FFAG'14 Workshop, BNL, 22-26 September 2014

End-to-end polarized bunch transport in eRHIC

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Contents

1 Reminder, dynamical effects of synchrotron radiation

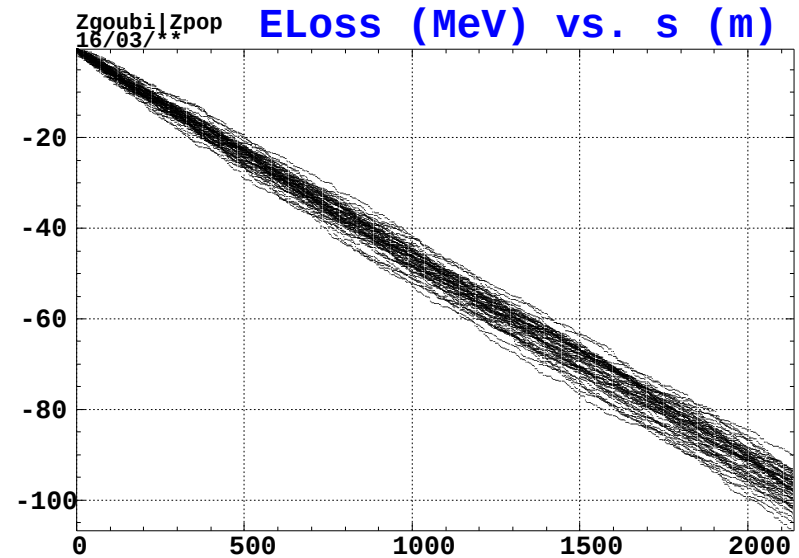
1.1 Particle dynamics

- **Electrons circulating in eRHIC arcs loose energy by synchrotron radiation (SR)**

Over a trajectory arc $\Delta\theta$, with constant curvature $1/\rho$, relative average energy loss :

$$\frac{\overline{\Delta E}}{E} = 1.879 \times 10^{-15} \frac{\gamma^3}{\rho} \Delta\theta$$

Energy loss over one eRHIC turn at top energy, starting E=21.16 GeV :



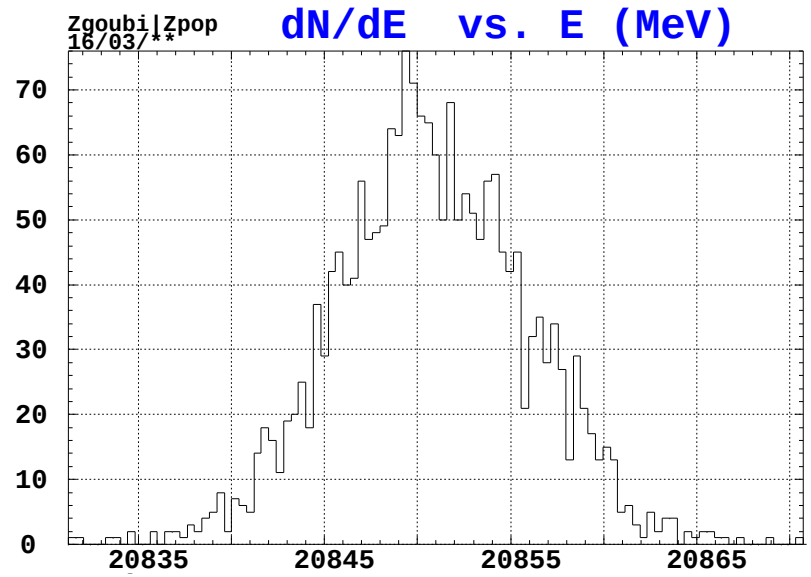
Example : **Energy loss over 1 ring turn, ≈ 3835 m, starting energy =21.16 GeV : $\overline{\Delta E} \approx 100$ MeV.**

• **SR is a stochastic process, photons fluctuate in number and energy, thus inducing**

- energy spread,

$$\frac{\sigma_E}{E} = 3.794 \times 10^{-14} \frac{\gamma^{5/2}}{\rho} \sqrt{\Delta\theta}$$

Energy spread σ_E after 6 turns from 7.944 to 21.16 GeV :



Example : $\sigma_E \approx 5.2$ **MeV after 11 passes in eRHIC from 7.94 to 21.16 GeV.**

- and bunch lengthening$^{(*)}$,

$$\sigma_l = \left(\frac{\sigma_E}{E}\right) \left[\frac{1}{L_{\text{bend}}} \int_s^{s_f} \left(D_x(s) T_{51}(s_f \leftarrow s) + D'_x(s) T_{52}(s_f \leftarrow s) - T_{56}\right)^2 ds\right]^{1/2}$$

(the integral is taken over the bends).

Example : **bunch lenghtening $\sigma_l \approx 5\,\mu$m upon 21.16 GeV turn (6 arcs).**

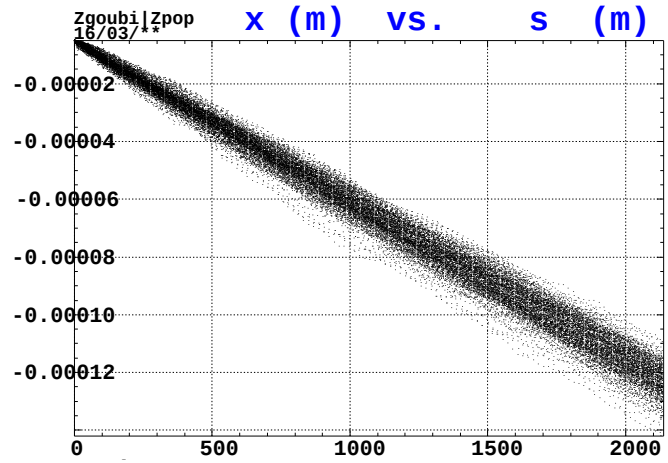
• **The energy loss causes a spiraling of the beam centroid.**

Over a distance $[s_i, s_f]$ **:**

$$\left[\begin{array}{c}\overline{x(s_f)}\\ x'(s_f)\end{array}\right] = T(s_f \leftarrow s_i) \times \left[\begin{array}{c}\overline{x(s_i)}\\ x'(s_i)\end{array} + \frac{\sigma_E}{E}\left\{\begin{array}{c}<U>\\ <V>\end{array}\right\}\right]$$

$$\left[\begin{array}{c}U(s_e)\\ V(s_e)\end{array}\right] = \left[\begin{array}{c}D_x(s_i) - \int_{s_i}^{s_e} \frac{T_{12}(s \leftarrow s_i)}{\rho(s)}\, ds\\ D'_x(s_i) + \int_{s_i}^{s_e} \frac{T_{11}(s \leftarrow s_i)}{\rho(s)}\, ds\end{array}\right]$$

Inward bunch spiraling, 21 GeV arc



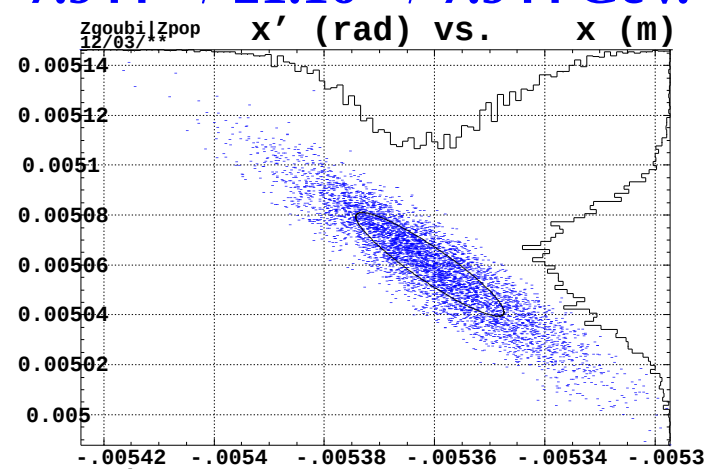
Example : $\Delta x = -0.12$ **mm at 21.16 GeV, over 2138 m distance, 6 arcs.**

• **and horizontal emittance growth**$^{(*)}$**,**

$$\Delta\sigma(s_f) = T(s_f \leftarrow s_i) \times$$

$$\left(\frac{\sigma_E}{E}\right)^2 \left[\begin{array}{cc}<U^2> & <UV>\\ <UV> & <V^2>\end{array}\right] \times \tilde{T}(s_f \leftarrow s_i)$$

(x,x') phase space after 21 passes
7.944 → 21.16 → 7.944 GeV.

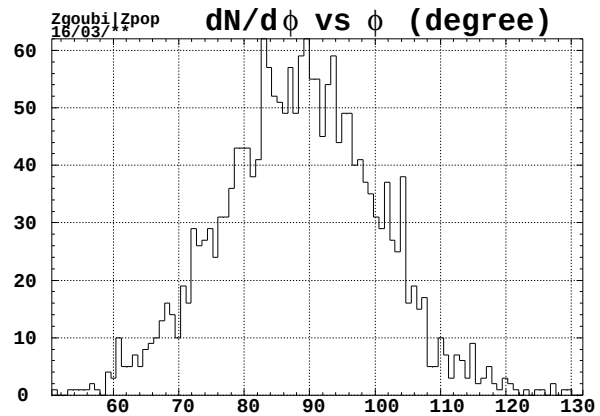


Example : **SR induced emittance is** $\beta\gamma\epsilon_x = 1.7\,\pi\mu$**m, after 21 passes in eRHIC, from 7.94 to 21.16 and back down to 7.94 GeV.**

1.2 Spin dynamics

• **In a similar manner, averages and second momenta build up upon stochastic SR. Some simple considerations can serve as a guidance, as follows.**

• **Spin precession around the magnetic field vector amounts to** $\phi = G\gamma\theta$



wherein

θ **is the particle deflection angle.**

$G = 1.1610^{-3}$**, anomalous gyromagnetic factor**

γ **is the Lorentz relativistic factor**

Example : over the 21.16 GeV turn, spin undergoes $G\gamma$ **=48 precessions around the vertical axis.**

• **A change in energy upon emission of a photon will cause a change in spin precession angle compared to the unperturbed case,**

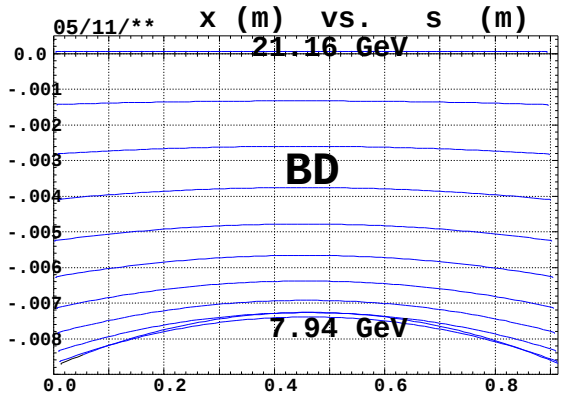
$$\{\delta E, \Delta\theta \text{ from } \Delta E\} \Rightarrow \Delta\phi$$

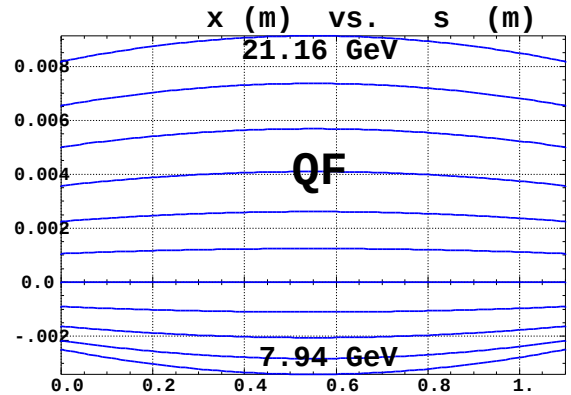
Example : figure above : after 11-pass acceleration, SR induced spin diffusion is $\sigma_\phi \approx 12$ **degrees.**

2 Brief review of eRHIC FFAG lattice, second ring

• **Note in passing : stepwise methods allow detailed insight in magnetic and geometrical aspects of lattices.**

