Center for Frontiers in Nuclear Science

Studies on the QCD Energy Momentum Tensor and Ioffe Time behavior of PDFs and GPDs

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BROOKHAVEN NATIONAL LABORATORY

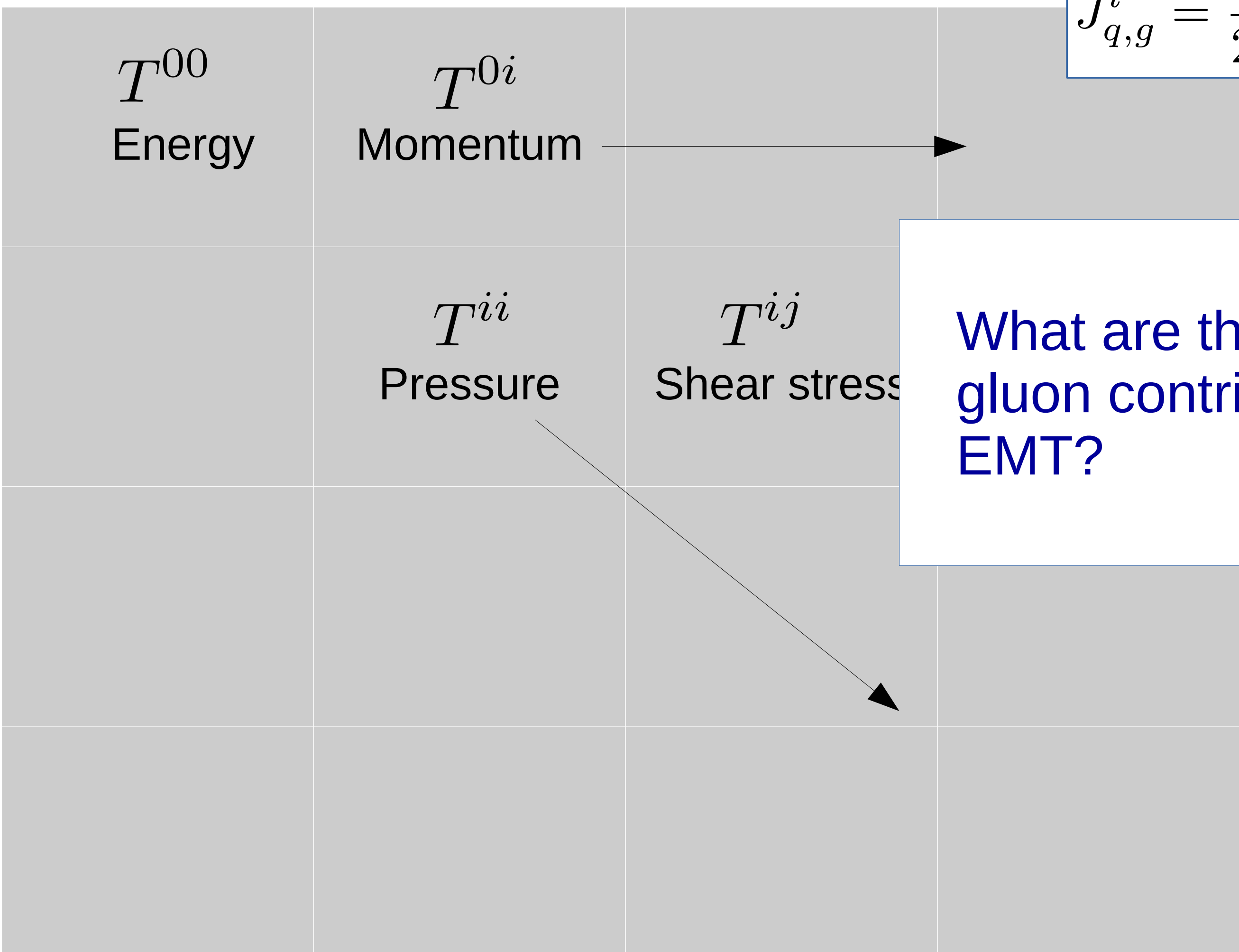
QCD Energy Momentum Tensor

T^{00} Energy	T^{0i} Momentum		
	T^{ii} Pressure	T^{ij} Shear stress	

QCD Energy Momentum Tensor

$$J^i_{q,g} = \frac{1}{2}\epsilon^{ijk}\int d^3x\,\left(T^{0k}_{q,g}x^j - T^{0j}_{q,g}x^k\right)$$

Angular Momentum



What are the separate quark and gluon contributions to the QCD EMT?

Energy Momentum Tensor Parameterization

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}$$

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T_q^{\mu\nu} = i\bar{\psi}\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\psi$$

$$T_g^{\mu\nu} = -F^{\mu\Lambda} F^{\nu}_{\Lambda} - \frac{g^{\mu\nu}}{4} F^2$$

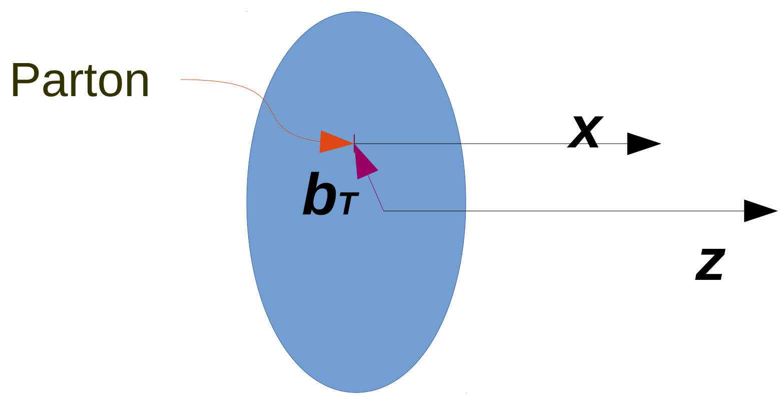
- The quark and gluon contributions to the energy momentum tensor are parameterized by the gravitational form factors.

$$\langle P' | T_{q,g}^{\mu\nu} | P \rangle =$$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu} i\sigma^{\nu)\alpha}\Delta_{\alpha}}{2M} + C_{q,g}\frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g} M \eta^{\mu\nu}\right] u(P)$$

Quark and gluon contributions not conserved separately

Generalized Parton Distributions and their connection to the EMT



Generalized Parton Distributions allow spatial imaging of quarks and gluons inside the nucleon

The Mellin moments of GPDs give the gravitational form factors

$$\int dx\, xH(x,\xi,t) = A_{20} + 4\xi^2 C_{20}$$

$$\int dx\, xE(x,\xi,t) = B_{20} - 4\xi^2 C_{20}$$

X Ji (1997)

A closer look at the quark quark correlator defining PDFs and GPDs

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0} = \bar{U}(p',\Lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\Delta}}{2M}E(x,\xi,t)\right]U(p,\Lambda)$$

Ioffe time $\quad P \cdot z = \nu$

$$\langle p', \Lambda' \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, \Lambda\rangle = \mathcal{M}(P\cdot z, \Delta\cdot z, z^2)$$ GPDs

$$\langle p, \Lambda' \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, \Lambda\rangle = \mathcal{M}(P\cdot z, z^2)$$ PDFs

For $z^2 = 0$ one obtains the correlator defining usual PDFs and GPDs

Measures how far off the light cone the correlator is

Outline

- QCD Energy Momentum Tensor
  - Mass decomposition of the proton
  - Equation of State of Neutron Stars at short distances
- Ioffe time behavior of Parton Distribution Functions and Generalized Parton Distributions

