Center for Frontiers in Nuclear Science

QCD Jet and TMD Resummation

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CFNS-UCLA Fellow since Sep 2019

Collaborate with Prof. Zhongbo Kang

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Brief CV

PhD: Peking University, Beijing, 2009-2014

PD: University of Bern, Bern, 2014-2017

PD: CERN, Theory Department, Geneva, 2017-2019

PD: UCLA, Los Angeles, 2019-now

Research Interests

QCD factorization and resummation

EFT: Soft-Collinear Effective Theory

Collider Phenomenology (Higgs, Jet, Top quark, …)

Precision calculation

Non-global logs: Interjet Energy Flow

Single log observable ($\alpha_s^n L^n$)

$$\Sigma_\Omega(Q_\Omega, Q) = \frac{1}{\sigma}\int_0^{Q_\Omega} dE_t \,\frac{d\sigma}{dE_t} \qquad (1)$$

Two sources of logarithms: $E_t =$ **Energy in Ω**

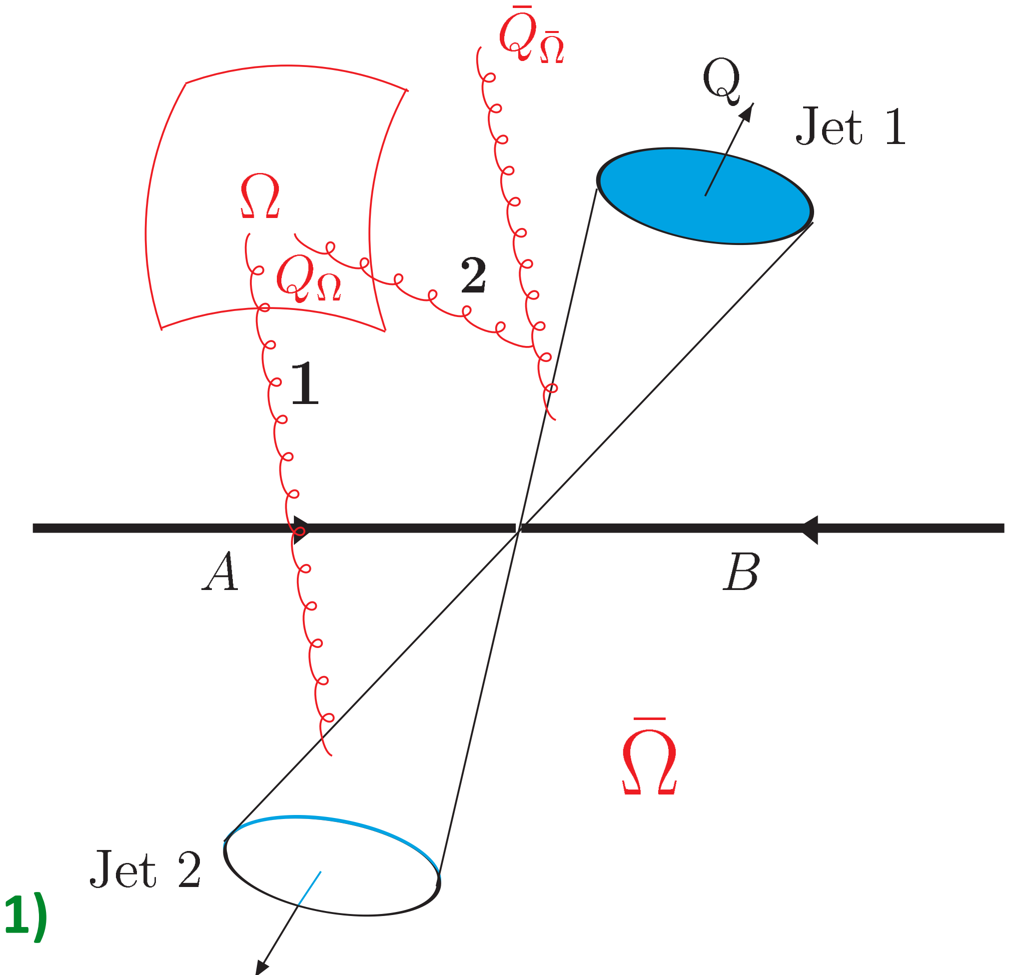

Fig from Sterman `04

1. From primary emissions

$$\alpha_s \int_{Q_\Omega}^{Q} \frac{d\omega_1}{\omega_1} \sim \alpha_s \ln\frac{Q}{Q_\Omega} \qquad (2)$$

2. From secondary emissions (Dasgupta & Salam '01)

$$\alpha_s^2 \int_{Q_\Omega}^{Q} \frac{d\omega_2}{\omega_2} \int_{\omega_2}^{Q} \frac{d\omega_1}{\omega_1} \sim \alpha_s^2 \ln^2 \frac{Q}{Q_\Omega} \qquad (3)$$

Non-global logs: Interjet Energy Flow

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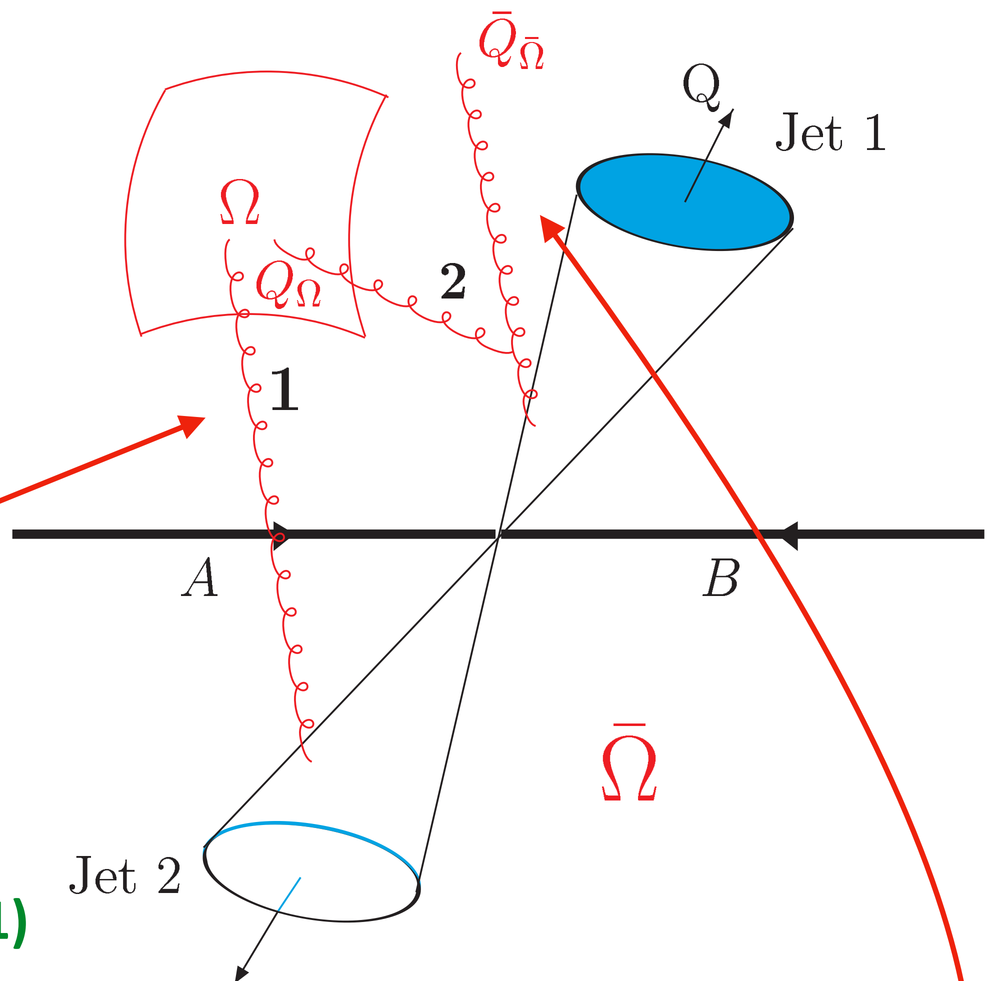

Fig from Sterman `04

- **for a): Number of jets not fixed: nonlinear evolution** (Banfi, Marchesini, Smye (2002)) LL in E/Q, large-N_c, **approximate evolution equation for distribution Σ is non-linear!**

- **Origin of the nonlinearity**
  - ∂_E requires a "hard" gluon k.
  - New hard gluon acts as new, recoil-less source.
  - Large-N limit: $\bar{q}(a)G(k)q(b)$ sources $\rightarrow \bar{q}(a)q(k) \oplus \bar{q}(k)q(a)$.
  - "Global" event shape eliminates extra hard gluon.
  - But fixing an event shape limits the number of events.
  - We are far from a full understanding.

