## Wavelength dependent PSFs



## Chromaticity

## Sources

- atmosphere
- differential chromatic refraction (DCR)
- seeing
- telescope optics
- sensor

Techniques for study

- analytic formulae
- GalSim "ring test" simulations
- photon-by-photon Monte Carlo (PhoSim)


## Differential Chromatic Refraction




## Where the photons land on the focal plane



## Where the photons land on the focal plane



## PSF centroid shifts

$$
\bar{R}=\frac{\int p(\lambda) R(\lambda) \mathrm{d} \lambda}{\int p(\lambda) \mathrm{d} \lambda}
$$

Centroid shifts don't directly affect galaxy shapes, but do make registration of multiple exposures tricky.

If images are registered using G5v stars, then centroids of other types of objects shift.

The shift depends on zenith angle and hour angle.

Distribution of zenith and hour angles depends on declination.

## Shifts in PSF centroids due to DCR



## Shifts in PSF centroids due to DCR



## Shifts in PSF centroids due to DCR



## PSF second moment shifts

DCR increases zenith-direction second moments of all objects.

Amount of increase depends on SED.

Amplitudes and directions of relative second moment shifts depend on zenith angle and hour angle.

Distribution of zenith and hour angles depends on declination.


## Effect of misestimating the PSF (c.f. Paulin-Henriksson+08)

Ellipticity definition

$$
\epsilon_{1}=\frac{I_{x x}^{\mathrm{gal}}-I_{y y}^{\mathrm{gal}}}{I_{x x}^{\mathrm{gal}}+I_{y y}^{\mathrm{gal}}}
$$

Corrected second moment

$$
I^{\mathrm{gal}}=I^{\mathrm{obs}}-I^{\mathrm{psf}, \mathrm{~g}}
$$

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Corrected second moment

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I^{\mathrm{gal}}=I^{\mathrm{obs}}-I^{\mathrm{psf}, \mathrm{~g}}
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PSF misestimate

$$
\Delta I^{\mathrm{psf}}=I^{\mathrm{psf}, *}-I^{\mathrm{psf}, \mathrm{~g}}
$$

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$$

PSF misestimate

$$
\Delta I^{\mathrm{psf}}=I^{\mathrm{psf}, *}-I^{\mathrm{psf}, \mathrm{~g}}
$$

Propagate into ellipticity

$$
\epsilon_{1} \rightarrow \frac{\left(I_{x x}^{\mathrm{gal}}+\Delta I_{x x}^{\mathrm{psf}}\right)-\left(I_{y y}^{\mathrm{gal}}+\Delta I_{y y}^{\mathrm{psf}}\right)}{\left(I_{x x}^{\mathrm{gal}}+\Delta I_{x x x}^{\mathrm{psf}}\right)+\left(I_{y y}^{\mathrm{gal}}+\Delta I_{y y}^{\mathrm{psf}}\right)}
$$

## Effect of misestimating the PSF (c.f. Paulin-Henriksson+08)

Ellipticity definition

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\epsilon_{1}=\frac{I_{x x}^{\mathrm{gal}}-I_{y y}^{\mathrm{gal}}}{I_{x x}^{\mathrm{gal}}+I_{y y}^{\mathrm{gal}}}
$$

Corrected second moment
$I^{\mathrm{gal}}=I^{\mathrm{obs}}-I^{\mathrm{psf}, \mathrm{g}}$

PSF misestimate

$$
\Delta I^{\mathrm{psf}}=I^{\mathrm{psf}, *}-I^{\mathrm{psf}, \mathrm{~g}}
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Propagate into ellipticity

$$
\epsilon_{1} \rightarrow \frac{\left(I_{x x}^{\mathrm{gal}}+\Delta I_{x x}^{\mathrm{psf}}\right)-\left(I_{y y}^{\mathrm{gal}}+\Delta I_{y y}^{\mathrm{psf}}\right)}{\left(I_{x x}^{\mathrm{gal}}+\Delta I_{x x}^{\mathrm{psf}}\right)+\left(I_{y y}^{\mathrm{gal}}+\Delta I_{y y}^{\mathrm{psf}}\right)}
$$