Mass decomposition of the proton

$$T^{\mu\nu} = \boxed{T^{\mu\nu}_{q,kin} + T^{\mu\nu}_{g,kin}} + T^{\mu\nu}_{m} + T^{\mu\nu}_{a}$$

Traceless

X Ji (1995)

$$M = \frac{\langle P| \int d^3\mathbf{x} T^{00}(0,\mathbf{x})|P\rangle}{\langle P|P\rangle} \equiv \langle T^{00}\rangle$$

Rest frame

$$\langle \bar{T}^{00}\rangle = 3/4M$$

Traceless

$$\langle \hat{T}^{00}\rangle = 1/4M$$

Trace part

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

* $\langle P'|(T^{\mu\nu}_{q,g})_R|P\rangle =$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

$\big\downarrow$ Trace

Quark and gluon components separated

$$\langle P'|(T^R_{q,g})^\alpha_\alpha|P\rangle = 2M^2\left(A^R_{q,g}(\mu) + 4\bar{C}^R_{q,g}(\mu)\right)$$

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

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Trace

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* $$T^{\mu\nu} = \frac{1}{2}\bar{\psi}i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^\nu_\alpha$$

Trace

$$T^\mu_\mu = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

$$T^\mu_\mu \stackrel{?}{=} T^\mu_{\mu,q} + T^\mu_{\mu,g}$$

Quark and gluon contributions to the trace anomaly

$$T^\mu_\mu \stackrel{?}{=} T^\mu_{\mu,q} + T^\mu_{\mu,g}$$

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^\nu_\alpha$$

Trace

$$T^\mu_\mu = (1+\gamma_m) m\bar{\psi}\psi + \frac{\beta}{2g} F^2$$

$$(T^\alpha_{g\alpha})_R = A^R_g(\mu) + 4\bar{C}^R_g(\mu)$$

$$\langle P | \frac{\alpha_s}{2\pi}\left(-\frac{11 C_A}{6}(F^2)_R + \frac{14 C_F}{3}(m\bar{\psi}\psi)_R\right)|P\rangle$$

gluons

$$(T^\alpha_{q\alpha})_R = A^R_q(\mu) + 4\bar{C}^R_q(\mu)$$

$$\langle P | (m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}(F^2)_R + \frac{4 C_F}{3}(m\bar{\psi}\psi)_R\right)|P\rangle$$

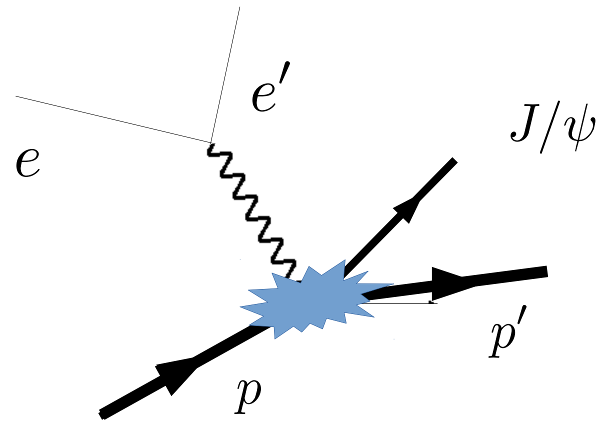
quarks

Y Hatta, AR, K Tanaka, JHEP 1812 (2018) 008

By studying the Gravitational form factors A and $\bar{C}$ we will know the quark and gluon contributions to the trace anomaly separately.

Experimental measurement of the trace anomaly

$$ep \to e'\gamma^* p \to e'p'J/\psi$$

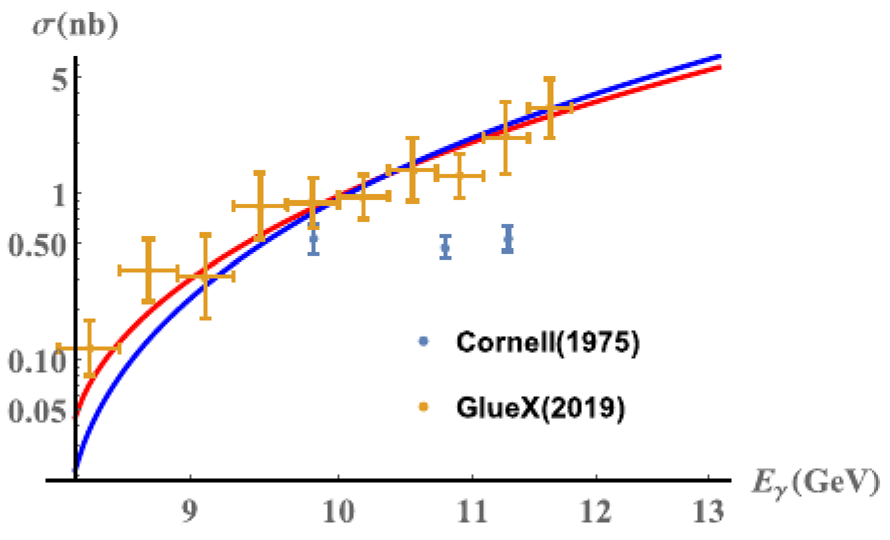


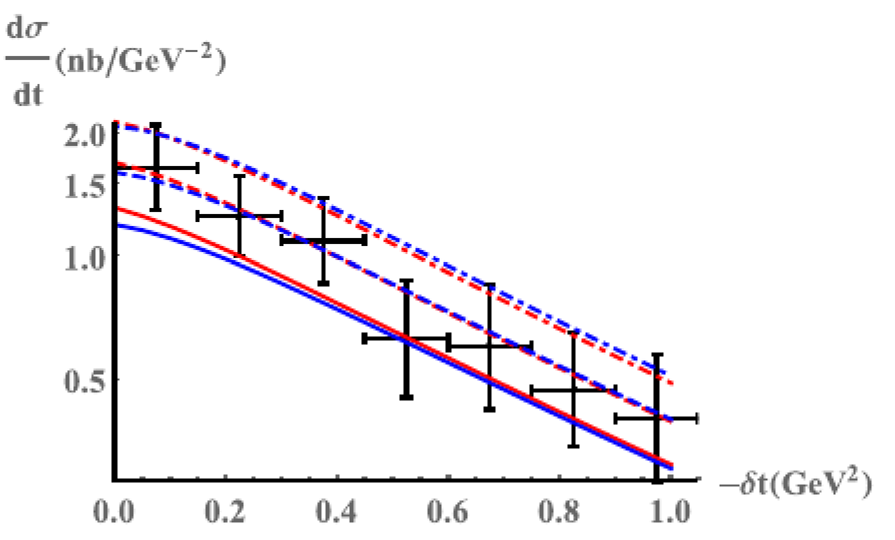
Y Hatta, DL Yang PRD98 (2018)

Y Hatta, A Rajan, DL Yang PRD100 (2019)

The cross-section involves a term that explicitly depends on the trace anomaly.

