Center for Frontiers in Nuclear Science

NLO impact factor for photon+dijet production in e+A DIS at small x

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BROOKHAVEN NATIONAL LABORATORY

Outline

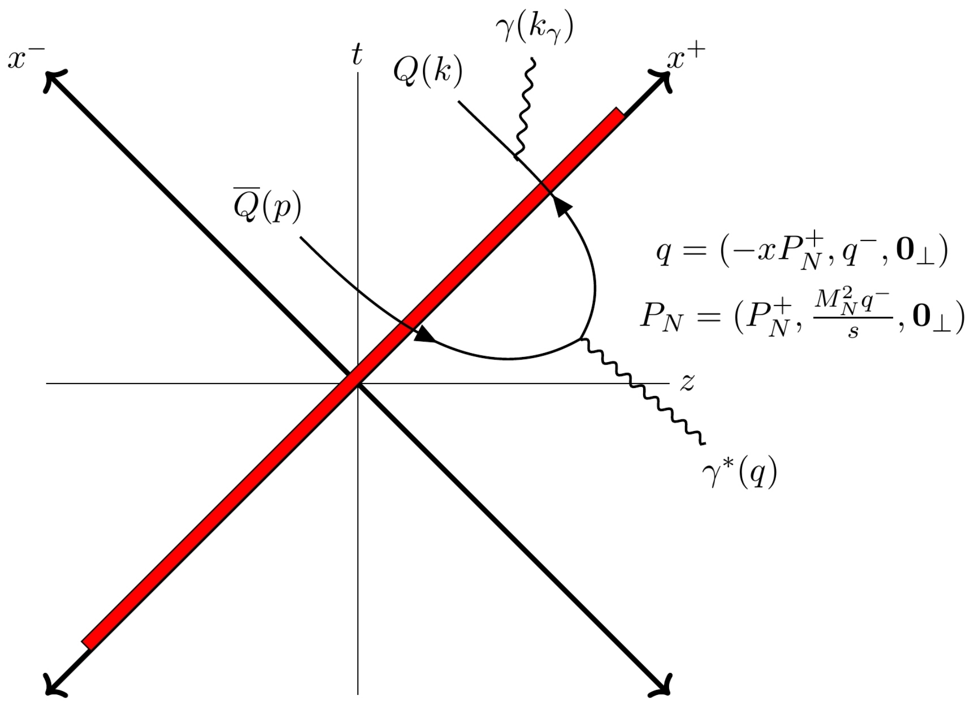
Probing the CGC with photon+dijets

- inclusive photon+dijet production in CGC EFT, highlights
- why is it interesting?
- CGC power counting, LO results
- structure of higher order computations, small-x evolution, NLO results

Summary and outlook

The process: $e + A \to e + q\bar{q} + \gamma + X$ (inclusive)

KR, Venugopalan, arXiv: 1802.09550, 1911.04530, 1911.04519



The right moving nucleus with large P_N^+ has its x^- extent Lorentz contracted

$$q^- \to \infty \ , \ P_N^+ \to \infty$$

fixed $Q^2 \gg \Lambda^2_{QCD}$, $s \to \infty$, $x \to 0$

Regge-Gribov small-x kinematics

- **Clean** initial and final states
- Can be measured at a future **E**lectron **I**on **C**ollider (**EIC**)

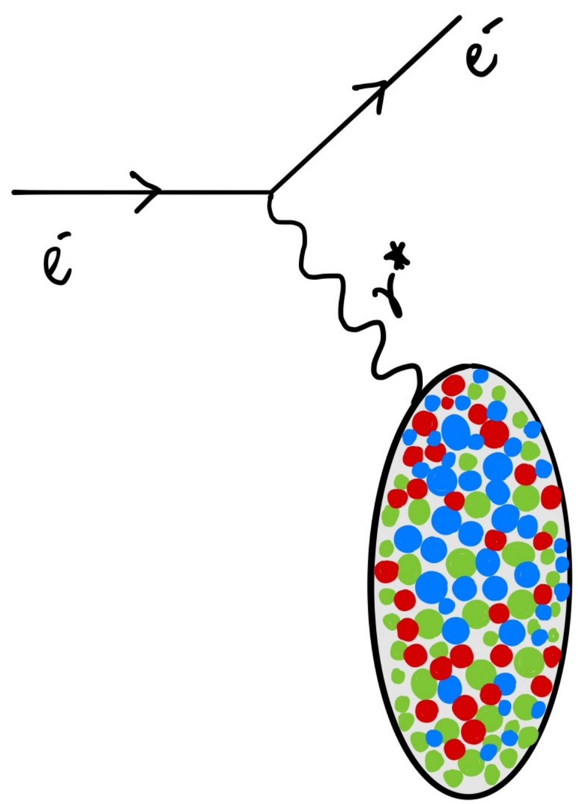
- Computed for the **first time** in the **Color Glass Condensate (CGC)** effective field theory

- Dijets+photon to NLO accuracy
  - $k_\gamma \to 0$ → **Inclusive dijet**
  - Integrate out $q/\bar{q}$ → **Inclusive photon + jet**
  - Integrate out $q\bar{q}$ → **Inclusive photon**
  - Integrate out $q\bar{q}\gamma$ → **Fully inclusive DIS**

Momentum space methods - efficient, can go to higher loops…

Why is it interesting?

At very high energies ($x < 0.01$), the γ^* samples strongly correlated gluonic matter instead of probing individual quarks and gluons



Gluon growth tamed by recombination and shadowing processes

$\boxed{\textbf{Non-linear dynamics !!!}}$

Emergence of semi-hard "saturation" momentum scale, $Q_S(x)$

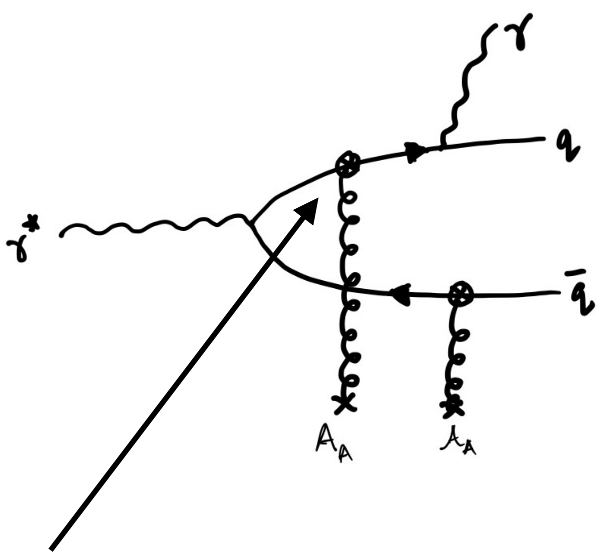
$\boxed{\textbf{Weak coupling: } \alpha_S(Q_s) \ll 1, \textbf{ for } Q_s(x) \gg \Lambda_{QCD}}$

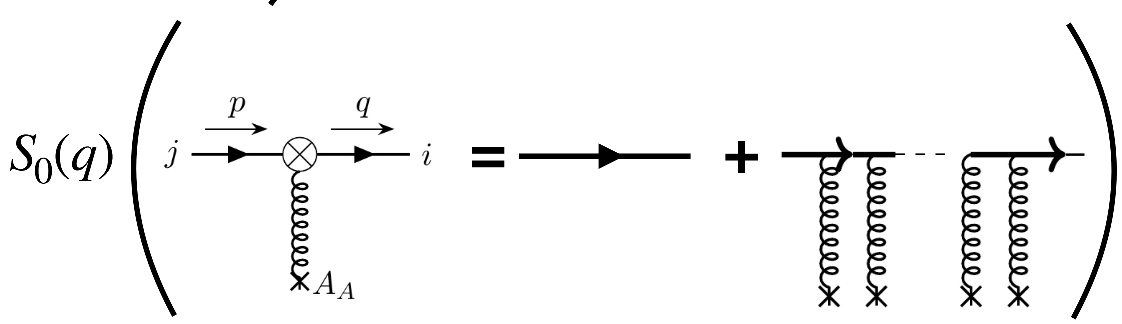
Physics intrinsically non-perturbative due to strong fields

Understanding the color charge interactions of the γ^* with the gluons via the $q\bar{q}$ dipole gives information on the distribution of saturated gluons $\longrightarrow$ **CGC EFT**

LO, NLO and all that..

