From Actions to Answers: Flavour physics from Lattice QCD

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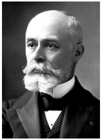
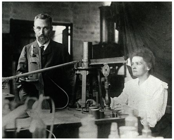
- Actions & introduction
- Kaon physics
- B physics
- Some work done since joining BNL

Historical case for Intensity Frontier

Radioactivity discovered by Becquerel, Pierre and Marie Curie; 1903 Nobel Prize.

- Categorised by Rutherford: α (helium nucleus), β (electron) and γ (photon)
- Correspond (loosely) to strong, weak, and electromagnetic forces
- Beta particles shown to be electrons by Becquerel (1900)
- Rutherford & Soddy (1901) demonstrated acompanied by *transmutation* of elements


The 1903 “standard model” conserved nuclear charge
Broken by rare β decay mediated by massive new particles

Standard Model

Standard model of particle physics developed by 1972 represents our best understanding of nature, and in my view humankind's greatest intellectual achievement

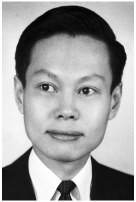
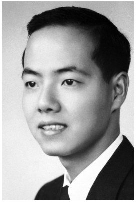
$$\boxed{SU(3)_{\mathrm{color}} \otimes SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{Y}}}$$

Beta decay: $u \leftrightarrow d$ quark flavour change mediated by massive W-boson

- Higgs mechanism (1964 Englert,Higgs; Nobel Prize 2013)
- W,Z,H,γ theoretically understood (1968; Glashow, Weinberg, Salam; Nobel prize 1979)
- W,Z produced in SPS at CERN (1983; Rubbia/Van der Meer Nobel Prize)
- Quark flavours (u,d,s,c,b,t) completed with top discovery (Fermilab 1995)
- Higgs Boson found by the LHC in 2012

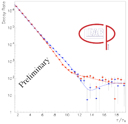
It took us 87 years to bridge the energy gap with accelerators !

Symmetry breaking in the Weak interactions

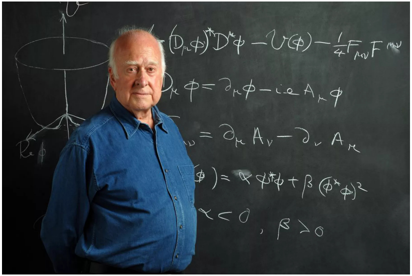
- Yang (BNL), Lee (CU) 1957 Nobel Prize
  $K^+ = \theta^+ = \tau^+$
  both $P = \pm 1$ decay modes $(\pi\pi)$ / $(\pi\pi\pi)$
  - Weak interactions may break parity (also noted Weak $T = CP$ untested)
- C. S. Wu parity violation experiment

- BNL AGS (1964) established CP violation
  Cronin & Fitch 1984 Nobel Prize
- K_L *and* K_S both decay CP=1 $\pi\pi$ and CP=-1 $\pi\pi\pi$ final states

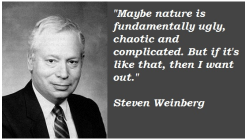


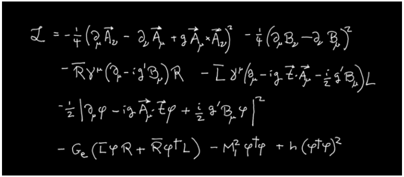
- CPlear experiment: two exponents in $\pi\pi$ deposition

Some of my favourite papers of all time!



Broken symmetries and the masses of gauge bosons

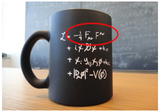


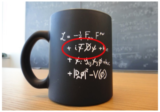


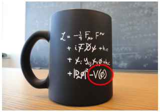
A model of leptons

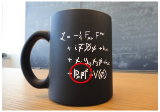
Standard model Lagrangian

Gauge field strength $F^{\mu\nu}F_{\mu\nu} = G^{\mu\nu a}G^a_{\mu\nu} + W^{\mu\nu i}W^i_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}$
W, G non-abelian gauge fields: Yang Mills, BNL









Fermion kinetic term & gauge interactions:

$$\mathcal{L}_{\text{fermion}} = \sum_{\text{flavours } f} \left\{ \overline{\psi}^f_{\mathrm{L}} i \not{D} \psi^f_{\mathrm{L}} + \overline{\psi}^f_{\mathrm{R}} i \widetilde{\not{D}} \psi^f_{\mathrm{R}} \right\}$$

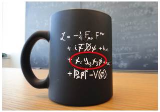
Left and right handed interact *differently*

Mexican hat $\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{pmatrix}$
P.W. Higgs (Edinburgh), Brout, Englert (+ Guralnik, Hagen, Kibble).