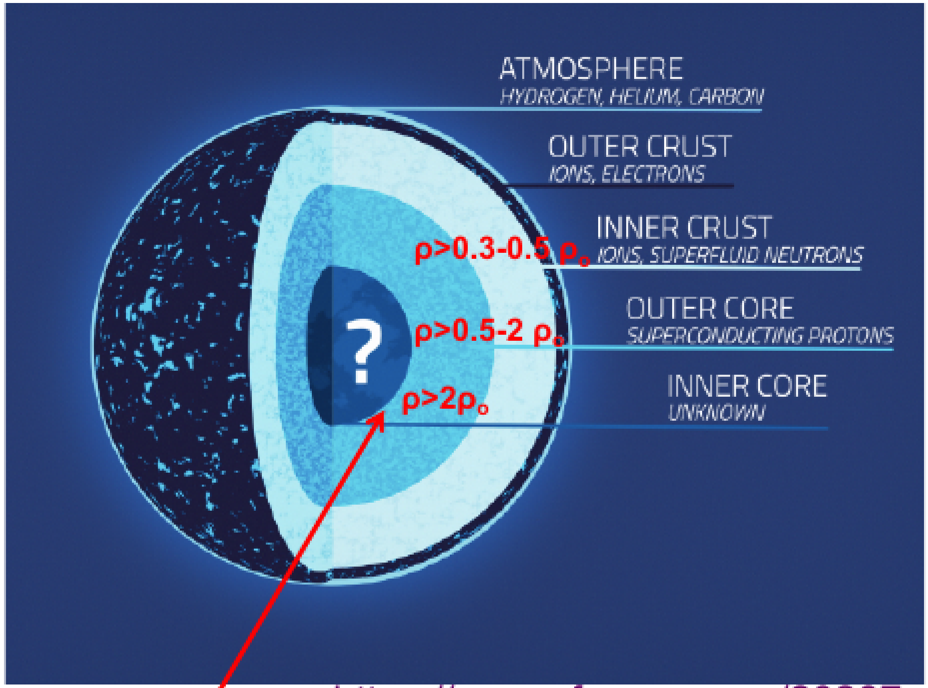
Use input from latest lattice QCD calculations of gluon gravitational form factors and fit to GlueX collaboration data.





Equation of state of neutron stars at short distances

Neutron Stars



https://svs.gsfc.nasa.gov/20267

Gravitational collapse is countered by pressure generated by nuclear forces

Quark content of neutron stars is an open question !!

TOV Equations

energy density

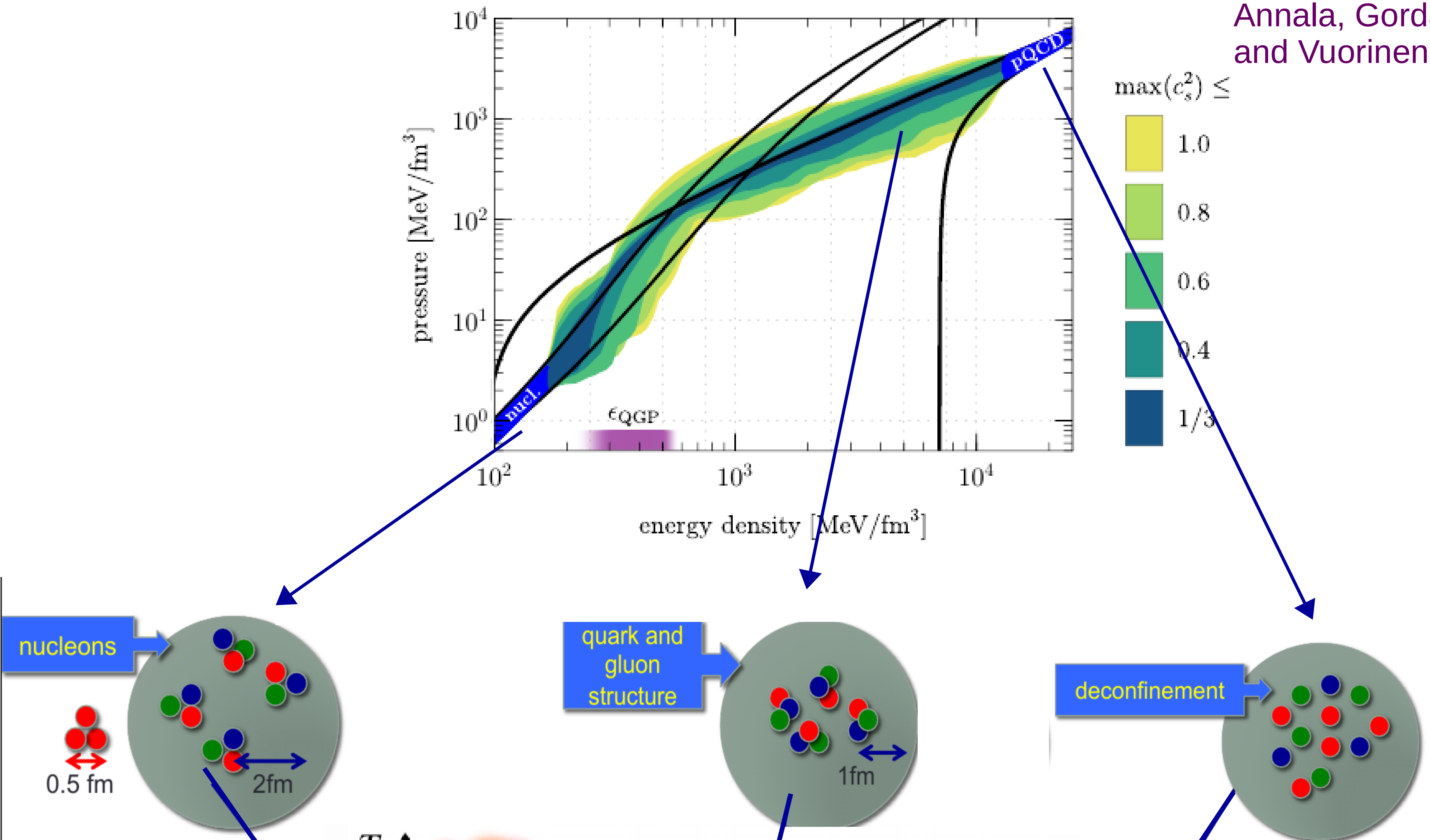
pressure

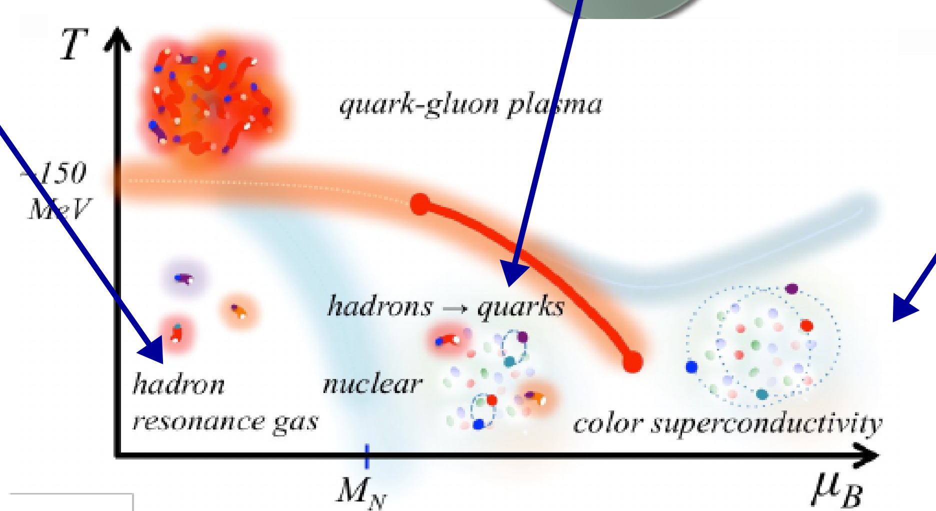
matter distribution

$$\frac{dp(r)}{dr} = -\frac{G}{r}\left(\boxed{\varepsilon(r)} + \boxed{p(r)}\right)\left(m(r) + 4\pi r^3 p(r)\right)\left(r - 2\,G\,\boxed{m(r)}\right)^{-1}$$

$$\frac{d\boxed{m(r)}}{dr} = 4\pi r^2 \boxed{\varepsilon(r)}$$

Annala, Gorda, Kurkela, Nattila and Vuorinen arxiv:1903.09121





G. Baym et al. arXiv:1707.04966

GPD moments and the EMT

- Mellin moments of GPDs give the gravitational form factors that parameterize the energy momentum tensor.

