The University of Manchester

Dynamic Aperture of FFAGs

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2014-09-23

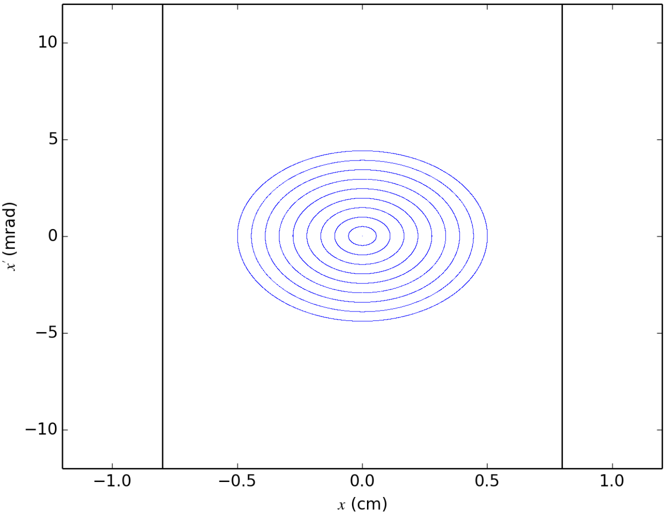
Introduction

- Defining a practical and rigorous Dynamic Aperture
- Measuring DA in simulation
- Misalignments
- Case studies
  - Pamela
  - Ultra-compact Isochronous
  - NuStorm

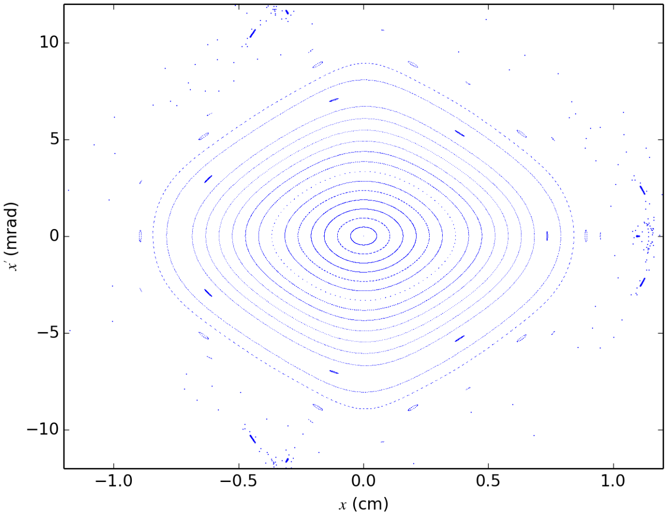
Defining Dynamic Aperture

- A particle accelerator is usually designed to have stable motion
- A particle with a small deviation from the closed orbit will have bounded motion around the closed orbit
- Due to non-linearities for large deviations the motion may not be bounded
- So there exists a maximum stable amplitude, this is the dynamic aperture
- There is no reliable analytic method to find the DA. Must track particles for a large number of turns

An ideal beam



An real beam

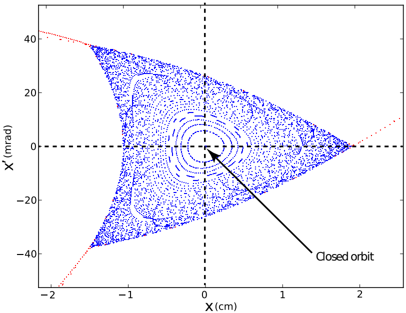


Practical Dynamic Aperture

- In many practical case we are only concerned that the motion is stable enough
  - In light sources, synchrotron radiation dominates long term motion (few thousand turns)
  - In FFAGs acceleration cycle may be short, 10s to 1000s
  - Muon beams may have half-lives of 10s or 100s of turns
- Tracking for a small number of turns saves lots of CPU time
- So we want the largest amplitude that will be transported for a sufficient number of turns
- Typically aim for better than 1% precision

Phase space

- At high amplitudes phase space can be distorted away from the ellipse seen for linear motion
- A search for the edge of stability can depend on direct of search
- We want to know the largest matched ellipse that can be injected
- So must search in $\pm x$ and $\pm x'$