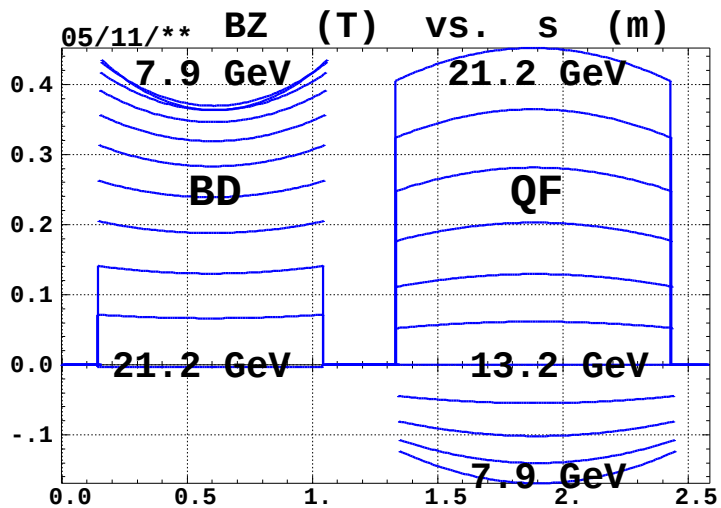
• **Orbits in defocusing quad and focusing quad :**





• **Field along orbits :**

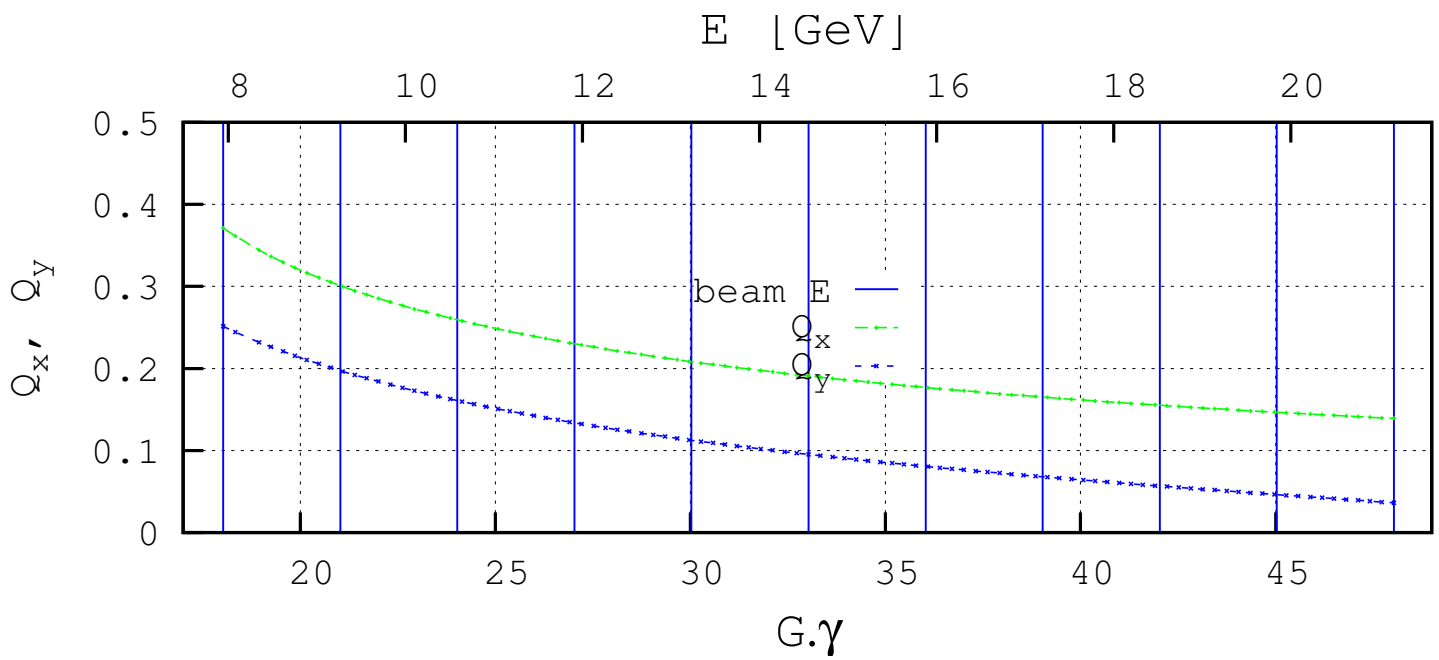
Trajectory curvature varies continuously across the magnets.



• **While we are there, still : hard edged magnet models will be used throughout, no such thing as "fringe fields" in the present study.**

- **Cell tunes**

A continuous $G\gamma$ scan of the high energy FFAG eRHIC cell (7.944 $\rightarrow$ 21.16 GeV) :



- **Matching polynomials :**

$$Q_x(G\gamma) = -0.002262268 + 8.60741/G\gamma - 120.24058/(G\gamma)^2 + 1545.2231/(G\gamma)^3$$
$$Q_y(G\gamma) = -0.140723255 + 11.113263/G\gamma - 153.6170/(G\gamma)^2 + 1457.3424/(G\gamma)^3$$

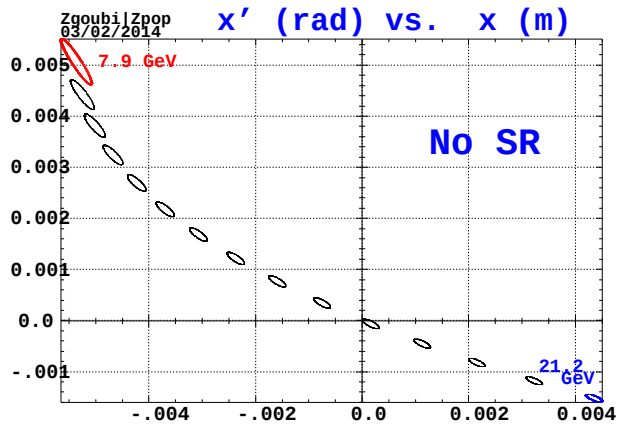
- **These polynomials can be used to investigate spin resonance conditions as**

$G\gamma \pm Q_{x,y}(G\gamma) =$**integer**

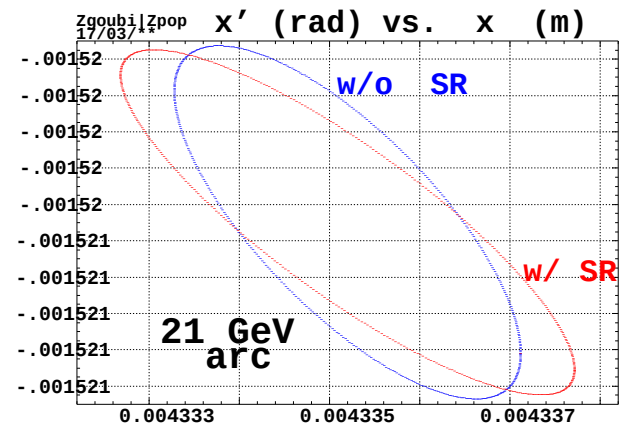
- **Optical functions**

Horizontal phase-space, matched ellipses at middle of cell drift,

E : $7.9 \rightarrow 21.2$ **GeV, step** 0.95 **GeV.**



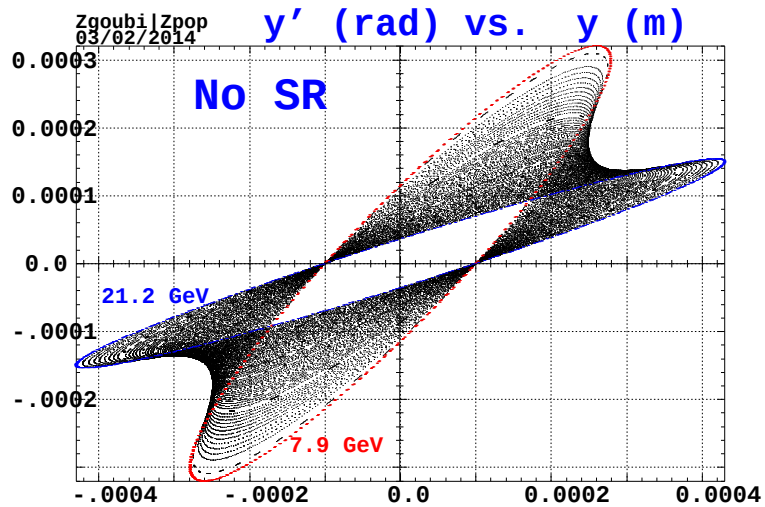
SR induced beam mis-match :



Middle of drift.

Matched vertical ellipses,

E : $7.9 \rightarrow 21.2$ **GeV, step** 0.14 **GeV.**



- **Tight comparisons have been performed between codes : SYNCH, MUON1, MADX[-PTC]. The conclusion is : good in general, however some discrepancies are observed, serious ones in some cases !**

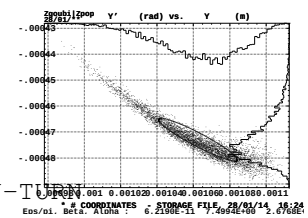
3 Dynamical effects of SR in the second ring

3.1 Turn by turn, starting emittances zero at start of turn

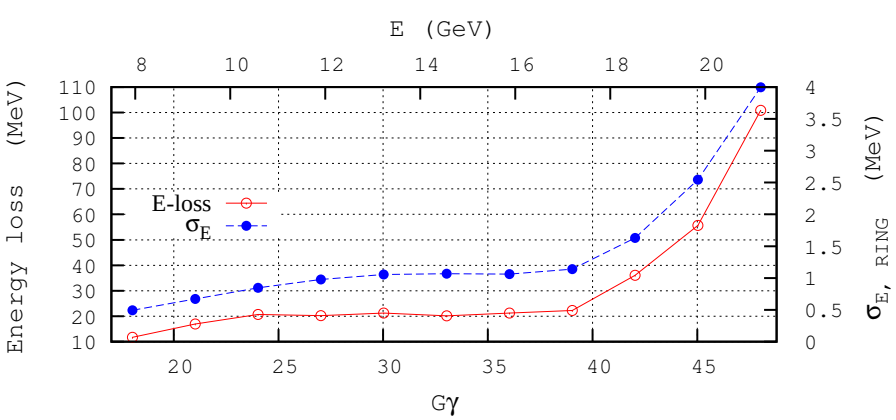
- **The "adiabatic DS" guides the centroid energy onto the appropriate FFAG orbit in the arc. Thus the bunch is launched in the middle of the long straigth with transverse coordinates zero.**
- **The game here : a bunch is tracked for a single turn. Its outcoming data (energy loss, energy spread) are compared to theoretical expectations (slides # 2, 3).**

$G\gamma$	Beam E (GeV)	Aver. bend radius in QF, BD l_{QF}/θ_{QF}, l_{BD}/θ_{BD} (m)	Trajctry deflection in BD, QF θ_{QF}, θ_{BD} (mrad)	THEORETICAL ONE SEXTANT $\overline{\Delta E}$ (MeV)	σ_E (MeV)	RING $\overline{6\alpha\Delta E}$ (MeV)	$\sqrt{6\alpha}\,\sigma_E$ (MeV)	T-B-T TRACKING $\overline{\Delta E_T}$ (MeV)	σ_{E_T} (MeV)	$\sigma_{\epsilon_x/\pi}$ (pm)
18.0291	7.944	-68.525, 172.486	-13.134, 6.377	1.770	0.185	11.9	0.48	12.3	0.50	24.3
21.0292	9.266	-79.267, 239.265	-11.354, 4.597	2.327	0.251	15.7	0.66	16.3	0.68	0.265
24.0293	10.588	-92.718, 372.852	-9.707, 2.950	2.750	0.313	18.5	0.81	19.4	0.83	0.367
27.0295	11.910	-110.041, 773.486	-8.179, 1.422	2.978	0.363	20.1	0.94	21.2	1.00	74.5
30.0296	13.232	-133.183, ∞	-6.758, 0.001	3.023	0.393	20.4	1.02	21.5	1.08	0.995
33.0297	14.554	-165.662, -830.886	-5.433, -1.324	2.998	0.397	20.2	1.03	21.1	1.08	0.287
36.0298	15.876	-214.552, -429.384	-4.195, -2.562	3.150	0.390	21.2	1.01	21.7	1.05	1.01
39.0300	17.198	-296.486, -295.618	-3.036, -3.721	3.880	0.443	26.1	1.15	25.6	1.13	2.26
42.0301	18.520	-462.056, -228.751	-1.948, -4.809	5.769	0.657	38.9	1.71	36.5	1.55	0.531
45.0302	19.842	-972.768, -188.637	-0.925, -5.831	9.597	1.068	64.7	2.77	59.2	2.62	3.65
48.0303	21.164	23700.462, -161.898	0.038, -6.795	16.362	1.678	110.3	4.35	99.7	3.98	5.10

