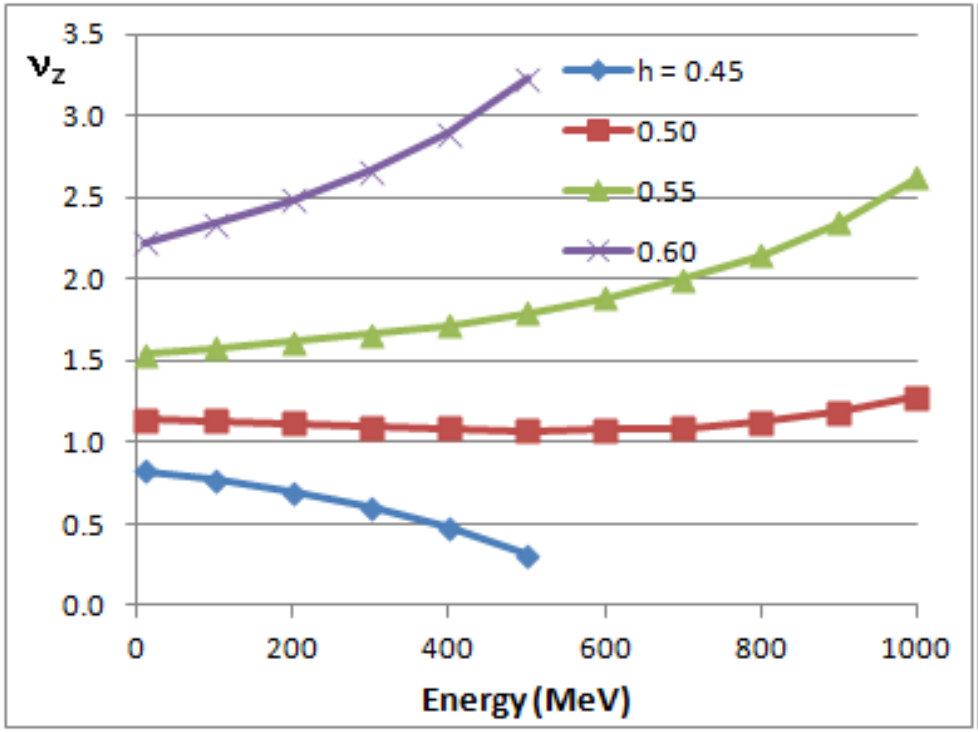


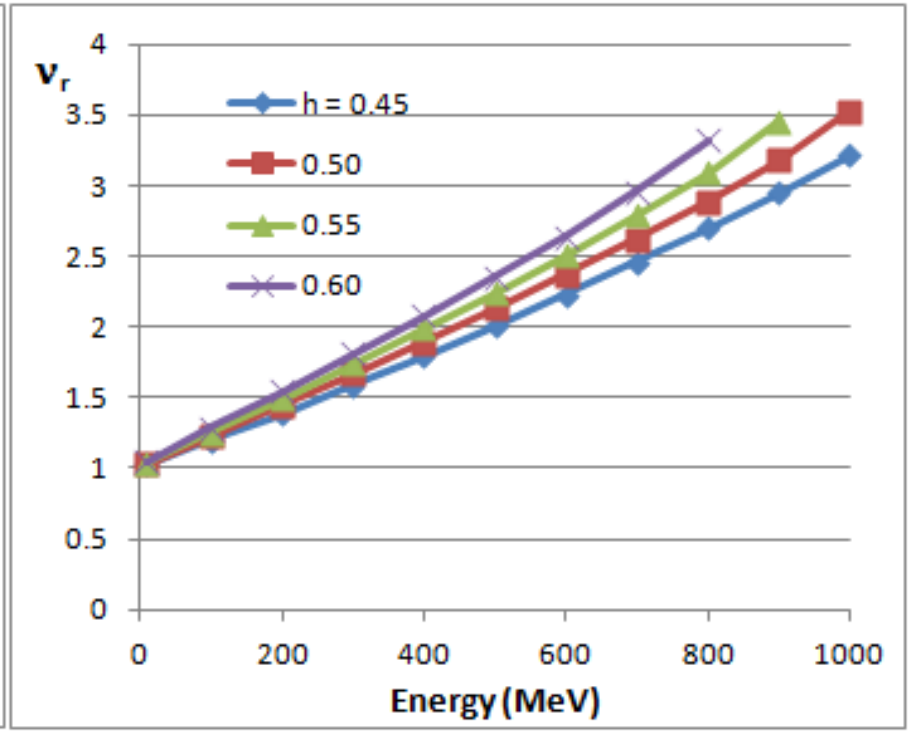
Increasing the drift

- reduces the vertical tune ν_z
- has little effect on the vertical tune's energy variation
- has little effect on the horizontal tune ν_r.