$|D_\mu \langle\phi\rangle|^2$: Weak boson mass terms $\Rightarrow W, Z$, unbroken γ

Yukawa interactions: origin of flavor physics

Lepton sector:



$$\lambda_e \overline{e}_R \vec{\phi}^{\,\dagger} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L + \text{h.c.} = m_e \overline{e} e$$

Most general gauge invariant Yukawa term puts couplings in flavor matrices F

$$(\overline{e \mu \tau})_R F_L \phi^\dagger L_L + (\overline{u c t})_R F_Q \phi^\dagger \mathbf{Q_L} + (\overline{d s b})_R F'_Q \phi^T (i\sigma_2) \mathbf{Q_L} + h.c.$$

- Simplifies after singular value decomposition $F = UDV$.
  - No (standard model) flavour mixing in lepton sector
  - Neutrino oscillation exists and *is* new physics
- Cabibbo Kobayashi Maskawa CKM matrix: (Nobel Prize 2008)
  - Cannot simultaneously diagonalise F_Q and F'_Q
  - $F'_Q = V_{CKM} D V^\dagger_{CKM}$
  - Unitary relative rotation between weak interaction and mass basis for *dsb*

CKM unitarity

Cabibbo-Kobayashi-Maskawa (CKM) matrix controls quark flavour changing
Wolfenstein parametrisation

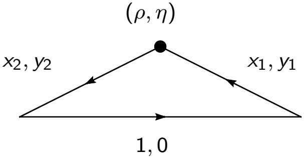
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$V^*_{ki} V_{kj} = (V^\dagger V)_{ij} = \delta_{ij}$: 9 relations, e.g.

$$V^*_{ub} V_{ud} + V^*_{cb} V_{cd} + V^*_{tb} V_{td} = 0 \quad \Leftrightarrow \quad 1 + \underbrace{\frac{V^*_{tb} V_{td}}{V^*_{cb} V_{cd}}}_{\in \mathbb{C}} + \underbrace{\frac{V^*_{ub} V_{ud}}{V^*_{cb} V_{cd}}}_{\in \mathbb{C}} = 0$$

$$\Leftrightarrow \quad (1,0) + (x_2, y_2) + (x_1, y_1) = 0$$

Identify these as vectors in the complex plane forming a *unitarity triangle*:



CKM and CP violation

- $i\eta$ is only source of CP violation in the standard model
  - CP violation is one of the Sakharov conditions for baryogenesis
  - SM CPV insufficient to explain our universe - new sources expected
- Different processes determined different constraints on $\rho + i\eta$ vertex of *unitarity triangle*
- Consistency check these to find new sources

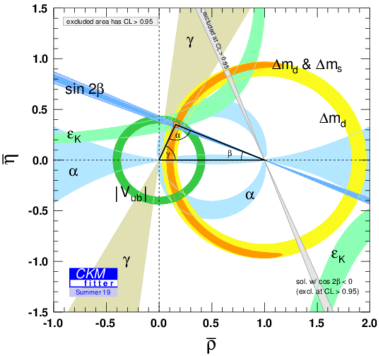
UPDATED RESULTS ON THE CKM MATRIX

Including results presented up to
Summer 19

Preliminary

Dec 15th, 2019

The CKMfitter Group



- **Theoretical input of hadronic weak matrix elements is required**

Importance of Lattice QCD

CKMfitter 2019

1 Inputs

2 Lattice QCD averages

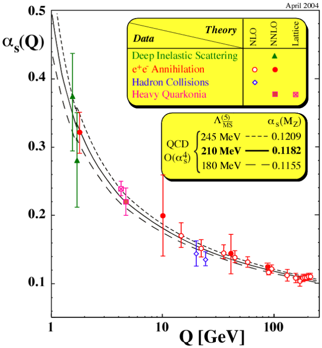
Several hadronic inputs are required for the fits presented by CKMfitter, and we mostly rely on lattice QCD simulations to estimate these quantities. The presence of results from different collaborations with various statistics and systematics make it all the more necessary to combine them in a careful way. We explain below the procedure that we have chosen to determine these lattice averages.