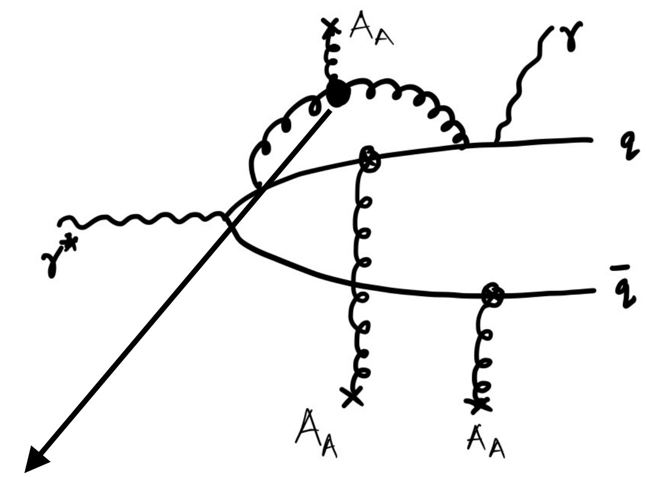
An example LO graph for $\gamma^* \to q\bar{q}\gamma$ in the CGC

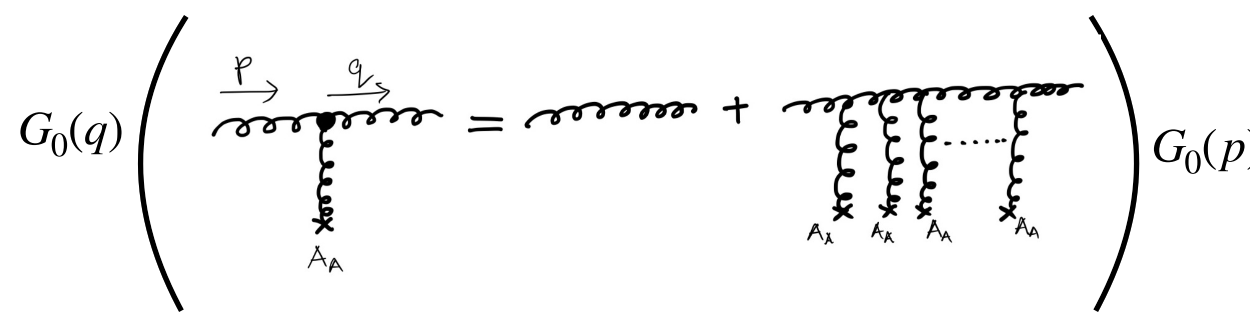




$$\propto \tilde{U}(\mathbf{x}_\perp) = P_-\left(\textbf{exp}\left\{-ig\int_{-\infty}^{+\infty} \mathrm{d}z^- A_{\mathrm{A}}^{+,a}(z^-,\mathbf{x}_\perp)\, t^a\right\}\right)$$

An example NLO graph for $\gamma^* \to q\bar{q}\gamma$ in the CGC





$$\propto U(\mathbf{x}_\perp) = P_-\left(\textbf{exp}\left\{-ig\int_{-\infty}^{+\infty} \mathrm{d}z^- A_{\mathrm{A}}^{+,a}(z^-,\mathbf{x}_\perp)\, T^a\right\}\right)$$

Wilson lines: Include all possible eikonal interactions with the background field

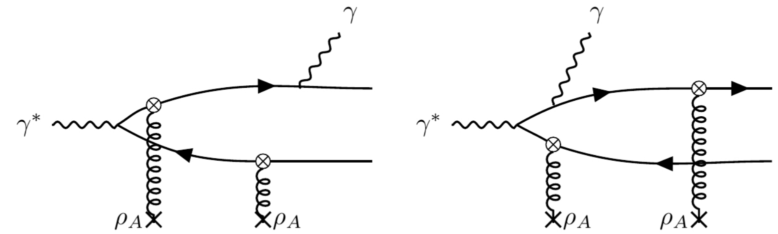
Background field: Solutions of $D_\nu F^{\nu\mu,a}(x) = \delta^{\mu+}\boxed{\rho_A^a(x^-,\boldsymbol{x}_\perp)} \rightarrow \sim \mathcal{O}(1/g)$

QFT in the presence of sources…

Large-x partons = static color sources

LO impact factor results

KR, Venugopalan, arXiv: 1802.09550



$+q \leftrightarrow \bar{q}$

Triple differential cross-section

Lepton tensor

$$\frac{\mathrm{d}^3\sigma^{\mathrm{LO}}}{\mathrm{d}x\,\mathrm{d}Q^2\mathrm{d}^6K_\perp\mathrm{d}^3\eta_K} = \frac{\alpha_{em}^2 q_f^4 y^2 N_c}{512\pi^5 Q^2}\frac{1}{(2\pi)^4}\frac{1}{2}L^{\mu\nu}\tilde{X}^{\mathrm{LO}}_{\mu\nu}$$

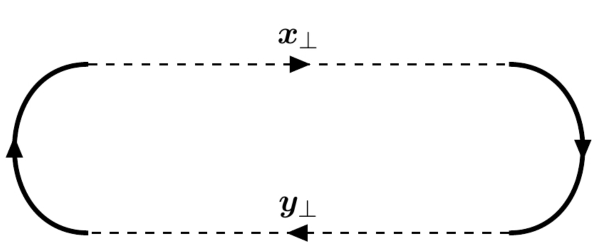
Hadron tensor

$$\tilde{X}^{\mathrm{LO}}_{\mu\nu} \propto \Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp) \longrightarrow$$ Non-perturbative input on strongly correlated gluons

$$\Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp) = 1 - D_{xy} - D_{y'x'} + Q_{y'x';xy}$$

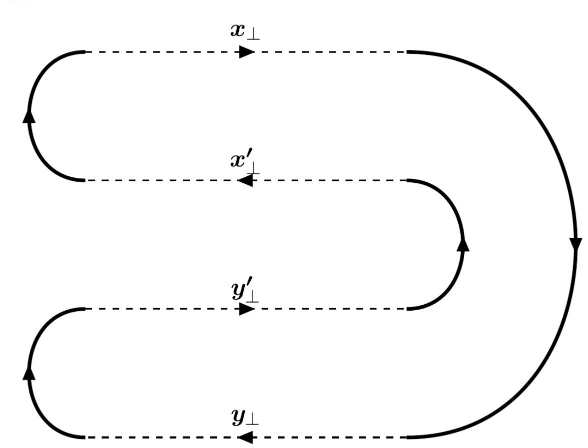
Dipole Wilson line correlator

$$D_{xy} = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp)\right)\right\rangle$$



Quadrupole Wilson line correlator

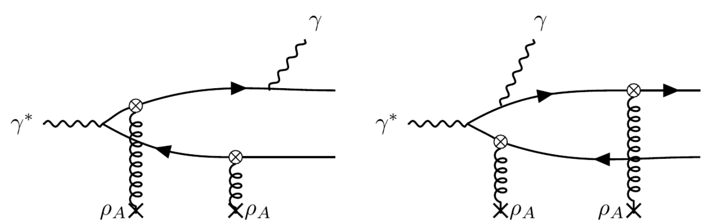
$$Q_{xy;zw} = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp)\tilde{U}(\boldsymbol{z}_\perp)\tilde{U}^\dagger(\boldsymbol{w}_\perp)\right)\right\rangle = Q_{zw;xy}$$



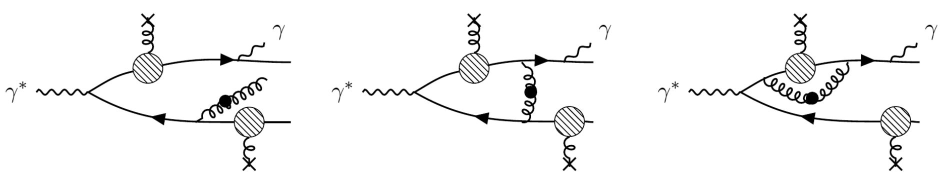
Ubiquitous building blocks of high energy QCD

Structure of higher order computations

Precision DIS experiments demand higher order computations in Regge asymptotics (small x) in close analogy to how higher order computations in the Bjorken limit provided powerful tests of pQCD.