Define second-moment radius (squared)

$$
r_{\mathrm{gal}}^{2}=I_{x x}^{\mathrm{gal}}+I_{y y}^{\mathrm{gal}}
$$

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$$

Algebra

$$
\epsilon_{1} \rightarrow \epsilon_{1}\left(1-\frac{\Delta I_{x x}^{\mathrm{psf}}+\Delta I_{y y}^{\mathrm{psf}}}{r_{\text {gal }}^{2}}\right)+\frac{\Delta I_{x x}^{\mathrm{psf}}-\Delta I_{y y}^{\mathrm{psf}}}{r_{\text {gal }}^{2}}+\mathcal{O}(\Delta I)^{2}
$$

## Effect of misestimating the PSF (c.f. Paulin-Henriksson+08)

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$$
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Algebra

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$$

Algebra

$$
\begin{aligned}
& \epsilon_{1} \rightarrow \epsilon_{1}(1-\underbrace{\frac{\Delta I_{x x}^{\mathrm{psf}}+\Delta I I_{y y}^{\mathrm{psf}}}{r_{\text {gal }}^{2}}})+\underbrace{\frac{\Delta I I_{x x}^{\mathrm{psf}}-\Delta I_{y y}^{\mathrm{psf}}}{r_{\text {gal }}^{2}}}_{\text {Shear }}+\mathcal{O}(\Delta I)^{2} \\
& \text { Shear calibration biases } \\
& \hat{\gamma}_{i}=\gamma_{i}\left(1+m_{i}\right)+c_{i} \\
& \epsilon_{2} \rightarrow \epsilon_{2}\left(1-\frac{\Delta I_{x x}^{\mathrm{psf}}+\Delta I_{y y}^{\mathrm{psf}}}{r_{\mathrm{gal}}^{2}}\right)+\frac{2 \Delta I_{x y}^{\mathrm{psf}}}{r_{\mathrm{gal}}^{2}}+\mathcal{O}(\Delta I)^{2}
\end{aligned}
$$

## Effect of misestimating the PSF (c.f. Paulin-Henriksson+08)

Ellipticity definition

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\epsilon_{1}=\frac{I_{x x}^{\text {gal }}-I_{y y}^{\mathrm{gal}}}{I_{x x x}^{\mathrm{gal}}+I_{y y}^{\mathrm{gal}}}
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Algebra

$$
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\text { generic, but assumes } \\
\text { unweighted second moments }
\end{array}
\end{aligned}
$$

## Effect of DCR on PSF and inferred galaxy ellipticity

$$
\begin{gathered}
\text { PSF centroid shift } \\
\bar{R}=\frac{\int p(\lambda) R(\lambda) \mathrm{d} \lambda}{\int p(\lambda) \mathrm{d} \lambda}
\end{gathered}
$$

PSF second moment shift

$$
V=\frac{\int p(\lambda)(R(\lambda)-\bar{R})^{2} \mathrm{~d} \lambda}{\int p(\lambda) \mathrm{d} \lambda \quad \text { Plazas } \& \text { Bernstein } 2012}
$$

## Effect of DCR on PSF and inferred galaxy ellipticity

> PSF centroid shift
> $\bar{R}=\frac{\int p(\lambda) R(\lambda) \mathrm{d} \lambda}{\int p(\lambda) \mathrm{d} \lambda}$

PSF second moment shift

$$
V=\frac{\int p(\lambda)(R(\lambda)-\bar{R})^{2} \mathrm{~d} \lambda}{\int p(\lambda) \mathrm{d} \lambda \quad \text { Plazas } \& \text { Bernstein } 2012}
$$