Parameter	Value ± Error(s)	Reference	Errors GS	Errors TH
$\overline{m}_c(m_c)$	$(1.2982 \pm 0.0013 \pm 0.0120)$ GeV	[4]	$\star$	$\star$
$\overline{m}_t(m_t)$	$(165.26 \pm 0.11 \pm 0.30)$ GeV	[35]	$\star$	$\star$
$\alpha_s(m_Z)$	$0.1181 \pm 0 \pm 0.0011$	[2]	-	$\star$
B_K	$0.7567 \pm 0.0021 \pm 0.0123$	[4]	$\star$	$\star$
κ_ϵ	$0.940 \pm 0.013 \pm 0.023$	[37, 38]	$\star$	$\star$
η_{ut}	$0.402 \pm 0 \pm 0.007$	[39]	-	$\star$
η_{tt}	$0.55 \pm 0 \pm 0.024$	[39]	-	$\star$
$\eta_B(\overline{\mathrm{MS}})$	$0.5510 \pm 0 \pm 0.0022$	[40, 41]	-	$\star$
f_{B_s}	$(228.8 \pm 0.7 \pm 1.9)$ MeV	[4]	$\star$	$\star$
B_s	$1.327 \pm 0.016 \pm 0.030$	[4]	$\star$	$\star$
f_{B_s}/f_{B_d}	$1.205 \pm 0.004 \pm 0.006$	[4]	$\star$	$\star$
B_s/B_d	$1.007 \pm 0.013 \pm 0.014$	[4]	$\star$	$\star$
f_K	$(155.6 \pm 0.2 \pm 0.6)$ MeV	[4]	$\star$	$\star$
f_K/f_π	$1.1973 \pm 0.0008 \pm 0.0014$	[4]	$\star$	$\star$
f_D	$(249.2 \pm 0.3 \pm 0.7)$ MeV	[4]	$\star$	$\star$
f_{D_s}/f_D	$1.1782 \pm 0.0006 \pm 0.0033$	[4]	$\star$	$\star$
$f_+^{K\to\pi}(0)$	$0.9681 \pm 0.0014 \pm 0.0022$	[4]	$\star$	$\star$
$f_+^{D\to\pi}(0)$	$0.621 \pm 0.018 \pm 0.012$	[4]	$\star$	$\star$
$f_+^{D\to K}(0)$	$0.765 \pm 0.010 \pm 0.012$	[4]	$\star$	$\star$
$\zeta(\Lambda_p \to p\mu^-\overline{\nu}_\mu)_{q^2>15}/\zeta(\Lambda_p \to \Lambda_c\mu^-\overline{\nu}_\mu)_{q^2>7}$	$1.471 \pm 0.096 \pm 0.290$	[4]	$\star$	$\star$

- Phenomenological influence lattice gauge theory is *going through a step change*
- LGT is systematically improvable
  - Computers and algorithms are systematically improving
  - u,d,s loop effects fully included (2000's)
  - No light quark mass extrapolations eliminated (2010's)
  - QED and isospin breaking for 0.1% accuracy (2020)
  - Direct 2+1+1+1f B simulation by 2025-2030??
- Resistance is futile!

Hadronic physics is non perturbative

- Asymptotic freedom: 2004 Nobel prize (Gross, Wilczek, Politzer)
- Flip side of asymptotic freedom is IR slavery *Wilczek*



- 't Hooft Euclidean instanton action

$$e^{-S_g} = e^{-\frac{8\pi|Q|}{g^2}}$$

- Atiyah-Singer index theorem $\Rightarrow$ associated fermionic topological zero modes
- Intrinsically non-perturbative

$$\frac{d^n}{d(g^2)^n}\, e^{-\frac{1}{g^2}}\Big|_{g^2=0} = 0$$

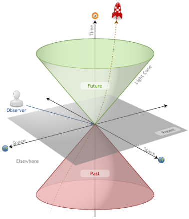
- There are non-perturbative dynamics weak coupling perturbation theory simply cannot explain

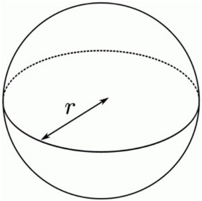
Euclidean space correlation functions

Importance sampled numerical path integration: must Wick rotate to Euclidean space

$$\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS_G^M} e^{iS_F^M} \to \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-S_G^E} e^{-S_F^E}$$

so that probability amplitude becomes a *partition function probability measure*.





$$G^M(\vec{r}, t) = \frac{1}{4\pi r}\delta(r - t),$$

$$G^M(\vec{p}, t) = \frac{ie^{i\omega_p t}}{2\omega_p}.$$

$$G^E(\vec{r}, t) = \frac{1}{4\pi^2(r^2 + t^2)},$$

$$G^E(\vec{p}, t) = \frac{e^{-\omega_p t}}{2\omega_p}.$$

Euclidean Lattice Field Theory

- Gauge invariant regularisation (Wilson); First numerical simulations (Creutz, BNL)
  - Non-perturbative but introduces discretisation and statistical errors
  - Computationally expensive