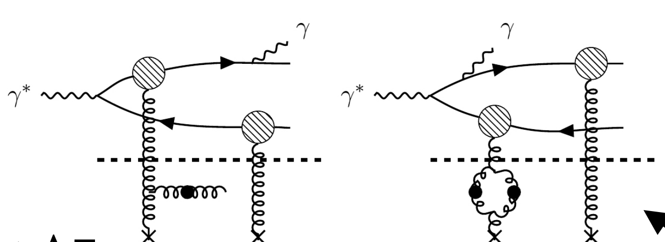
LO



Gluon modes

$l^- > \Lambda_0^-$

NLO projectile side



$l^- < \Lambda_0^-$

NLO target side

Fixed order →

Resummation ↓

LO	α_S^0						
NLO	$\alpha_S L_x$	α_S					
N^{2}LO	$\alpha_S^2 L_x^2$	$\alpha_S^2 L_x$	α_S^2				
N^{3}LO	$\alpha_S^3 L_x^3$	$\alpha_S^3 L_x^2$	$\alpha_S^3 L_x$	α_S^3			
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
N^{k}LO	$\alpha_S^k L_x^k$	$\alpha_S^k L_x^{k-1}$	……	……	……	α_S^k	
	↓ LLx	↓ $NLLx$	↓ N^2LLx			↓ N^kLLx	

Also computing this

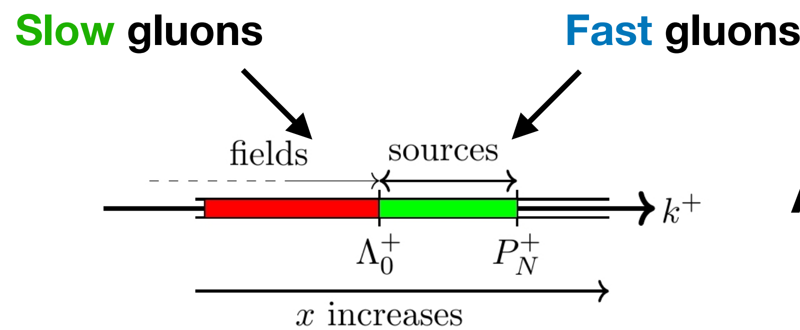
Go one step beyond…

Match leading log pieces
(LLx JIMWLK)

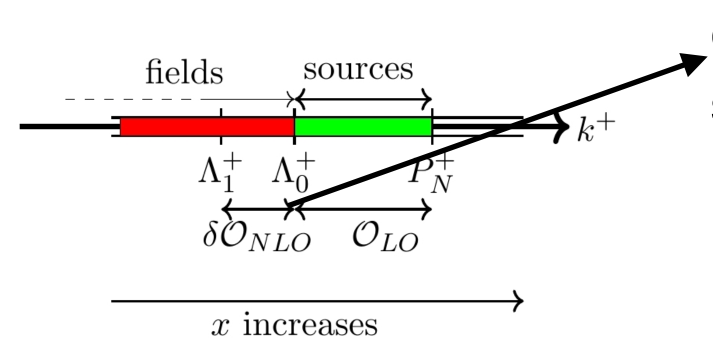
Small-x evolution: The Wilsonian RG ideology

Jalilian-Marian, Kovner, Leonidov, Weigert, hep-ph/9701284
Jalilian-Marian, Kovner, Weigert, hep-ph/9709432
Iancu, Leonidov, McLerran, hep-ph/0102009, hep-ph/0011241
Ferreiro, Iancu, Leonidov, McLerran, hep-ph/0109115

Evolve to smaller x



At LO, the separation scale Λ_0^+ is arbitrary



Quantum effects (large logarithms) induced by slow gluons

What are fields at a certain energy can be sources at higher energies….

All quantum effects absorbed in a redefinition of statistical distribution of sources $W_{\Lambda^+}[\rho_A]$

JIMWLK RGE: $$\frac{\partial W_{\Lambda^+}}{\partial(\ln(\Lambda^+))} = \mathcal{H}_{\text{JIMWLK}}\, W_{\Lambda^+}$$

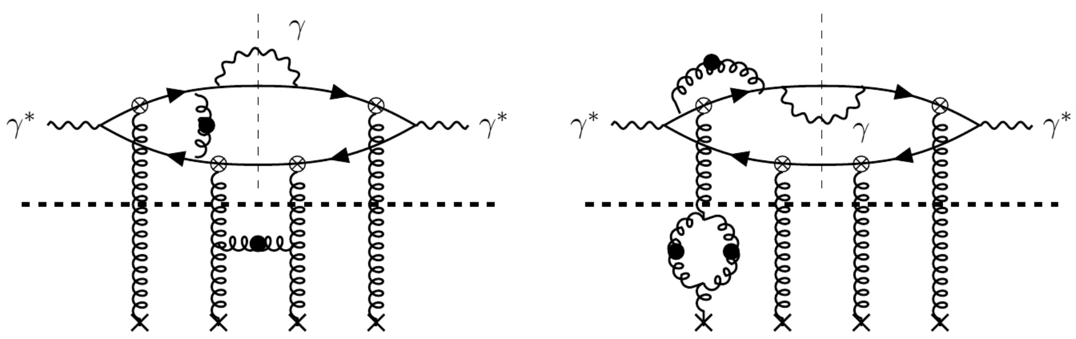
ρ_A **and** $W_{\Lambda^+}[\rho_A]$ **reproduce the effects of the fast gluons**

JIMWLK eqn. resums all powers of $\alpha_S \ln(1/x)$ and $Q_s/p_\perp$ arise in loop corrections

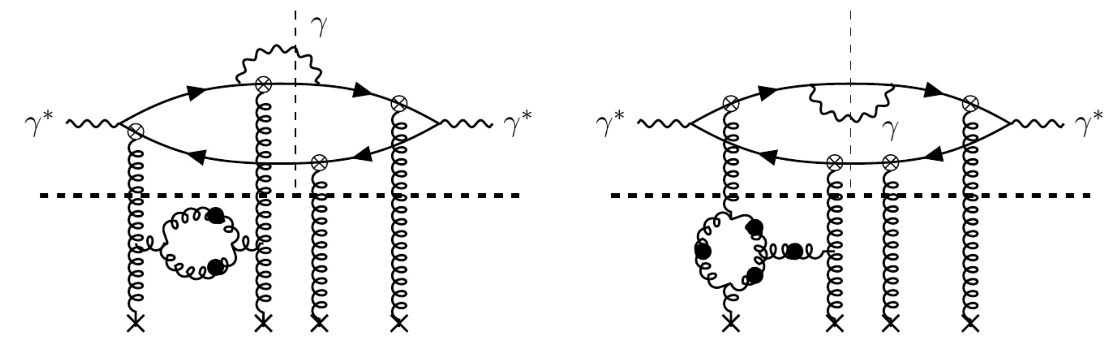
DIS dijet+photon at NLO+NLLx

KR, Venugopalan, arXiv: 1911.04530, 1911.04519

NNLO:1



NNLO:2



Formally NNLO(=O((α_S)^2) in CGC power counting

Collect leading log in x (LLx) pieces $\sim\ \alpha_S\ \ln(\Lambda_1^-/\Lambda_0^-)$ + NLO pieces~ α_S in the photon+dijet impact factor	Collect next-to-leading-log (NLLx) pieces $\sim\ \alpha_S^2\ \ln(\Lambda_1^-/\Lambda_0^-)$ + LO pieces~ α_S^0 in the photon+dijet impact factor

$$\langle \mathrm{d}\sigma^{\mathrm{jet}}\rangle_{\mathrm{NLO+NLL}x} = \int [\mathcal{D}\rho_A]\left\{W^{NLLx}[\rho_A]\,\mathrm{d}\hat{\sigma}^{\mathrm{jet}}_{\mathrm{LO}}[\rho_A] + W^{LLx}[\rho_A]\,\mathrm{d}\hat{\sigma}^{\mathrm{jet;finite}}_{\mathrm{NLO}}[\rho_A]\right\}$$

$$\simeq \int [\mathcal{D}\rho_A]\left(W^{NLLx}[\rho_A]\left\{\mathrm{d}\hat{\sigma}^{\mathrm{jet}}_{\mathrm{LO}}[\rho_A] + \boxed{\mathrm{d}\hat{\sigma}^{\mathrm{jet;finite}}_{\mathrm{NLO}}[\rho_A]}\right\} + O(\alpha_S^3 \ln(\Lambda_1^-/\Lambda_0^-))\right)$$

This is known

$$W^{NLLx}_{\Lambda_0^-}[\rho_A] = \left\{1 + \ln(\Lambda_1^-/\Lambda_0^-)(\mathcal{H}_{LO} + \mathcal{H}_{NLO})\right\}$$

This is what we compute

Accuracy of our result

NLO BK
- Balitsky, Chirilli, arXiv: 0710.4330
- Kovchegov, Weigert, hep-ph/0609090

NLO JIMWLK
- Kovner, Lublinsky, Mulian, arXiv:1310.0378
- Grabovsky, arXiv:1307.5414
- Caron-Huot, arXiv:1309.6521
- Balitsky, Chirilli, arXiv: 1309.7644
- Balitsky, Grabovsky, arXiv: 1405.0443
- Lublinksy, Mulian, arXiv: 1610.03453

Can be tested to $\sim 10\,\%$ accuracy at an EIC

Obtaining the “finite” impact factor at NLO

Choice of gauge: $A^- = 0$

Intermediate steps of the calculation contains soft, collinear, ultraviolet and rapidity divergences (from the spurious gluon pole at $p^- = 0$)

$$G^0_{\mu\nu;ab}(p) = \frac{i}{p^2 + i\varepsilon}\Big(- g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p^-}\Big)\delta_{ab}, \quad n^\mu = \delta^{\mu +}$$

- **Use dimensional regularization in $d = 2 - \epsilon$ dimensions to regularize UV divergences**
- **Regulate spurious gluon pole by imposing cutoff at initial scale of longitudinal momentum,**

$$\Lambda_0^- = \frac{Q_0^2}{2x_0 P_N^+} = \frac{Q_0^2}{x_0} . \frac{x}{Q^2} q^-$$

- **Logarithms in Λ_0^- can be absorbed in a redefinition of the weight functional $W_{\Lambda_0^-}[\rho_A]$**

$$\Big(1 + \ln(\Lambda_1^-/\Lambda_0^-)\mathcal{H}_{\rm LO}\Big) W_{\Lambda_0^-}[\rho_A] = W_{\Lambda_1^-}[\rho_A]$$

Equivalently by LLx JIMWLK evolution of the LO result

- **Use a jet algorithm to absorb remaining collinear singularities after real-virtual cancellation. We use a cone algorithm and work in the small cone approximation (R<<1) to analytically obtain the result in the form A log (R) +B .**

We have extracted the remaining genuine α_S suppressed pieces which constitute the NLO impact factor. Some of these have analytical expressions while rest have to calculated numerically.

