Non-Gaussianity post Planck and BICEP2

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BICEP2

- Inflation (and its alternatives) predict initial curvature perturbations with a nearly scale invariant power spectrum
$$(k^3/2\pi^2)P_\zeta(k) = \Delta_\zeta^2(k/k_0)^{n_s-1}$$
- Some models of inflation (but not its alternatives) predict initial gravity wave perturbations with power spectrum
$$(k^3/2\pi^2)P_{\rm gw}(k) = r\Delta_\zeta^2(k/k_0)^{n_t}$$
- CMB temperature constraint on r is fairly strong but sample variance limited: r < 0.11 (95% CL, Planck)
- B-mode polarization experiments can ultimately provide much better constraints, but very low noise is required
- 17 Mar 2014: BICEP2 reports 7σ detection, $r = 0.2^{+0.07}_{-0.05}$!

BICEP2

Inflationary observables: $\{\Delta_\zeta, n_s, r\}$ defined by

$$(k^3/2\pi^2)P_\zeta(k) = \Delta_\zeta^2 (k/k_0)^{n_s - 1}$$

$$(k^3/2\pi^2)P_{\rm gw}(k) = r\Delta_\zeta^2 (k/k_0)^{n_t}$$

In single-field slow-roll inflation

$$S = \int d^4x \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right)$$

the observables $\{\Delta_\zeta, n_s, r\}$ are related to $V(\phi)$ by:

$$\Delta_\zeta^2 = \frac{1}{8\pi^2\epsilon}\frac{H^2}{M_{\rm Pl}^2}$$

$$n_s = 1 - 6\epsilon + 2\eta$$

$$r = 16\epsilon$$

where $\epsilon = \dfrac{M_{\rm Pl}^2}{2}\left(\dfrac{V'}{V}\right)^2$

$$\eta = M_{\rm Pl}^2\left(\frac{V''}{V}\right)$$

BICEP2

Post-BICEP2, everything is pinned down!

$$\Delta_\zeta^2 = \frac{1}{8\pi^2\epsilon}\frac{H^2}{M_{\rm Pl}^2} \qquad n_s = 1 - 6\epsilon + 2\eta \qquad r = 16\epsilon$$

$$\Delta_\zeta^2 = 2.2 \times 10^{-9}$$

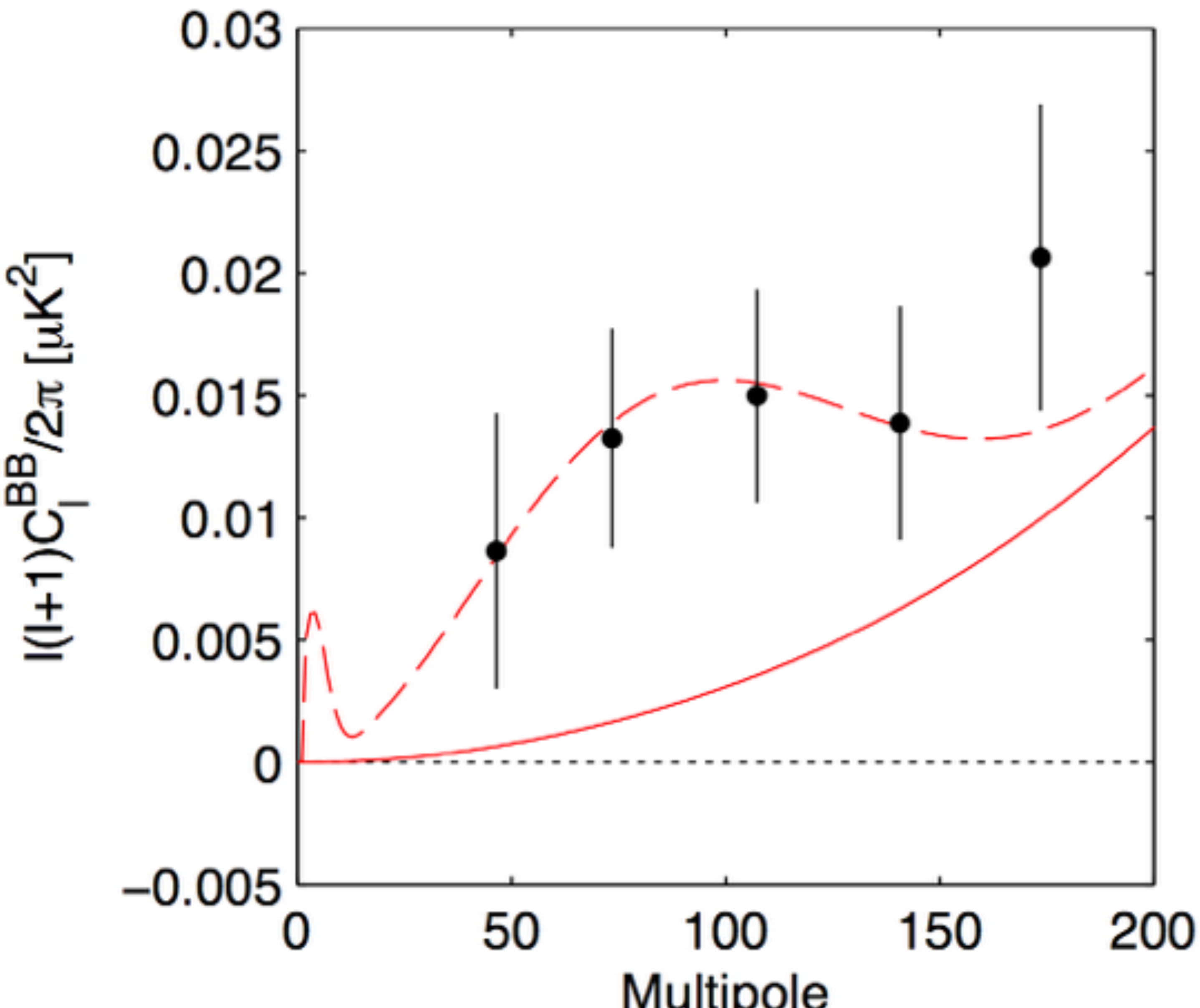
$\Rightarrow$ Energy scale of inflation $V^{1/4} = 0.009 M_{\rm Pl} = 2.2 \times 10^{16}\ \mathrm{GeV}$