Partition function becomes a real, statistical mechanical probability weight (at zero chemical potential)

$$Z = \int d\bar{\psi} d\psi dU e^{-S_G[U] - S_F[\bar{\psi},\psi,U]}$$

$$S_G \sim \int d^4x \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad ; \quad S_F \sim \int d^4x \bar{\psi}(D_L - m)\psi$$

Discretise space-time

$$\int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\bar{\psi}(x)A_{xy}\psi(y)} = \det A$$

- Importance sample Gauge configurations using Markov Chain Monte Carlo
- Fermion action is a *sparse matrix* finite difference operator
- Replace anti-commuting fermion determinant with complex *pseudofermion* integral

$$det[D_L - m]^2 = \int d\phi^* d\phi e^{-\phi^\dagger \left([D_L - m]^\dagger [D_L - m]\right)^{-1} \phi}$$

- Multiple choices for fermion action D_L (discretisation scheme)

RBC-UKQCD approach: domain wall fermions

1981 **Callan/Harvey**: 5-d solutions bind massless chiral modes to the 4-d "wall".
in presence of gauge fields creates the correct chiral anomaly
Consider Euclidean five dimensional free fermions with step function mass term $m(x_5) = m\theta(x_5)$

$$S_f = \bar{\psi}\left(\gamma_\mu \partial_\mu + \gamma_5 \partial_5 + m(x_5)\right)\psi$$

Massless 4-d free fermions have chiral modes

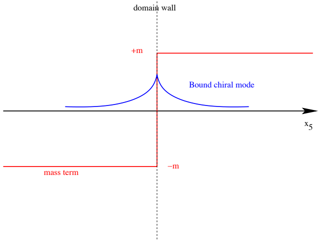
$$\psi = e^{ip_\mu x_\mu} u_\pm ,$$

$$u_\pm = \frac{1 \pm \gamma_5}{2} u_\pm .$$

Separable positive chirality 5-d solutions have

$$\psi = e^{ip_\mu x_\mu} e^{-m|x_5|} u_+$$

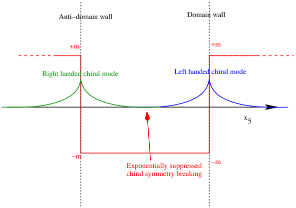
The negative chirality solutions are not normalisable.



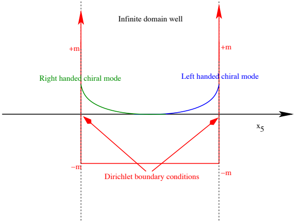
$\boxed{\text{A massless chiral mode is bound to a domain wall}}$

Domain walls

1994 **Kaplan**; 1995 **Shamir's**
Trick works with a *discrete* 5d Wilson action (gauged in 4d).



Two walls, separate chiralities



"infinite square well" suppresses tails

- Exact chiral symmetry constrains no $O(a)$, $O(a^3)$, ... discretisation errors both *on and off-shell*
- V-A does what it says on the tin: no unphysical mixings in four quark operators, O(a) mixing
- First numerical simulation in BNL (Blum & Soni 1997)
- QIS : relation to topological insulators and majorana states on a 1d quantum wire (Kitaev)
- **PB:** connection between Furman & Shamir current and Luscher's on shell chiral symmetry, MDWF conserved currents
- **PB:** implementation adopted by UKQCD, RBC, JLQCD

Leptonic decay constants

$$\langle \pi^+(p) | J_\lambda(V - A) \rangle = i f_\pi p_\lambda$$

$$\Gamma_l = \frac{m_\pi}{8\pi} \left(|V_{ud}| f_\pi G_F \right)^2 m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2} \right)^2$$

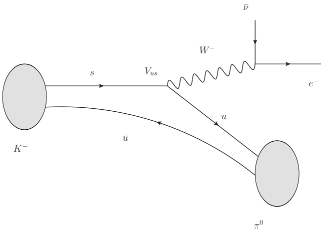
$$f_K / f_\pi = 1.1945(45)$$

Phys.Rev. D76 (2007) 014504,
Phys.Rev. D78 (2008) 114509,
Phys.Rev. D83 (2011) 074508,
Phys.Rev. D87 (2013) 094514,
Phys.Rev. D93 (2016) no.7 074505,
Phys.Rev. D93 (2016) no.5, 054502

- (Marciano) One of two robust LGT approaches to V_{us} and λ in the Wolfenstein parametrisation
- Also determine SU(2) and SU(3) chiral lagrangian LECs, quark masses.
- On-going work: include QED and $m_u \neq m_d$ effects to enable 10^{-3} precision
  PoS LATTICE2016 (2016) 172,
  JHEP 1709 (2017) 153,
  EPJ Web Conf. 175 (2018) 06024,
  PoS LATTICE2018 (2019) 267