NLO impact factor results

KR, Venugopalan, arXiv: 1911.04530, 1911.04519

Triple differential cross-section to NLO+NLLx

$$\frac{\mathrm{d}^3\sigma^{\mathrm{LO+NLO+NLL}x;\mathrm{jet}}}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}^6K_\perp\mathrm{d}^3\eta_K} = \frac{\alpha_{em}^2 q_f^4 y^2 N_c}{512\pi^5 Q^2}\frac{1}{(2\pi)^4}\frac{1}{2}L^{\mu\nu}\tilde{X}_{\mu\nu}^{\mathrm{LO+NLO+NLL}x;\mathrm{jet}}$$

Hadron tensor to NLO+NLLx

$\mathbf{p}_{J\perp}$, $\mathbf{p}_{K\perp}$ $\longrightarrow$ **Jet momenta**

$$\tilde{X}_{\mu\nu}^{\mathrm{LO+NLO+NLL}x;\mathrm{jet}} = \int[\mathcal{D}\rho_A]\,W_{x_{\mathrm{Bj}}}^{NLLx}[\rho_A]\left[\left(1+\frac{2\alpha_S C_F}{\pi}\left\{-\frac{3}{4}\ln\left(\frac{R^2|\boldsymbol{p}_{J\perp}|\,|\boldsymbol{p}_{K\perp}|}{4z_J z_K Q^2 e^{\gamma_E}}\right)+\frac{7}{4}-\frac{\pi^2}{6}\right\}\right)\tilde{X}_{\mu\nu}^{\mathrm{LO;jet}}[\rho_A]+\tilde{X}_{\mu\nu;\mathrm{finite}}^{\mathrm{NLO;jet}}[\rho_A]\right]$$

$$\begin{aligned}\langle\mathrm{d}\sigma^{\mathrm{jet}}\rangle_{\mathrm{NLO+NLL}x} &= \int[\mathcal{D}\rho_A]\left\{W^{NLLx}[\rho_A]\,\mathrm{d}\hat{\sigma}_{\mathrm{LO}}^{\mathrm{jet}}[\rho_A]+W^{LLx}[\rho_A]\,\mathrm{d}\hat{\sigma}_{\mathrm{NLO}}^{\mathrm{jet;finite}}[\rho_A]\right\}\\ &\simeq \int[\mathcal{D}\rho_A]\left(W^{NLLx}[\rho_A]\left\{\mathrm{d}\hat{\sigma}_{\mathrm{LO}}^{\mathrm{jet}}[\rho_A]+\mathrm{d}\hat{\sigma}_{\mathrm{NLO}}^{\mathrm{jet;finite}}[\rho_A]\right\}+O(\alpha_S^3\ln(\Lambda_1^-/\Lambda_0^-))\right)\end{aligned}$$

Summary and outlook

- We performed a first computation of inclusive photon+dijet production in e+A DIS at small x in the CGC framework to NLO in α_S .
- The simple structure of the dressed quark and gluon propagators in the “wrong” light cone gauge enables higher order computations in momentum space using otherwise standard covariant perturbation theory (pQCD) techniques.
- The NLO impact factor in combination with extant results on NLO JIMWLK provide the ingredients towards extending the precision to $\mathcal{O}(\alpha_S^3 \ln(1/x))$ accuracy.

Upcoming work

- Theoretical studies of $\gamma + q\bar{q}$ under different limits (soft gluon theorem and its violation, double soft limit…)
- Obtain NLO results for associated sub-channels (Inclusive dijet and its correlation limit, photon+jet and photon final states..)
- Numerical evaluation of these contributions

- Extend precision to NNLO (two loops) starting with inclusive dijet
  - Independent non-trivial derivation of the NLO JIMWLK result
  - Scheme dependence, running coupling, collinear log resummation..
  - New features !!!

Formidable task but feasible on the required timescales

Publications and preprints:

1. Kaushik Roy and Raju Venugopalan, *NLO impact factor for inclusive photon+dijet production in $e + A$ DIS at small x* ; **arXiv:1911.04530 [hep-ph]** . **Submitted to PRD**
2. Kaushik Roy and Raju Venugopalan, *Extracting many-body correlators of saturated gluons with precision from inclusive photon+dijet final states in deeply inelastic scattering* ; **arXiv:1911.04519 [hep-ph]** . **Submitted to PRL**
3. Kaushik Roy and Raju Venugopalan, *Progress in higher order computations of prompt photon production in $e + A$ DIS at small x* ; **PoS DIS2018 (2018) 053**.
4. Kaushik Roy and Raju Venugopalan, *Inclusive photon production in electron-nucleus scattering at small x* ; **JHEP 1805 (2018) 013**; **arXiv:1802.09550 [hep-ph]** .

Talks given at conferences and schools:

1. *NLO impact factor for photon+dijet production in e+A DIS at small x*, 9^{th} International Conference on **P**hysics **O**pportunities at an **E**lec**T**ron-**I**on-**C**ollider (**POETIC9**), Lawrence Berkeley Laboratory, CA, USA (September 2019)
2. *QCD in the Regge-Gribov Limit: An Introduction to the Color Glass Condensate (CGC) EFT*, **Department of Physics Seminar (Invited)**, Indian Institute of Technology (IIT) Madras, Chennai, India (April 2019)
3. *Inclusive prompt photon+dijet production in e+A DIS at small x as a probe of gluon saturation*, **The Myriad Colorful Ways Of Understanding Extreme QCD Matter**, International Center for Theoretical Sciences (ICTS), Bengaluru, India (April 2019)
4. *The NLO impact factor for photon+dijet production in e+A DIS at small x and JIMWLK factorization*, **3^{rd} Meeting of the TMD Collaboration**, Duke University, NC, USA (November 2018)

Posters:

NLO impact factor for inclusive photon+dijet production in e+A DIS at small x, **Initial Stages 2019**, Columbia University, NY, USA

Thank you…

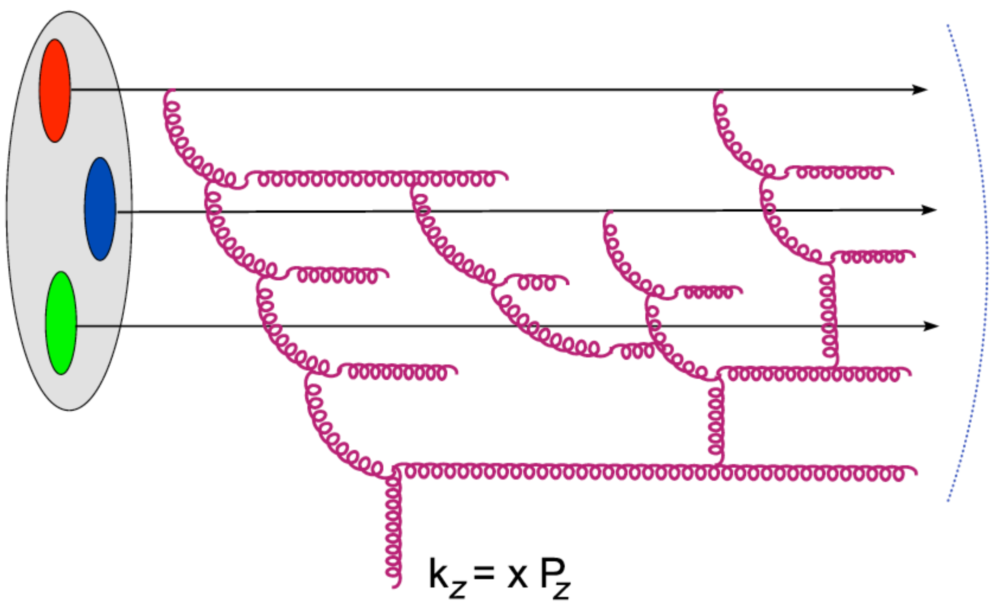
Back up slides

McLerran, Venugopalan, hep-ph/9309289
Iancu, Leonidov, McLerran, hep-ph/0011241
Iancu, Venugopalan, hep-ph/0303204

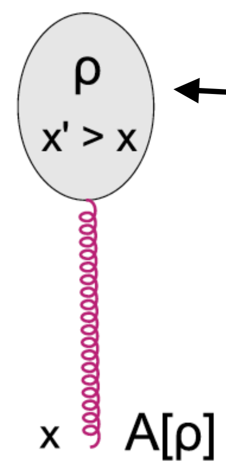
High energy QCD is classical !!