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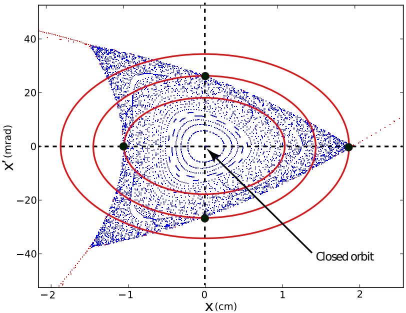
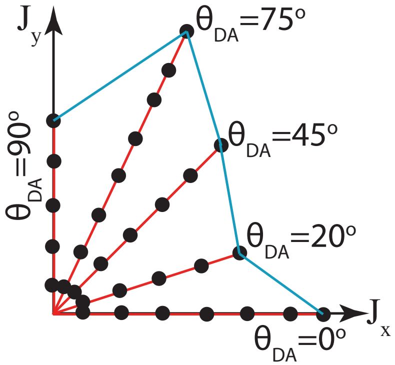


Real space

- If we measure phase space in x and y independently we risk missing losses due to coupled resonances
- Can define a realspace angle θ. $\theta = 0$ is horizontal, $\theta = 90^\circ$ is vertical.
- For a given action J_{DA}, the action in each plane is
  - $J_x = J_{DA} \cos(\theta_{DA})$
  - $J_y = J_{DA} \sin(\theta_{DA})$



Method

- For each of a range of realspace angles, step out in action J_{DA}
- At each action get J_x and J_y
- Use the Courant-Snyder parameters to find x, x', y and y' extents of ellipse
- Make a set of particles with combinations of positive and negative offsets from closed orbit
  - $(x, 0, y, 0), (0, x', y, 0), (-x, 0, y, 0)$ ...
- Track particles for N turns
- If there are no losses step out in action

Technical details

This is implemented in the new get_cell_properties module in PyZgoubi. A framework built around the tracking code Zgoubi.

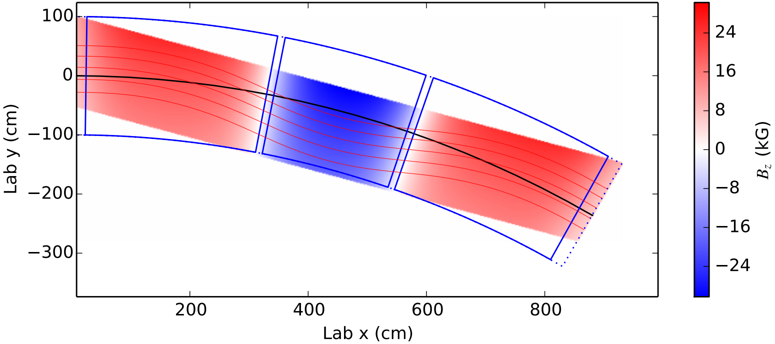
- Simulations for Dynamic aperture can be slow.
- For 1000 turns, of a small ring (36 magnets), at reasonable step size (1cm), need of order 1 hour
- To see how DA varies with lattice set up we may need to do 100s of DA simulations
- 48 core machines in the Manchester accelerator group
- Manchester EPS Condor pool, up to 3000 cores overnight across the Uni (but used by many users)

Misalignment

- When an accelerator is built there will be misalignments between the components
- These errors can excite resonances reducing DA
- Also can shift tune and closed orbit, effecting dynamics
- For a given misalignment tolerance create a set of random lattice
- Model horizontal and vertical misalignments as truncated Gaussian
- Need 50 - 100 seeds to find sensible (e.g. 95%) boundaries on requirements

Fringe field challenge

- In some FFAG designs fringe field cause an additional challenge
- To overlap fringe fields in Zgoubi multiple magnets can be placed in the same element e.g DIPOLES, FFAG
- But now it is hard to misalign independently
- Sometimes not an issue, for example a triplet may be constructed as a single unit