Semileptonic form factors

Transition	CKM element
$K \to \pi e \bar{\nu}$	V_{us}
$B \to \pi e \bar{\nu}$	V_{ub}
$B \to D e \bar{\nu}$	V_{cb}
$D \to K e \bar{\nu}$	V_{cs}
$D \to \pi e \bar{\nu}$	V_{cd}



e.g. Beta decay of the Kaon leads to V_{us}

$$\langle \pi(p_\pi) | J_\mu (V - A) | K(p_K) \rangle = f_+(q^2)(p_K + p_\pi)_\mu + f_-(q^2)(p_K - p_\pi)_\mu$$

$$\mathcal{M} = V_{us} \frac{G_F}{\sqrt{2}} \left[f_+(q^2)(p_K + p_\pi)_\mu + f_-(q^2)(p_K - p_\pi)_\mu \right] \bar{u}_{\nu_e}(p_2) \gamma^\mu (1 - \gamma_5) v_e(p_1)$$

Kaon semileptonic form factors

- Kaon semileptonic form factor, Pion form factor
  PhysRevLett.100.141601 (2007),
  JHEP 0807:112,2008,
  Eur.Phys.J.C69:159-167,2010,
  JHEP08(2013)132,
  JHEP06(2015)164.

$$f_+^{K\pi}(0) = 0.9685(34)_{\mathrm{stat}}(14)_{\mathrm{finitevolume}}$$

- Pion form factor, charge radius etc..
- Introduction of twisted boundary conditions to obtain arbitrary q^2
- On-going work:
  - Introduce QED and $m_u \neq m_d$ effects for 0.1% precision
  - Map precisely full q^2 range of form factor sensitive to lepton flavour violation

Combine with experimental rate:

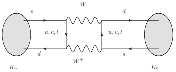
$$|V_{us}| = 0.2233(5)_{\mathrm{experiment}}(9)_{\mathrm{lattice}}$$

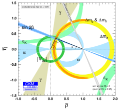
Combined with V_{us} from super-allowed nuclear β-decay, first row unitarity check:

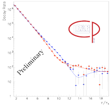
$$1 - |V_{ud}|^2 - |V_{us}|^2 = 0.0010(4)_{V_{ud}}(2)_{V_{us}^{\mathrm{exp}}}(4)_{V_{us}^{\mathrm{lat}}} = 0.0010(6)$$

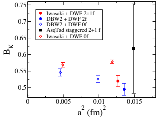
Indirect CP violation in $K^0 - \overline{K}^0$ *mixing* : short-distance contributions to ϵ_K

$$
\begin{array}{ccccccc}
K_L & \sim & \bar{\epsilon} K_1 & + & K_2 & \leftarrow & O(\bar{\epsilon}) \stackrel{\text{fast}}{\rightarrow} \pi\pi \\
K_S & \sim & K_1 & + & \bar{\epsilon} K_2 & \leftarrow & O(1) \stackrel{\text{fast}}{\rightarrow} \pi\pi
\end{array}
\qquad
\epsilon_K = \frac{A(K_L \to \pi\pi)_{I=0}}{A(K_S \to \pi\pi)_{I=0}}
$$









$$
M_{12}^* \propto G_F^2 \left[(V_{cs}^* V_{cd})^2 S_0(x_c, x_c) + 2(V_{cs}^* V_{cd})(V_{ts}^* V_{td}) S_0(x_c, x_t) + (V_{ts}^* V_{td})^2 S_0(x_t, x_t) \right] \langle K^0 | \bar{s}\gamma_\mu(1-\gamma_5) d \bar{s}\gamma_\mu(1-\gamma_5) d | \bar{K}^0 \rangle
$$

Imaginary part gives a hyperbola constraint: $\eta(1-\rho) = \text{constant}$

$$\hat{B}_K = 0.720(13)(37) \quad \rightarrow \quad \hat{B}_K = 0.7499(24)(150)$$

$$B_K^{SMOM} = 0.5341 \pm 0.0018$$

- $\overline{MS}$ result: 2% perturbative error dominant (3 GeV matching scale)
- **Reduced to 0.4% error in SMOM scheme**
- most precise result in FLAG 2019 average

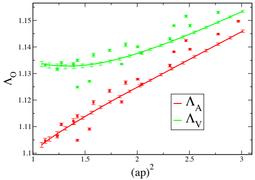
Phys.Rev.Lett. 100 (2008) 032001
Phys.Rev. D78 (2008) 114509
Phys.Rev. D84 (2011) 014503
Phys.Rev. D87 (2013) 094514
Phys.Rev. D93 (2016) no.7, 074505