EFFECT ON ν_z, ν_r OF VARYING THE HILL WIDTH h

($H_1/B_c = 2$, $H_2/B_c = 0.5$, $\underline{\delta = 0.3}$, $N = 8$)





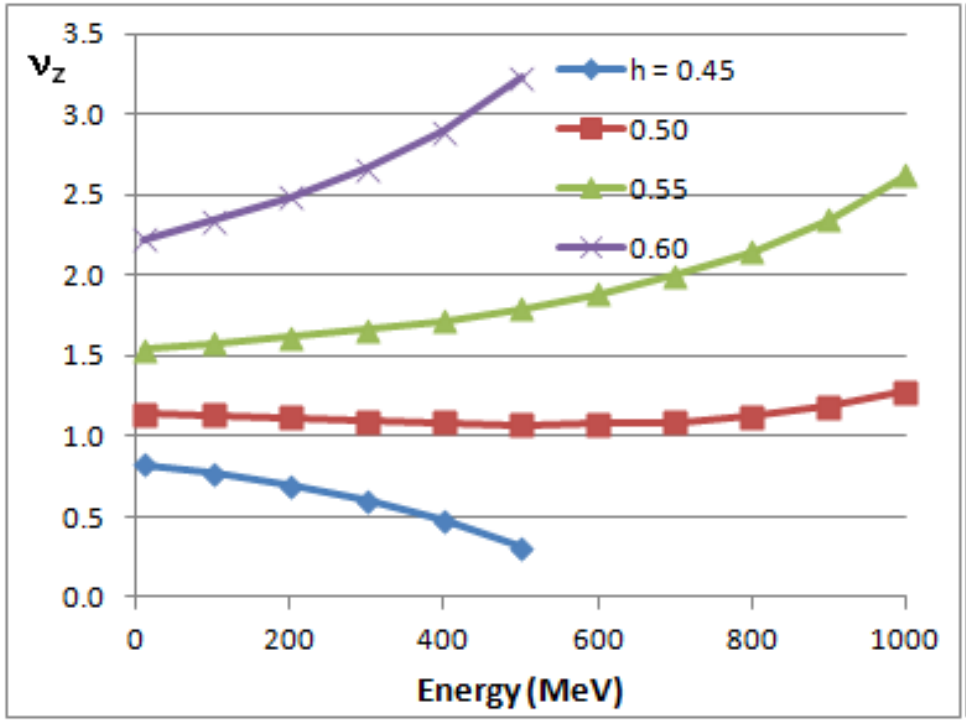
Increasing the hill width

- increases the vertical tune strongly, especially at higher energies
- increases the growth rate of the radial tune ν_r with energy

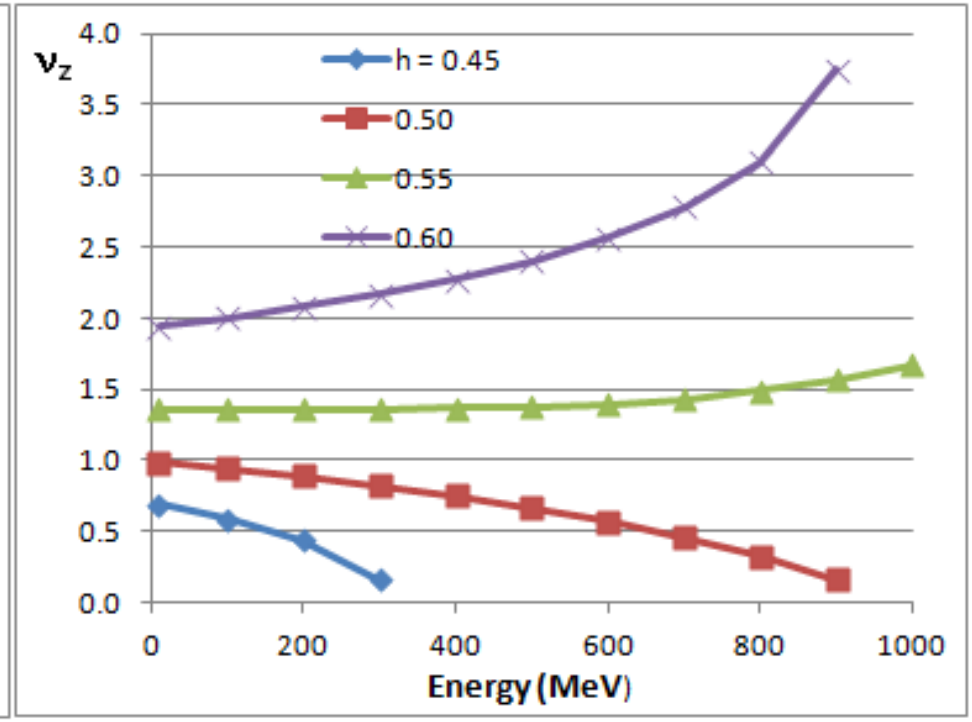
COMPLEMENTARY EFFECTS OF h AND H_2 ON ν_z, ν_r

(H_1/B_c = 2, , δ = 0.3, N = 8)

H_2/B_c = 0.5



H_2/B_c = 0.4



In each case, solutions with near constant ν_z can be found - but choosing higher h with lower H_2 and v require lower field strengths.

H_2/B_c	h	v	B_h/B_c	B_v/B_c
0.4	0.54	0.16	5.4	-7.6
0.5	0.50	0.20	6.3	-9.5

SUMMARY

Adding a γ^2 component to the hill field in a radial sector machine - and subtracting an offsetting γ^2 component from the valley fields to maintain isochronism

- increases the flutter and gives stronger alternating gradients
- can provide sufficient vertical focusing to reach at least 1 GeV
- offers a possible alternative to spiral sectors
- though the radial tune grows even faster with energy: $\nu_r > \gamma$.

The scheme is also viable with drift spaces between the sectors.

Alternative variations in the hill and valley field profiles might be even more advantageous!