Bayesian Shear Estimation

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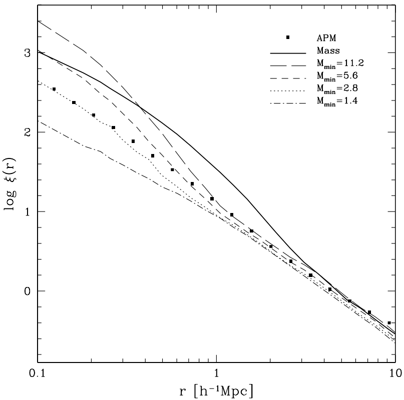
Weak Shear

- Weak shear is smaller than the intrinsic variance in shapes.
- We average the shear from many sources to extract a signal.
- Look for correlations in the shear field, or between shear and known object positions.

Weak Shear

- Correlations in the shear/matter field hold information about the **Dark Matter** distribution. The Cold Dark Matter theory predicts these correlations and the corresponding shear correlations.
- The theory does **not** predict the structure of individual dark matter halos.
- A weak shear measurement in the direction of a single object is just a poorly measured, difficult to interpret shear correlation function.

Berlind & Weinberg 2002



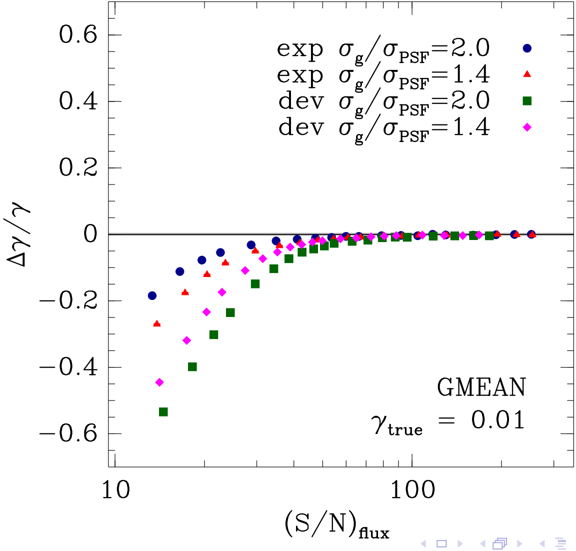
Shear Measurements

- Tradition: measure shapes and average them.
- Forward-modeling: Fit a model that is convolved by an estimate of the PSF. Limited by how well one can model the galaxy and PSF (e.g. Miller et al., Bernstein & Armstrong, many others).
- Moments: Measure moments and correct them for the PSF. Use a weight function to suppress the noise. Derive how that measurement responds to smearing by the PSF and shearing (e.g. Kaiser, Squires, & Broadhurst, Bernstein & Jarvis, Melchior, Bernstein & Armstrong). Working in Fourier space can help with the deconvolution (Bernstein).
- These methods can be made to work well, as long as the S/N is still pretty high, say 50 or higher.

Noise Bias

- Non-linear fitting in the presence of noise is biased, both the maximum likelihood and expectation value: using the mean shape won't work (Hirata, Refregier, etc). Results in a **calibration** error.
- This is generally known in statistics, but not yet solved for the particular problem of shear. Badly aggravated by the PSF "deconvolution".
- The noise also causes problems for moment based methods.

Using Expectation Value $< e >$ for Shear

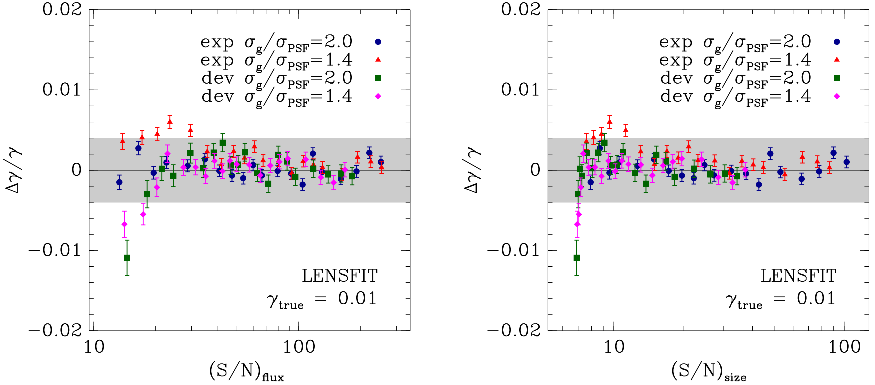


Bayesian Shear Measurement

Miller et al. 2007

- Miller et al. 2007 (`LENSFIT`): Use priors on the parameters and explore a constrained posterior surface (Prior × Likelihood).
- Attempt to derive how the shear estimate (the shape) is affected by the noise and prior. Called the **response** or sensitivity.
- No rigorous expression is given for the mean shear of a population given the ellipticities and responses. Miller et al. average them separately and divide.
- Miller et al. 2013 find large biases in simulations, of order 10% at $(S/N)_{\mathrm{flux}} \sim 10$.

LENSFIT Tests



- I did my own tests of `LENSFIT` with strong structural priors.
- Very fast code needed to explore full likelihood. Use Gaussian mixtures to approximate `exp,dev,PSF` profiles (Hogg et al.). Fast analytic convolutions.
- Bias vs $(S/N)_{\mathrm{size}}$ has more universal form than vs $(S/N)_{\mathrm{flux}}$.