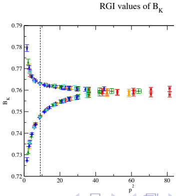
Continuous momenta on a discrete lattice

	boundary conditions	Fourier modes
Periodic	$\psi(L) = e^{i0\pi}\psi(0)$	$p = 2n\frac{\pi}{L}$
Antiperiodic	$\psi(L) = e^{i1\pi}\psi(0)$	$p = (2n+1)\frac{\pi}{L}$
Twisted	$\psi(L) = e^{i\alpha\pi}\psi(0)$	$p = (2n+\alpha)\frac{\pi}{L}$

- Form factors:
  - directly calculate $q^2 = 0$ without interpolation
    Nucl.Phys.Proc.Suppl. 129 (2004) 358-360,
    JHEP 0705 (2007) 016,
    Phys.Rev.Lett. 100 (2008) 141601
- Non-perturbative renormalisation:
  - enable rigorous control of lattice artefacts in NPR
  - decouple non-perturbative scale evolution from matrix element measurement
    Phys.Rev. D85 (2012) 014501,
    Phys.Rev. D83 (2011) 114511
- Confident that if required NP running to M_Z, through flavour thresholds, possible using step scaling techniques I developed
- **Non-peturbative anomalous dimensions as continuous function of p^2**

Phys.Rev. D88 (2013) no.11, 114506

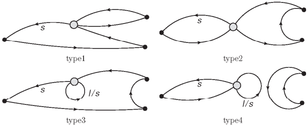




Direct CP violation: $K \to \pi\pi$ decay amplitudes

Phys.Rev.Lett. 115 (2015) no.21, 212001
Phys.Rev. D91 (2015) no.7, 074502
Phys.Rev.Lett. 110 (2013) no.15, 152001
Phys.Rev.Lett. 108 (2012) 141601 Ken Wilson Lattice Award
Phys.Rev. D84 (2011) 114503

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i(\mu)$$



$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{ \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon} \left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right] a \right\} = 1.38(5.15)(4.59) \times 10^{-4}$$

$$\mathrm{Re}A_0 = 4.66(1.0)(1.26) \times 10^{-7}\mathrm{GeV} \qquad \mathrm{Re}A_2 = 1.50(4)(14) \times 10^{-8}\mathrm{GeV}$$

$$\mathrm{Im}A_0 = -1.90(1.23)(1.08) \times 10^{-11}\mathrm{GeV} \qquad \mathrm{Im}A_2 = -6.99(20)(84) \times 10^{-13}\mathrm{GeV}$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{expt} = 16.5(2.6) \times 10^{-4}$$

10% accuracy relative to expt result is possible

Significant update expected next month: Seminar talk by Chris Kelly(BNL)

- New horizontal constraint on unitarity triangle. Directly constrains imaginary part η.
- Theory has lagged experiment by thirty years !

BSM Kaon mixing with physical quark masses

Non-SM effective 4q operators (e.g. SUSY) could induce new indirect CP contributions
Non-perturbative matrix elements needed to connect any observation to a new mass scale

$$O_1 = (\bar{s}_a \gamma_\mu (1-\gamma_5) d_a)(\bar{s}_b \gamma_\mu (1-\gamma_5) d_b)$$

$$O_2 = (\bar{s}_a (1-\gamma_5) d_a)(\bar{s}_b (1-\gamma_5) d_b)$$

$$O_3 = (\bar{s}_a (1-\gamma_5) d_b)(\bar{s}_b (1-\gamma_5) d_a)$$

$$O_4 = (\bar{s}_a (1+\gamma_5) d_a)(\bar{s}_b (1+\gamma_5) d_b)$$

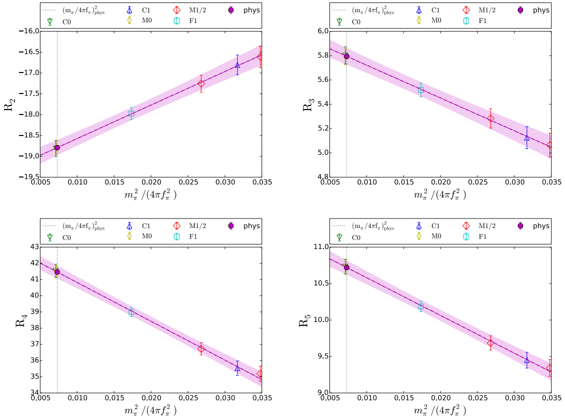
$$O_5 = (\bar{s}_a (1+\gamma_5) d_b)(\bar{s}_b (1+\gamma_5) d_a).$$

$$B_1(\mu) = \frac{\langle \bar{K^0} | O_1(\mu) | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}$$

$$B_i(\mu) = \frac{(m_s(\mu) + m_d(\mu))^2}{N_i m_K^2 f_K^4} \langle \bar{K^0} | O_i(\mu) | K^0 \rangle, \ i > 1$$