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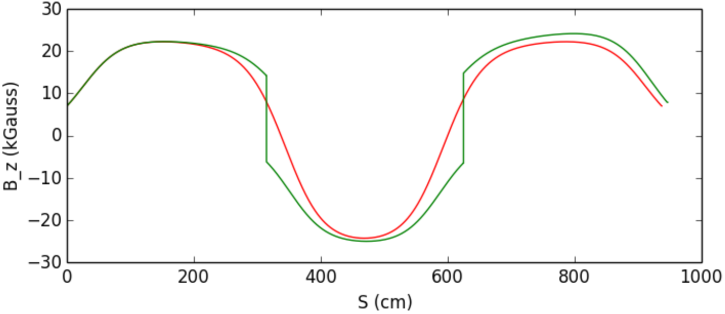
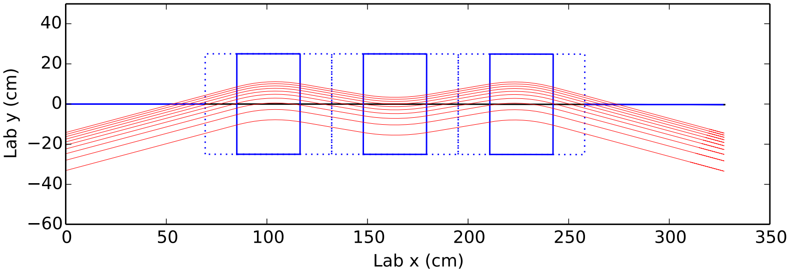


Pamela studies

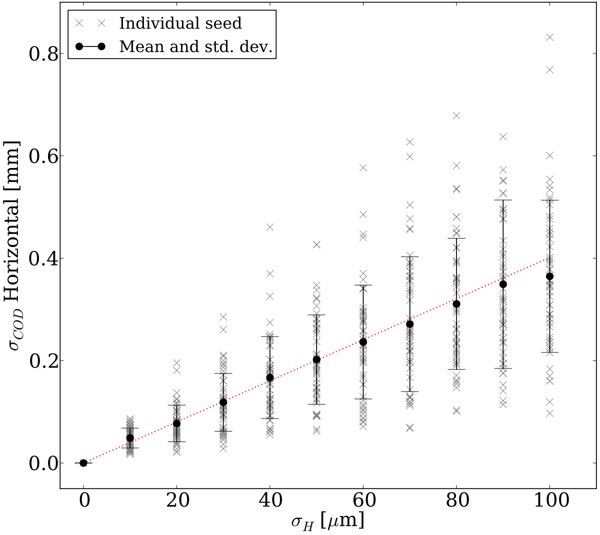
- 12 Cell super-conducting FDF triplet
- Designed as a compact proton therapy source [1]



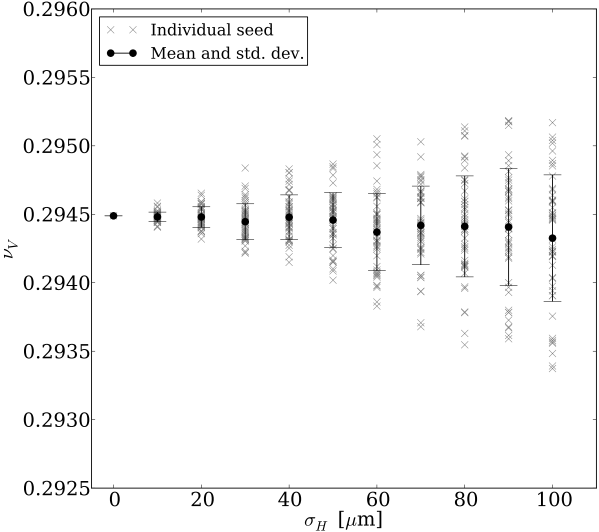
- 50 seeds, misalignement in steps of μm