G. Sterman, CTEQ school 2006

An Effective Field Theory for non-global jet processes

Becher, Neubert, Rothen, DYS ’15 PRL

The first all-order factorization and resummation formula:

$$\sigma(Q, Q_\Omega) \sim \sum_{m=2}^{\infty} \prod_{i=1}^{m} \int \frac{d\Omega(\vec{n}_i)}{4\pi} \mathrm{Tr}_c \left[\mathcal{H}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q, \mu) \mathcal{S}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q_\Omega, \mu) \right]$$

Soft particles can resolve individual hard partons, leading to a multi-Wilson-line structure

At the LL level, our evolution equation can reduce to the nonlinear BMS eq

Not restricted to LL, beyond LL resummation see Balsiger, Becher & DYS ’19

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Color Trace Hard scale Soft scale

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\# of jet not fixed

Integrate the angles for hard partons

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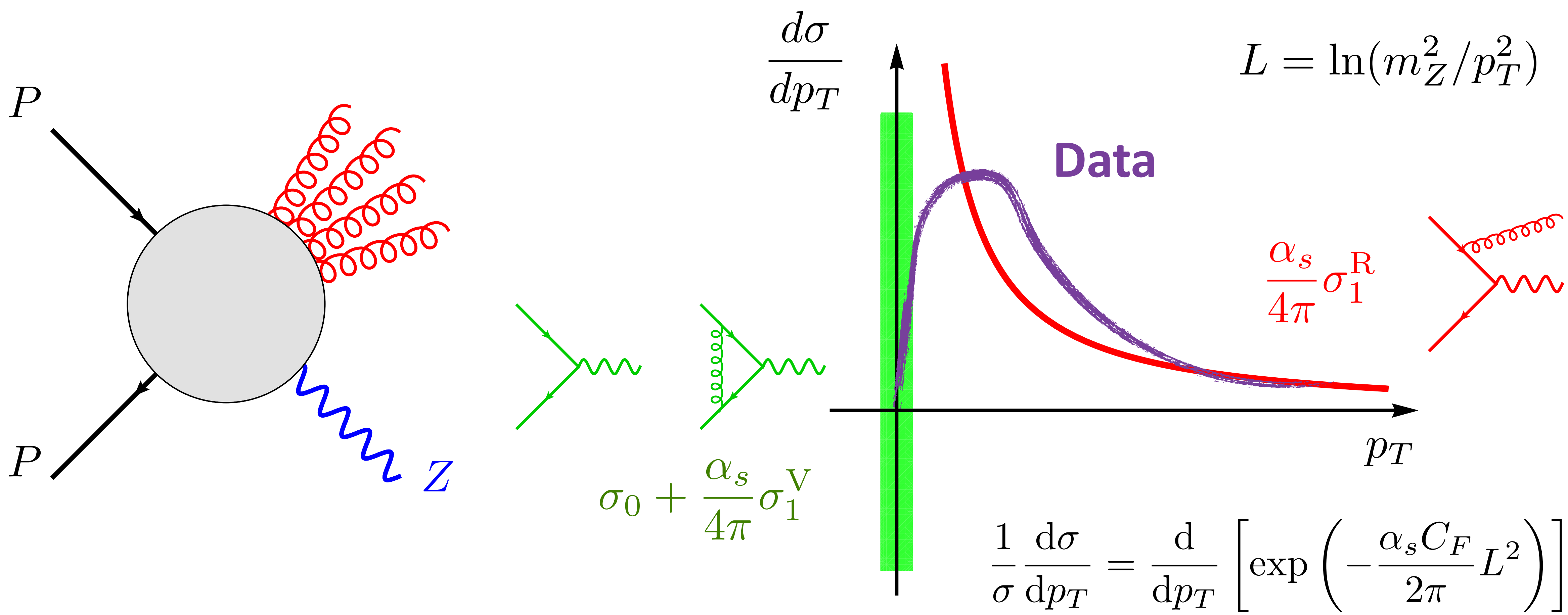
TMD factorization and resummation



- **Drell-Yan type processes: (Z, W, Higgs, VV, …)**
  - **the all order structure is first understood by CSS at 1985**
  - **computer codes: CUTE, DYRES/DYTurbo, MATRIX, NangaParbat, RADISH, RESBOS, reSolve, SCETlib ……**
  - **N³LL for single-boson processes (CUTE, RADISH, SCETlib,…)**

TMD factorization is violated in di-jet/di-hadron production

Collins, Qiu `07; Collins `07, Vogelsang, Yuan `07; Rogers, Mulders `10, …

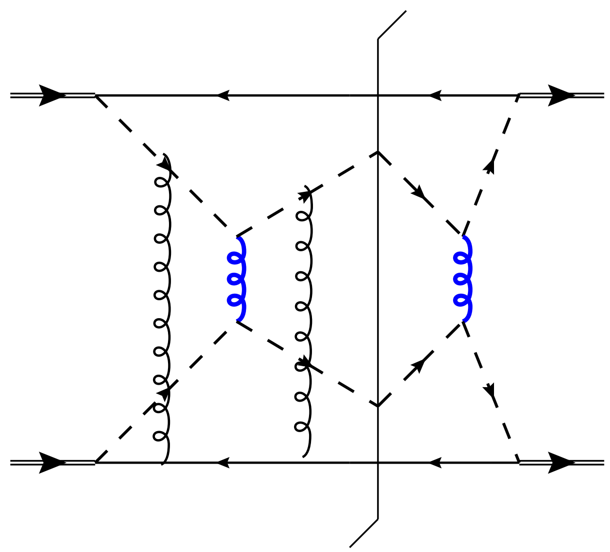


FIG. 8 (color online). The exchange of two extra gluons, as in this graph, will tend to give nonfactorization in unpolarized cross sections.

We remark that, because the TMD factorization breaking effects are due to the Glauber region where all components of gluon momentum are small, the interactions responsible for breaking TMD factorization are associated with large distance scales.

Rogers, Mulders `10

- **The first step: study TMD factorization without Glauber region**
- **Tools: Soft-Collinear Effective Theory** (Bauer, Pirjol, Stewart *et.al.* '01, '02)
  - **Assign scaling behavior to fields**
  - **Expand Lagrangian to leading power**
  - **Resummation with Renormalization Group**

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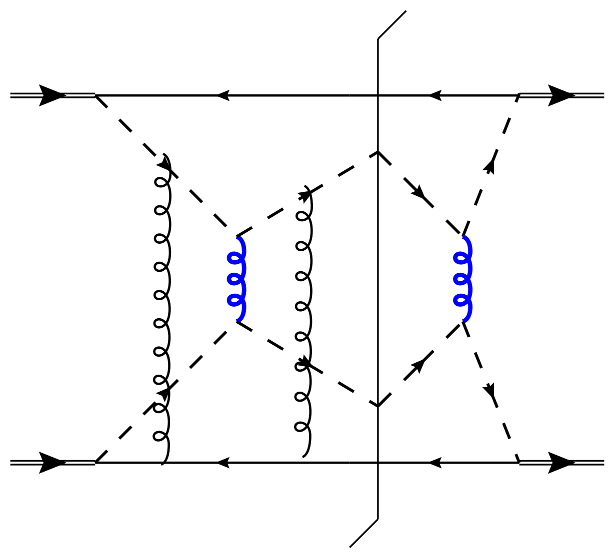

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  - **Assign scaling behavior to fields** $\mathcal{L}_{\text{Collinear}} + \mathcal{L}_{\text{Soft}} + \cdots + \cancel{\mathcal{L}_{\text{Glauber}}}$
  - **Expand Lagrangian to leading power**
  - **Resummation with Renormalization Group**