BSM Kaon mixing with physical quark masses

$$R_i\left(\frac{m_P^2}{f_P^2}, a^2, \mu\right) = \left[\frac{f_K^2}{m_K^2}\right]_{Exp.} \left[\frac{m_P^2}{f_P^2} \frac{\langle \bar{P}|O_i(\mu)|P\rangle}{\langle \bar{P}|O_1(\mu)|P\rangle}\right]_{Lat.}$$

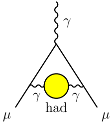


- 1% - 2% error in a mom scheme
- 3% - 6% error in $\overline{MS}$

Hadronic vacuum polarisation

3.7σ tension in muon $a_\mu = \frac{g_\mu - 2}{2}$ between theory and BNL E821 results
Fermilab E989 and JPARC experiments will reduce experimental error by 4x

- Unphysical masses:
  Phys.Rev. D85 (2012) 074504
  JHEP 1604 (2016) 063
  Phys.Rev.Lett. 121 (2018) no.20, 202003
- Physical quark masses
  Phys.Rev.Lett. 121 (2018) no.2, 022003,
  Phys.Rev.Lett. 116 (2016) no.23, 232002
- Isospin breaking effects:
  JHEP 1709 (2017) 153



Pure lattice result:

$$a_\mu^{\mathrm{HVP}} = 715.4(16.3)(9.2) \times 10^{-10}$$

Combined Lattice / R-ratio ($e^+e^- \to \mathrm{hadrons}/e^+e^- \to \mu^+\mu^-$) fits:

$$a_\mu^{\mathrm{HVP}} = 692.5(1.4)(0.5)(0.7)(2.1) \times 10^{-10}$$

Eliminated mass extrapolation systematic, statistical error improvements
Near-term lattice results *will improve*
Preferable to have a "theory" result that has *only* QCD Lagrangian as input

Charm and B physics with domain wall fermions

- Discretisation errors a serious issue for charm and bottom quarks.
- Chiral action: no unphysical operator mixing, no $O(a)$ operator counter terms
- Extrapolate SU(3) breaking ratios f_{D_s}/f_D, f_{B_s}/f_B, $\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_B^2 B_B}$ from charm region.
  - Common curvature and correlation makes extrapolation of SU(3) breaking simpler
- Simultaneous a^2, m_{ud} and $\frac{1}{m_{D_s}}$ extrapolation (exploit HQ symmetry)

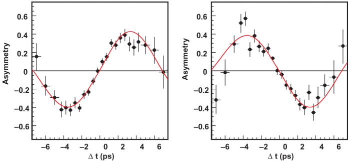
Goals

- Ratios of CKM matrix elements are useful in UT fit: e.g.

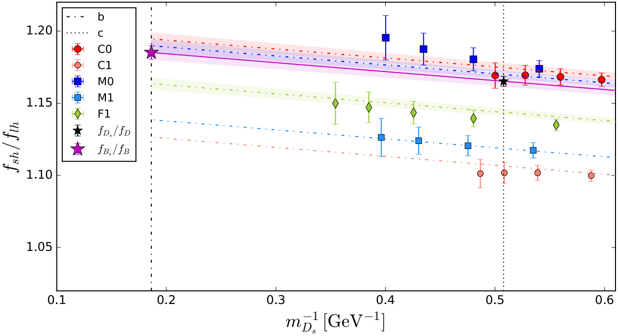
$$\frac{|V_{ub}|}{|V_{cb}|} \Rightarrow |\rho + i\eta|$$

- Time dependent CP asymmetry in $B_{(s)}\overline{B}_{(s)}$ pairs give $V_{tb}^* V_{ts}$ and $V_{tb}^* V_{td}$

$$\frac{M_{B_d}\Delta M_s}{M_{B_s}\Delta M_d} = \left|\frac{V_{ts}}{V_{td}}\right|^2 \xi^2 \Rightarrow |1 - \rho - i\eta|$$