[1] Thanks to Suzie Sheehy and David Kelliher

Pamela Closed Orbit Shift

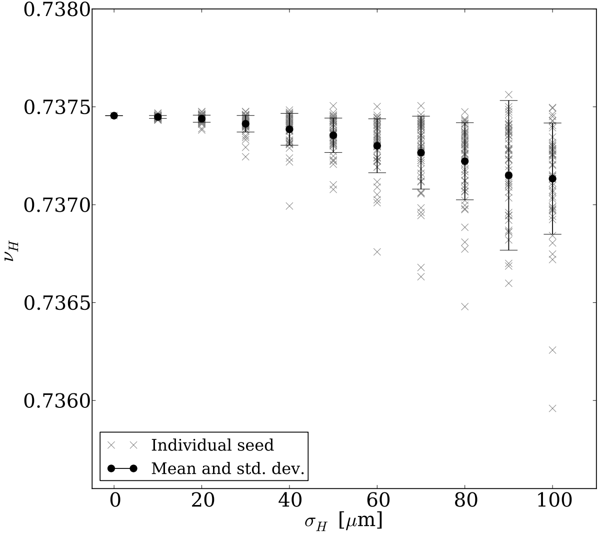


Pamela Tune



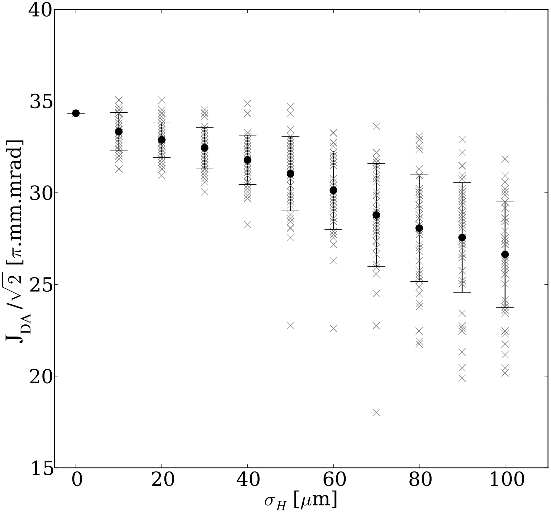
Individual seed
Mean and std. dev.
0.2960
0.2955
0.2950
0.2945
0.2940
0.2935
0.2930
0.2925
ν_V
0
20
40
60
80
100
σ_H [μm]

Pamela Tune

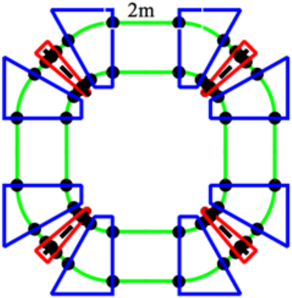


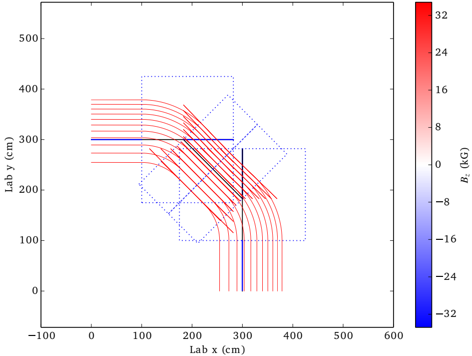
0.7380
0.7375
0.7370
0.7365
0.7360
ν_H
Individual seed
Mean and std. dev.
0
20
40
60
80
100
σ_H [μm]

