Optimal cross-correlation estimators

Theory / Joint Probes session

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Angular galaxy correlations

- The observed galaxy angular number density is a sum of the intrinsic clustering of galaxies, distortions due to lensing magnification, and shot noise,

$$\delta(\mathbf{x}) = \delta_g(\mathbf{x}) + \delta_\mu(\mathbf{x}) + \epsilon \tag{1}$$

- The galaxy angular (cross-)correlation functions in tomographic bins have in principle the following contributions:

$$w(\theta) = w_{gg}(\theta) + w_{g\mu}(\theta) + w_{\mu g}(\theta) + w_{\mathrm{SN}} \tag{2}$$

- With photo-z errors, it can be difficult to separate these contributions due to 'soft' tomographic bin boundaries.

Optimal redshift weighting (1)

Each term contributing to the cross-correlation can be written as a projection of the 3D matter or galaxy correlation function, assuming Limber's approximation, flat-sky approximation, and zero spatial curvature,

$$w_{XY}(\theta) = \int_0^{\chi_\infty} d\chi \, W_X(\chi) W_Y(\chi) \, \xi(\chi\theta), \tag{3}$$

where $W_X(\chi)$ is either a redshift distribution or lensing kernel for a given sample, and,

$$\xi(r) \equiv \int \frac{k\, dk}{2\pi} P(k)\, J_0(kr), \tag{4}$$

where $P(k)$ is the 3D matter or galaxy power spectrum and J_0 is the zeroth order Bessel function and we have neglected redshift space distortions.

Optimal redshift weighting (2)

The Limber equation is a *Fredholm integral equation of the 1st kind*,

$$w_{XY}(\theta) = \int_0^{\chi_\infty} d\chi\, W_X(\chi) K(\chi, \theta), \tag{5}$$

where $K(\chi, \theta) \equiv W_Y(\chi)\xi(\chi\theta)$.

Use eigenvectors of K to describe the optimal solution space.

But, $K(\chi, \theta)$ is not Hermitian $(K(\chi, \theta) \neq K(\theta, \chi))$.

A better kernel is found by considering the square of $K(\chi, \theta)$,

$$C(\chi, \chi') \equiv \int d\theta K(\chi, \theta)\, K(\chi', \theta), \tag{6}$$

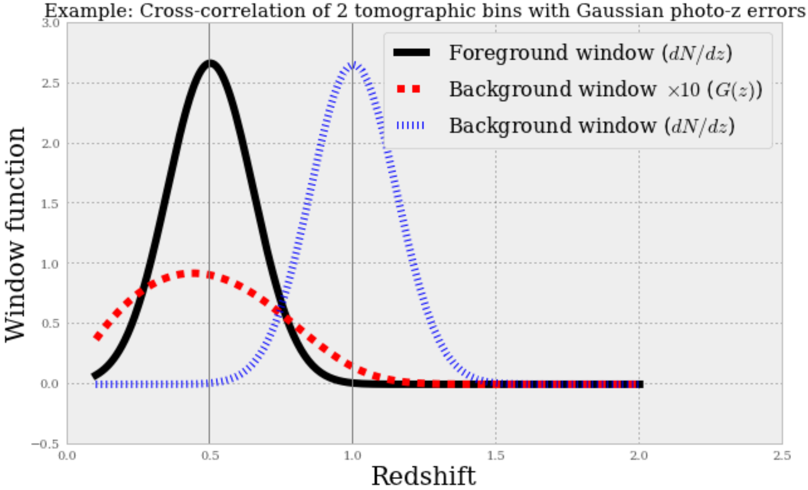
so that,

$$\int d\chi\, C(\chi, \chi')\, \psi(\chi) = \lambda \psi(\chi'). \tag{7}$$

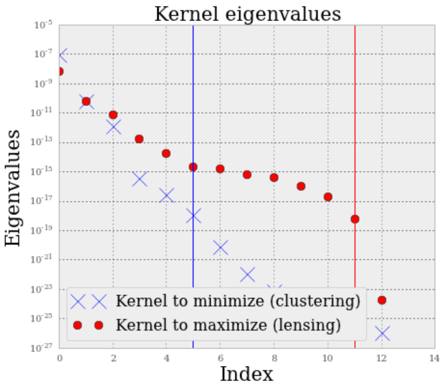
Algorithm for optimal estimator calculation