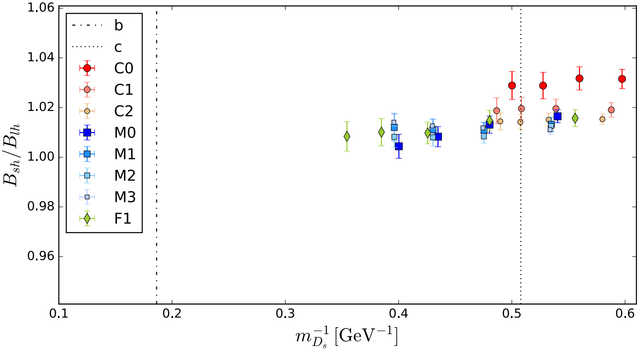


Belle time dependent CP asymmetry



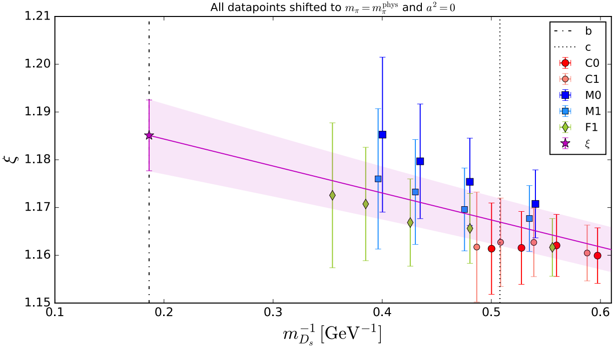
Leptonic decay constants

Charm and B physics with domain wall fermions



Bag parameters

Charm and B physics with domain wall fermions



B and D meson interests

On-going work

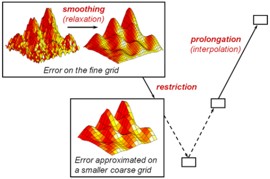
- Continue work on f_D, f_{D_s} and ξ to add third lattice spacing, better extrapolation
- Determine $B \to D$, $B_s \to K$, $B \to \pi$ form-factors.
  - V_{cb} and V_{ub}
- Hard radiative decays: $B \to l\nu\gamma$
- Third, 2.8 GeV physical point lattice is exciting opportunity

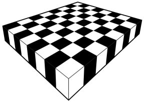
Multiscale physics and multiscale algorithms

Multiscale simulation required for heavy quark physics

$\overline{m}_u(2\mathrm{GeV})$	2.3 MeV
$\overline{m}_d(2\mathrm{GeV})$	4.7 MeV
$\overline{m}_s(2\mathrm{GeV})$	90.3 MeV
$\Lambda^{(4)}_{QCD}$	294 MeV
m^{+}_{π}	139 MeV
m_N	938 MeV
$\overline{m}_c(\overline{m}_c)$	1.28 GeV
$\overline{m}_b(\overline{m}_b)$	4.164 GeV
m_Υ	9.46 GeV
M_Z	91.2 GeV
M_W	80.4 GeV
M_X	$\geq$ 1 TeV

- Project to low dimensional basis that captures the lowest fermion modes, by using segments of the lowest fermion modes
- Faithfully represent the original matrix on a coarser space





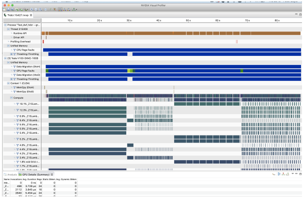
- Gauge symmetry means naive smoothing cannot work

	Fine	Coarse	CoarseCoarse	Evecs
$\lambda_{\min}$	1.0e-6	1.0e-6	1.0e-6	1.0e-6
$\lambda_{\max}$	60	11	5.0	4.0e-3

- Improve the condition number by lowering the cut-off as you go coarser
- Preserve low spectrum and handle this precisely

Multigrid : very recent results

- 128 nodes on DOE Summit computer
- CPU-GPU portable code significant element of USQCD ECP project



- Test system: $48^3 \times 96 \times 24$ $am = 0.00078$
- Can now setup *and* solve faster than baseline

Algorithm	Fine Matmuls	Time
CGNE	11400	440s
Solve	2400	240s
Setup	2500	160s
Setup+solve	4900	400s

- First recursive multi grid for Chiral fermions
- Goal: enable a new generation of 2+1+1f chiral Fermion simulations in the 3-5 GeV range on DOE exascale computers

Summary and Outlook

- Hadronic theoretical uncertainty is unavoidable in experimental search for any new physics involving quarks
  - Lattice gauge theory is the best tool we have for confronting this
- Phenomenological influence of lattice gauge theory is *going through a step change*
  - many systematics have been eliminated
  - sub-percent scale accuracy achieved on many quantities
- QED and isospin breaking for 0.1% accuracy (2020)
- Direct 2+1+1+1f B simulation by 2025-2030??
- Progress has been made against long standing theoretical challenges
  - including the long standing $\Delta_I = 1/2$ rule