$$\int dx\, xH(x,\xi,t) = A_{20} + 4\xi^2 C_{20} \qquad \int dx\, xE(x,\xi,t) = B_{20} - 4\xi^2 C_{20}$$

$$\langle P'|T^{\mu\nu}|P\rangle =$$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

Energy

Pressure

$$T^{ij} = \left(\frac{r^i r^j}{r^2} - \frac{1}{2}\delta_{ij}\right)s(r) + \delta_{ij}p(r)$$

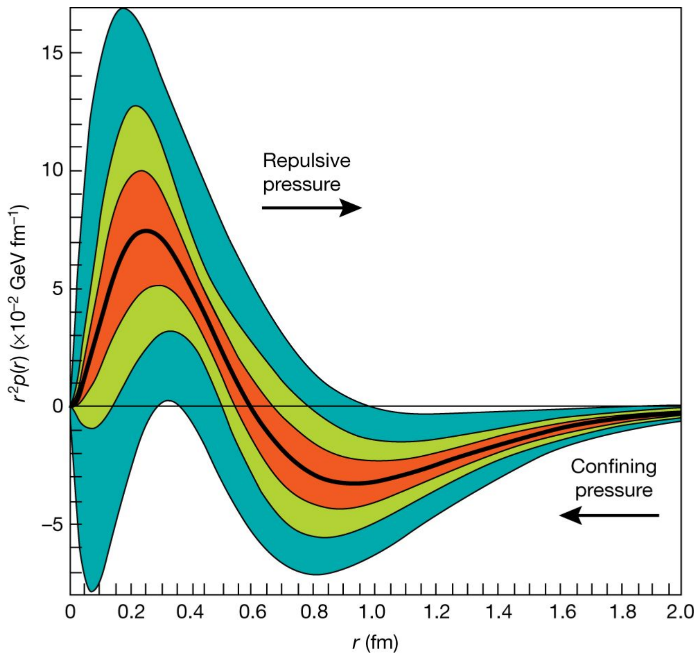
shear pressure

Landau&Lifshitz, Vol.7

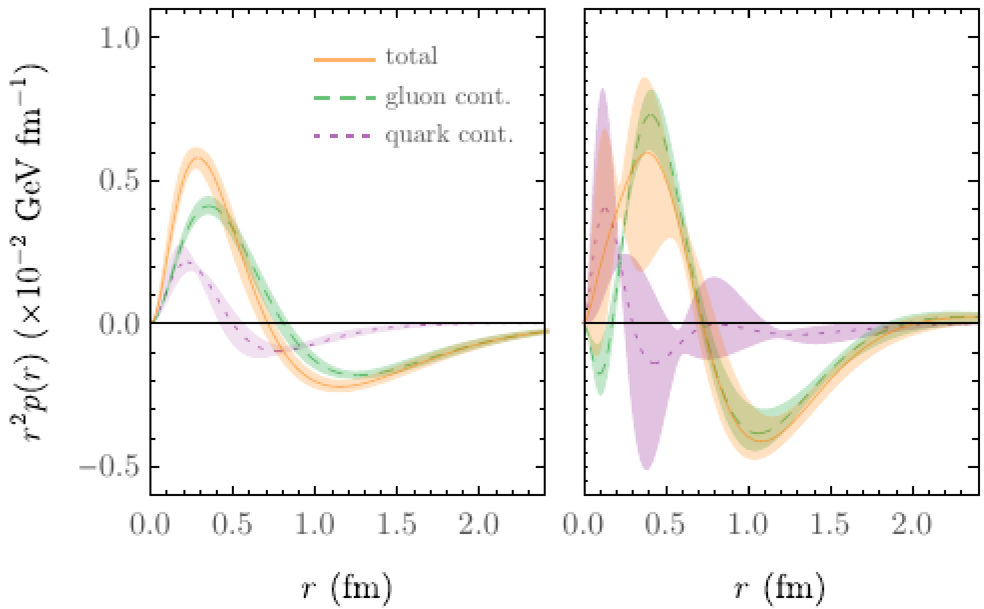
M. Polyakov, hep-ph/0210165

M. Polyakov, P. Schweitzer, arXiv:1805.06596

Pressure Distribution inside the Proton

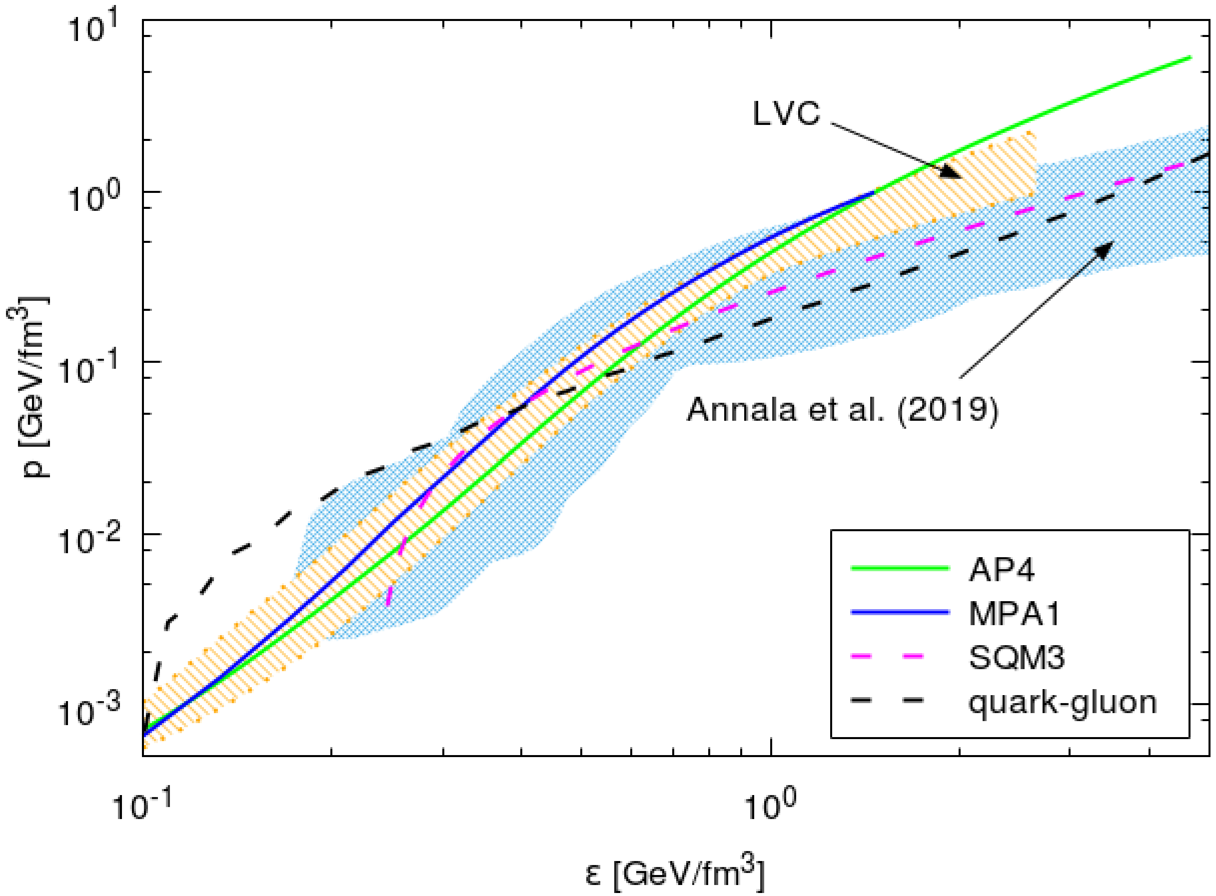


Burkert, Elouadrhiri, Girod (Nature, 2018)