PSF misestimate for DCR, declare x to be the zenith direction

$$
\Delta I_{x x}^{\mathrm{psf}, \mathrm{DCR}}=\Delta V \quad \Delta I_{x y}^{\mathrm{psf}, \mathrm{DCR}}=\Delta I_{y y}^{\mathrm{psf}, \mathrm{DCR}}=0
$$

## Effect of DCR on PSF and inferred galaxy ellipticity

> PSF centroid shift
> $\bar{R}=\frac{\int p(\lambda) R(\lambda) \mathrm{d} \lambda}{\int p(\lambda) \mathrm{d} \lambda}$

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$$

PSF misestimate for DCR, declare x to be the zenith direction
$\Delta I_{x x}^{\mathrm{psf}, \mathrm{DCR}}=\Delta V$

$$
\Delta I_{x y}^{\mathrm{psf}, \mathrm{DCR}}=\Delta I_{y y}^{\mathrm{psf}, \mathrm{DCR}}=0
$$

Shear calibration biases for DCR

$$
m_{1}^{\mathrm{DCR}}=m_{2}^{\mathrm{DCR}}=\frac{-\Delta V}{r_{\mathrm{gal}}^{2}} \quad c_{1}^{\mathrm{DCR}}=\frac{\Delta V}{2 r_{\text {gal }}^{2}} \quad c_{2}^{\mathrm{DCR}}=0
$$

## Shifts in PSF second moments due to DCR cf. Plazas \& Bernstein 2012



## Shifts in PSF second moments due to DCR cf. Plazas \& Bernstein 2012



## Shifts in PSF second moments due to DCR cf. Plazas \& Bernstein 2012



## LSST OpSim airmass (zenith angle) distribution

OpSim run 3.61

- Operations simulator: simulate observation cadence, weather, incorporate past observing, other priorities
- zenith angle 25th/50th/ 75th percentiles:
- 29, 37, 42 degrees
- much ongoing work to optimize survey
- first moment of DCR
scales like $\tan \left(\mathbf{Z}_{\mathrm{a}}\right)$
- second moment of DCR scales like $\tan ^{2}\left(z_{a}\right)$


## LSST OpSim airmass distribution

OpSim run 3.61


## LSST OpSim zenith angle distribution

OpSim run 3.61


## LSST OpSim zenith and parallactic angle distribution

OpSim run 3.61

Where the zenith is in field coordinates

video available at:
github.com/DarkEnergyScienceCollaboration/chroma/blob/master/bin/opsim/zenith_parallactic.mp4

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Where the zenith is in field coordinates

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Where the zenith is in field coordinates

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## Chromatic seeing due to atmospheric turbulence

Kolmogorov turbulence predicts that the atmosphere smears out blue photons more than red photons:

$$
F W H M \propto \lambda^{-1 / 5}
$$

Qualitatively similar to diffraction limit:
$\mathrm{FWHM} \propto \lambda^{+1}$


## Chromatic seeing and effect on inferred galaxy ellipticity

PSF second moments dependence on the SED:

$$
I^{\mathrm{psf}}=I^{\mathrm{psf}, \lambda_{0}} \frac{\int p(\lambda)\left(\lambda / \lambda_{0}\right)^{-2 / 5} \mathrm{~d} \lambda}{\int p(\lambda) \mathrm{d} \lambda}
$$

## Chromatic seeing and effect on inferred galaxy ellipticity

PSF second moments dependence on the SED:

$$
\begin{aligned}
& I^{\mathrm{psf}}=I^{\mathrm{psf}, \lambda_{0}} \frac{\int p(\lambda)\left(\lambda / \lambda_{0}\right)^{-2 / 5} \mathrm{~d} \lambda}{\int p(\lambda) \mathrm{d} \lambda} \\
& r_{\mathrm{psf}}^{2}=r_{\mathrm{psf}, \lambda_{0}}^{2} \frac{\int p(\lambda)\left(\lambda / \lambda_{0}\right)^{-2 / 5} \mathrm{~d} \lambda}{\int p(\lambda) \mathrm{d} \lambda}
\end{aligned}
$$