1. Solve for the eigenfunctions of the lensing and clustering symmetric source kernels (K) and define which physical effect is to be maximized (e.g. lensing) and which is to be nulled (e.g. clustering).
2. Find components of the eigenfunctions of the source kernel to be maximized (e.g. lensing) that are in the null space of the source kernel to be nulled (e.g. clustering) using Gram-Schmidt orthogonalization. Construct a basis set from these components for the final weight functions.
3. Solve for a combination of basis functions that optimizes the signal-to-noise ratio to construct the pair weights for the cross-correlation function estimator.

2-bin example: dN/dz models

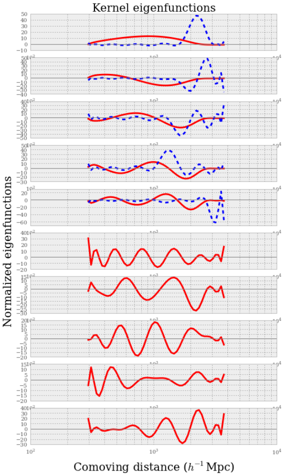


Kernel eigenvalue spectrums

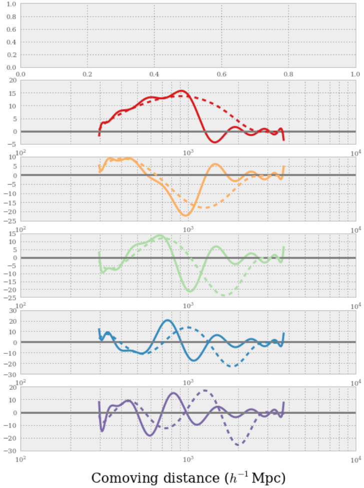


Only 4-5 eigenfunctions are useable due to numerical errors.