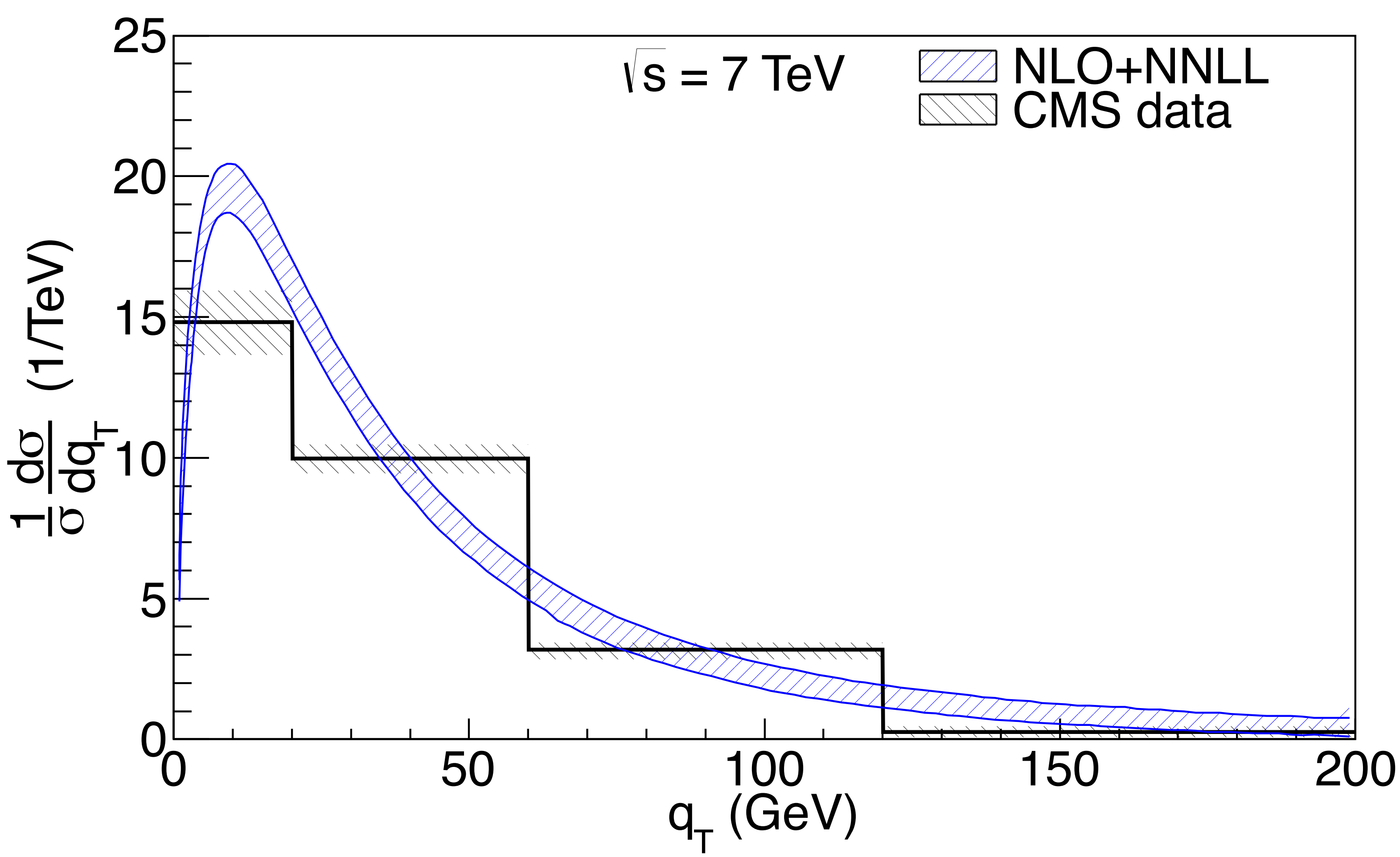
TMD resummation for top quark pair production: SCET + HQET

(Li, Li, DYS, Yang, Zhu, '13 PRL)

- **NNLL predictions for top quark pair production in the small transverse momentum region.**

$$\frac{d^4\sigma}{dq_T^2 dy dM d\cos\theta} = \sum_{i=q,\bar{q},g} \frac{8\pi\beta_t}{3sM} \frac{1}{2} \int x_T dx_T J_0(x_T q_T) \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{i\bar{i}}(x_T^2,\mu)} B_{i/N_1}(\xi_1, x_T^2, \mu) B_{\bar{i}/N_2}(\xi_2, x_T^2, \mu)$$

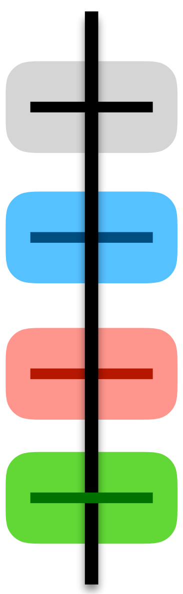
$$\times \operatorname{Tr}[\boldsymbol{H}_{i\bar{i}}(M, m_t, \cos\theta, \mu) \boldsymbol{S}_{i\bar{i}}(L_\perp, M, m_t, \cos\theta, \mu)].$$



Jet radius and TMD resummation for boson-jet correlation

(Chien, DYS & Wu ’19)

$$N_1(P_1) + N_2(P_2) \to \underbrace{\text{boson}(p_V) + \text{jet}(p_J)}_{q_T} + X$$



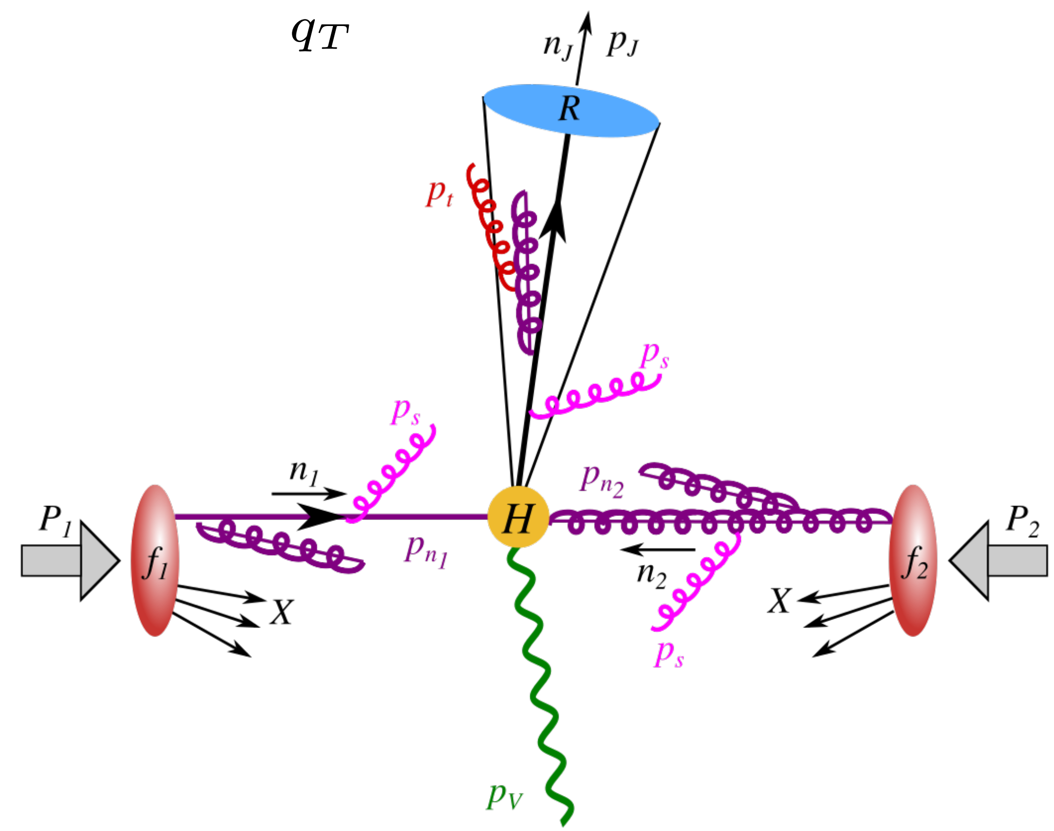
$p_h \sim Q(1,1,1)$

$p_{n_J} \sim p_T^J\,(R^2, 1, R)_{n_J \bar{n}_J}$

$p_{n_1} \sim (q_T^2/Q, Q, q_T)_{n_1 \bar{n}_1}$

$p_s \sim (q_T, q_T, q_T)$

$p_t \sim q_T\,(R^2, 1, R)_{n_J \bar{n}_J}$



- **Collinear-Soft (Coft) modes:** $p_t^\mu \sim q_T (R^2, 1, R)_{n_J \bar{n}_J}$ **for the jet radius R resummation** (Becher, Neubert, Rothen & DYS ’15; Chien, Hornig & Lee ’15; Kolodrubetz, Pietrulewicz, Stewart, Tackmann & Waalewijn ’16; Buffing, Kang, Lee & Liu `18; ……)
- **Multi-Wilson-Line operators describe radiations along the jet direction for NGLs resummation** (Caron-Hout ’15; Becher, Neubert, Rothen & DYS ’15; ……)

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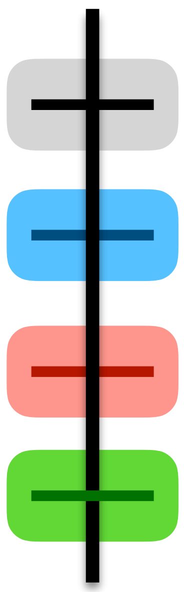

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$p_{n_1} \sim (q_T^2/Q, Q, q_T)_{n_1\bar{n}_1}$

$p_s \sim (q_T, q_T, q_T)$

$p_t \sim q_T\,(R^2,1,R)_{n_J\bar{n}_J}$

Factorization formula (neglecting glauber modes):

$$\frac{d\sigma}{d^2q_T d^2p_T d\eta_J dy_V} = \sum_{ijk}\int \frac{d^2x_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{x}_T} \mathcal{S}_{ij\to Vk}(\vec{x}_T,\epsilon)\mathcal{B}_{i/N_1}(\xi_1,x_T,\epsilon)\mathcal{B}_{j/N_2}(\xi_2,x_T,\epsilon)$$
$$\times \mathcal{H}_{ij\to Vk}(\hat{s},\hat{t},m_V,\epsilon)\sum_{m=1}^{\infty}\langle \boldsymbol{\mathcal{J}}_m^k(\{\underline{n_J}\}, R\,p_J,\epsilon)\otimes \boldsymbol{\mathcal{U}}_m^k(\{\underline{n_J}\}, R\,\vec{x}_T,\epsilon)\rangle$$