CGC = classical effective field theory in the non-linear regime of QCD describing **dynamical** gluon fields (**small x** partons) created by **static** color sources (**large x** partons).

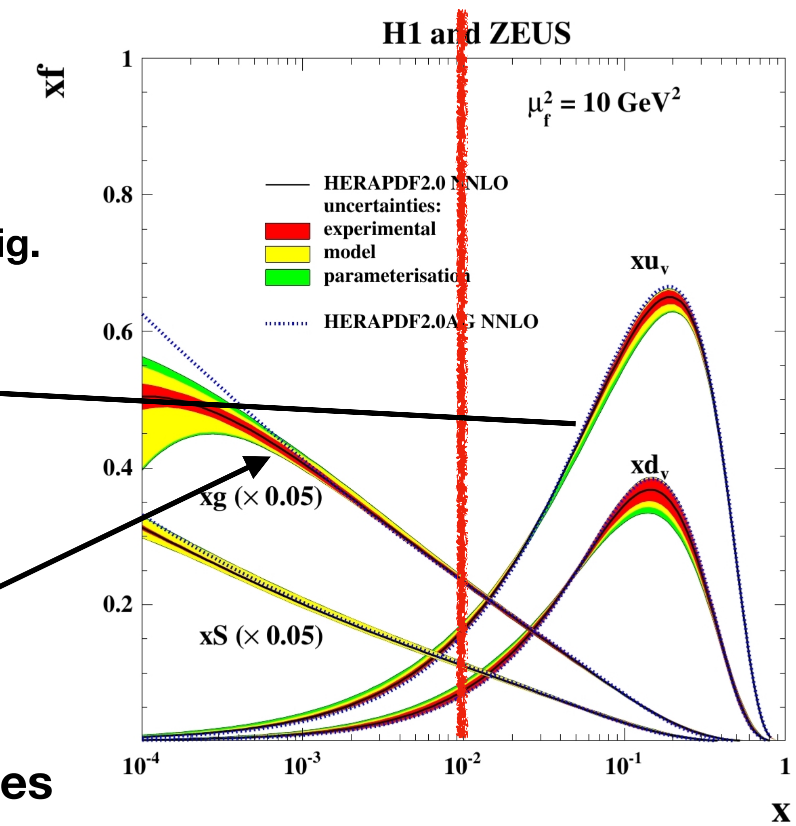
$$D_\nu F^{\nu\mu,a}(x) = \delta^{\mu+}\rho^a_A(x^-, \boldsymbol{x}_\perp)$$



Valence quarks+soft gluons frozen in some random config.



"slow" gluon modes



Lifetimes strongly ordered by time dilation: $\Delta t \simeq \dfrac{2xp_z}{k_\perp^2}$

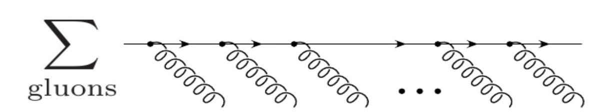
- **Gauge invariant, stochastic weight functional** $W_x[\rho]$ **gives probability of finding a configuration** ρ
- $W_x[\rho]$ **is built by integrating out soft gluon fluctuations in layers of x**
- **Initial condition at low energy (**$x_0 \sim 0.01$**) -> McLerran-Venugopalan (MV) model**

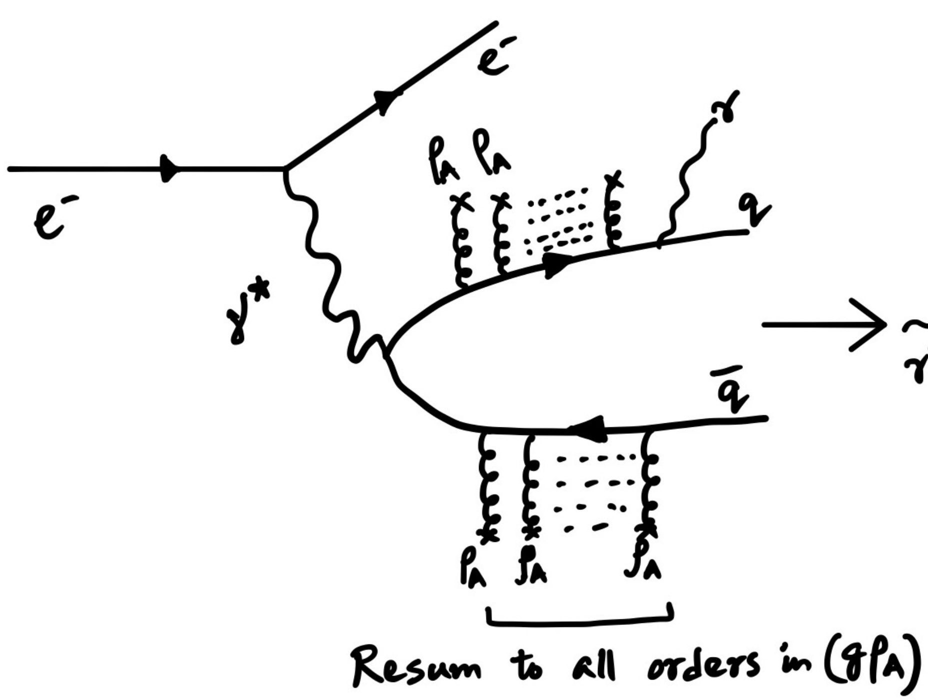
CGC power counting

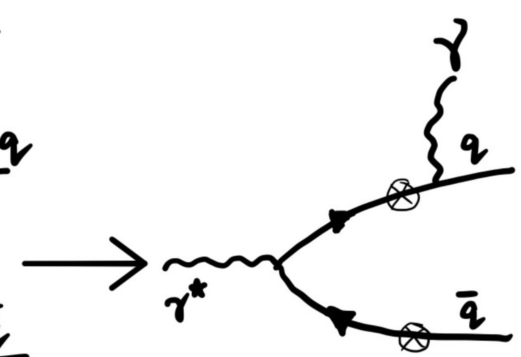
- In the saturation regime, gluon occupation number $\propto \langle \rho_A \rho_A \rangle \sim \mathrm{O}(1/\alpha_S)$ implying strong classical sources, $\rho_A \sim \mathrm{O}(1/g)$.
- Must resum eikonal interactions, to all orders at each fixed order in α_S .
- So, in general for an observable, O, the following perturbative expansion holds

$$\mathrm{O} = \sum_{n=0} c_n \alpha_S^n \quad \text{where} \quad c_n = \sum_{j=1}^{\infty} d_{nj} (g\rho_A)^j$$

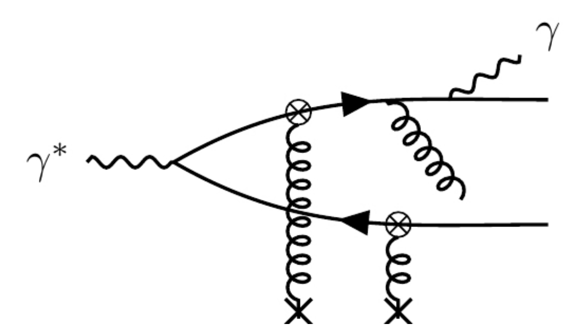
QCD at infinite twist…







LO process



An NLO process

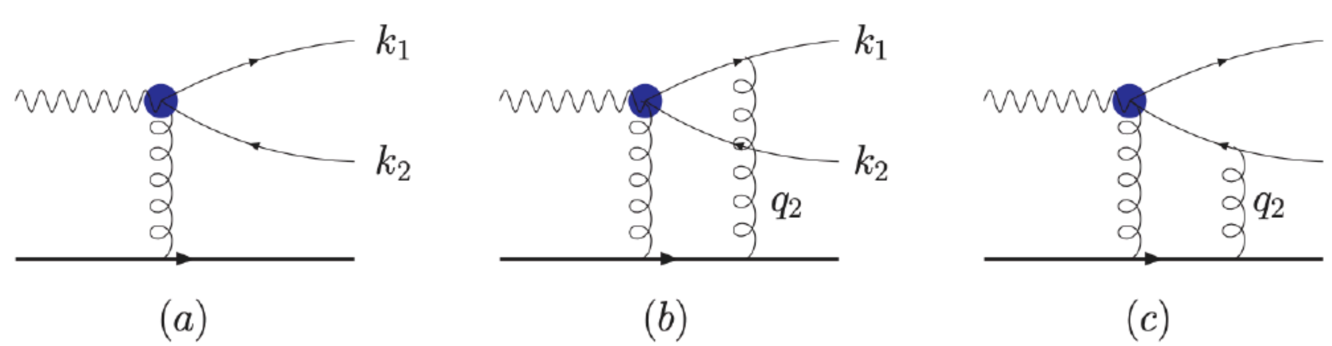
Interesting limits:

KR, Venugopalan, arXiv: 1802.09550

In the limit of $k_\gamma \to 0$ the amplitude satisfies the Low-Burnett-Kroll theorem

$$\mathcal{M}_\mu(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p},\boldsymbol{k}_\gamma) \to -(eq_f)\epsilon^*_\alpha(\boldsymbol{k}_\gamma,\lambda)\left(\frac{p^\alpha}{p.k_\gamma} - \frac{k^\alpha}{k.k_\gamma}\right)\mathcal{M}^{NR}_\mu(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p})$$

Polarization vector **x** Vectorial structure depending only on momenta of emitted particles **x** Non-radiative DIS amplitude



Recover existing results on inclusive dijet production in DIS

$$\frac{\mathrm{d}\sigma^{L,T}}{\mathrm{d}^3k\mathrm{d}^3p} = \alpha\, q_f^2 N_c \delta(q^- - p^- - k^-) \int \frac{\mathrm{d}^2\boldsymbol{x}_\perp}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{x}'_\perp}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{y}_\perp}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{y}'_\perp}{(2\pi)^2}\; e^{-i\boldsymbol{k}_\perp.(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)} e^{-i\boldsymbol{p}_\perp.(\boldsymbol{y}_\perp - \boldsymbol{y}'_\perp)}$$
$$\times \sum_{\alpha,\beta} \psi^{L,T}_{\alpha\beta}(q^-, z, |\boldsymbol{x}_\perp - \boldsymbol{y}_\perp|)\; \psi^{L,T*}_{\alpha\beta}(q^-, z, |\boldsymbol{x}'_\perp - \boldsymbol{y}'_\perp|) \times \Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp, \boldsymbol{y}'_\perp)$$

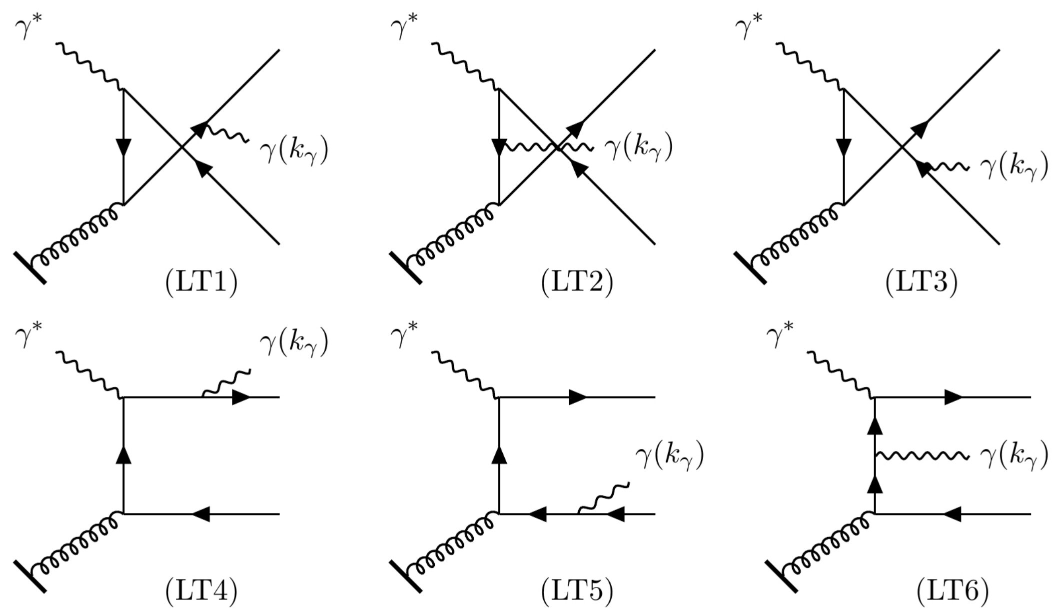
Dominguez, Marquet, Xiao, Yuan, arXiv:1101.0715

Important check because it is sensitive to the Weizsäcker-Williams UGD in the back-to-back correlation limit $|\boldsymbol{k}_\perp + \boldsymbol{p}_\perp| \ll |\boldsymbol{k}_\perp - \boldsymbol{p}_\perp|/2$.

Golden channel for probing and understanding the WW distribution at a future EIC or LHeC!!

In the limit of large $P_\perp$, we recover leading twist $k_\perp$ and collinear factorization expressions

$k_\perp$



Unintegrated gluon distribution (UGD)

$$\tilde{X}^{\mathrm{LT}}_{\mu\nu} = \frac{2\alpha_S}{N_c} \frac{\phi_{Y_A}(\boldsymbol{P}_\perp)}{\boldsymbol{P}_\perp^2} \, \Theta_{\mu\nu}(\boldsymbol{P}_\perp)$$

Modified trace over Dirac matrices in this limit

$$\tilde{X}^{\mathrm{coll.}}_{\mu\nu} = \frac{2\alpha_S \pi^2}{N_c} \, (2\pi)^2 \delta^{(2)}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp + \boldsymbol{k}_{\gamma\perp}) \, x_A f_{g,A}(x_A, Q^2) \lim_{\boldsymbol{P}_\perp \to 0} \frac{\Theta_{\mu\nu}(\boldsymbol{P}_\perp)}{\boldsymbol{P}_\perp^2}$$

Collinear factorized result directly sensitive to nuclear gluon distribution at small x

Agreement with small x limit of results by Aurenche *et. al.*

Aurenche, Douiri, Baier, Fontannaz Schiff, Z. Phys. C 24, 309 (1984)

NLO impact factor for photon+dijet in eA DIS

KR, Venugopalan, arXiv: 1911.04530, 1911.04519

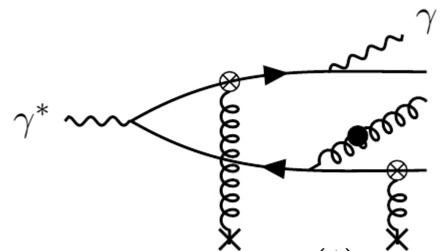
Extant NLO results in literature

Fully Inclusive DIS
- Balitsky, Chirilli, arXiv: 1009.4729
- Beuf, arXiv: 1606.00777, 1708.06557
- Hanninen, Lappi, Paatelainen, arXiv: 1711.08207

Diffractive DIS dijet → Boussarie, Grabovsky, Szymanowski, Wallon, arXiv:1606.00419

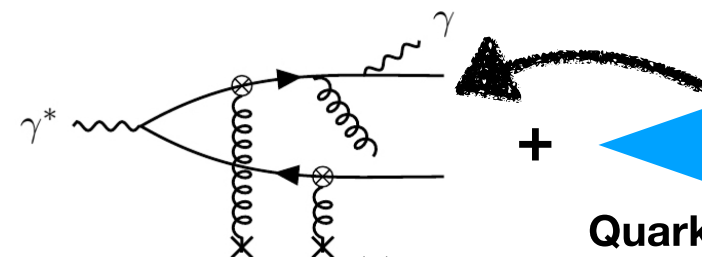
First computation of photon+dijet in eA DIS at small x

Real gluon emission:



$T_R^{(1)}$

+



$T_R^{(2)}$

+ Quark - Antiquark

$T_R^{(3)}$

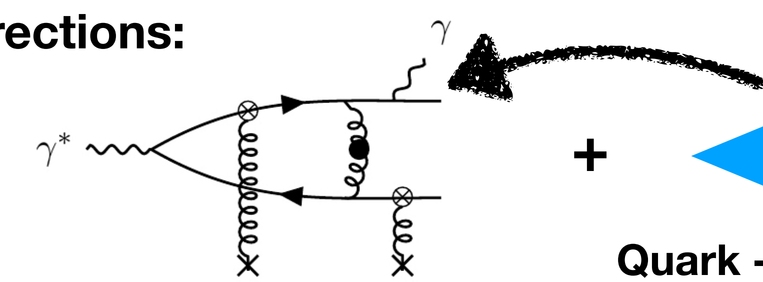
Gluon can possibly cross the "shock wave"

Gluon does not cross the "shock wave"

20x20=400 contributions in the squared amplitude

Virtual emission:

a. Vertex corrections:

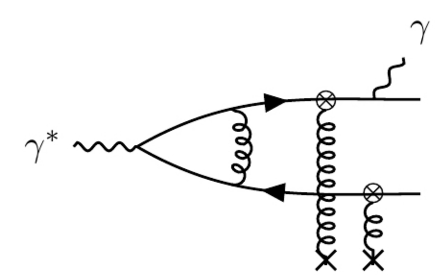


$T_V^{(1)}$

+ Quark - Antiquark

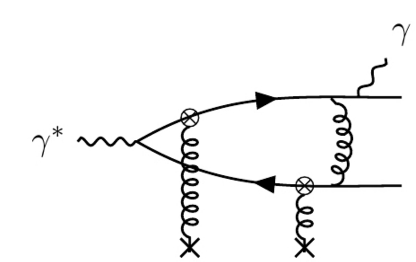
$T_V^{(2)}$

+



$T_V^{(3)}$

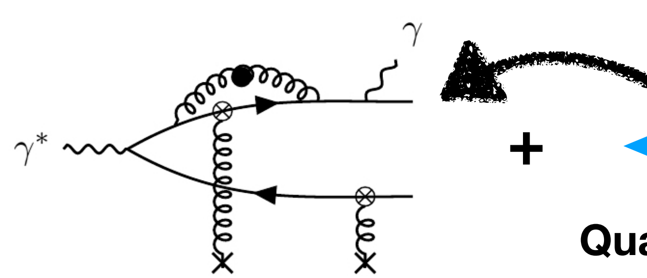
+



$T_V^{(4)}$

6x4=24 diagrams in total interfering with LO processes

b. Self-energy corrections:

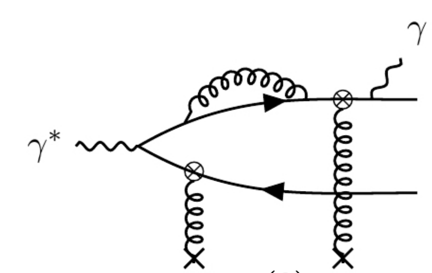


$T_S^{(1)}$

+ Quark - Antiquark

$T_S^{(2)}$

+



$T_S^{(3)}$

(6x2=12)+24=36 diagrams in total interfering with LO processes

KR, Venugopalan, arXiv: 1911.04530, 1911.04519

Deriving the JIMWLK evolution from the projectile side for a non-trivial process

Hadron tensor at LO:

$$X^{\rm LO}_{\mu\nu} = \mathcal{C}^{\rm LO}_{\mu\nu} \otimes \Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp|\Lambda_0^-) \qquad \Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp, \boldsymbol{y}'_\perp) = 1 - D(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) - D(\boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp) + Q(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$$

Hard coefficient

In the soft gluon limit which generates logarithms in x, we obtain the following for our hadron tensor at NLO

$$\begin{aligned}
X^{\rm NLO}_{\mu\nu;LLx} = C^{\rm LO}_{\mu\nu} \otimes \ln(\Lambda_1^-/\Lambda_0^-) \Bigg[& \frac{\alpha_S N_c}{2\pi^2}\Big\{\mathcal{K}_B(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{z}_\perp)\boxed{D_{xy}} + \left(\begin{smallmatrix}\boldsymbol{x}_\perp \to \boldsymbol{y}'_\perp \\ \boldsymbol{y}_\perp \to \boldsymbol{x}'_\perp\end{smallmatrix}\right)\Big\} - \frac{\alpha_S N_c}{(2\pi)^2}\mathcal{K}_1(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp; \boldsymbol{z}_\perp)\boxed{Q_{xy}} \\
& - \frac{\alpha_S N_c}{2\pi^2}\Big\{\mathcal{K}_B(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{z}_\perp) D_{xz} D_{zy} + \left(\begin{smallmatrix}\boldsymbol{x}_\perp \to \boldsymbol{y}'_\perp \\ \boldsymbol{y}_\perp \to \boldsymbol{x}'_\perp\end{smallmatrix}\right)\Big\} + \frac{\alpha_S N_c}{(2\pi)^2}\Bigg(\Big\{\mathcal{A}(\boldsymbol{x}_\perp, \boldsymbol{y}'_\perp, \boldsymbol{y}_\perp, \boldsymbol{x}'_\perp; \boldsymbol{z}_\perp)\boxed{D_{xx'}D_{y'y}} + \boldsymbol{x}_\perp \leftrightarrow \boldsymbol{y}'_\perp \\
& + \Big\{\mathcal{K}_2(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{x}'_\perp; \boldsymbol{z}_\perp)\boxed{D_{xz} Q_{zy;y'x'}} + \left(\begin{smallmatrix}\boldsymbol{x}_\perp \to \boldsymbol{y}'_\perp \\ \boldsymbol{y}_\perp \to \boldsymbol{x}'_\perp\end{smallmatrix}\right)\Big\} + \Big\{\mathcal{K}_2(\boldsymbol{x}'_\perp, \boldsymbol{x}_\perp, \boldsymbol{y}'_\perp; \boldsymbol{z}_\perp)\, D_{zx'} Q_{y'z;xy'} + \left(\begin{smallmatrix}\boldsymbol{x}_\perp \to \boldsymbol{y}'_\perp \\ \boldsymbol{y}_\perp \to \boldsymbol{x}'_\perp\end{smallmatrix}\right)\Big\}\Bigg)
\end{aligned}$$

Building blocks: Non-trivial combinations of dipole and quadrupole Wilson line correlators

Dominguez, Mueller, Munier, Xiao, arXiv: 1108.1752

$\mathscr{A}, \mathscr{K}_{1,2}$ **are evolution kernels composed of several BFKL kernels** $\mathcal{K}_B = \dfrac{(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)^2}{(\boldsymbol{x}_\perp - \boldsymbol{z}_\perp)^2(\boldsymbol{y}_\perp - \boldsymbol{z}_\perp)^2}$

Remarkably, this whole thing can be simply written as

$$X^{\rm NLO}_{\mu\nu;LLx} = C^{\rm LO}_{\mu\nu} \otimes \ln\left(\frac{\Lambda_1^-}{\Lambda_0^-}\right) H^{\rm LO}_{\rm JIMWLK}\, \Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp|\Lambda_0^-)$$

Leads immediately to the JIMWLK evolution equation:

$$W_{\Lambda_1^-}[\rho_A] = \left(1 + \ln\left(\frac{\Lambda_1^-}{\Lambda_0^-}\right) H^{\rm LO}_{\rm JIMWLK}\right) W_{\Lambda_0^-}[\rho_A]$$

$$\frac{\partial}{\partial\,(\ln \Lambda^-)}(\mathrm{d}\sigma^{LO}) = \langle H_{\rm JIMWLK}\,(\mathrm{d}\sigma^{LO})\rangle$$

$$H^{\rm LO}_{\rm JIMWLK} = \frac{1}{2}\int_{\boldsymbol{u}_\perp,\boldsymbol{v}_\perp} \frac{\delta}{\delta\alpha^a(\boldsymbol{u}_\perp)}\,\eta^{ab}(\boldsymbol{u}_\perp,\boldsymbol{v}_\perp)\,\frac{\delta}{\delta\alpha^b(\boldsymbol{v}_\perp)}\,,$$

$$\eta^{ab}(\boldsymbol{u}_\perp,\boldsymbol{v}_\perp) = \frac{1}{\pi}\int_{\boldsymbol{z}_\perp}\frac{1}{(2\pi)^2}\frac{(\boldsymbol{u}_\perp-\boldsymbol{z}_\perp).(\boldsymbol{v}_\perp-\boldsymbol{z}_\perp)}{(\boldsymbol{u}_\perp-\boldsymbol{z}_\perp)^2\,(\boldsymbol{v}_\perp-\boldsymbol{z}_\perp)^2}\left[\mathbb{1}+U^\dagger(\boldsymbol{u}_\perp)U(\boldsymbol{v}_\perp)-U^\dagger(\boldsymbol{u}_\perp)U(\boldsymbol{z}_\perp)-U^\dagger(\boldsymbol{z}_\perp)U(\boldsymbol{v}_\perp)\,\right]^{ab}.$$

$$U(\boldsymbol{x}_\perp) = P_-\left(\exp\left\{-ig\int_{-\infty}^{+\infty}{\rm d}z^-\,\alpha^a(z^-,\boldsymbol{x}_\perp)T^a\right\}\right),$$

$$\frac{\delta U(\boldsymbol{x}_\perp)}{\delta\alpha^a(\boldsymbol{z}_\perp)} = -ig\delta^{(2)}(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp)U(\boldsymbol{x}_\perp)T^a\,,\quad \frac{\delta U^\dagger(\boldsymbol{x}_\perp)}{\delta\alpha^a(\boldsymbol{z}_\perp)} = ig\delta^{(2)}(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp)T^aU^\dagger(\boldsymbol{x}_\perp)\,,$$