Detmold and Shanahan (PRL, 2019)

Equation of State of Neutron Stars



AR, T Gorda, S Liuti, K Yagi arxiv:1812.01479

$$\epsilon_{q,g}(r) = \int \frac{d^2 \mathbf{\Delta}_T}{(2\pi)^2} e^{i \mathbf{\Delta}_T \cdot \mathbf{b}} A_2^{q,g}(t),$$

$$p_{q,g}(r) = \int \frac{d^2 \mathbf{\Delta}_T}{(2\pi)^2} e^{i \mathbf{\Delta}_T \cdot \mathbf{b}} \, 2\, t\, C_2^{q,g}(t)$$

$$\sum_{\Lambda,\lambda} \rho^q_{\Lambda\lambda}(\mathbf{b}) = H_q(\mathbf{b}^2) = \int \frac{d^2 \mathbf{\Delta}_T}{(2\pi)^2} e^{i \mathbf{\Delta}_T \cdot \mathbf{b}} A_1^q(t).$$

Reconstructing Parton Distribution Functions and Generalized Parton Distributions from Ioffe time behavior

Ioffe Time Distributions

- Due to invariance under Lorentz transformations, the matrix element depends on two scalars

$$\mathcal{M}\left(\underline{(pz)}, z^2\right) = \langle p|\bar{\psi}(0)\gamma^+\psi(z)|p\rangle$$

Ioffe time

Fourier Transform

$$\int_{-1}^{1} dx e^{-ix(pz)}\mathcal{P}(x, z^2) = \mathcal{M}\left((pz), z^2\right)$$

On the light cone

$$\mathcal{P}(x, 0) = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-ix\nu}\mathcal{M}(\nu, 0)$$