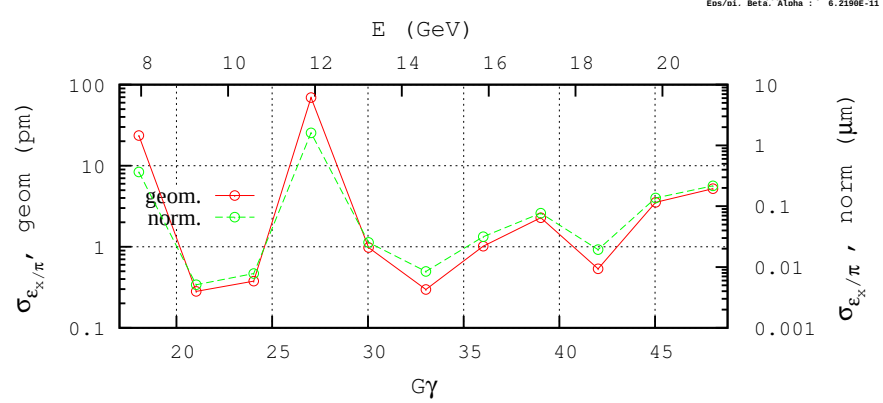
α **accounts for actual deviation per sextant** $< 2\pi/6$ **:**
$\alpha = (2\pi\ /\ 6)/\,(138[\text{cells per arc}] \times \text{deviation per cell}) \approx 1.123$



ENERGY LOSS AND SPREAD, TURN-BY-TURN



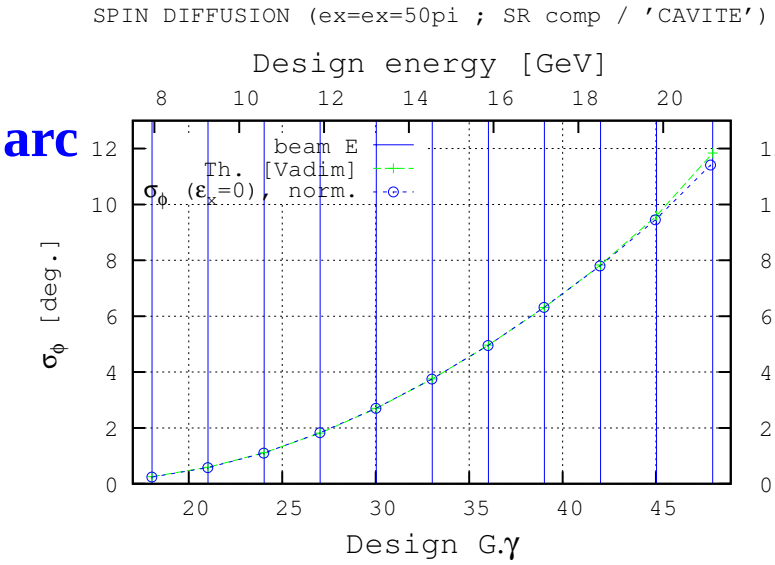
HORIZONTAL EMMITTANCE INCREASE, TURN-BY-TURN



3.2 Spin motion, quick overview

• **Expected precession of spins in horizontal plane :**

$\phi = G\gamma\theta = G\gamma(\theta_{\rm QF} + \theta_{\rm BD})$ **per cell** $\xrightarrow{\times 138}$ **per arc**

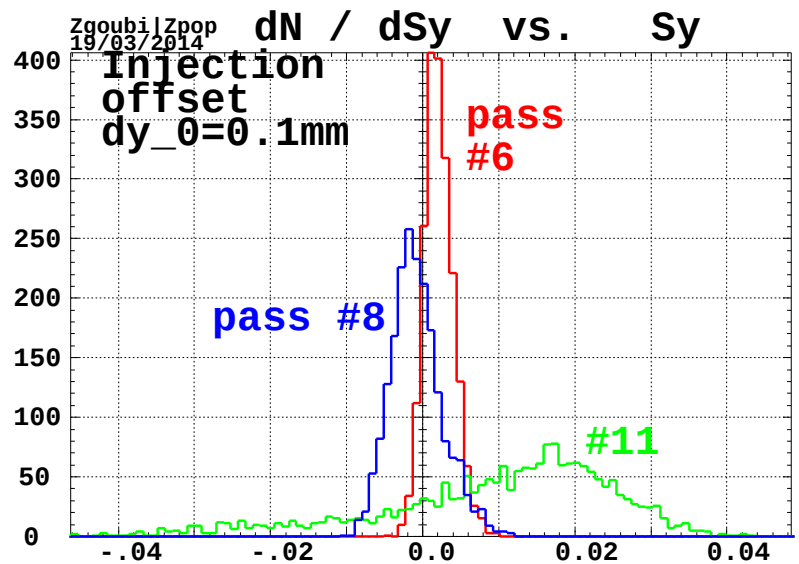


• **Stochastic change of energy due to SR causes diffusion of spins in horizontal plane**

Simple just-6-arcs case : agreement with theory (case of beam with zero vertical emittance) is very good

≈12 degrees spin spreading after 11 recirculations from 7.9 to 21 GeV.

• **As to vertical motion : betatron motion and orbit add vertical spin component and diffusion, typical :**



• **Benchmarking efforts using existing analytical formalism are part of the game.**

3.3 End-to-end tracking, starting emittances 50π μm normalized

- **The game here : a 50π norm. emittance bunch (≈ 3.1 nm at 7.944 GeV) is launched on axis in the long straigth (thus D=D'=0), with energy 7.944 GeV, and then tracked over 21 turns.**
- **SR is compensated (actually poorly, this is being worked on / automatized using theoretical energy loss) by adjusted linac energy, different at each pass.**
- **Beam energy, energy spread, horizontal emittance, are recorded at each pass.**

ZGOUBI FILE AVAILABLE THERE :
/home/owl/fmeot/zgoubi/struct/bnl/eRHIC/140618_fullRingFromSBrooks/[etc.]

START OF FILE :

