The Halo Alignment Model for Intrinsic Alignments

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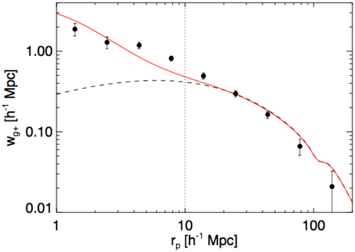
Linear alignment model

Most popular model: galaxies align along large-scale potential gradients,

$$\gamma^I = -\frac{C_1}{4\pi G}\left(\nabla_x^2 - \nabla_y^2, 2\nabla_x\nabla_y\right) S\left(\Psi_P\right) \qquad \Psi_P \sim k^2\delta_{\rm lin}(k) \quad (1)$$

This implies,

$$P_{\tilde{\gamma}^{\rm I}}^{EE}(k), P_{\delta,\tilde{\gamma}^{\rm I}}(k) \sim P_\delta^{\rm lin}(k) \quad (2)$$



Halos are reasonably fit by self-similar ellipsoids

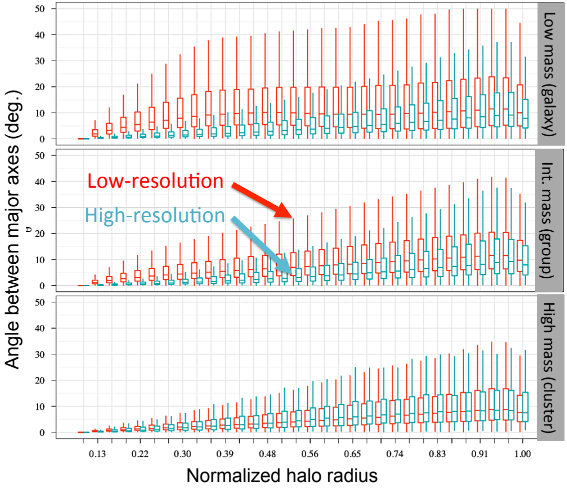
The triaxial halo shape enclosing a volume $V = \frac{4}{3}\pi r^3$ is defined by the surface of constant $R(r)$ where

$$R^2(r) \equiv \frac{x^2}{s^2(r)} + \frac{y^2}{q^2(r)} + z^2, \tag{3}$$

x, y, z are defined in the principal-axis frame of the ellipsoid, which has axis unit vectors $\hat{\mathbf{e}}_a(r), \hat{\mathbf{e}}_b(r), \hat{\mathbf{e}}_c(r)$ with respect to a fixed cartesian coordinate system and axis lengths $a(r) \leq b(r) \leq c(r)$ with $s \equiv a/c$ and $q \equiv b/c$.

Light traces mass?

Maybe. But need to resolve halo shapes at small fractions of their virial radii.



Halo model for 3D intrinsic ellipticity field

Sum over the density-weighted intrinsic ellipticity distribution in triaxial dark matter halos at positions $\mathbf{r}_i$ that are described by mass m_i, concentration c_i, 3D orientation $\boldsymbol{\mathcal{E}}_i \equiv (\hat{\mathbf{e}}_a, \hat{\mathbf{e}}_b, \hat{\mathbf{e}}_c)$, and axis lengths $\mathbf{a}_i \equiv (a, b, c)$,

$$\begin{aligned} \tilde{\gamma}^{\mathrm{I}}(\mathbf{r}) &= \frac{1}{\bar{n}_g} \sum_i \gamma^I(\mathbf{r} - \mathbf{r}_i, \boldsymbol{\mathcal{P}}_i) N_{g,i} u(\mathbf{r} - \mathbf{r}_i, \boldsymbol{\mathcal{P}}_i) \\ &= \sum_i \int d\boldsymbol{\mathcal{P}} \int d^3\mathbf{r}' \, \delta\left(\boldsymbol{\mathcal{P}} - \boldsymbol{\mathcal{P}}_i\right) \delta^{(3)}\left(\mathbf{r}' - \mathbf{r}_i\right) \\ &\quad \times \frac{N_{g,i}}{\bar{n}_g} \gamma^I\left(\mathbf{r} - \mathbf{r}', \boldsymbol{\mathcal{P}}\right) \, u\left(\mathbf{r} - \mathbf{r}' | \boldsymbol{\mathcal{P}}\right), \end{aligned} \tag{4}$$

where $\boldsymbol{\mathcal{P}}_i \equiv (m_i, c_i, \boldsymbol{\mathcal{E}}_i, \mathbf{a}_i)$ denotes the properties describing a triaxial dark matter halo, and $\gamma^I(\mathbf{r})$ is the intrinsic galaxy ellipticity at position $\mathbf{r}$.