Ioffe Time Distributions

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Ioffe Time Distributions

$$T_c + iT_s = \int_0^1 dx\, f(x) e^{i(x\nu)}$$

$$\nu = (Pz)$$

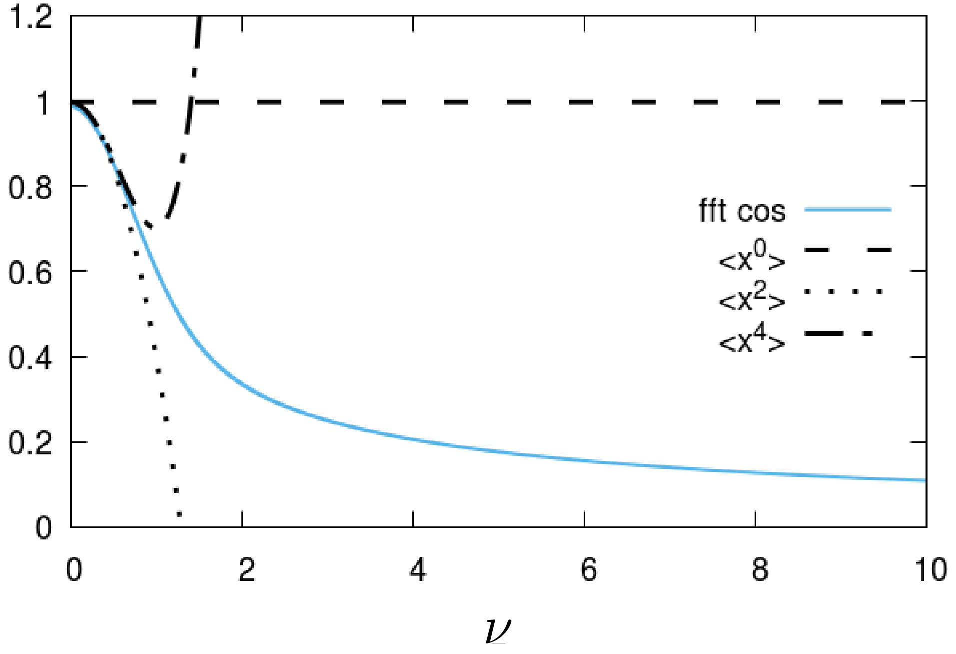
$$M_n = \int dx\, x^{n-1}\, f(x)$$

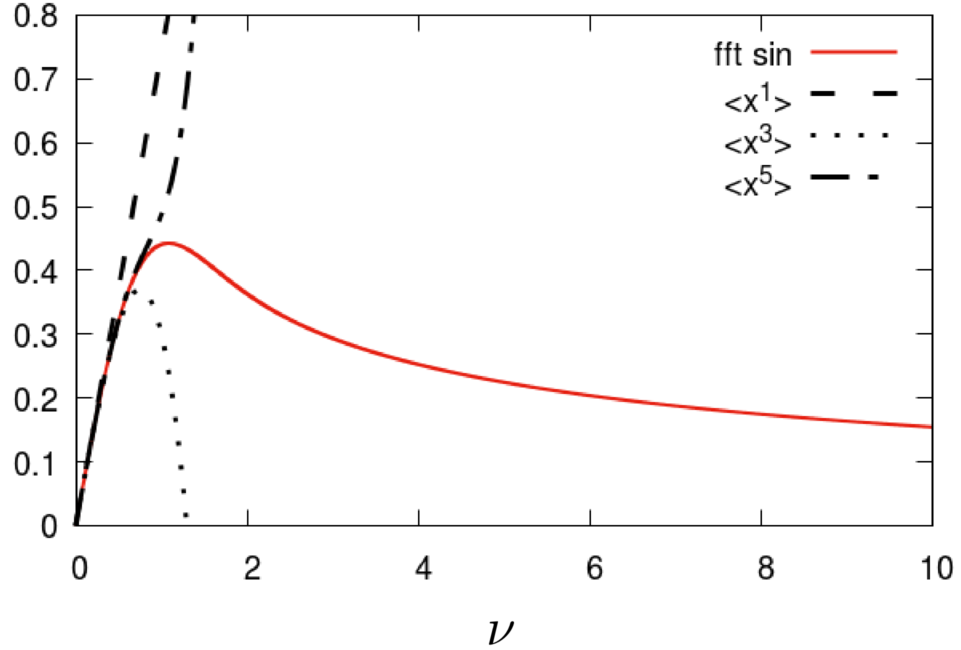
Taylor expansion for small ν

$$T_s(x) = \int_0^1 dx f(x) sin(x\nu) = M_2\nu - \frac{1}{3!}M_4\nu^3 + \ldots$$

$$T_c(x) = \int_0^1 dx f(x) cos(x\nu) = M_1 - \frac{1}{2!}M_3\nu^2 + \ldots$$

Describing Ioffe Time Distributions using Mellin Moments

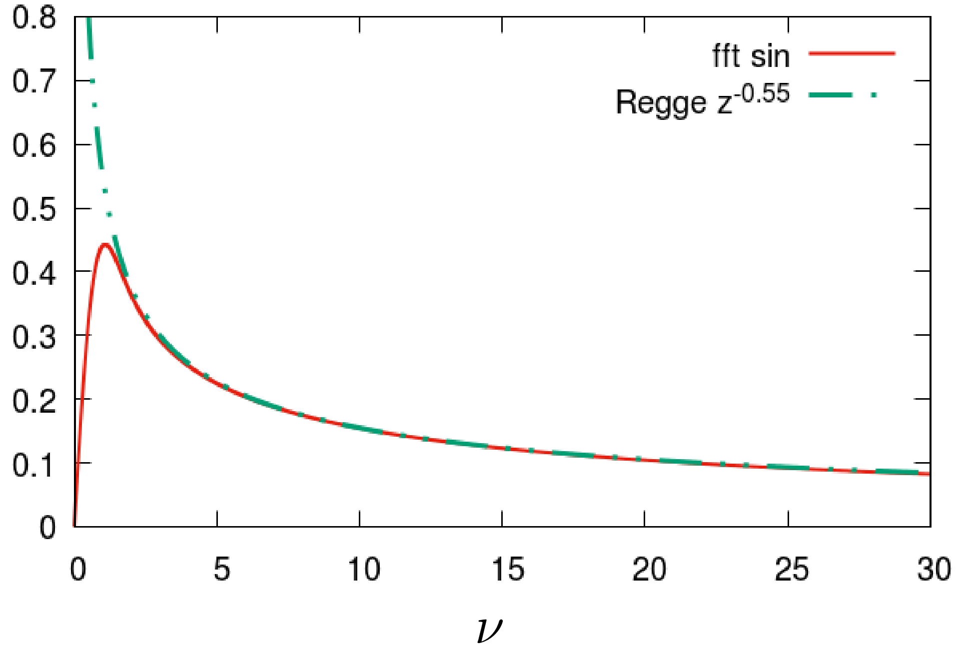




$$T_c(\nu) = M_1 - \frac{1}{2!}M_3\nu^2 + \dots$$

$$T_s(\nu) = M_2\nu - \frac{1}{3!}M_4\nu^3 + \dots$$

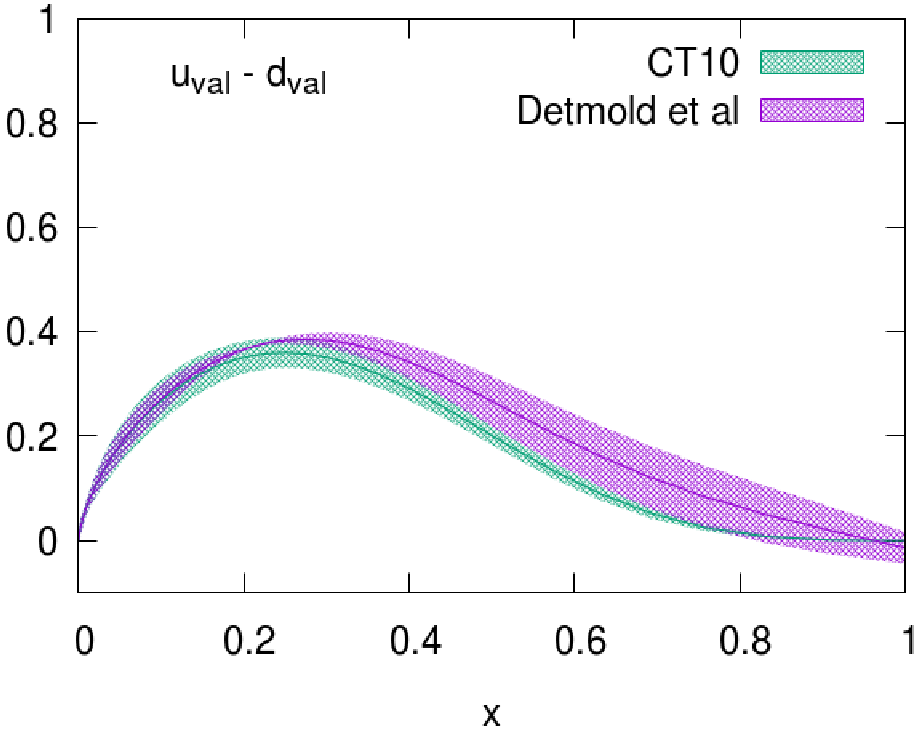
From x space to Ioffe Time ν