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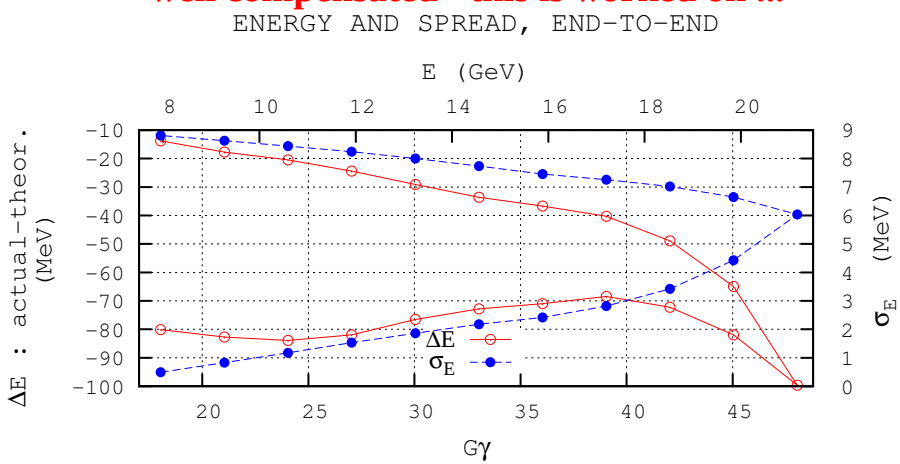
 Generated by MADX -> Zgoubi translator
 'MCOBJET'
57.36635309d3           p= 17198000000.0000  eV/c,
3
5000
2 2 2 2 1 1
0. 0. 0. 0. 0. 0.462255816  'o'  ! 0.4619439+ 1/2loss@1st turn
 2.703371  1.859656  3.15e-09   6.25
-2.592712  2.421706  3.15e-09   6.25
0. 1.  0.     6.25
123456 234567 345678
 'PARTICUL'
0.51099892 1.60217653e-19 1.15965218076e-3 0.0 0.0
 'SPNTRK'
1
 'FAISTORE'
zgoubi.fai #EndRing
1
 'FAISCEAU'
 'SRLOSS'
1   srLoss
MULTIPOL
1  123456
 'SCALING'
1  1
MULTIPOL
-1
57.36635309
1
```

END OF ZGOUBI FILE :

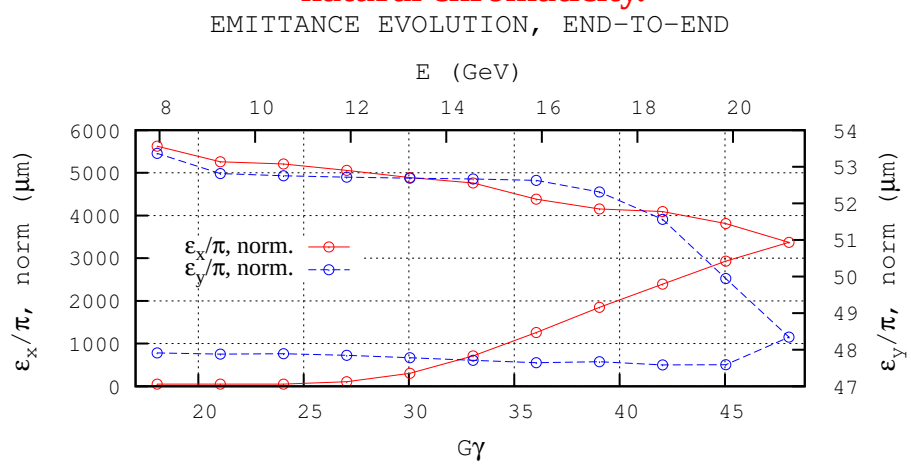
```
 'MARKER'  #EndRing
 'FAISCEAU'
 'CAVITE'
3
0  0
1.334423e9  1.57079632679
 'REBELOTE'
20  0.1 99 1
1
CAVITE  20  1.337399e9   1.33934e9    1.34012e9    1.34018e9
 1.34057e9    1.34324e9    1.35119e9    1.36838e9    1.40021e9
  -1.24379e9  -1.27562e9  -1.29281e9   -1.30076e9   -1.30343e9
 -1.30382e9  -1.30388e9  -1.30388e9  -1.30466e9  -1.306601e9
-1.309577e9
 'SPNPRT'
 'FAISCEAU'
 'END'
```

• 1st case, for the record : the bunch is just let go (no bunch recentering, nowhere)

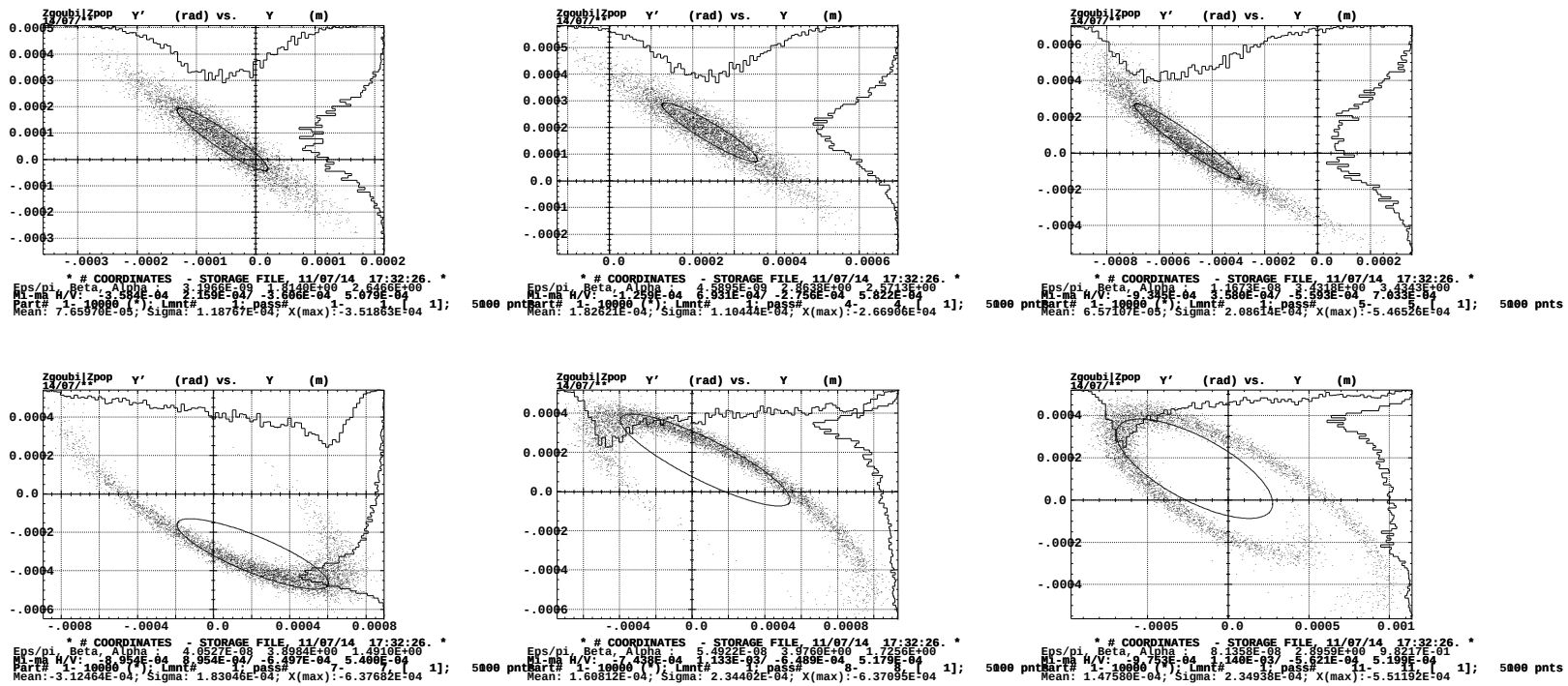
The red markers show that the energy loss is not very well compensated - this is worked on ...



Vertical emittance behaves well, increase is $\lesssim 10\%$.
Horizontal shows apparent increase, an effect of the large natural chromaticity.



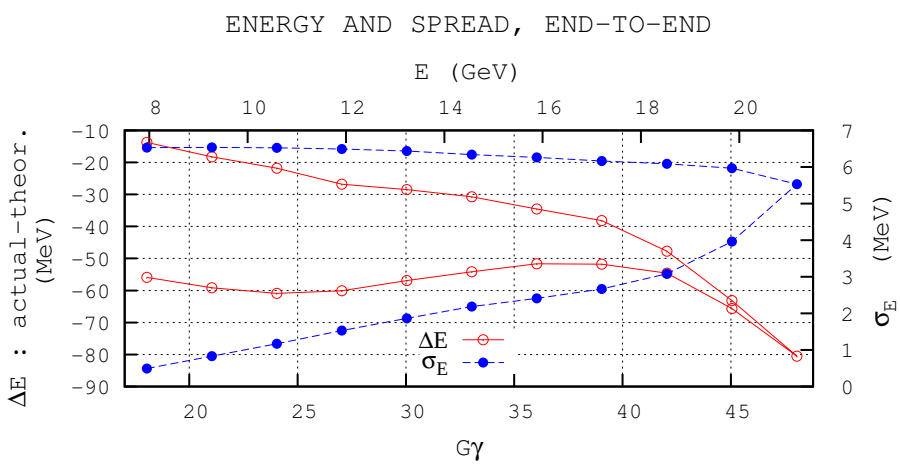
Horizontal phase-space, pass 1, 4, 5, 7, 8 and pass 11

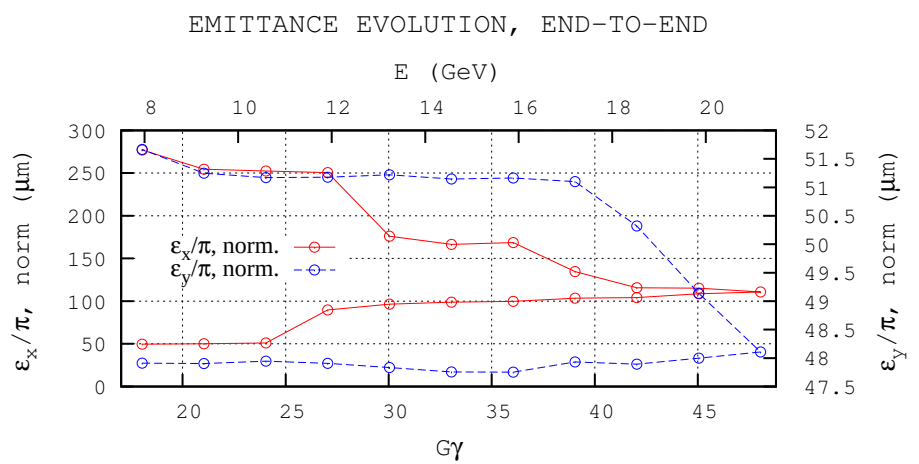


• 2nd case : bunch recentered on linac axis on successive turns - 5000 particles

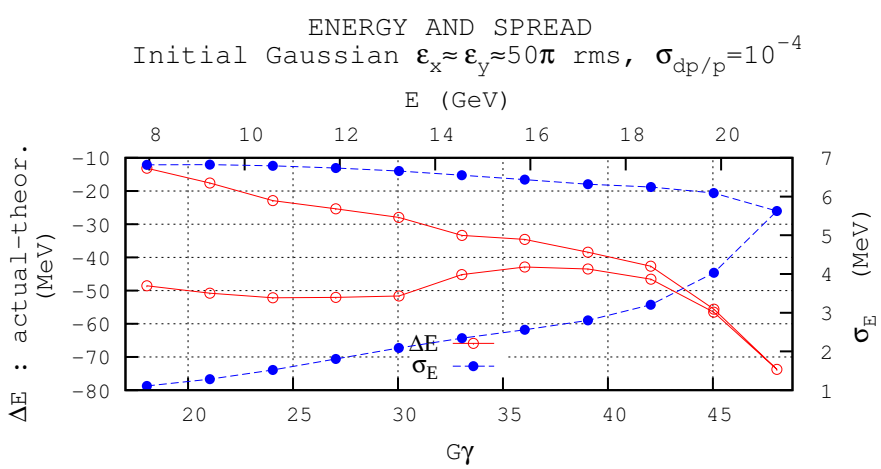
Horizontal emittance increase is strongly reduced compared to previous case (no recentering) :