Pamela Dynamic Aperture



Ultra-compact isochronous lattice

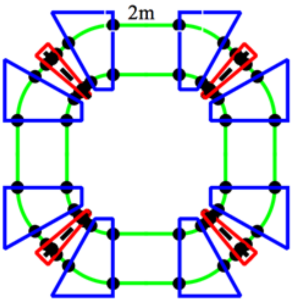


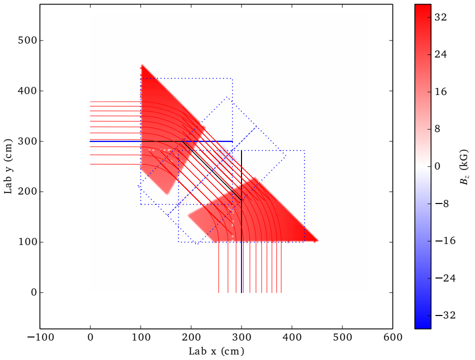


- 0.2 - 1 GeV Proton FFAG [2]
- 4.5 m radius

[2] Thanks to Carol Johnstone

Ultra-compact isochronous lattice





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Isochronous Condition

- Dipole fields, i.e. cyclotrons, maintain isochronicity at nonrelativistic energies - that is, at nonrelatistic energies velocity is proportional to momentum and path length is proportional to momentum in a constant B field, therefore path length is proportional to velocity.
- Isochronism can be imposed on the orbits in FFAGs into nonrelativistic energies by requiring the path length remain proportional to velocity, which has an increasingly nonlinear dependence on momentum. The average B field which determines path length as a function of momentum must therefore increase nonlinearly in this energy regime.
- The aperture in an isochronous machine is completely determined by the radius of the machine specified at any single energy

Isochronous Condition

- $\bar{R}_{ext} - \bar{R}_{inj} = Aperture$
- $\bar{R}_{inj} = \frac{\beta_{inj}}{\beta_{ext}}\bar{R}_{ext}$
- $\left(1 - \frac{\beta_{inj}}{\beta_{ext}}\right)\bar{R}_{ext} = Aperture$

Put in the numbers for $\approx$ 4 m extraction and 0.2-1 GeV

- $Aperture = \left(1 - \frac{0.566}{0.875}\right)4m$
- $= 1.4$

Ultra-compact isochronous results

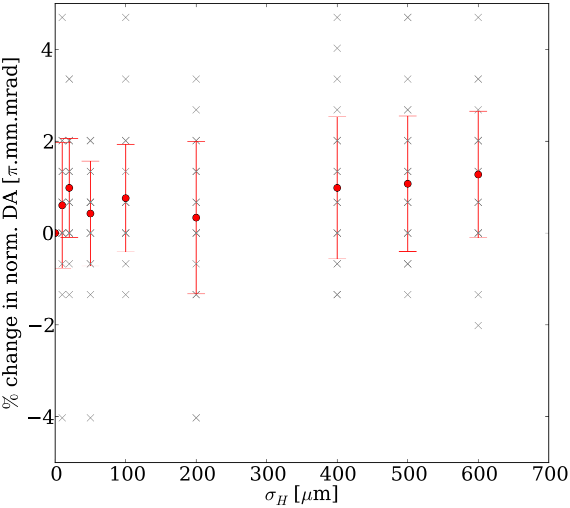
- DA measurements for 500 turns
- Large DA

Normalised DA (mm mmrad)

	H	45	V
200 MeV	234760	462	335
800 MeV	491171	1484	1142

- Robust against misalignments

Ultra-compact isochronous results



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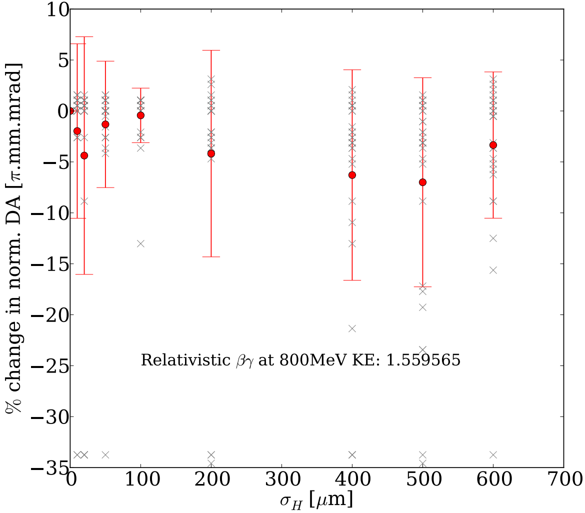
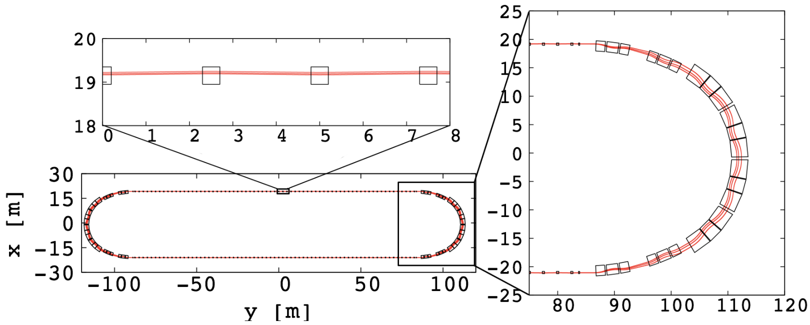