Kernels appearing in the JIMWLK derivation

$$\begin{aligned}
\mathcal{A}(\boldsymbol{x}_\perp,\boldsymbol{y}'_\perp,\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp;\boldsymbol{z}_\perp) &= \mathcal{K}_B(\boldsymbol{x}_\perp,\boldsymbol{y}'_\perp;\boldsymbol{z}_\perp)-\mathcal{K}_B(\boldsymbol{y}'_\perp,\boldsymbol{y}_\perp;\boldsymbol{z}_\perp)+\mathcal{K}_B(\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp;\boldsymbol{z}_\perp)-\mathcal{K}_B(\boldsymbol{x}'_\perp,\boldsymbol{x}_\perp;\boldsymbol{z}_\perp)\,,\\
\mathcal{K}_1(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp;\boldsymbol{z}_\perp) &= \mathcal{K}_B(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{z}_\perp)+\mathcal{K}_B(\boldsymbol{y}_\perp,\boldsymbol{y}'_\perp;\boldsymbol{z}_\perp)+\mathcal{K}_B(\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp;\boldsymbol{z}_\perp)+\mathcal{K}_B(\boldsymbol{x}'_\perp,\boldsymbol{x}_\perp;\boldsymbol{z}_\perp)\,,\\
\mathcal{K}_2(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp;\boldsymbol{z}_\perp) &= \mathcal{K}_B(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{z}_\perp)-\mathcal{K}_B(\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp;\boldsymbol{z}_\perp)+\mathcal{K}_B(\boldsymbol{x}'_\perp,\boldsymbol{x}_\perp;\boldsymbol{z}_\perp)\,.
\end{aligned}$$

KR, Venugopalan, arXiv: 1802.09550

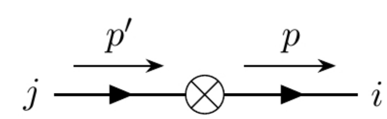
CGC inputs: Shockwave gluon propagator

- **Convenient to work in the "wrong" light-cone gauge $A^- = 0$ for the kinematics of this problem. (Gauge links appearing in PDF definitions are unity in the conventional LC gauge $A^+ = 0$.)**
- **Resulting momentum space expression is simple and similar to the shockwave fermion propagator.**

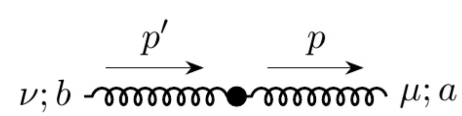
McLerran, Venugopalan, hep-ph/9402335
Ayala, Jalilian-Marian, McLerran, Venugopalan, hep-ph/9501324
Balitsky, Belitsky, hep-ph/0110158

$$G^{\mu\nu;ab}(p,p') = G_0^{\mu\rho;ac}(p)\,\mathcal{T}_{\rho\sigma;cd}(p,p')\,G_0^{\sigma\nu;db}(p')$$

$G_0^{\mu\nu;ab}$ **: Free gluon propagator in $A^- = 0$ gauge**



$$\mathcal{T}_{ij}(p,p') = (2\pi)\delta(p^- - p'^-)\gamma^-\mathrm{sign}(p^-)\int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp).\boldsymbol{z}_\perp}\ \tilde{U}_{ij}^{\mathrm{sign}(p^-)}(\boldsymbol{z}_\perp)$$



$$\mathcal{T}_{\mu\nu;ab}(p,p') = -2\pi\delta(p^- - p'^-)\times(2p^-)g_{\mu\nu}\ \mathrm{sign}(p^-)\int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp).\boldsymbol{z}_\perp}\ U_{ab}^{\mathrm{sign}(p^-)}(\boldsymbol{z}_\perp)$$

Vertex structures identical to quark-quark-reggeon and gluon-gluon-reggeon in Lipatov's Reggeon EFT

Bondarenko, Lipatov, Pozdnyakov, Prygarin , arXiv: 1708.05183
Hentschinski, arXiv: 1802.06755

McLerran-Venugopalan (MV) Model:

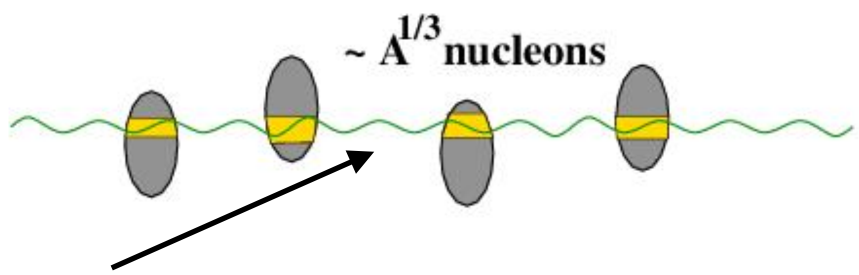
McLerran, Venugopalan, hep-ph/9309289, hep-ph/9311205, hep-ph/9402335

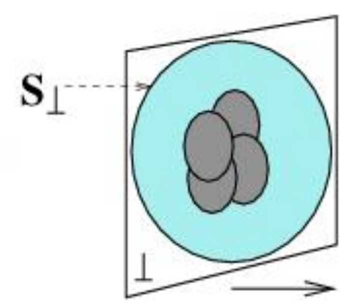
Originally formulated for a large nucleus in the IMF

$$|\Psi\rangle = c_1|qqq\rangle + c_2|qqqq\rangle + \ldots + c_n|qqqggg\ldots q\bar{q}gg\rangle$$

Higher Fock-components dominate the hadronic wave function at small x. Predominantly gluons.

Based on natural separation of energy (time) scales into ‘wee’ (small x) and valence (large x) partons





$$\lambda wee \approx \frac{1}{k^+} = \frac{1}{xP^+} >> \lambda val. = \frac{Rm_p}{P^+} \Rightarrow x << A^{-1/3}$$

‘Wee’ partons ‘see’ a large density of color ‘sources’ at small transverse resolutions

- Slow/`wee’ partons (small-x) have short lifetimes, $\Delta x^+ \equiv \frac{xP_N^+}{m_\perp^2}$ and must be treated as standard gauge fields.
- Fast/valence partons (large-x) appear to live forever on the time-scale of wee partons.
- Consider them as static sources of color charge, $J^\mu(x) = \delta^{\mu+}\delta(x^-)\rho(\boldsymbol{x}_\perp)$.
- When `wee’ partons couple to a large number of these `random’ color sources simultaneously, we can consider the charge distribution to be classical.
- Gauge fields of small-x gluons ‘eikonally’ couple to J^+.

Iancu, Venugopalan, hep-ph/0303204

Total charge in transverse area

$$Q^a = \int_{\Delta S_\perp} \mathrm{d}^2\boldsymbol{x}_\perp \rho^a(\boldsymbol{x}_\perp) = \int_{\Delta S_\perp} \mathrm{d}^2\boldsymbol{x}_\perp \int \mathrm{d}x^- \rho^a(x^-, \boldsymbol{x}_\perp)$$

Equal LC time charge correlators

$$\langle \rho_a(x^-, \boldsymbol{x}_\perp)\rho_b(y^-, \boldsymbol{y}_\perp)\rangle_A = \delta_{ab}\delta^{(2)}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)\delta(x^- - y^-)\lambda_A(x^-)$$

$$\int \mathrm{d}x^- \lambda_A(x^-) = \mu_A^2 \equiv \frac{g^2 A}{2\pi R_A^2}$$

$$\mu_A^2 \propto A^{1/3}$$

For large nucleus, $\alpha_S(\mu^2) \ll 1$

Average color charge squared of valence quarks per unit transverse area per color

Nuclear wavefunction at small x is perturbative (not totally); must treat non-linear effects due to large gluon density to all orders

Criterion for gluon recombination = $\rho\sigma_{gg\to g} \geq 1$

$$\rho \sim \frac{xG(x,Q^2)}{\pi R^2} \qquad \sigma_{gg\to g} \sim \frac{\alpha_S}{Q^2}$$

$$Q^2 \leq Q_S^2 \equiv \frac{\alpha_S\,(xG(x,Q^2))}{\pi R^2}$$

Saturation momentum scale = avg. color charge squared of gluons/rapidity/transverse area

In MV model, the non-trivial correlators shown in the previous slide are generated by the weight functional

$$W_{\Lambda_0^+}[\rho_A] = \mathcal{N}\, exp\Big\{ -\frac{1}{2}\int \mathrm{d}x^- \mathrm{d}^2\boldsymbol{x}_\perp \frac{\rho_A(x^-,\boldsymbol{x}_\perp)\rho_A(x^-,\boldsymbol{x}_\perp)}{\lambda_A(x^-)}\Big\}$$

Gauge invariant (since local) and Gaussian in ρ_A

Valid by construction for a large nucleus and for kinematical range

$$\Lambda^2_{QCD} \ll Q^2 (= 1/\Delta S_\perp) \ll \Lambda^2_{QCD} A^{1/3}$$