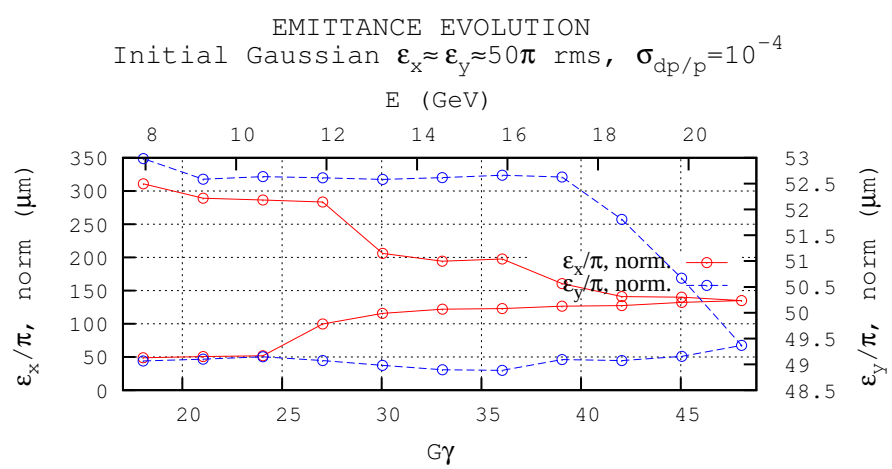
- Case of initial $\epsilon_x \approx \epsilon_y \approx 50\,\pi$mm.mrad and dp/p=0 :





- Case of initial $\epsilon_x \approx \epsilon_y \approx 50\,\pi$mm.mrad and $dp_0/p_0 = 10^{-4}$:

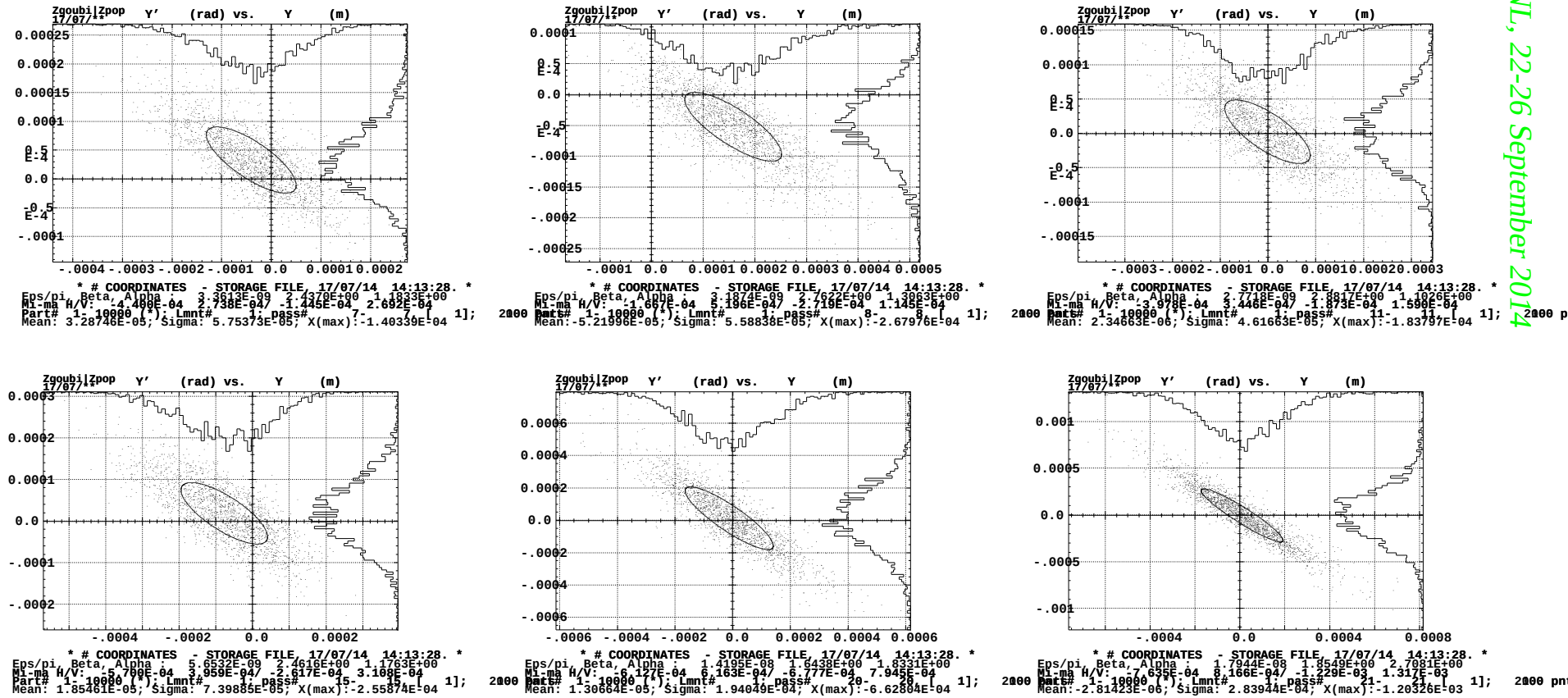




Sample tracking outcomes in the 2nd case (bunch recentered at the linac, at each pass) :

dilution effect of chromaticity is strongly reduced, no longer visible.

Horizontal phase-space, pass 7, 8, 11, 15, 20, 21 :



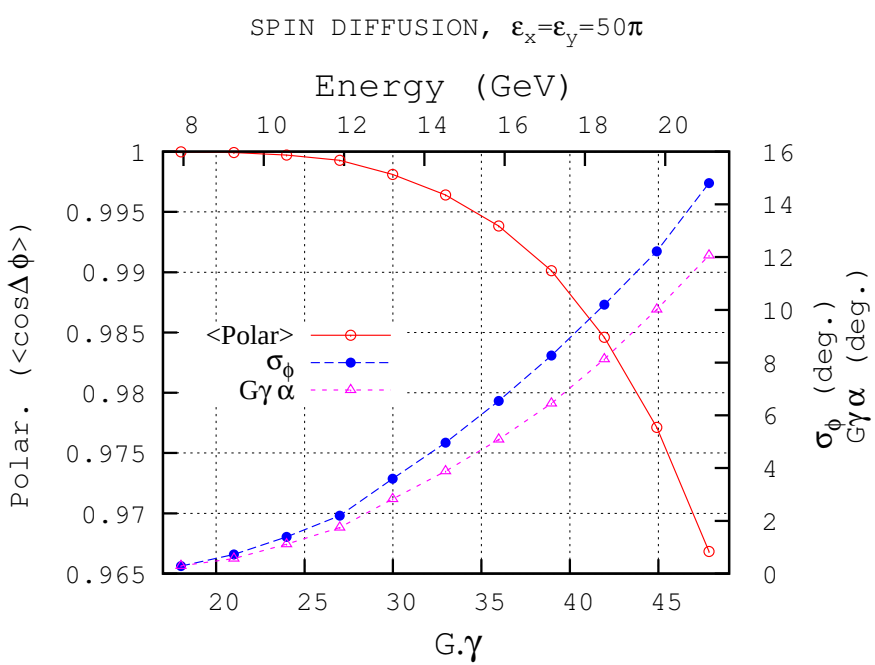
POLARIZATION. The plot below shows, at each pass :

- the average of the projection of spins on the local average spin direction, $< \cos \Delta\phi >$

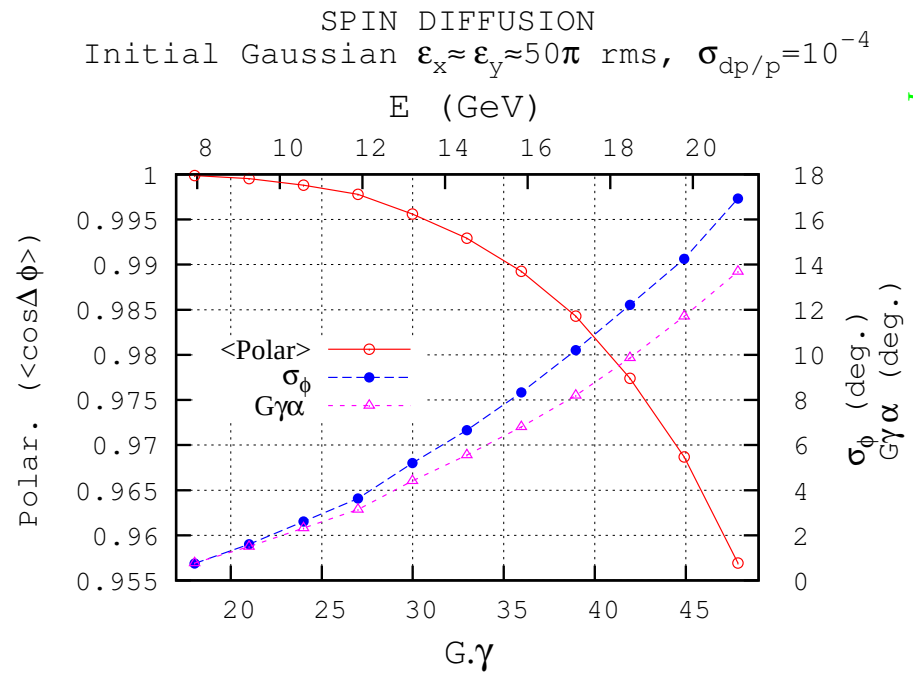
- the rms value of individual spin angles with the local average spin direction, σ_ϕ

- the cumulated spin rotation, $G\gamma\alpha$**.**

Case initial $\epsilon_x \approx \epsilon_y \approx 50\,\pi$**mm.mrad and** $dp_0/p_0 = 0$



Case initial $\epsilon_x \approx \epsilon_y \approx 50\,\pi$**mm.mrad and** $dp_0/p_0 = 10^{-4}$



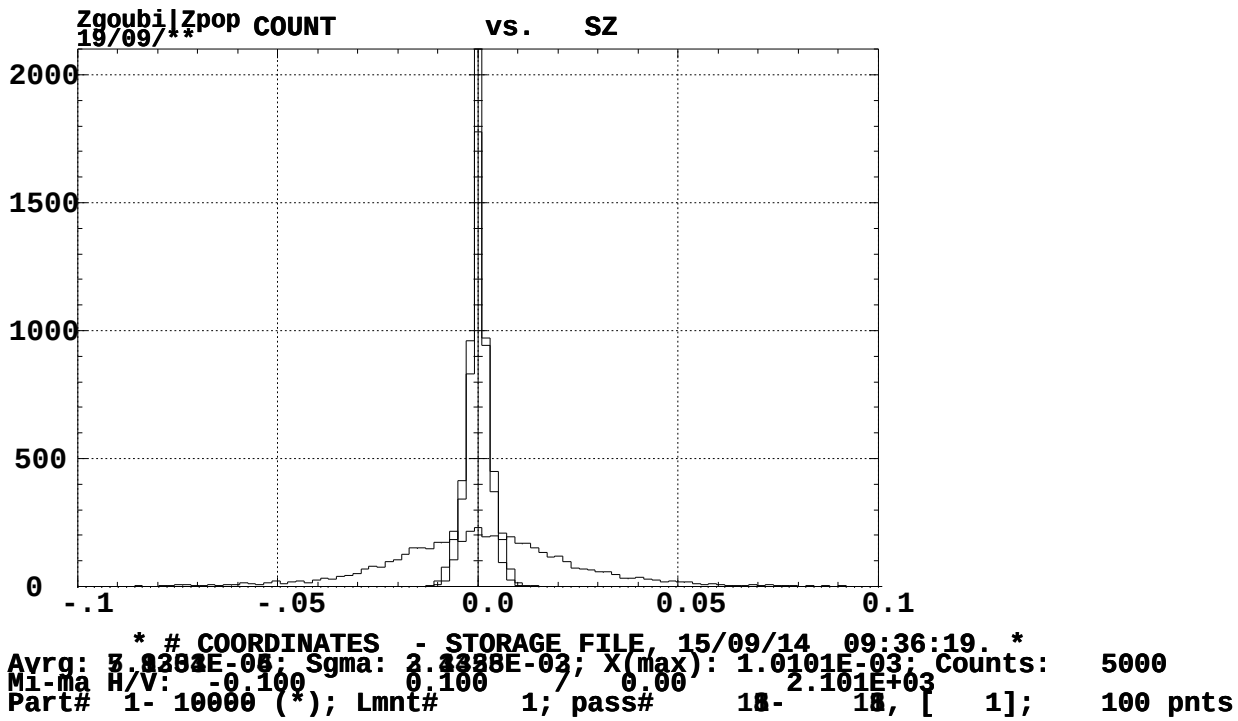
Native beam momentum spread induces additional spreading of spins (from left plot to right plot).

Vertical component of spins :

The polarization is initially in the horizontal plane,

however the vertical betatron motion causes spins to diffuse out of the horizontal plane under the effect of horizontal field components.

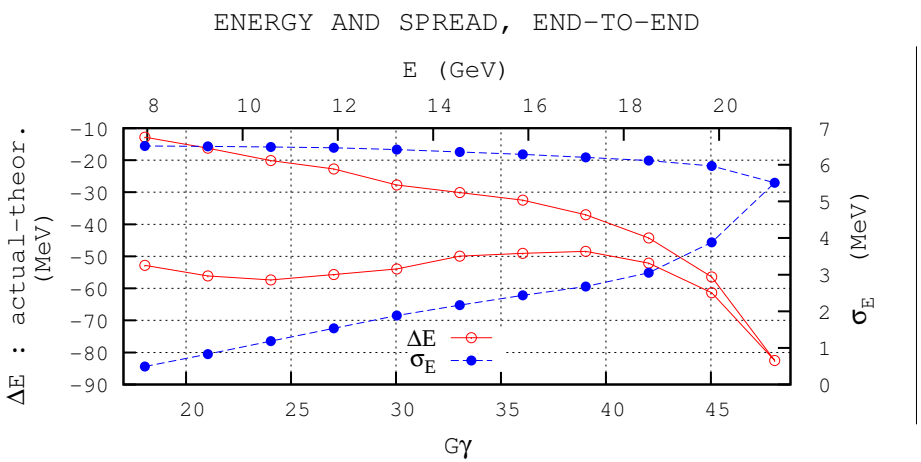
After passes 5, 8, 11

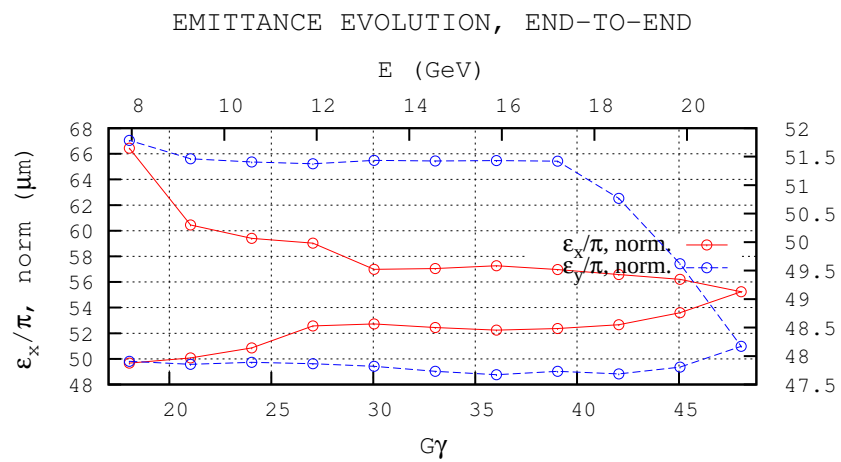


• 3rd case : bunch recentered (on the optical axis in the long straight) at each arc - 5000 particles

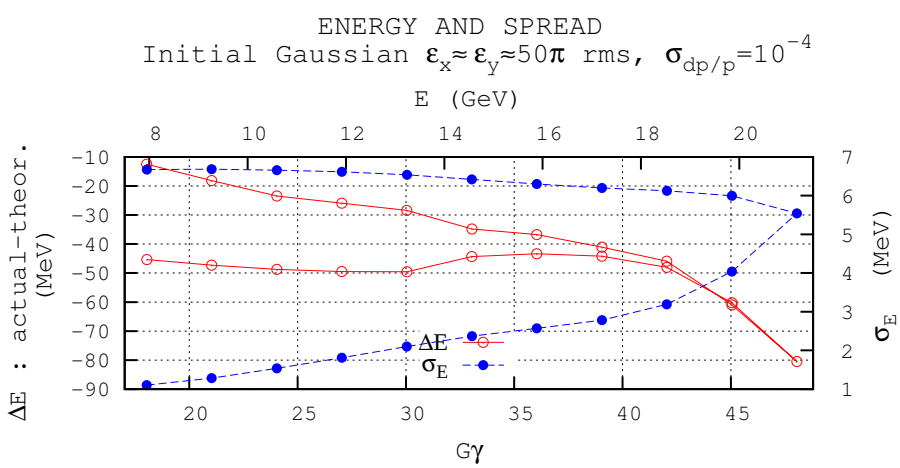
This causes a strong reduction of horizontal emittance increase over the 21 passes.