Bernstein & Armstrong 2013

- Shape is not shear.
- While the posterior surface for the *shape* of single galaxy is complex, the posterior surface for the *mean shear* of a large ensemble must approach a Gaussian according to the central limit theorem. This is both useful and actually true!
- Assuming Gaussianity, weak shear, and knowledge of underlying distribution of shapes for the ensemble (the prior), one can derive an unbiased estimator for the mean shear of the ensemble.
- Nothing is lost: in the limit of weak shear, need to use an ensemble statistic anyway, and theory only predicts the ensemble statistics, e.g. correlation functions.
- This is a good idea, but needed an implementation, so I worked it into my existing code.

Bernstein & Armstrong

Assume a small shear $\mathbf{g}$. The posterior probability for the shear estimated from many galaxies is

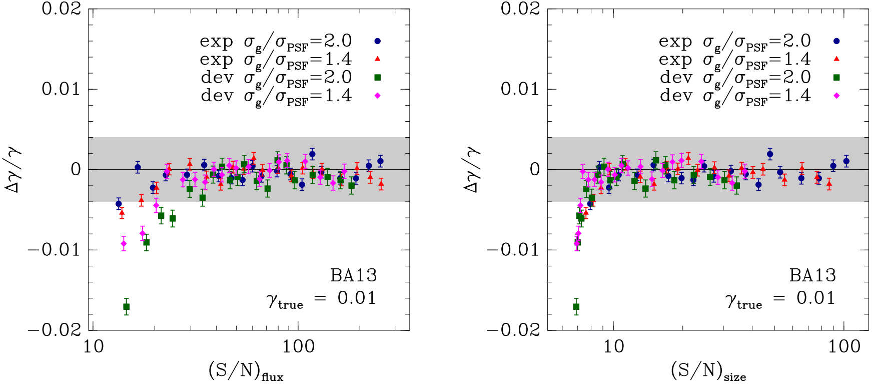
$$P(\mathbf{g}|\mathbf{D}) = \frac{P(\mathbf{g})P(\mathbf{D}|\mathbf{g})}{P(\mathbf{D})} \tag{1}$$

$$= P(\mathbf{g}) \prod_i \frac{P(\mathbf{D}_i|\mathbf{g})}{P(\mathbf{D}_i)} \tag{2}$$

$$P(\mathbf{D}_i|\mathbf{g}) \approx P_i + \mathbf{g}\cdot\mathbf{Q}_i + \frac{1}{2}\mathbf{g}\cdot\mathbf{R}_i\cdot\mathbf{g} \tag{3}$$

$$-\ln P(\mathbf{g}|\mathbf{D}) \approx (\text{stuff}) - \mathbf{g}\cdot\sum_i \frac{\mathbf{Q}_i}{P_i} + \frac{1}{2}\mathbf{g}\cdot\left[\sum_i \left(\frac{\mathbf{Q}_i\mathbf{Q}_i^T}{P_i^2} - \frac{\mathbf{R}_i}{P_i}\right)\right]\cdot\mathbf{g} \tag{4}$$

Bernstein & Armstrong Tests



- Assuming we can find the right model.
- Sufficient accuracy for DES at $(S/N)_{\mathrm{size}} > 10$.

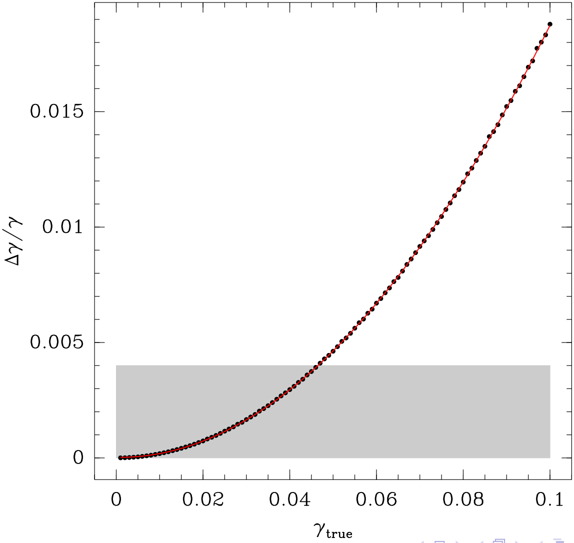
Bernstein & Armstrong Tests

- Can push to lower $(S/N)_{\mathrm{size}}$ than `LENSFIT`.
- Bias varies with $(S/N)_{\mathrm{size}}$ in a simpler way than `LENSFIT`.
- Bias vs $(S/N)_{\mathrm{size}}$ even more universal. Sufficient accuracy for DES at $(S/N)_{\mathrm{size}} > 10$.

Limitations

- I'm assuming I can find the right model. Not true in real data. Might be OK for DES (Kacprzak et al. 2013) but perhaps not LSST.
- Taylor expansion breaks down at higher shear.
- TODO:
    - Why is there bias at all? Is it the likelihood sampling method?
    - For real data need a functional form for the distribution of shapes that is twice differentiable.
    - Bernstein & Armstrong propose a model-independent technique using moments in Fourier space, but not yet implemented. Gary and Bob plan to do it. Student at Stony Brooke as well (Madhavacheril).

Bernstein & Armstrong



The Code

- The package is `ngmix`. This version is a rewrite using `Python`+ `Numba`.
- On github `https://github.com/esheldon/ngmix`.
- The old version `gmix_image` doesn't require `Numba`. Also available, but is no longer maintained.
- The code is running on DES data, measuring multi-epoch and multi-band shears and fluxes.
- Need to parametrize shape distribution with twice differentiable function.

Summary

- The error in most standard shear estimation techniques is dominated by noise bias.
- Modern techniques such as Bernstein & Armstrong can work well enough for current surveys.
- Implemented and running on DES.
- Future analyses using LSST data may need a model-independent approach.