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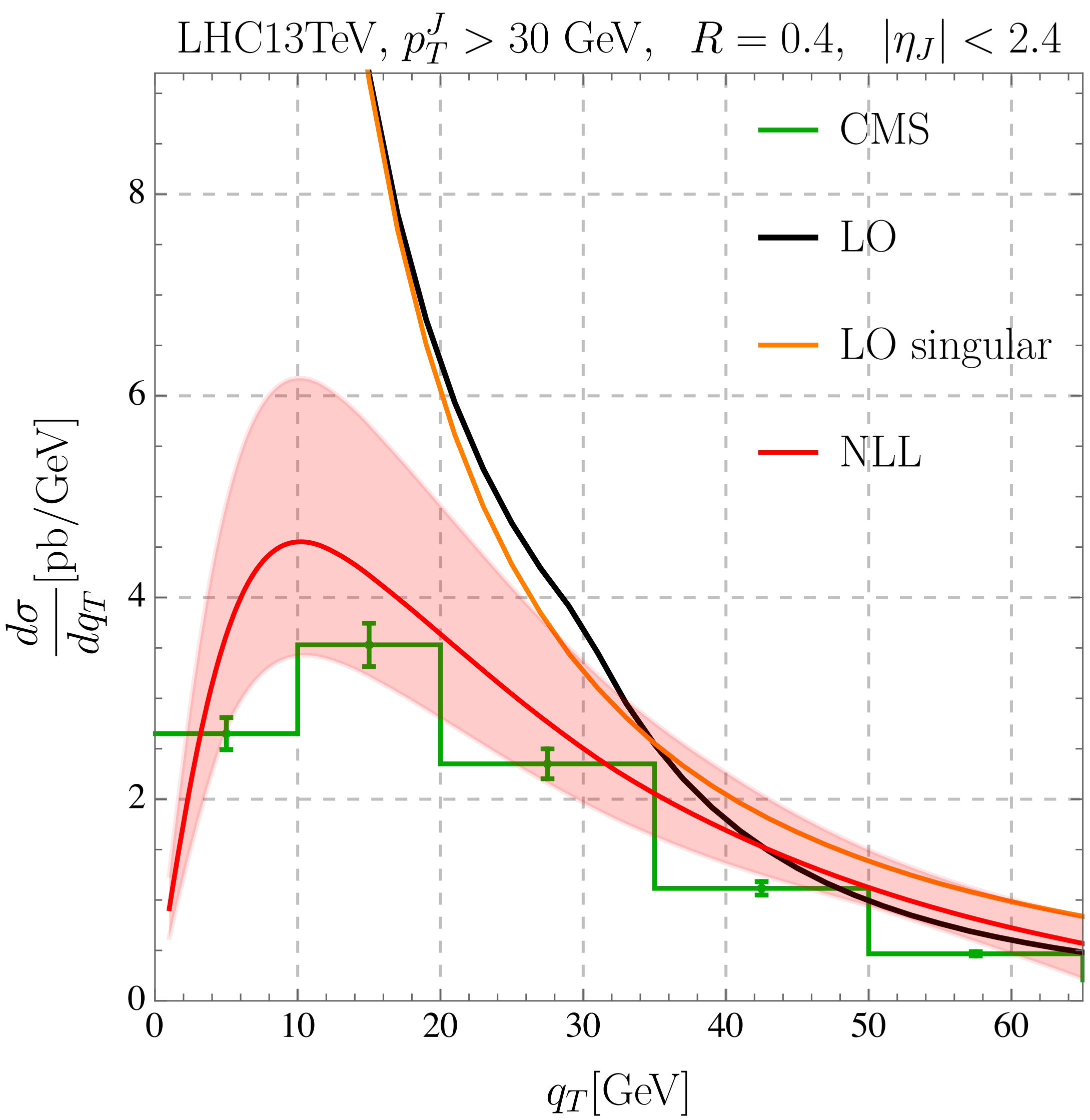
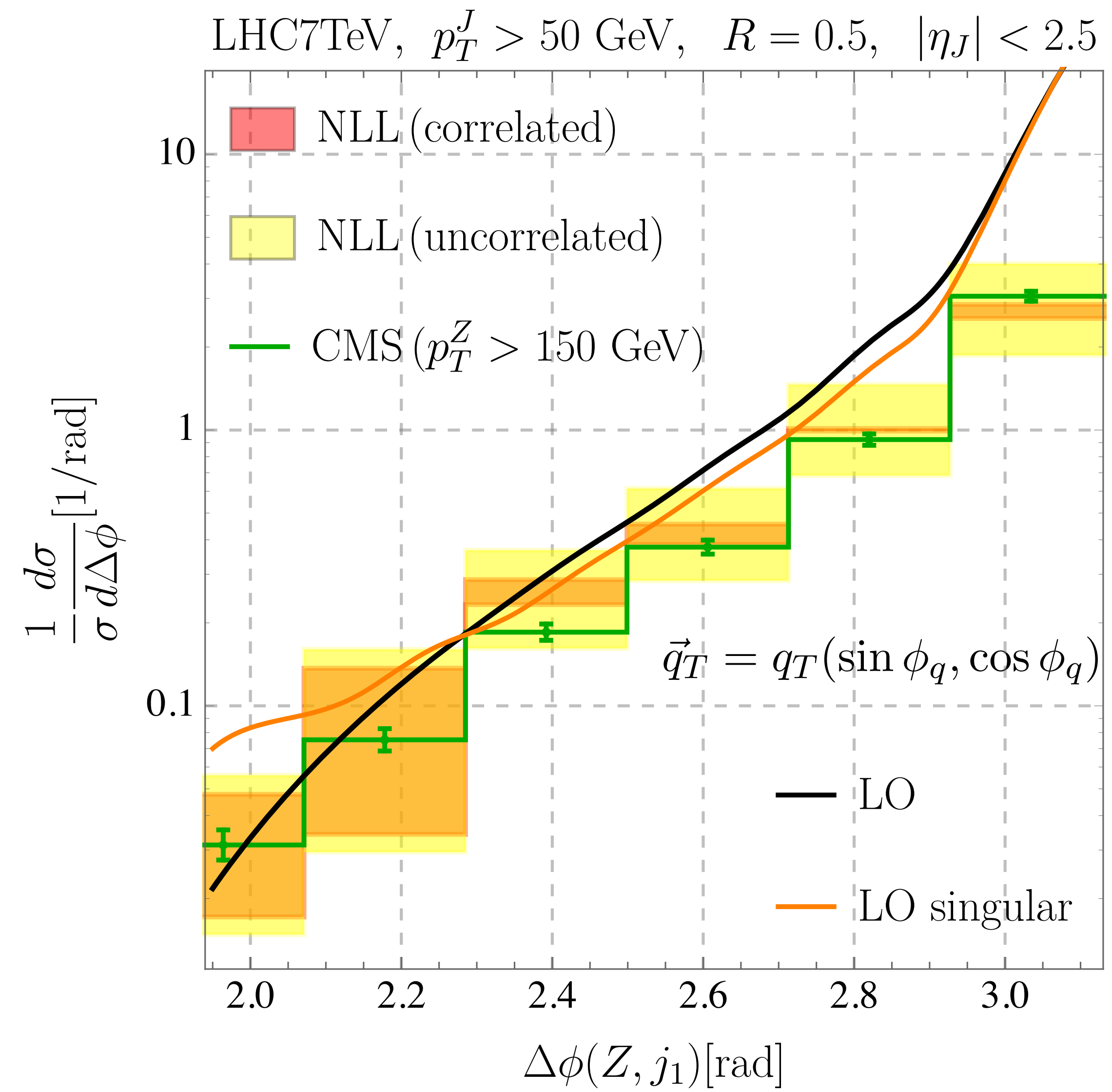
Numerical results





- **NLL resummation is consistent with the LHC data (q_T & ΔΦ)**
- ***TMD factorization violation* effects are suppressed at the LHC.**
  - **e.g. perturbative logs from Glauber regions beyond NNLO**

TMD resummation and Gluon Sivers function at the EIC

- **E.g. heavy quark pair production** $ep^{\uparrow} \to e' c\bar{c} X$
- **Open charm production is an ideal probe to tag the Photon-Gluon-Fusion processes, can be used to probe spin structures for the gluon TMD (Boer *et al.* '11; Burton `12; Zheng, Aschenauer, Lee, Xiao, Yin `18, …)**