Contributions to the II 2-halo term

1-h, satellite-satellite: Schneider & Bridle (2010), Schneider+ (2013) – not large

1-h, central-satellite: "anti-Holmberg effect" – need to model anisotropic spatial distribution of satellites within a halo.

2-h, central-central: **Main term for Halo Alignment Model**

2-h, satellite-satellite: Should be small due to symmetry

2-h, central-satellite: Near zero due to symmetry

Contributions to the GI 2-halo term

1-h, satellite-δ: Likely small given small 1-h, sat-sat term

1-h, central-δ: Zero for spherical halos – Likely important on sub-Mpc scales for elliptical halos.

2-h, central-δ: **Main term for Halo Alignment Model**

2-h, satellite-δ: Should be small due to symmetry

Seed correlation function

Smith & Watts (2005):

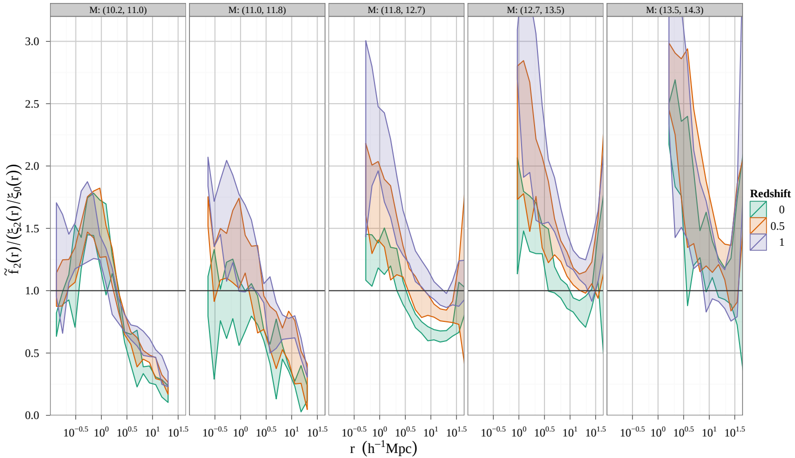
$$p(\mathbf{x}_1, \mathbf{x}_2, \mathcal{P}_1, \mathcal{P}_2) = p(\mathbf{x}_1, \mathcal{P}_1) p(\mathbf{x}_2, \mathcal{P}_2) \left(1 + \xi^{\rm seed}(\mathbf{x}_1 - \mathbf{x}_2, \mathcal{P}_1, \mathcal{P}_2)\right)$$

Schneider, Frenk, Cole (2012):

$$\xi^{\rm seed}_{hX}(\mathbf{r}, \mu, m_1, m_2) = \xi_{hX}(r, m_1, m_2) \left(1 + \tilde{f}_2(r, m_1, m_2) P_2(\mu)\right), \tag{5}$$

where $\mu \equiv \hat{\mathbf{e}}_{c,1} \cdot \hat{\mathbf{e}}_{c,2}$ for the II terms and $\mu \equiv \hat{\mathbf{e}}_{c,1} \cdot \hat{\mathbf{r}}$ for the GI terms.

Halo-mass correlation quadrupole



2-h GI 3D power spectrum

$$P^{\text{2-halo}}_{\tilde{\gamma}^{\mathrm{I}},\delta}(\mathbf{k}) = \int dm_1 \, \frac{\left\langle N_g^c | m_1 \right\rangle}{\bar{n}_g} n(m_1) \int d\mathcal{E}_1 \, \bar{\gamma}(\mathcal{E}_1 | m_1) \times \int d^3 r \, e^{i\mathbf{k}\cdot\mathbf{r}} \xi^{\mathrm{seed}}(\mathbf{r}, \mu, m_1) \cos(2(\phi_e - \phi_r))$$

We evaluate this expression using the multipole expansion of the plane wave.