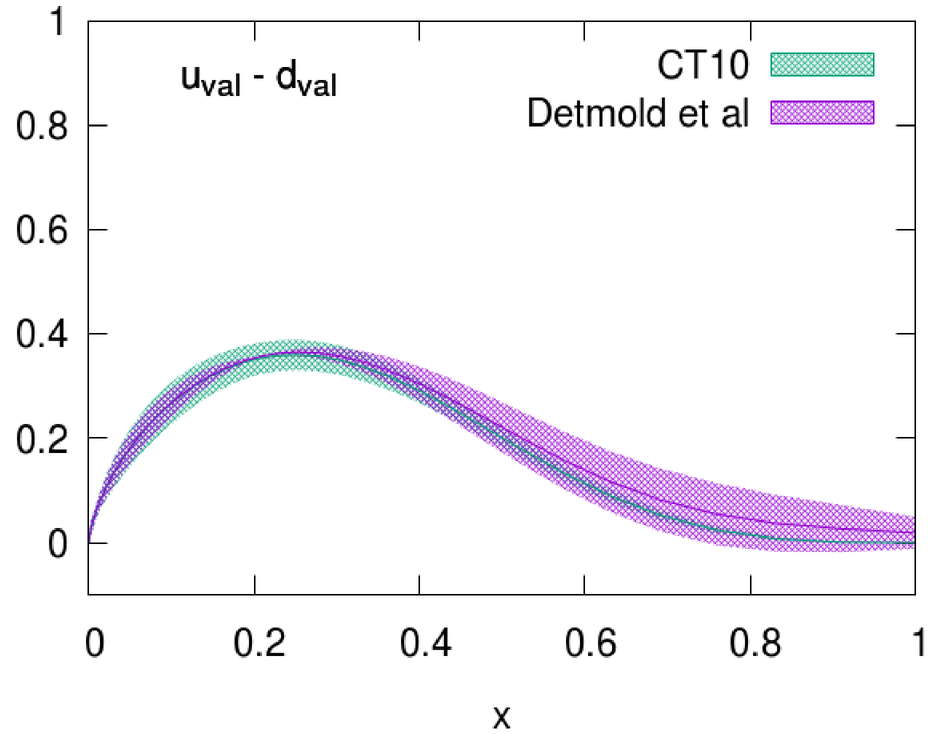
$\nu = (Pz)$

Large z / small x behavior dominated by Regge factor

Reconstructing PDFs from Mellin moments



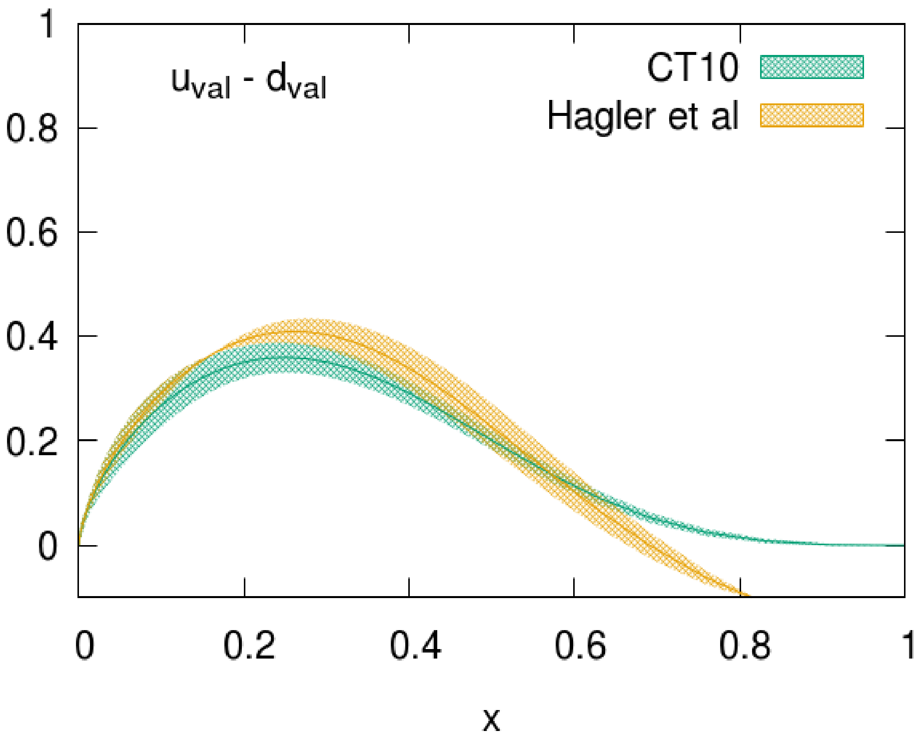
3 moments



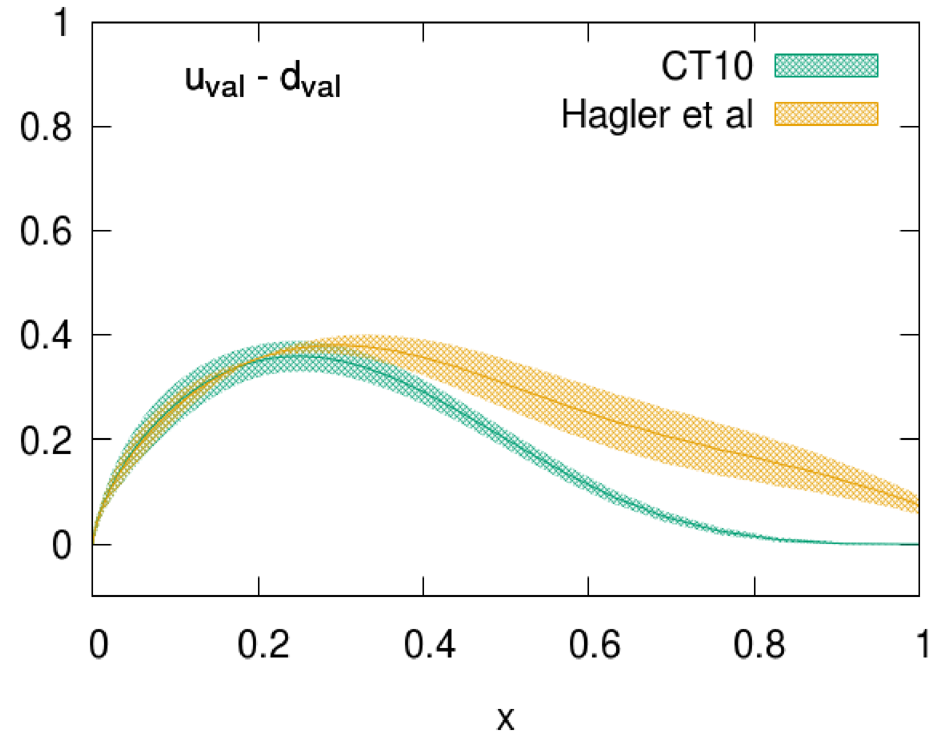
4 moments

W Detmold et al., Eur. Phys. J.3 (2001), Mod. Phys. Lett. A18 (2003)

Reconstructing PDFs from Mellin moments



2 moments



3 moments (third moment not extrapolated to physical pion mass)

LHPC (Ph. Hagler et al.) Phys. Rev. D. 77 (2008)

Extending to GPDs

The GPD moments are a polynomial in ξ

$$\int dx\, H(x,\xi,t) = A_{10}$$

$$\int dx\, x\, H(x,\xi,t) = A_{20} + 4\xi^2 C_{20}$$

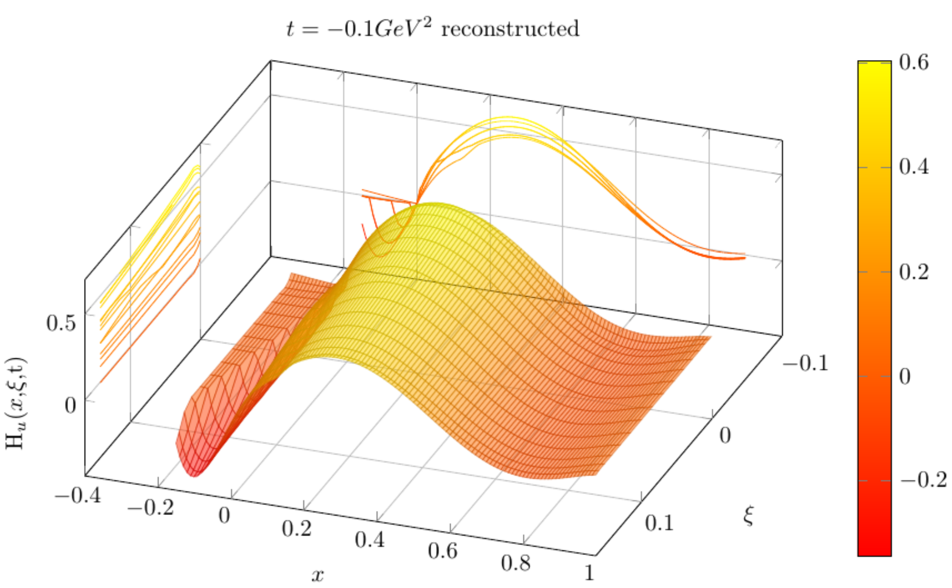
$$\vdots$$

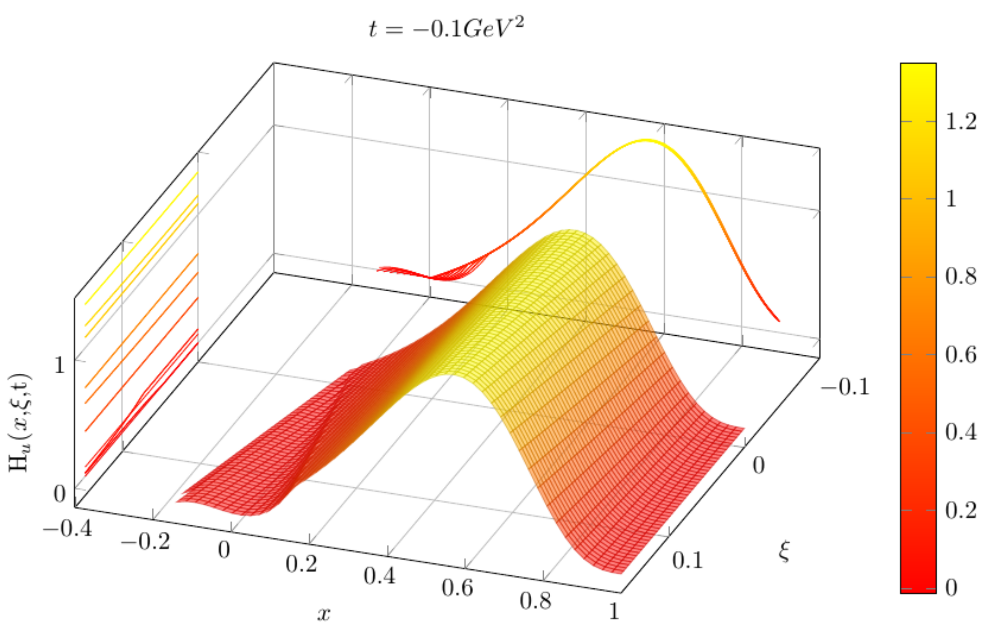
Generalized form factors are a function of t.