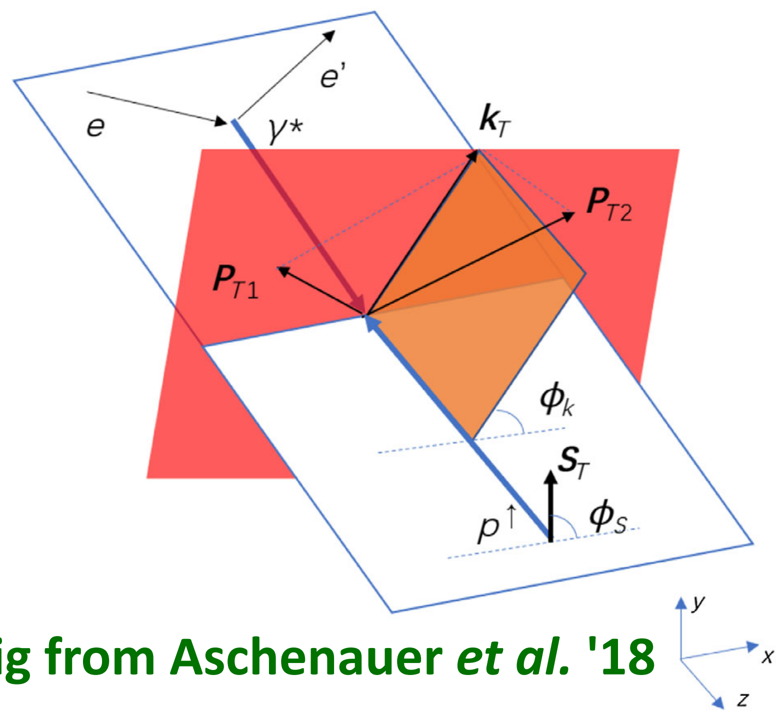


Fig from Aschenauer *et al.* '18

- **In the small k_T limit (Kang, Lee, DYS in progress)**

$$\vec{k}_\perp = \vec{k}_{c\perp} + \vec{k}_{s\perp}$$

Collinear: $k_{c\perp} \sim (k_\perp^2/Q, Q, k_\perp)$

Soft: $k_{s\perp} \sim (k_\perp, k_\perp, k_\perp)$

$$d\sigma_{eP\to eQQX} \sim \int \prod_i^2 d^2 k_{i\perp} H^{eg\to eQ\bar{Q}}\left(Q^2\right) \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} - \vec{k}_\perp) f_g\left(x_1, k_{1\perp}\right) S_{nn_Q n_{\bar{Q}}}\left(k_{2\perp}\right)$$

Research plans in the next year

- **Jet physics at the EIC: jet spectrum, Jet substructure and heavy flavor jet**

- **Global event shapes at the EIC: Non-perturbative power corrections**

Recent Publications

- *“Resummation of Boson-Jet Correlation at Hadron Colliders”* Chien, DYS & Wu JHEP1911(2019)025
- *“Momentum-space Threshold resummation in tW production at the LHC”* Li, Li, DYS, Wang JHEP1906(2019)125
- *“NLL` resummation of jet mass” Balsiger, Becher & DYS* JHEP1904(2019)020

Recent Talks

- “Jet TMD and Non-global logs” Parton Shower & Resummation 2019, Vienna, June 2019
- “Overview of state-of-the-art resummation techniques in jet physics” JetTools 2019, Bergen, May 2019
- “Soft gluon evolution at the amplitude level” Circular Electron-Positron Collider workshop, Oxford, April 2019
- “Soft gluon evolution beyond leading order” Soft-Collinear Effective Theory 2019, San Diego, March 2019

THANK YOU

RG evolution and resummation

- **Resummation formula:**

$$\frac{d\sigma}{d^2q_T d^2p_T d\eta_J dy_V} = \sum_{ijk}\int \frac{d^2x_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{x}_T} e^{\int_{\mu_h}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\Gamma^{\mathrm{H}_{ij\to Vk}}(\bar{\mu})}\mathcal{H}_{ij\to Vk}(\hat{s},\hat{t},m_V,\mu_h)$$
$$\times\left(\frac{x_T^2\,\hat{s}}{b_0^2}\right)^{-(C_i+C_j)F_\perp(\mu)} e^{\int_{\mu_b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\Gamma^{\mathrm{W}_{ij\to Vk}}(\bar{\mu})} S_{ij\to Vk}(\vec{x}_T,\mu_b)B_{i/N_1}(\xi_1,x_T,\mu_b)B_{j/N_2}(\xi_2,x_T,\mu_b)$$
$$\times\, e^{\int_{\mu_t}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\Gamma^{\mathrm{U}_k}(\bar{\mu})+\int_{\mu_j}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\Gamma^{\mathrm{J}_k}(\bar{\mu})}U^k_{\mathrm{NG}}(\mu_t,\mu_j),$$

- **Typical scales:** $\mu_h \sim Q, \qquad \mu_b \sim b_0/x_T, \qquad \mu_j \sim R\,p_T, \qquad \mu_t \sim R\,b_0/x_T,$

- **Non-global logs resummation:** (Becher, Neubert, Rothen & DYS '16 PRL)

$$U_{\mathrm{NG}}(\mu_t,\mu_j) \equiv \sum_{l=1}^{\infty}\left\langle \boldsymbol{\mathcal{J}}_l(\{\underline{n}'\},R\,p_T,\mu_j)\otimes\sum_{m\geq l}^{\infty}\boldsymbol{U}_{lm}(\{\underline{n}\},\mu_t,\mu_j)\,\hat{\otimes}\,\boldsymbol{\mathcal{U}}_m(\{\underline{n}\},R\,\vec{x}_T,\mu_t)\right\rangle$$

RG evolution and resummation

- **Resummation formula:** Logs from different scales

$$\frac{d\sigma}{d^2q_T d^2p_T d\eta_J dy_V} = \sum_{ijk}\int \frac{d^2x_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{x}_T} \boxed{e^{\int_{\mu_h}^{\mu}\frac{d\bar\mu}{\bar\mu}\Gamma^{\mathrm{H}_{ij\to Vk}}(\bar\mu)}} \mathcal{H}_{ij\to Vk}(\hat s,\hat t, m_V,\mu_h)$$
$$\times\left(\frac{x_T^2\,\hat s}{b_0^2}\right)^{-(C_i+C_j)F_\perp(\mu)} \boxed{e^{\int_{\mu_b}^{\mu}\frac{d\bar\mu}{\bar\mu}\Gamma^{\mathrm{W}_{ij\to Vk}}(\bar\mu)}} S_{ij\to Vk}(\vec{x}_T,\mu_b) B_{i/N_1}(\xi_1, x_T,\mu_b) B_{j/N_2}(\xi_2,x_T,\mu_b)$$
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Rapidity logs resummation

$$\times \boxed{\left(\frac{x_T^2\,\hat{s}}{b_0^2}\right)^{-(C_i+C_j)F_\perp(\mu)}} \boxed{e^{\int_{\mu_b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\Gamma^{\mathrm{W}_{ij\to Vk}}(\bar{\mu})}} S_{ij\to Vk}(\vec{x}_T,\mu_b) B_{i/N_1}(\xi_1,x_T,\mu_b) B_{j/N_2}(\xi_2,x_T,\mu_b)$$

$$\times \boxed{e^{\int_{\mu_t}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\Gamma^{\mathrm{U}_k}(\bar{\mu}) + \int_{\mu_j}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\Gamma^{\mathrm{J}_k}(\bar{\mu})}} U_{\mathrm{NG}}^{k}(\mu_t,\mu_j),$$

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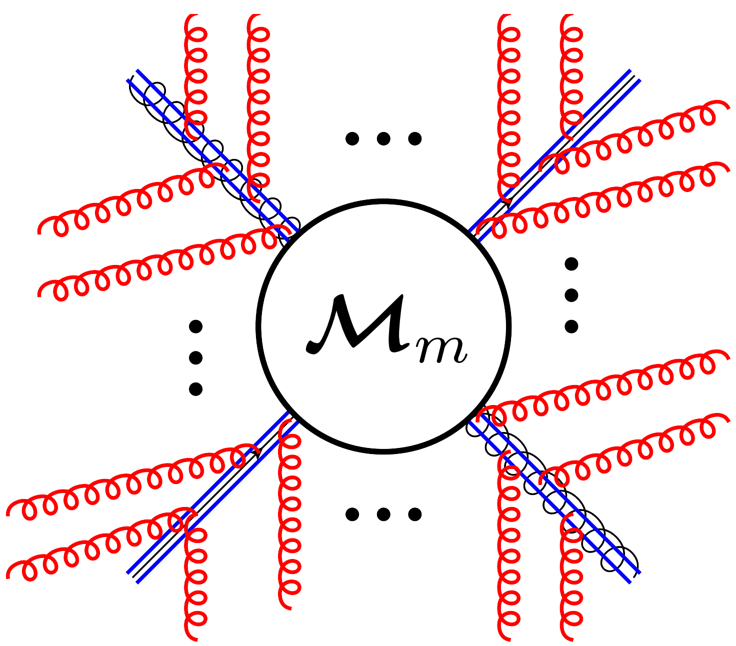
Factorization