2-h GI 3D power spectrum (multipoles)

The shear-density power spectrum expanded in multipoles becomes,

$$P_{\tilde{\gamma}^{\mathrm{I}},\delta}(\mathbf{k}) = \sum_{\ell=0}^{\infty} i^{\ell}(2\ell+1)\int dm_1\,\frac{\langle N_g^c|m_1\rangle}{\bar{n}_g}n(m_1)\,\bar{C}_\ell(\hat{k})\,F_\ell(k), \quad (6)$$

where,

$$\bar{C}_\ell(\hat{k}) \equiv \int d\boldsymbol{\mathcal{E}}_1\,\bar{\gamma}(\boldsymbol{\mathcal{E}}_1|m_1)\int d\hat{r}\,P_\ell(\hat{k}\cdot\hat{r})\,\cos(2(\phi_e-\phi_r))\,P_2(\hat{\mathbf{e}}_{c,1}\cdot\hat{r}), \quad (7)$$

and,

$$F_\ell(k) \equiv \int_0^\infty r^2 dr\, j_\ell(kr)\tilde{f}_2(r)\xi_{h\delta}(r), \quad (8)$$

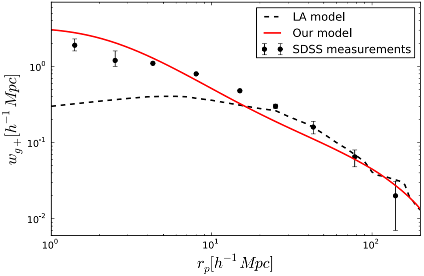
and all halo mass dependence is implicit.

New model predictions: scale dependence

The term,

$$F_\ell(k) \equiv \int_0^\infty r^2 dr\, j_\ell(kr) \tilde{f}_2(r) \xi_{h\delta}(r), \tag{9}$$

determines the scale dependence of the GI correlations from the quadrupole of the halo-mass correlation in *N*-body simulations.



New model predictions: amplitude vs z

$F_\ell(k)$ plus the term,

$$\bar{C}_\ell(\hat{k}) \equiv \int d\mathcal{E}_1\, \bar{\gamma}(\mathcal{E}_1|m_1) \int d\hat{r}\, P_\ell(\hat{k}\cdot\hat{r}) \cos(2(\phi_e - \phi_r))\, P_2(\hat{\mathbf{e}}_{c,1}\cdot\hat{r}), \tag{10}$$

determine the redshift-dependent amplitude of the GI correlations (only a fit parameter in the LA model).

$$\bar{\gamma}_c(\mathcal{E}|m) \equiv \int d\mathbf{a}\, p(\mathbf{a}|m_1, z)\, \left|\gamma_c^I(\mathcal{E}, \mathbf{a}, m_1)\right|, \tag{11}$$

- $p(\mathbf{a}|m_1, z)$ can be easily measured in N-body simulations by computing inertia tensors of halos (as functions of radius to mimic galaxy alignment variations)
- γ_c^I is derived from a **model for 3D galaxy morphologies** and geometric terms (see Sereno papers).

More on z-dependent amplitude

The distribution of minor-to-major axis ratios is z-dependent:

$$p(\tilde{s}) \propto \tilde{s}^{13.3-2.9z}\,(1-\tilde{s})^{9.4-1.8z} \qquad \tilde{s} \equiv s\left(\frac{m_{\rm vir}}{M_*(z)}\right)^{0.0375[\Omega(z)]^{0.16}} \tag{12}$$

The conditional distribution for intermediate-to-major axis ratios given s is mostly z-independent.

The remaining z-dependence in the IA amplitudes comes from:

- Linear growth function in the matter correlation
- Halo-mass alignment multipoles
  - Relative amplitude decreasing with z when compared to linear theory prediction

Summary

- The anisotropic cross-correlation of elliptical halos with mass in N-body simulations is relatively easy to measure and provides a bridge between the linear regime and the virialized regime.
- We can better fit the shape of the GI correlation in SDSS using a halo model with the halo-mass correlation quadrupole from simulations.
- The z-dependent **amplitude** of the IA correlations is a prediction of the halo alignment model, rather than a fit parameter.
- Halo alignment multipoles can synthesize results from simulations of varying resolutions; mocks can't.