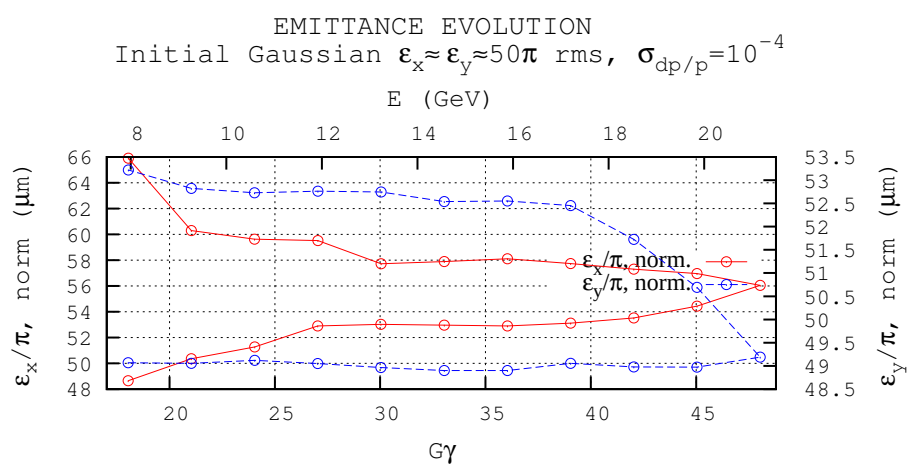
Case initial $\epsilon_x \approx \epsilon_y \approx 50\,\pi$mm.mrad and dp/p=0





Case initial $\epsilon_x \approx \epsilon_y \approx 50\,\pi$mm.mrad and $dp_0/p_0 = 10^{-4}$

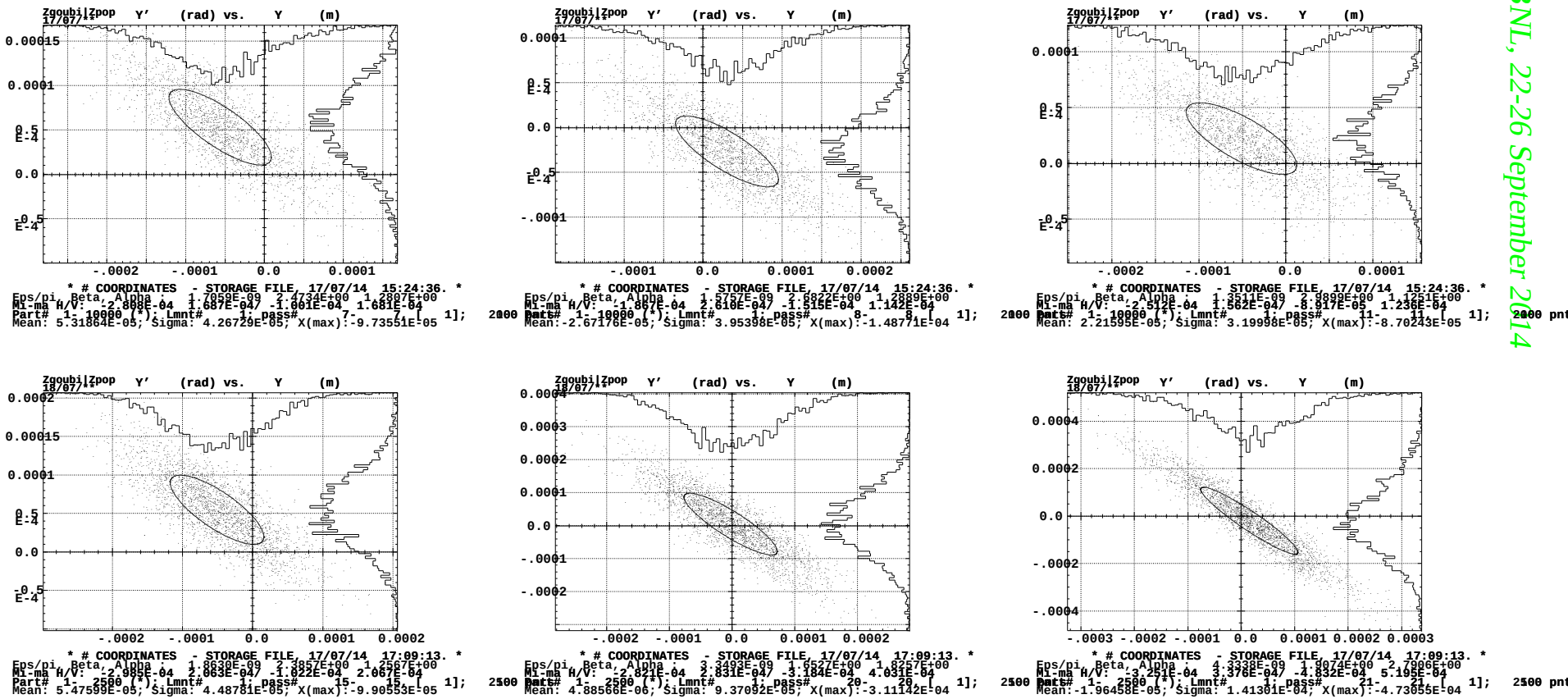




Sample tracking outcomes in the 3rd case (bunch recentered at each arc)

Horizontal phase-space, end of pass # 7, 8, 11, 15, 20, 21.

Case initial $\epsilon_x \approx \epsilon_y \approx 50\,\pi$mm.mrad and dp/p=0



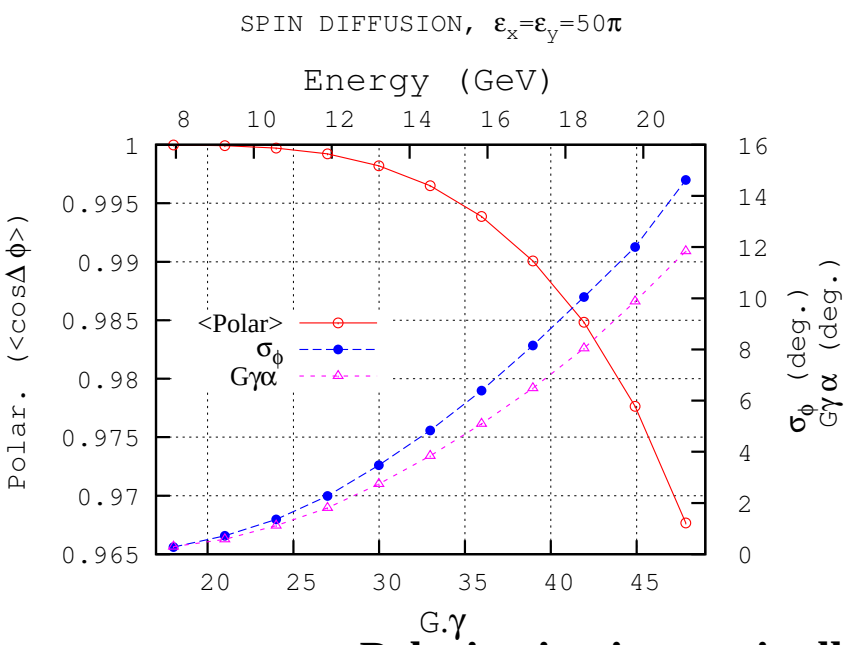
POLARIZATION. The plots below show, as a function of the pass :

- the average of the projection of spins on the local average spin direction, $< \cos \Delta\phi >$

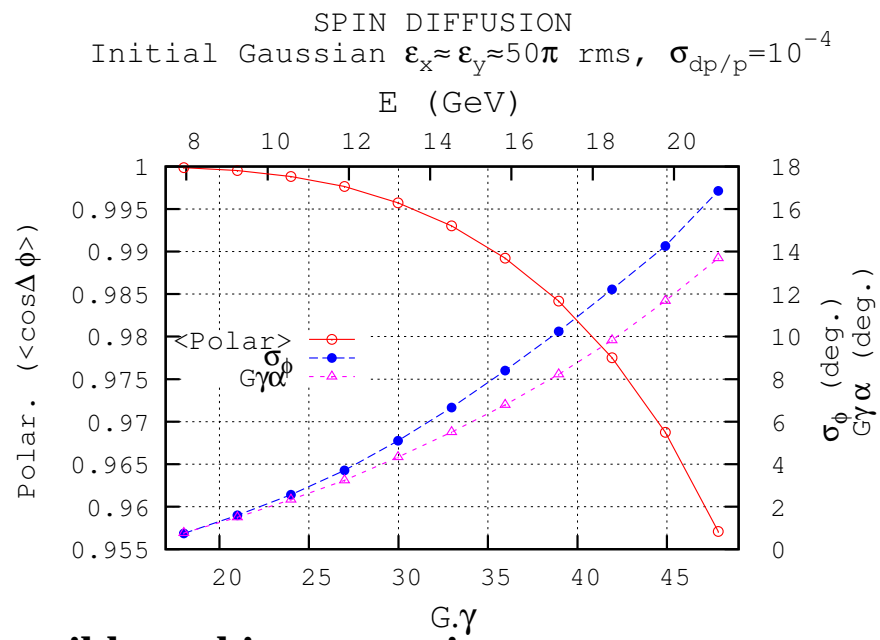
- the rms value of individual spin angles with the local average spin direction, σ_ϕ

- the cumulated spin rotation, $G\gamma\alpha$**. That value differs from zero because of a mys-tuning of the energy loss compensation.**

Case initial $\epsilon_x \approx \epsilon_y \approx 50\,\pi$**mm.mrad and** $dp_0/p_0 = 0$



Case initial $\epsilon_x \approx \epsilon_y \approx 50\,\pi$**mm.mrad and** $dp_0/p_0 = 10^{-4}$



Polarization is marginally sensible to this recentering.

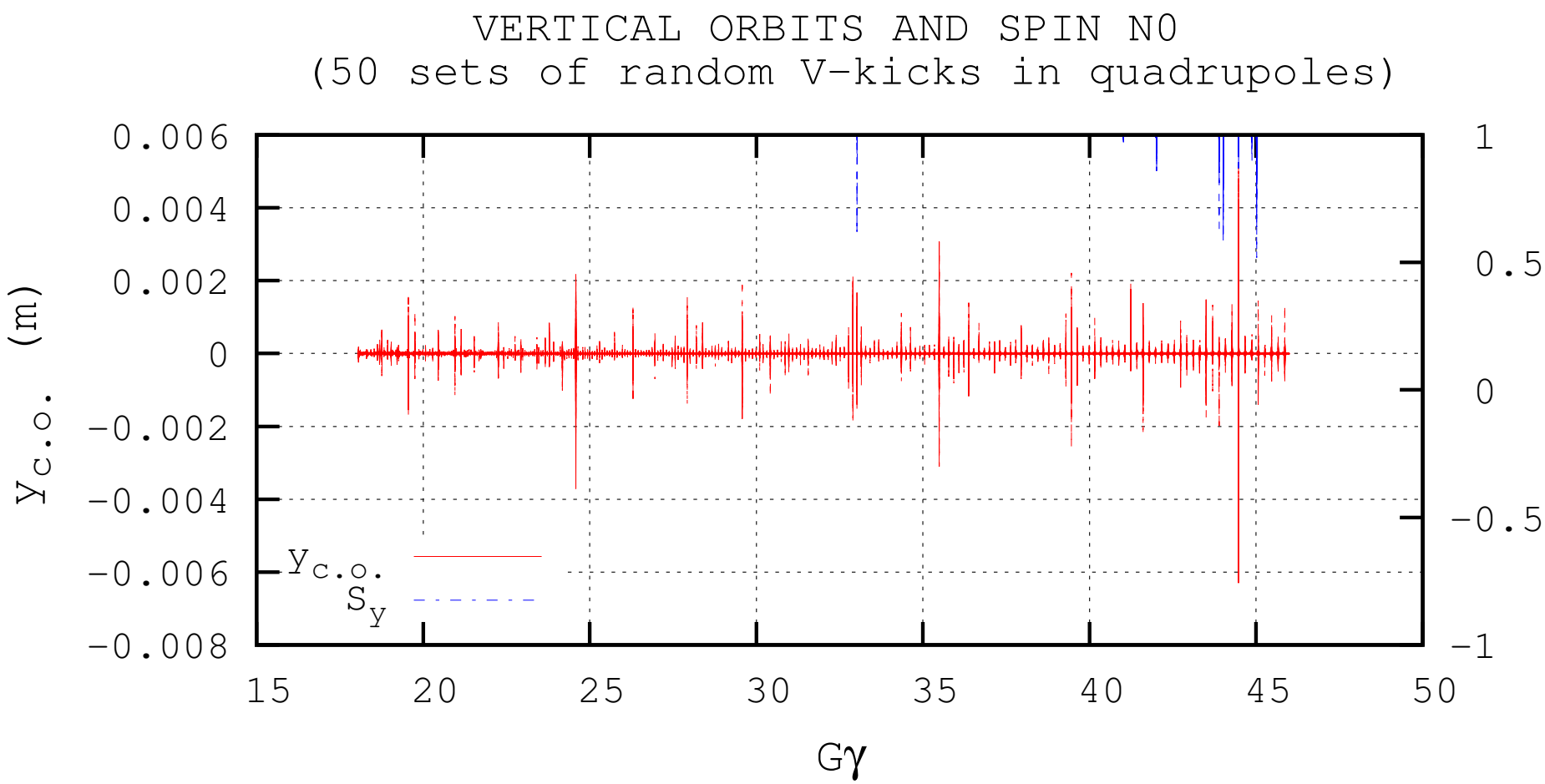
3.4 Spin $\vec{n}_0$ scans in the presence orbit vertical closed orbit (SR off)

- **Zgoubi's error generator is used to inject random dipole component in all cell quads, so to cause vertical orbit.**
- **Error injection is repeated with 50 different seeds.**
- **For each seed :**
  - **tracking is performed at 4000 different energies evenly distributed in [7.94,21.16GeV]**
  - **5000 particles tracked for each one of these 4000 energies.**

ERROR GENERATOR DATA IN ZGOUBI :