(Becher, Neubert, Rothen & DYS `15)

- The operator for soft emissions from an amplitude with m hard partons



hard scattering amplitude with m particles (vector in color space)

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\,\ldots\,\boldsymbol{S}_m(n_m)\,|\mathcal{M}_m(\{\underline{p}\})\rangle$$

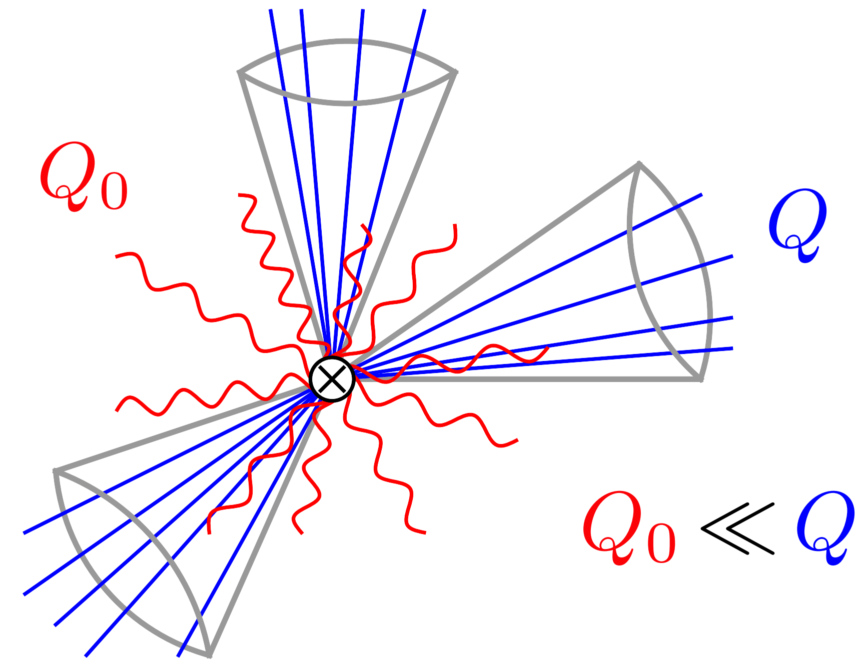
soft Wilson lines along the directions of the energetic particles (color matrices)

$$\boldsymbol{S}_i(n_i) = \mathbf{P}\exp\left(ig_s\int_0^\infty ds\, n_i\cdot A_s^a(sn_i)\,\boldsymbol{T}_i^a\right)$$

Factorization & Multi-Wilson line structures

(Caron-Huot '15 & Becher, Neubert, Rothen & DYS '15 PRL)

- **For k jets process at lepton collider**



$$d\sigma(Q, Q_0) = \sum_{m=k}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

$$\boxed{\{\underline{n}\} = \{n_1, n_2, \cdots, n_m\}}$$

- **Soft function:**

$$\boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu) = \sum\!\!\!\!\!\!\!\int_{X_s} \langle 0 | \, \boldsymbol{S}_1^\dagger(n_1) \, \ldots \, \boldsymbol{S}_m^\dagger(n_m) \, | X_s \rangle \langle X_s | \, \boldsymbol{S}_1(n_1) \, \ldots \, \boldsymbol{S}_m(n_m) \, | 0 \rangle \, \theta(Q_0 - E_{\text{out}})$$

- **Hard function: integrating over the energies of the hard particles, while keeping their direction fixed**

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) = \frac{1}{2Q^2} \sum_{\text{spins}} \prod_{i=1}^{m} \int \frac{dE_i \, E_i^{d-3}}{(2\pi)^{d-2}} \, |\mathcal{M}_m(\{\underline{p}\})\rangle \langle \mathcal{M}_m(\{\underline{p}\})| \, (2\pi)^d \, \delta\Big(Q - \sum_{i=1}^{m} E_i\Big) \, \delta^{(d-1)}(\vec{p}_{\text{tot}}) \, \Theta_{\text{in}}\big(\{\underline{p}\}\big)$$

- **$\otimes$ indicates integration over the direction of the energetic partons**

- **$\langle \cdots \rangle$ takes the color trace**

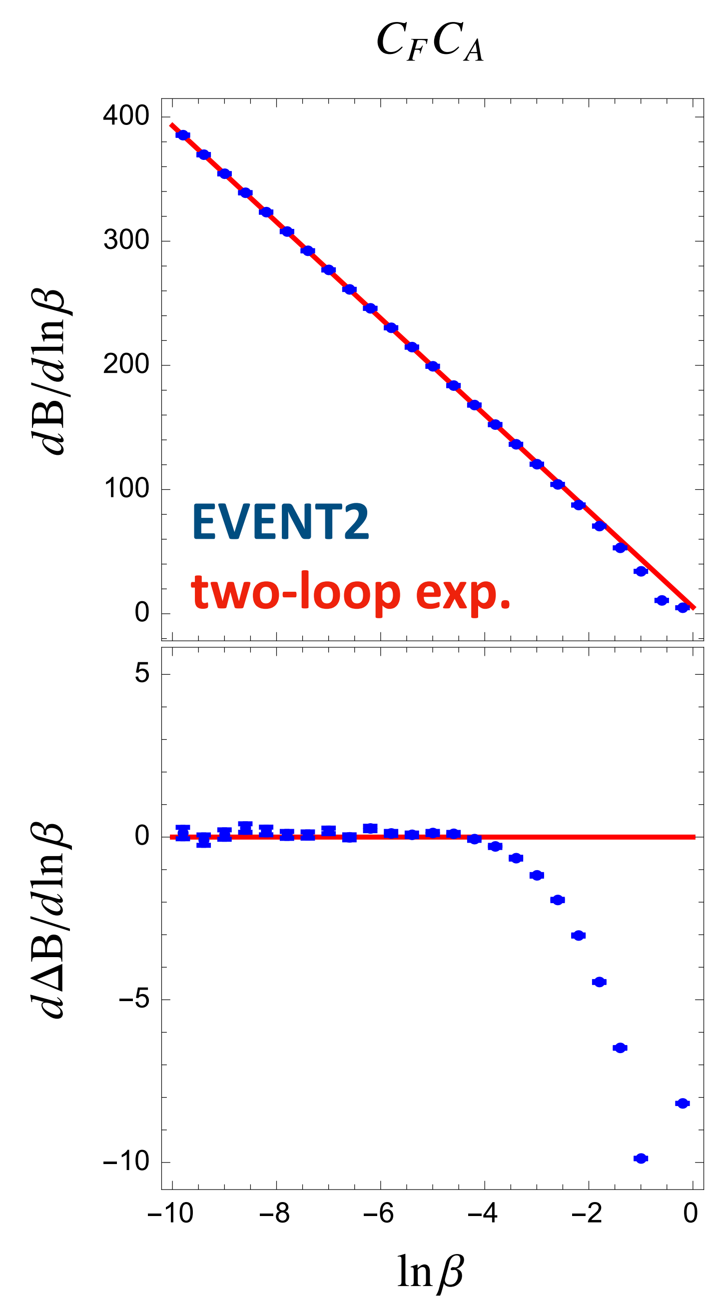
NNLO consistency check

(Becher, Neubert, Rothen & DYS '15)

$$\frac{\sigma(Q,Q_0)}{\sigma_0} = 1 + \frac{\alpha_s}{2\pi}A + \left(\frac{\alpha_s}{2\pi}\right)^2 \left(C_F^2 B_F + C_F C_A B_A + C_F T_F n_f B_f\right)$$