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\end{aligned}
$$

Shear calibration biases for chromatic seeing:

$$
\begin{aligned}
& m_{1}^{\text {seeing }}=m_{2}^{\text {seeing }}=-\frac{r_{\mathrm{psf}}^{2}}{r_{\mathrm{gal}}^{2}} \frac{\Delta r_{\mathrm{psf}}^{2}}{r_{\mathrm{psf}}^{2}} \\
& c_{i}^{\text {seeing }}=\frac{r_{\mathrm{psf}}^{2}}{r_{\mathrm{gal}}^{2}} \frac{\Delta r_{\mathrm{psf}}^{2}}{r_{\mathrm{psf}}^{2}} \epsilon_{i}^{\mathrm{psf}}
\end{aligned}
$$

## Chromatic seeing and effect on inferred galaxy ellipticity

PSF second moments dependence on the SED:

$$
\begin{aligned}
& I^{\mathrm{psf}}=I^{\mathrm{psf}, \lambda_{0}} \frac{\left.\int p(\lambda)\left(\lambda / \lambda_{0}\right)-2 / 5\right) \mathrm{d} \lambda}{\int p(\lambda) \mathrm{d} \lambda} \quad \begin{array}{l}
\text { Switch exponent to } \\
+2 \text { for diffraction limit }
\end{array} \\
& r_{\mathrm{psf}}^{2}=r_{\mathrm{psf}, \lambda_{0}}^{2} \frac{\int p(\lambda)\left(\lambda / \lambda_{0}\right)-2 / 5 \mathrm{~d} \lambda}{\int p(\lambda) \mathrm{d} \lambda}
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\end{aligned}
$$

## Chromatic seeing and effect on inferred galaxy ellipticity

PSF second moments dependence on the SED:

$$
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\text { Switch exponent to } \\
+2 \text { for diffraction limit }
\end{array} \\
& r_{\mathrm{psf}}^{2}=r_{\mathrm{psf}, \lambda_{0}}^{2} \frac{\int p(\lambda)\left(\lambda / \lambda_{0}\right)-2 / 5 \mathrm{~d} \lambda}{\int p(\lambda) \mathrm{d} \lambda} \quad \text { This term is worse } \\
& \text { Shear calibration biases for chromatic seeing: } \\
& m_{1}^{\text {seeing }}=m_{2}^{\text {seeing }}=-\frac{r_{\mathrm{psf}}^{2}}{r_{\mathrm{gal}}^{2}}\left(\frac{\Delta r_{\mathrm{psf}}^{2}}{r_{\mathrm{psf}}^{2}}\right)^{2} \\
& c_{i}^{\text {seeing }}=\frac{r_{\mathrm{psf}}^{2}}{r_{\mathrm{gal}}^{2}} \frac{\Delta r_{\mathrm{psf}}^{2}}{r_{\mathrm{psf}}^{2}} \epsilon_{i}^{\mathrm{psf}}
\end{aligned}
$$

## Chromatic seeing and effect on inferred galaxy ellipticity

PSF second moments dependence on the SED:

$$
\begin{aligned}
& I^{\mathrm{psf}}=I^{\mathrm{psf}, \lambda_{0} \frac{\left.\int p(\lambda)\left(\lambda / \lambda_{0}\right)-2 / 5\right)}{\int p(\lambda) \mathrm{d} \lambda} \quad \begin{array}{c}
\text { Switch exponent to } \\
+2 \text { for diffraction limit }
\end{array}} \begin{array}{ll}
r_{\mathrm{psf}}^{2}=r_{\mathrm{psf}, \lambda_{0}}^{2} \frac{\left.\int p(\lambda)\left(\lambda / \lambda_{0}\right)-2 / 5\right) \mathrm{d} \lambda}{\int p(\lambda) \mathrm{d} \lambda} \quad \begin{array}{l}
\text { This term is worse }
\end{array} \\
\text { for Euclid }
\end{array}
\end{aligned}
$$