```
 Generated by MADX -> Zgoubi translator
 'MCOBJET'     VARYZP
57.36635309d3            ! reference rigidity (kG.cm) =>   p= 17198000000.0000  eV/c,  G.gamma =     39028845.768
3.1                      ! .1 is for emittances normalized to BORO
1
2 2 2 2 1 1
0. 0. 0. 0. 0.   4.61943880E-01
 2.703371  1.859656  1.48563786487e-99   9    ! norm. emittance : 1.48563786487e-09 = 50e-6/gamma|_BORO
-2.592712  2.421706  1.48563786487e-99   9
0. 1.  0.     9
12345 23456 34567
 'PARTICUL'
0.51099892 1.60217653e-19 1.15965218076e-3 0.0 0.0
 'SPNTRK'
1
 'SRLOSS'
0   srLoss
MULTIPOL
1  123456
 'SCALING'
1  1
MULTIPOL
-1
57.36635309
1
 'ERRORS'          !!! Dipole error, with density uniform in +/-0.01kG
1 1 12345                         dB(kG)
MULTIPOL{}    1    BP A U 0.d0     0.01    9999
 'MARKER'  #StartRing
  --- RING SEQUENCE FOLLOWS ---
```

- **Case vertical orbit** ~ 0.1 **mm rms overall.**



4 Defect studies - multipole defects

4.1 Simulation conditions :

• **Simplified case of a ring made of just-6-arcs, no drifts.**

• **A 5000 particle bunch is launched for 21 passes. Starting x and y emittances are close to** 50π **mm.mrad norm.**

• **SR is compensated at the linac, namely, energy gain is 1.322 GeV + half the energy loss at the previous pass + half the energy loss at the next pass. Starting energy is 7.944 GeV + half the energy loss at pass #1.**

• **Prior to each new turn with a new energy, the bunch is placed, in position and angle (artificially, by an appropriate change of frame) on the theoretical FFAG orbit.**

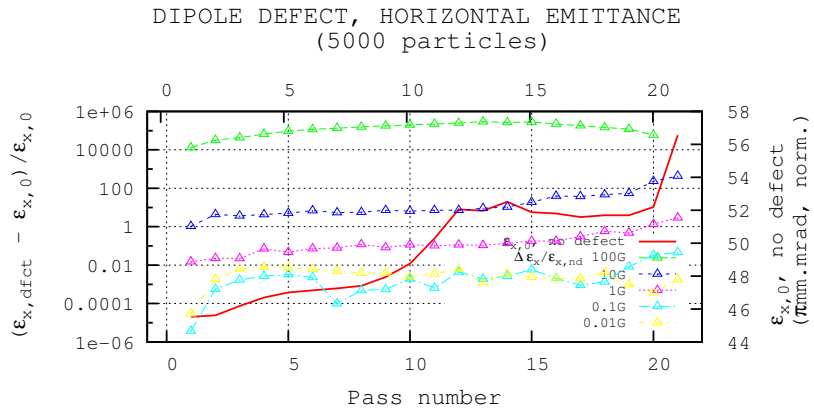
• **The game here consists in introducing multipole defects in the FFAG quadrupoles. Defects considered go from quadrupole to dodecapole, a single one at a time. The defect is sorted at random or each quad, in a uniform distribution with a given width. Various widths are tried to determine tolerances.**