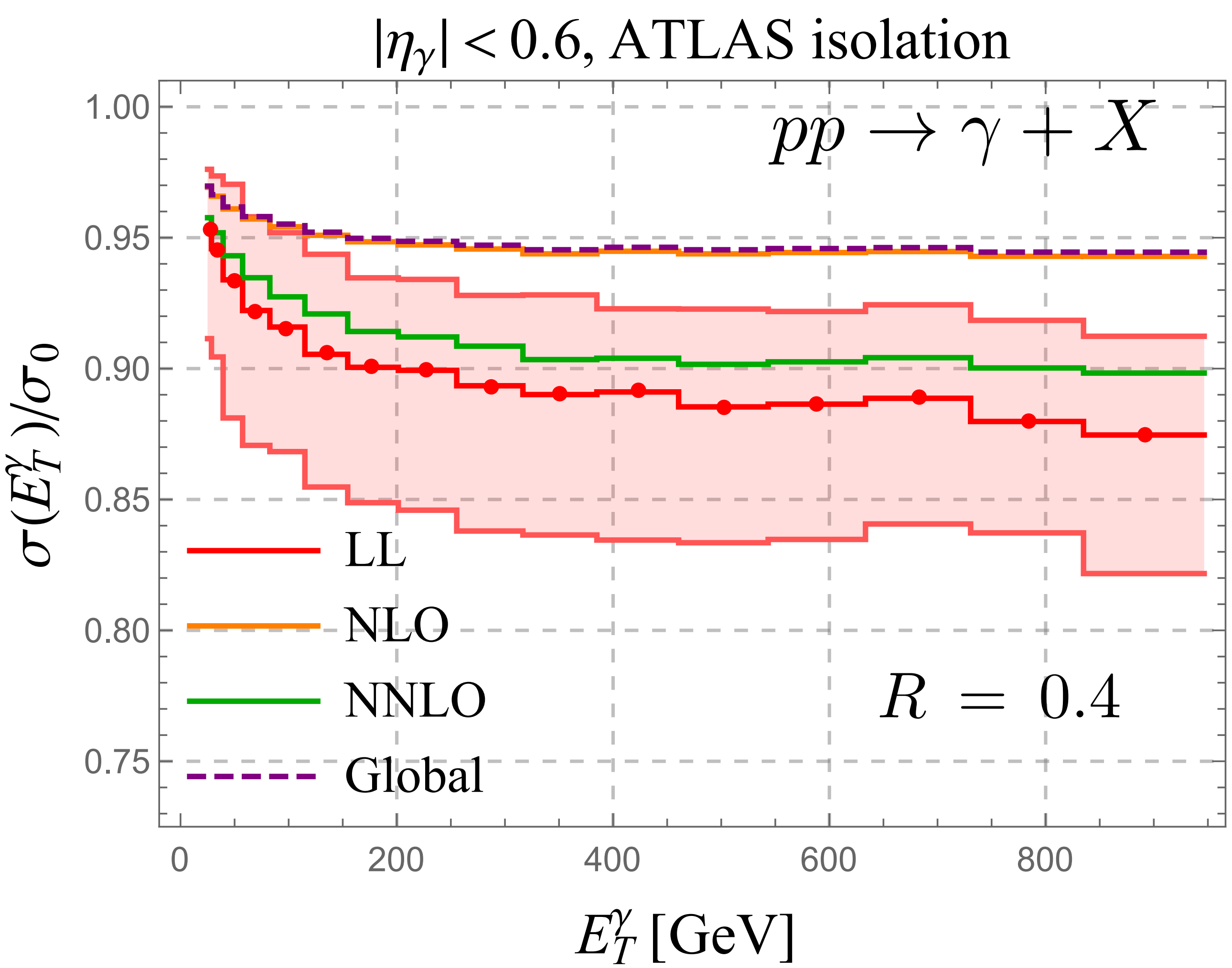
$$\begin{aligned}
B_A = {} & \frac{4}{3}\left[11\ln\delta - \frac{\pi^2}{2} + 3\,\mathrm{Li}_2(\delta^4)\right]\ln^2\beta + \frac{4}{3}\Bigg[11\ln^2\delta - \frac{67\ln\delta}{3} + \frac{4\delta^4\ln\delta}{(1-\delta^4)^2} + \frac{1}{1-\delta^4} \\
& + 36\ln\delta\ln^2\left(1-\delta^2\right) - 12\ln\delta\ln^2\left(1+\delta^2\right) + 22\ln\delta\ln\left(1-\delta^2\right) - 5\pi^2\ln\left(1-\delta^2\right) \\
& + 22\ln\delta\ln\left(1+\delta^2\right) - \pi^2\ln\left(1+\delta^2\right) - 4\ln^3\left(1+\delta^2\right) + 33\,\mathrm{Li}_2\left(-\delta^2\right) + 22\,\mathrm{Li}_2\left(\delta^2\right) \\
& + 48\ln\delta\,\mathrm{Li}_2\left(-\delta^2\right) - 12\ln\left(1-\delta^2\right)\,\mathrm{Li}_2\left(-\delta^2\right) - 36\ln\left(1+\delta^2\right)\,\mathrm{Li}_2\left(-\delta^2\right) \\
& + 12\ln 2\,\mathrm{Li}_2\left(-\delta^2\right) + 24\ln\delta\,\mathrm{Li}_2\left(\delta^2\right) + 24\ln\left(1-\delta^2\right)\,\mathrm{Li}_2\left(\delta^2\right) + 12\ln 2\,\mathrm{Li}_2\left(\delta^2\right) \\
& + 12\ln\left(1-\delta^4\right)\,\mathrm{Li}_2\left(1-\delta^2\right) - 6\,\mathrm{Li}_3\left(1-\delta^4\right) + 24\,\mathrm{Li}_3\left(1-\delta^2\right) - 36\,\mathrm{Li}_3\left(-\delta^2\right) \\
& - 36\,\mathrm{Li}_3\left(\delta^2\right) + 24\,\mathrm{Li}_3\left(\frac{\delta^2}{1+\delta^2}\right) - 12\,\zeta_3 - \frac{11\pi^2}{12} - \frac{1}{2} - \pi^2\ln 2 - \frac{3}{8}M_A^{[1]}(\delta)\Bigg]\ln\beta \\
& + c_2^A(\delta)\,,
\end{aligned}$$

$$Q_0 = Q\beta$$



Resummation effects in γ isolation at the LHC

(Balsiger, Becher, DYS,'18)



$$1 + \#\alpha_s R^2 \ln \epsilon_\gamma$$
$$+ \#\alpha_s^2 R^2 \ln R \ln^2 \epsilon_\gamma$$

- NLO: ~5% reduction, NNLO ~10%, resummed ~ 12%
- NGL dominates over global contribution: naive exponentiation (dashed)
- LL result suffers from large scale uncertainties

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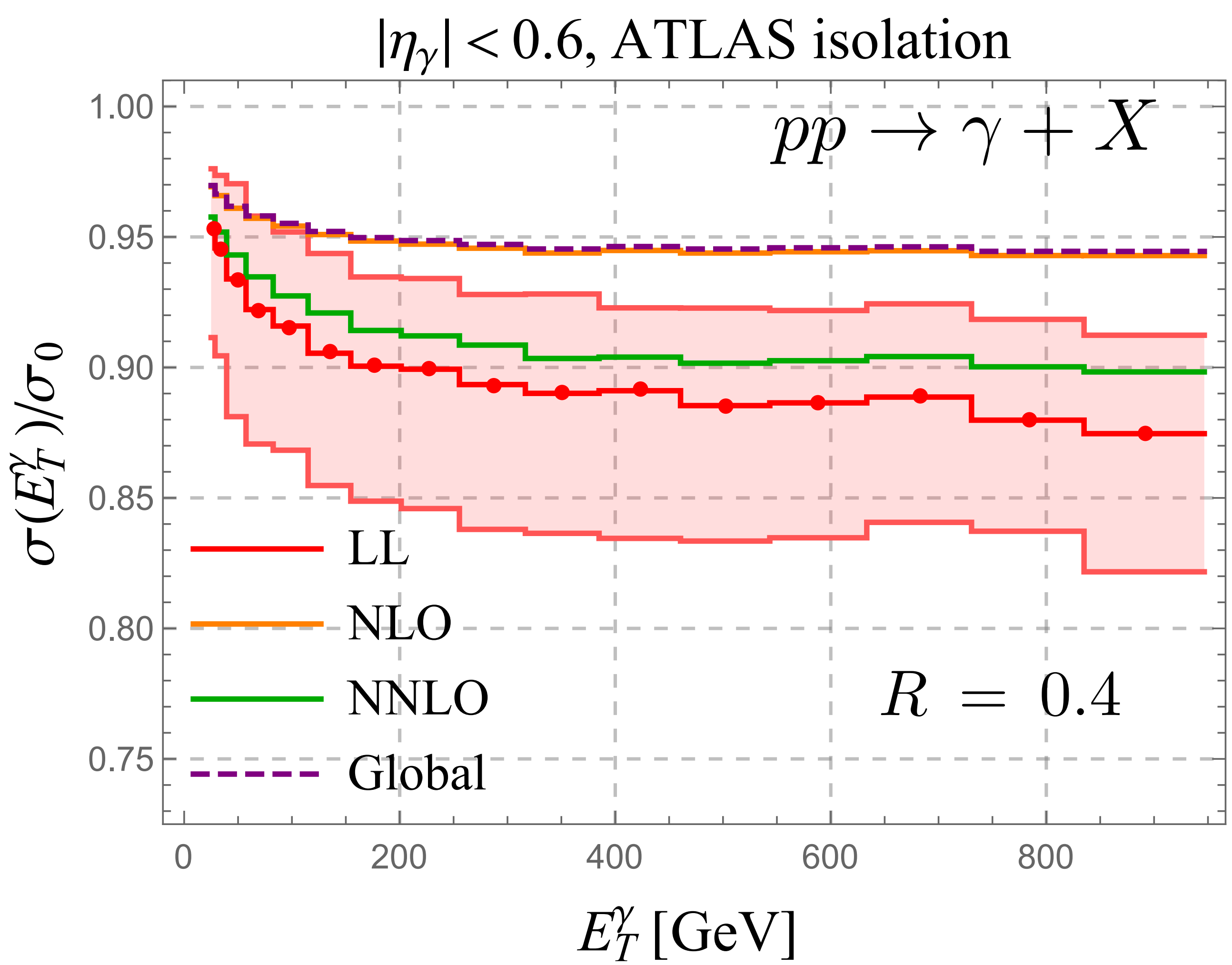