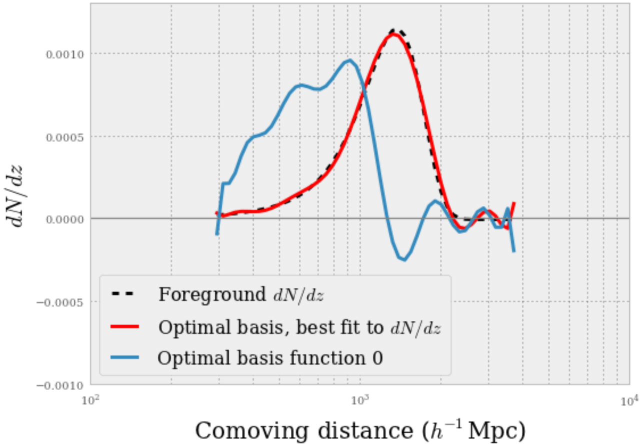
Kernel eigenfunctions



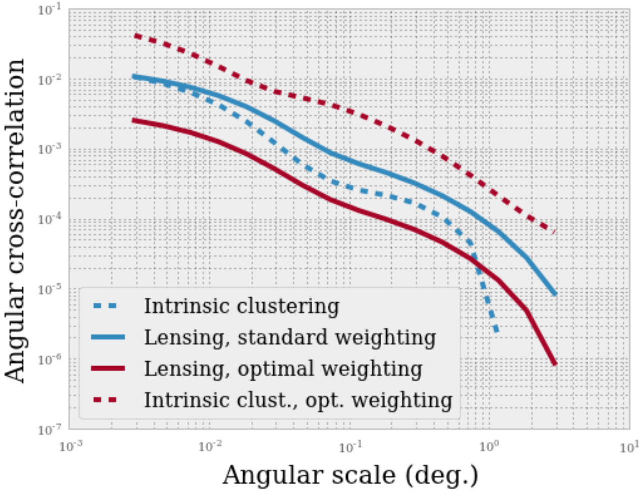
Kernel eigenfunctions after nulling



dN/dz with optimal foreground weights



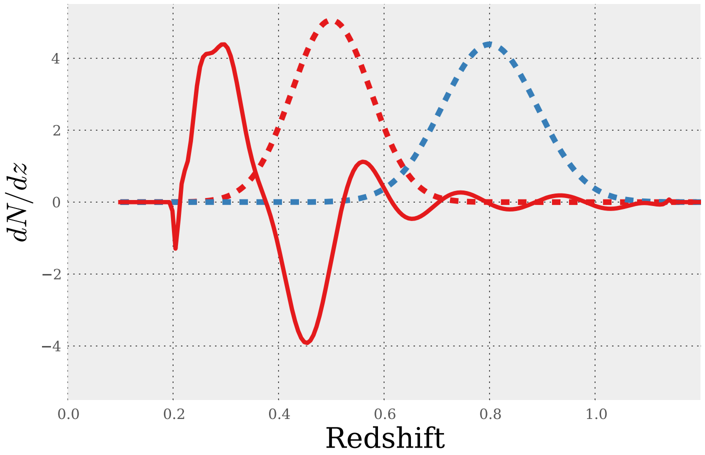
Optimized clustering correlation



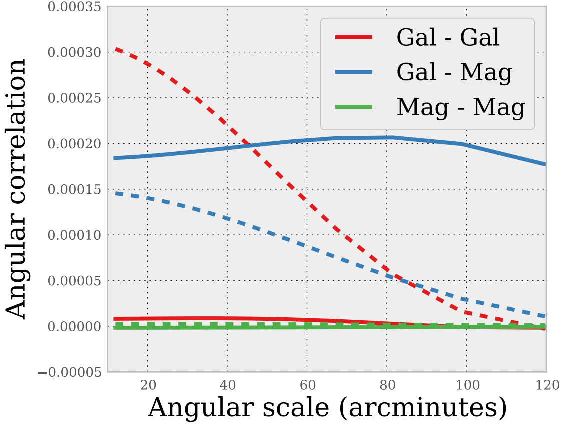
LSST tomography: Fiducial model

- $w(a) = w_0 + (1 - a)w_a$
- 4 tomographic bins
- $0.5 < z < 1.4$ – chosen to cover region most sensitive to w_a
- 3 photo-z nuisance parameters: $z_{\rm cen}$, σ_z, $f_{\rm outlier}$
  - $\sigma_z = 0.05(1 + z)$
  - $f_{\rm outlier} = 0.05$ for all bins
- linear galaxy clustering bias (all analysis limited to linear scales)
  - Consider zero bias and 1% bias with respect to ‘true’ values
- Marginalize over σ_8 and Ω_m

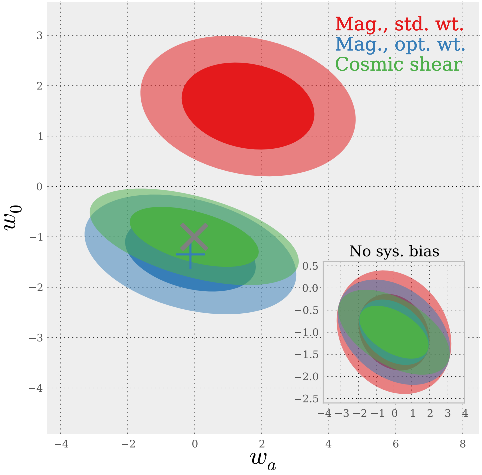
Optimal dN/dz for 1st foreground bin



Cross-correlations between 1st 2 bins



Forecasts on $w(z)$



Summary and Conclusions

- General method to optimize the SNR and minimize theoretical systematics in angular correlations.
- With optimization, the lensing magnification measured via cross-correlations of photo-z bins may yield a dark energy Figure of Merit up to 80% that from cosmic shear with the same set of galaxies and tomographic binning.
- The combination of optimized lensing and clustering correlations helps self-calibrate the linear galaxy bias.
- Typically, at least 10^6 sources are required in each foreground redshift bin to detect lensing magnification via cross-correlations with optimal redshift weighting (because many galaxies are down-weighted, increasing shot noise).