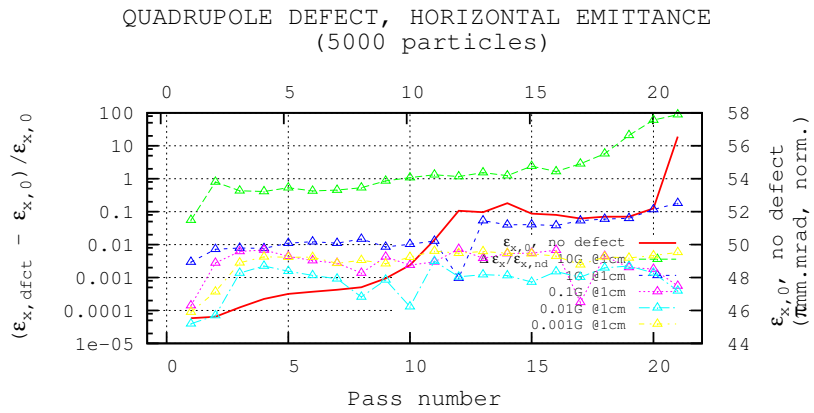
• **These multipole defects act on the orbit and on the focusing, by feed-down. Emitttance increase is expected from that.**

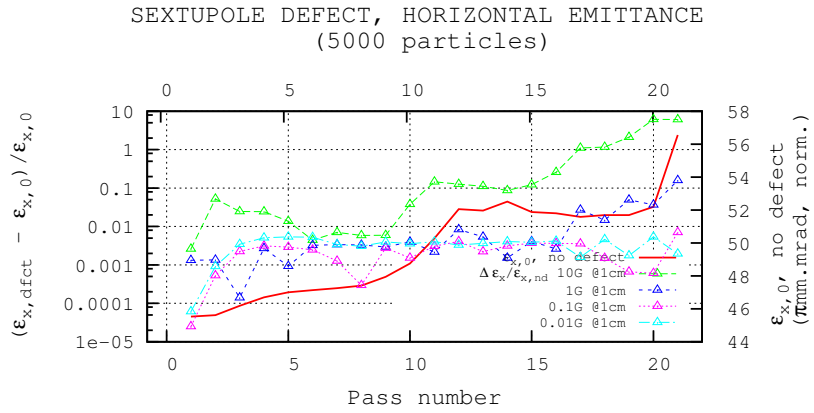
• **The plots that follow indicate the tolerable defect value that would lead to a tolerable xx% emittance increase.**

4.2 Simulation results

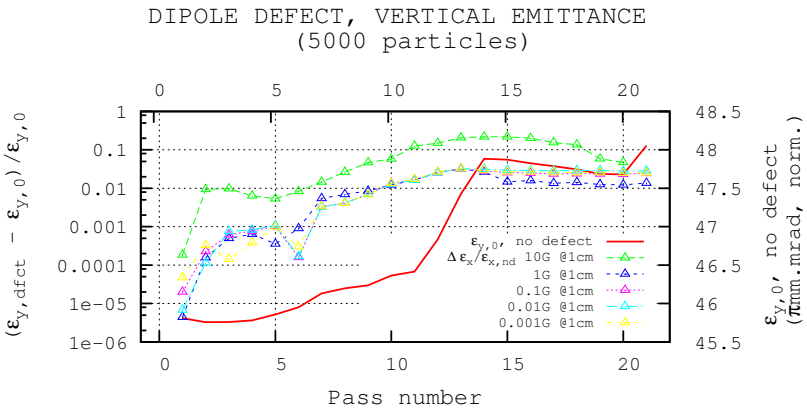
Dipole to sextupole defect, evolution of horizontal emittance

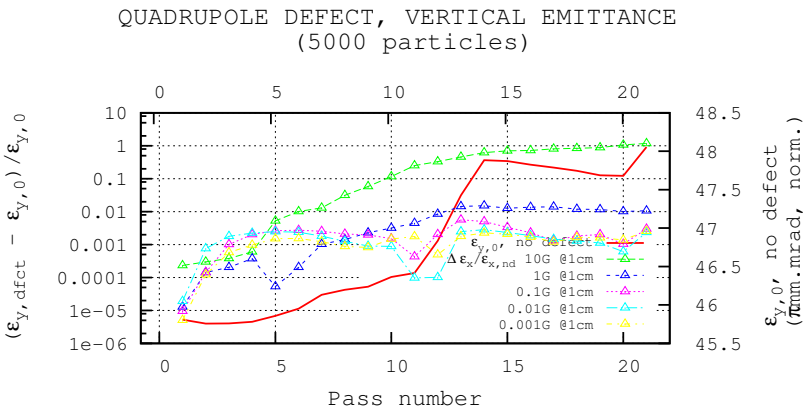


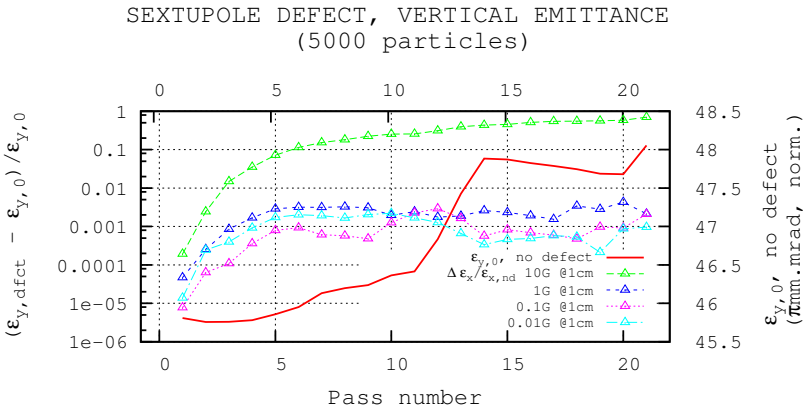




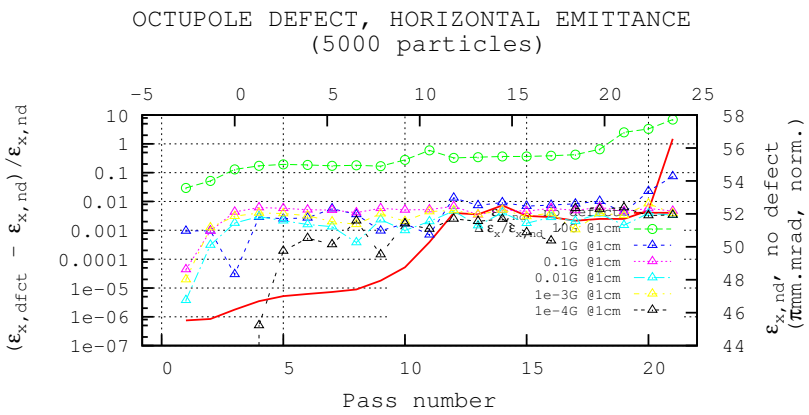
Dipole to sextupole defect, evolution of vertical emittance

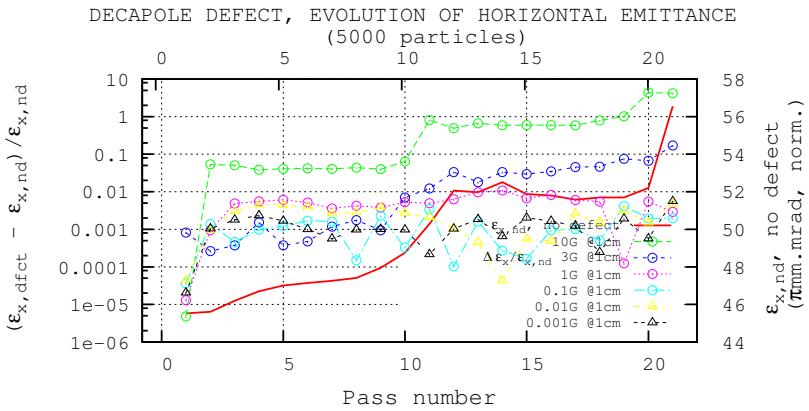


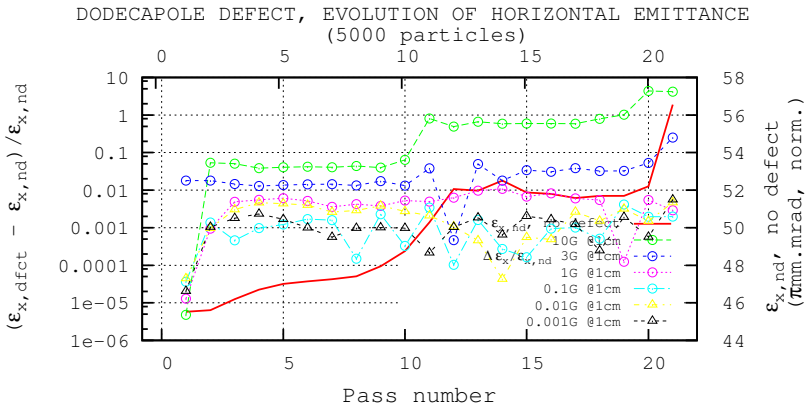




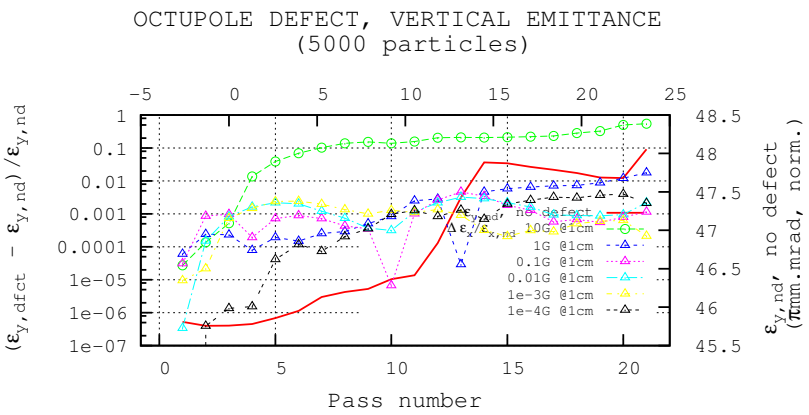
Octupole to dodecapole defect, evolution of horizontal emittance

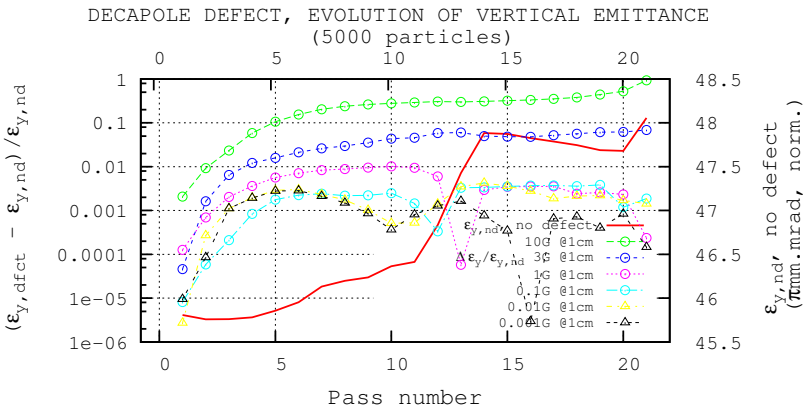


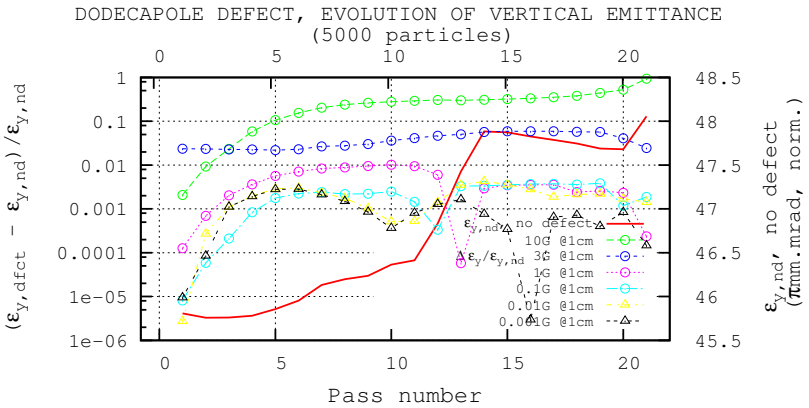




Octupole to dodecapole defect, evolution of vertical emittance

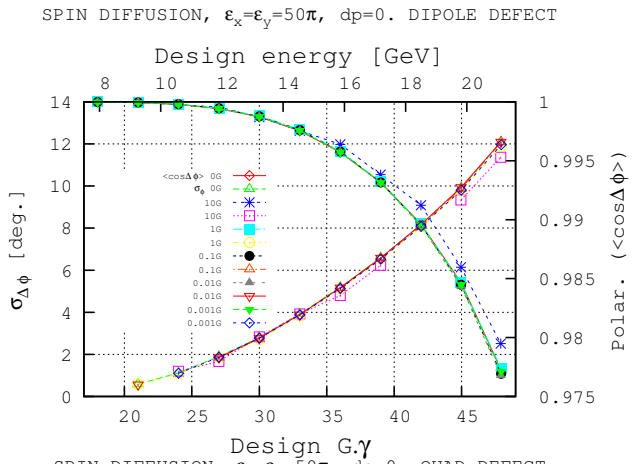


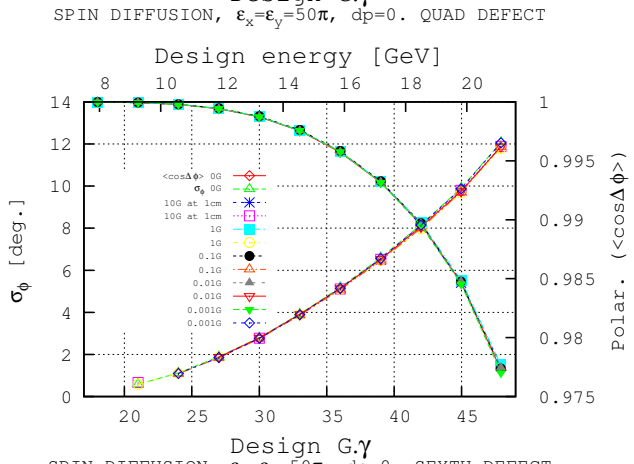


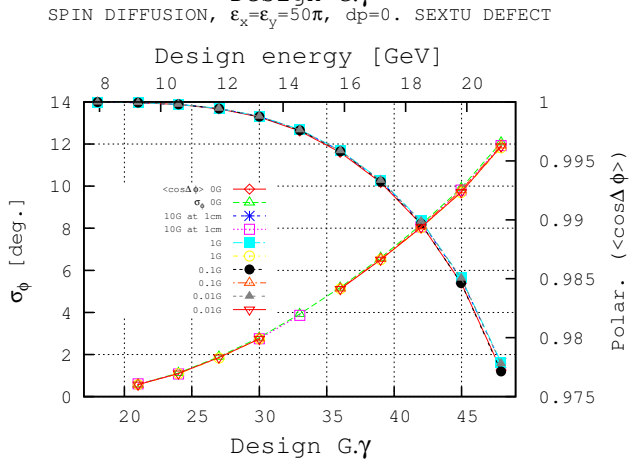


SPIN DIFFUSION

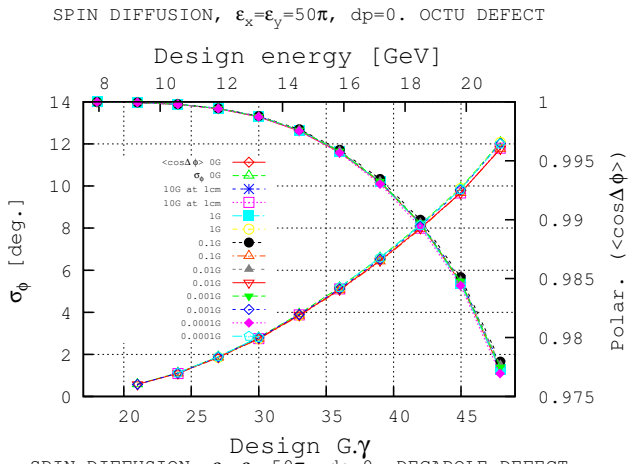
Dipole to sextupole defect

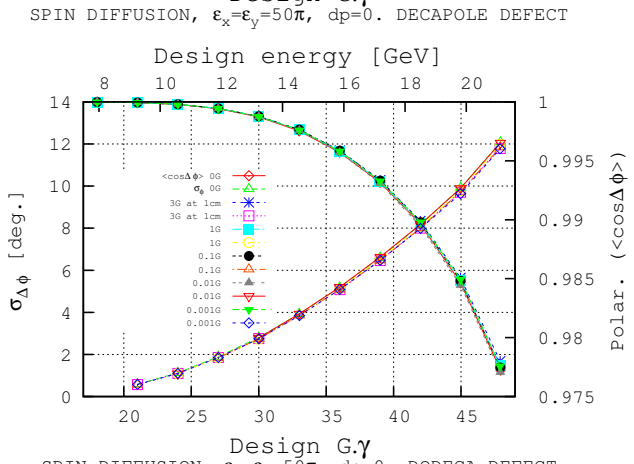


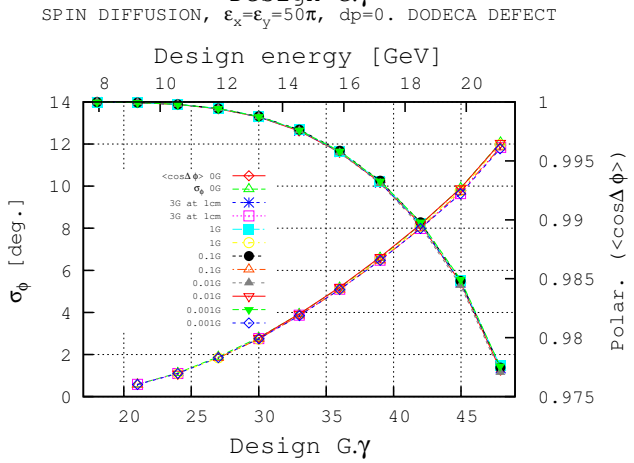




Octupole to dodecapole defect







Plans for future

- Carry on investigation of possible effects of spin resonances
- Work on energy loss compensation in [dedicated] cavities
- Complete end-to-end tracking, study/optimize beam and spin dynamics...
- Wait for the 30 T/m variant of the ring...
- Add spreader-recombiners, realistic linac, etc.
- Produce magnet multipole defect tolerances
- Re-do the above...
- Include the first ring, 1.334$\longrightarrow$6.622 GeV,
- and so forth...

THANK YOU FOR YOUR ATTENTION