Shear calibration biases for chromatic seeing:

$$
\begin{aligned}
& m_{1}^{\text {seeing }}=m_{2}^{\text {seeing }}=\left(-\frac{r_{\mathrm{psf}}^{2}}{r_{\mathrm{gal}}^{2}} \frac{\Delta r_{\mathrm{psf}}^{2}}{r_{\mathrm{psf}}^{2}} c_{\text {gal }}^{r_{\mathrm{psf}}^{2}} \frac{\Delta r_{\mathrm{psf}}^{2}}{r_{\mathrm{psf}}^{2}} \epsilon_{i}^{\mathrm{psf}} \quad \begin{array}{c}
\text { This term is worse } \\
\text { for LSST }
\end{array}\right.
\end{aligned}
$$

## Estimating $r^{2}{ }_{\text {PSF }} / r^{2}{ }^{\text {gal }}$ and LSST requirement on $\Delta r_{\mathrm{psf}}^{2} / r_{\mathrm{psf}}^{2}$

- Want to keep $|m| \lesssim 0.0015$
- Gaussian has $3 x$ smaller $r^{2}$ than Moffat of same FWHM
- Half light radius (HLR) underestimates $\mathrm{r}^{2}$ by factor of ~2-20 for Sersic profiles with n between I. 0 and 4.0
- Caveat: Formula assumes unweighted second moments

| PSF | gal | $\mathrm{r}^{2} \mathrm{PSF} / \mathrm{r}^{2}{ }_{\text {gal }}$ | $\Delta r_{\mathrm{psf}}^{2} / r_{\mathrm{psf}}^{2}$ |
| :---: | :---: | :---: | :---: |
| Chang priv. comm. |  | 1.5 | $10 \times 10^{-4}$ |
| OpSim Gaussian | CatSim <br> Sersic $r^{2}$ |  |  |
|  | Jouvel+09 |  |  |
| Gaussian $F W H M=0.7$ | $\begin{gathered} \text { COSMOS } \\ <H L R>=0.27 \end{gathered}$ | 2.5 | $6 \times 10^{-4}$ |
|  | Jouvel+09 |  |  |
| Moffat FWHM=0.7 | $\begin{gathered} \text { COSMOS } \\ <\text { HLR }>=0.27 \end{gathered}$ | 7.5 | $2 \times 10-4$ |
|  |  |  |  |

## Differences in PSF size due to chromatic seeing



## Differences in PSF size due to chromatic seeing



## Differences in PSF size due to chromatic seeing



## Differences in PSF size due to chromatic seeing



## Differences in PSF size for Euclid



Chromatic seeing in data
Boyd78: solar limb


visible wavelength, but done in really poor seeing!

Selby+79: speckle interferometry


IR only, really bad seeing ( $\sim 5$ ")

## Suggestions that power law slope is not $-\mathrm{I} / 5$

$\beta$, the turbulence power spectrum

Linfield+0l:Palomar testbed interferometer


Fig. 5.-Mean value of spectral slope ( $\beta$ ) for each night in 1999 with 10 or more scans. The error bars represent the $1 \sigma$ scatter about the mean value for that night. [See the electronic edition of the Journal for a color version of this figure.] index, predicted to be 5/3
$F W H M \propto \lambda^{1-2 / \beta}$

| $\beta$ | $1-2 / \beta$ |
| :---: | :---: |
| $5 / 3$ | -0.2 |
| 1.45 | -0.4 |
| 1.35 | -0.5 |

Similar sub-Kolmogorov slopes found by Bester+92, Buscher+95, Colavita\&Lane 200I, Short+03

## Additional sources of chromatic PSF size changes

## phoSim-3.3.2 FWHM measurements



## Corrections

Can de-bias PSF measurements if SEDs are known

Use photometry to constrain SED

- similar to a photometric redshift
- however, no catastrophic outliers!