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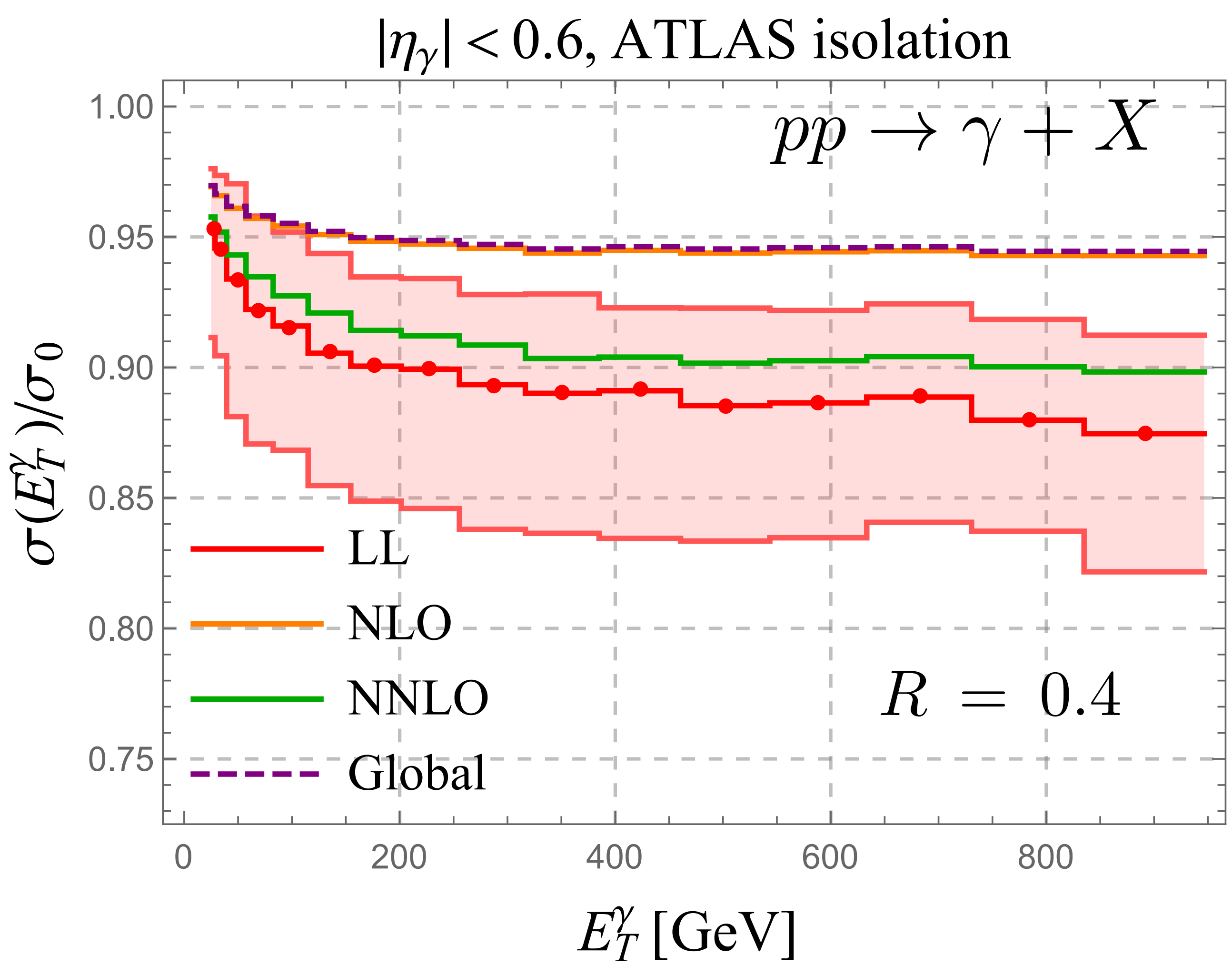

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RG evolution & Resummation

$$d\sigma(Q,Q_0) = \sum_{m=k}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu) \rangle$$

Wilson coefficients fulfill Renormalization Group equation

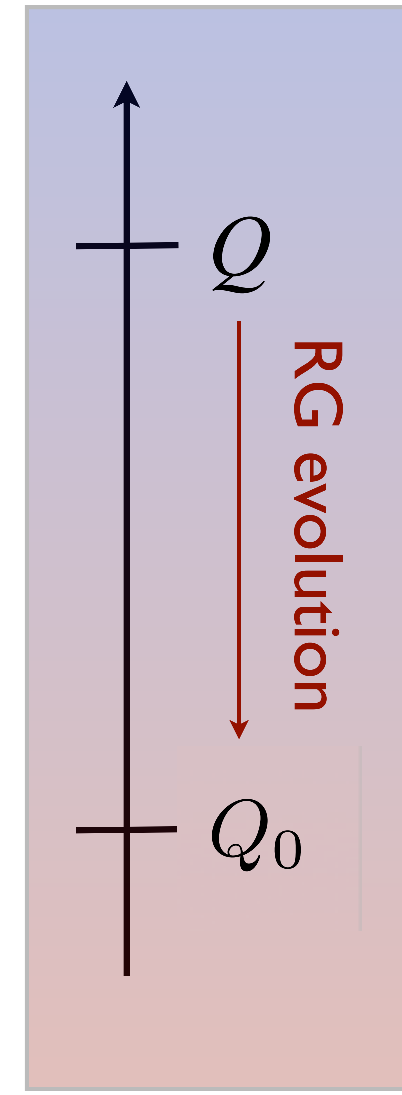
$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=2}^{m} \boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

1. **Compute** $\mathcal{H}_m$ **at characteristic high scale** $\mu_h \sim Q$

2. **Evolve** $\mathcal{H}_m$ **to the scale of low energy physics** $\mu_s \sim Q_0$

3. **Compute** $\mathcal{S}_m$ **at** $\mu_s \sim Q_0$

Resum large logarithms: $\alpha_s^n \ln^m \frac{Q}{Q_0}$



Infinite operators are mixed under RG evolution

- **Analytical method fails**
- **Leading-log RG evolution = parton shower**

Resummation in SCET

Evolving hard function from $\mu_h \sim Q$ **to** $\mu_s \sim Q_0$

$$\sigma(Q, Q_0) = \sum_{l=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_l(\{\underline{n}'\}, Q, \mu_h) \otimes \sum_{m \geq l}^{\infty} \boldsymbol{U}_{lm}(\{\underline{n}\}, \mu_s, \mu_h) \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu_s) \big\rangle$$

Resummation in SCET

Evolving hard function from $\mu_h \sim Q$ **to** $\mu_s \sim Q_0$

$$\boxed{\otimes = \int d\Omega_1 \cdots d\Omega_l} \qquad \boxed{\hat{\otimes} = \int d\Omega_{l+1} \cdots d\Omega_m}$$

$$\sigma(Q, Q_0) = \sum_{l=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_l(\{\underline{n}'\}, Q, \mu_h) \otimes \sum_{m \geq l}^{\infty} \boldsymbol{U}_{lm}(\{\underline{n}\}, \mu_s, \mu_h) \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu_s) \rangle$$

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$$\boldsymbol{U}(\{\underline{n}\}, \mu_s, \mu_h) = \mathbf{P} \exp \left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^H(\{\underline{n}\}, \mu) \right]$$

Resummation in SCET

Evolving hard function from $\mu_h \sim Q$ to $\mu_s \sim Q_0$

$$\otimes = \int d\Omega_1 \cdots d\Omega_l$$

$$\hat{\otimes} = \int d\Omega_{l+1} \cdots d\Omega_m$$

$$\sigma(Q, Q_0) = \sum_{l=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_l(\{\underline{n}'\}, Q, \mu_h) \otimes \sum_{m \geq l}^{\infty} \boldsymbol{U}_{lm}(\{\underline{n}\}, \mu_s, \mu_h) \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu_s) \rangle$$

$$\{\underline{n}'\} = \{n_1, \ldots, n_l\}$$

$$\{\underline{n}\} = \{n_1, \ldots, n_l, n_{l+1}, \ldots, n_m\}$$

$$\boldsymbol{U}(\{\underline{n}\}, \mu_s, \mu_h) = \mathbf{P} \exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^H(\{\underline{n}\}, \mu)\right]$$