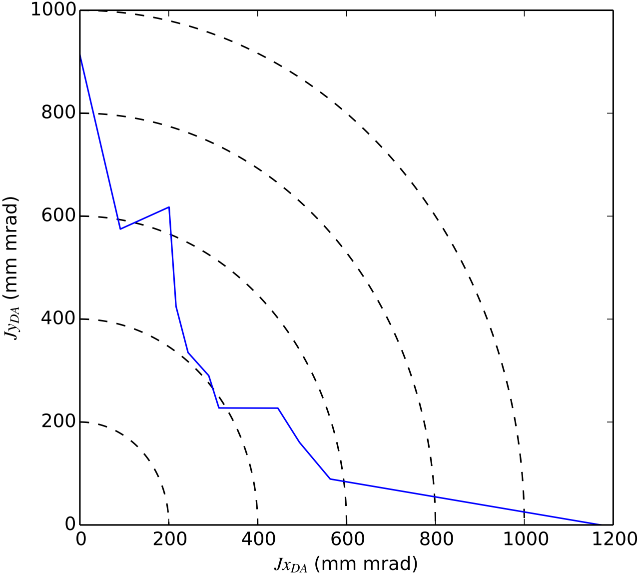
NuStorm FFAG lattice



- 500 m circumference FFAG racetrack [3]
- 3.8 GeV/c muons, 10% energy spread
- Scaling FFAG magnets in the arcs
- Straight FFAG magnets in the straight

[3] Thanks to JB Lagrange and Jaroslaw Pasternak

NuStorm FFAG lattice Dynamic aperture



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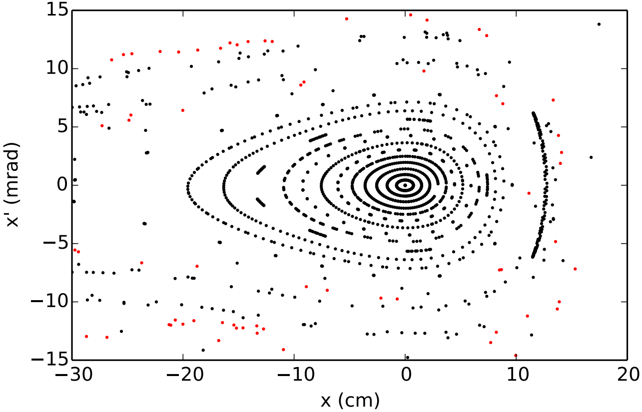
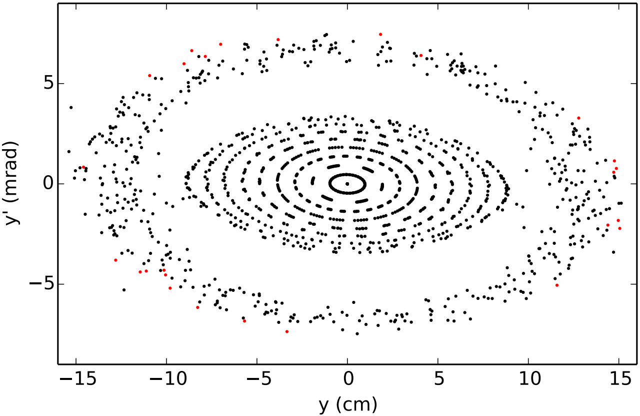


Horizontal phase space

NuStorm FFAG lattice



Vertical phase space

Conclusion

- Dynamic aperture is an important parameter for an accelerator
- Needs careful definition to avoid confusion
- PyZgoubi simplifies high level studies around DA
  - Misalignemnts
  - Scans and optimsations
- Good results from some large aperture lattices

PyZgoubi Tutorial

- On friday afternoon I am giving a tutorial on PyZgoubi, interface/framework for zgoubi
- It will save time if people can install PyZgoubi in advance
- I shall put some instructions on the agenda page
- If you have any problems come and see me