## Correct second moment shift using color



## Correct chromatic biases using machine learning

- Can better constrain SED using all 6 LSST photometry points.
- Train support vector regression algorithm on LSST colors + i-band.



## Correct chromatic biases using machine learning

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- Train support vector regression algorithm on LSST colors + i-band.



## Correct chromatic biases using machine learning

- LSST photometry works really well at constraining Euclid chromatic biases
- LSST sky overlaps proposed Euclid sky by $\sim 5000$ square degrees



## Correct chromatic biases using machine learning

- LSST photometry works really well at constraining Euclid chromatic biases
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## Galaxies with color gradients



- Let's pretend we know the composite bulge+disk SED perfectly. Then two effects are present:
- bulge and disk separate spatially
- would like to deconvolve bulge with bulge-PSF, and similarly the disk, but only have access to the composite bulge+disk PSF


## Bulge+Disk separation: parallel axis theorem

| Parallel Axis Theorem |  |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: |

$$
R_{c}=f_{b} R_{b}+f_{d} R_{d}
$$

$$
V_{\text {grad }}=V_{\text {nograd }}
$$

$$
\begin{aligned}
& +f_{b}\left(R_{b}-R_{c}\right)^{2} \\
& +f_{d}\left(R_{d}-R_{c}\right)^{2}
\end{aligned}
$$

bulge flux = disk flux, zenith=30


## Bulge+Disk separation: parallel axis theorem




## When analytic approaches are unavailable (e.g. color gradients)

## Simulate and use a ring test!



Nakajima \& Bernstein 2007

- Simulate ring of galaxy observations using "true" PSF, all sheared same amount
- Measure ellipticities of sheared galaxies using (incorrect) inferred PSF
- Shear estimate is average of reconstructed ellipticities. Ring ensures that average pre-shear ellipticity is 0 .
- Repeat for multiple shears and measure calibration parameters:

$$
\hat{\gamma}=\gamma^{t}(1+m)+c
$$

## Ring test results for Sersic gals and DCR



## Model fitting bias

## Gaussian $\mathrm{n}=0.5$

Residuals grow with increasing
Sersic index

Bias comes from improper modeling of PSF, not galaxy


## Model fitting bias

## Exponential $\mathrm{n}=\mathrm{I} .0$

Residuals grow with increasing
Sersic index

Bias comes from improper modeling of PSF, not galaxy


## Model fitting bias

## de Vaucouleurs n=4.0

Residuals grow with increasing
Sersic index

Bias comes from improper modeling of PSF, not galaxy


## Preliminary: toy model not sensitive to color gradients?

- Implement chromatic seeing and DCR with GalSim
- Similar to Voigt+ I 2 study for Euclid
- Fiducial parameters (from Simard +2002 catalog):
- second-moment radius $=0.27$ "
- $\quad r_{e, B} / r_{e, D}=I . \mid$
- $\mathrm{n}_{\mathrm{B}}=4, \mathrm{nD}_{\mathrm{D}}=\mathrm{l}$
- $\quad z=0.9$
- $\quad$ disk spec $=$ Sbc, bulge spec $=\mathrm{E}$
- $\mathrm{B} / \mathrm{T}=0.25$
- $\quad\left|e_{\mathrm{g}}\right|=0.2$
- sufficient catalog?
- what about realistic galaxies?


## Final thoughts

DCR depends on zenith angle and filter, chromatic seeing does not
Orders of magnitude for LSST chromaticity

|  | DCR (r-band) | DCR (i-band) | seeing |
| :---: | :---: | :---: | :---: |
| m | $\sim 10^{-2.5}$ | $\sim 10^{-3}$ | $\sim 10^{-1.5}$ |
| c | $\sim 10^{-3}$ | $\sim 10^{-3.5}$ | $\sim 0$ |

What next?

- Chromatic effects in optics and sensors
- Measure effects in real data (stellar FWHM vs color?)
- Galaxy color gradients (especially a realistic catalog)