LHPC (Ph. Hagler et al.), 2008

Detmold and Shanahan, 2018

ETMC (C. Alexandrou et al.), 2019





Pseudo PDFs in a phenomenological model

- Ab initio calculations on the lattice are hindered by its inablity to go on the lightcone

- Motivated the field of 'off the lightcone' objects such as quasi PDFs and pseduo PDFs that approach actual PDFs in a certain limit

X Ji (2013, 2014)
Lin et al (2014)
Alexandrou et al (2014)
...

Ioffe Time Distributions

- Due to invariance under Lorentz transformations, the matrix element depends on two scalars

$$\mathcal{M}\left((pz), z^2\right) = \langle p|\bar{\psi}(0)\gamma^+\psi(z)|p\rangle$$

Ioffe time

Fourier Transform

$$\int_{-1}^{1} dx e^{-ix(pz)} \mathcal{P}(x, z^2) = \mathcal{M}\left((pz), z^2\right)$$

On the light cone

$$\mathcal{P}(x, 0) = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-ix\nu}\mathcal{M}(\nu, 0)$$

Pseudo PDFs

- Pseudo PDFs generalize the lightcone PDFs onto space like intervals $z = (0,0,0,z_3)$

Radyushkin, Phys Rev D 96 (2017)

Orginos et al, Phys Rev D 96 (2017)

$$\mathcal{M}(\nu, z_3^2) \longrightarrow \mathcal{P}(x, z_3^2)$$

- Reduce z_3^2 dependence by taking ratios $\dfrac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)}$

- By rotational invariance, one can equivalently take an interval of the form $z = (0, z_1, z_2, 0)$

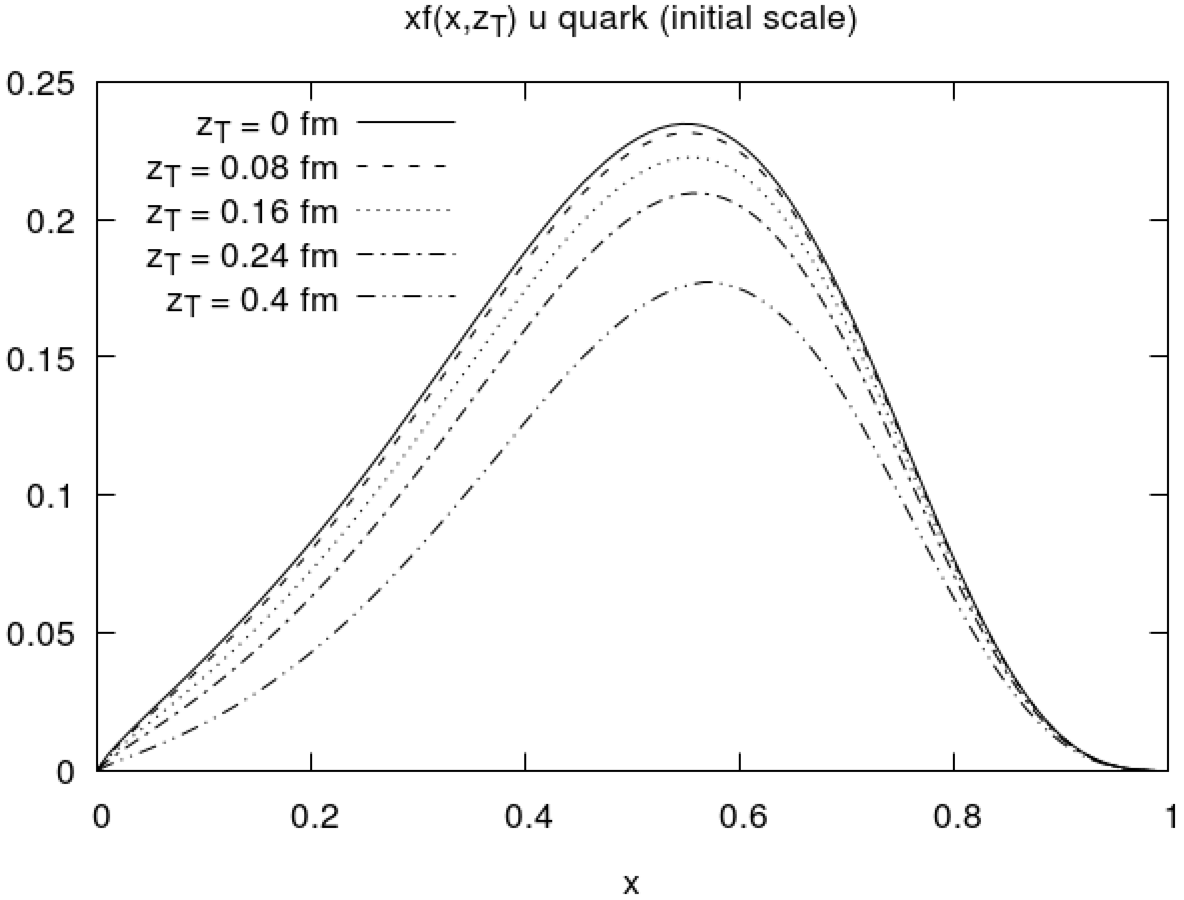
$$\mathcal{M}(\nu, z_T^2) \longrightarrow \mathcal{P}(x, z_T^2)$$

Pseudo PDFs in a diquark model

$$f(x, k_T^2) = \frac{(m + xM)^2 + k_T^2}{(k^2 - M_\Lambda^2)^4}$$

2D Fourier transform

$$\tilde{f}(x, z_T)$$



Extension to GPDs in progress!

PUBLICATIONS AND PREPRINTS

- Y. Hatta, A. Rajan and D. L. Yang, *“Near Threshold J/ψ and ϒ Photoproduction at JLab and RHIC”,* Phys.Rev. D**100** (2019) no.1, 014032
- *Y. Hatta, A. Rajan and K. Tanaka, “Quark and Gluon Contributions to the QCD Trace Anomaly”,JHEP 1812 (2018) 008*
- *A. Rajan, T. Gorda, S. Liuti and K. Yagi, “Bounds on the Equation of State of Neutron Stars from High Energy Deeply Virtual Exclusive Experiments” arxiv:1812.01479*

INVITED TALKS AND SEMINARS

- Unraveling the 3D structure of the proton, Triangle Nuclear Theory (TNT) Colloquium, Duke University, Durham, October 2019
- Ioffe Time Behavior of Generalized Parton Distributions and Parton Distribution Functions, 2nd Workshop on Parton Distibutions and Lattice Calculations (PDFLattice 2019), Michigan State University, September 2019
- Generalized Wandzura Wilczek Relations and Partonic Orbital Angular Momentum, QCD Evolution 2019, Argonne National Laboratory, May 2019
- Generalized Parton Distributions and Partonic Contributions to the QCD Energy Momentum Tensor, Theory Seminar, Jefferson Laboratory, March 2019

SUBMITTED TALKS AND SEMINARS

- Partonic orbital angular momentum and contributions to the trace anomaly, APS Division of Nuclear Physics Meeting, Arlington, October 2019
- Quark gluon interactions and partonic orbital angular momentum, 9th International Conference on Physics Opportunities at an ElecTron-Ion-Collider (POETIC 9), Lawrence Berkeley Laboratory, September 2019
- Quark and Gluon Contributions to the Proton Mass and Spin, 8th Workshop of the American Physical Society Topical Group on Hadronic Physics (APS GHP), Denver, April 2019

Summary and Outlook

- Gluons play a key role in the mass and pressure make up of the proton – even neutron stars!

- Large Ioffe time behavior extraction using models complements studies of x dependence of PDFs and GPDs on the lattice.

- Avenues for future explorations: nuclear GPDs for equation of state of neutron stars, scale dependence of gravitational form factors, extend Ioffe time and pseudo PDF studies on proton to the pion.