First two derivatives $\dfrac{V'}{V} = 0.16 M_{\rm Pl}^{-1}$, $\dfrac{V''}{V} = 0.017 M_{\rm Pl}^{-2}$

Field excursion per e-folding

BICEP2: gravity waves or dust?

At issue: whether the B-mode signal seen by BICEP2 must be cosmological gravity waves, or whether it can be explained as polarized thermal emission by dust grains in our galaxy



BICEP2 collaboration

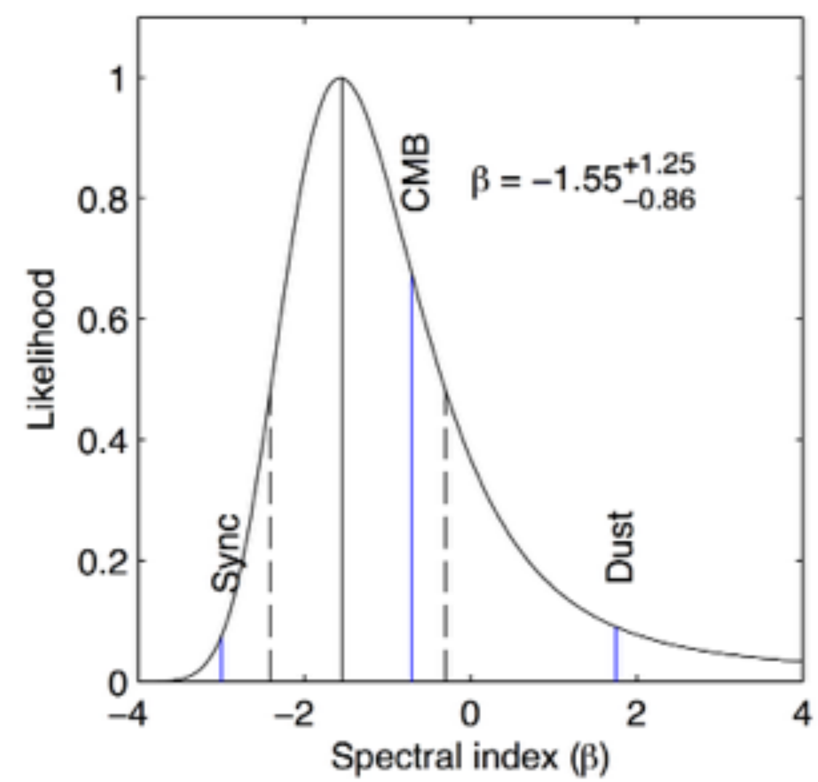
BICEP2: gravity waves or dust?

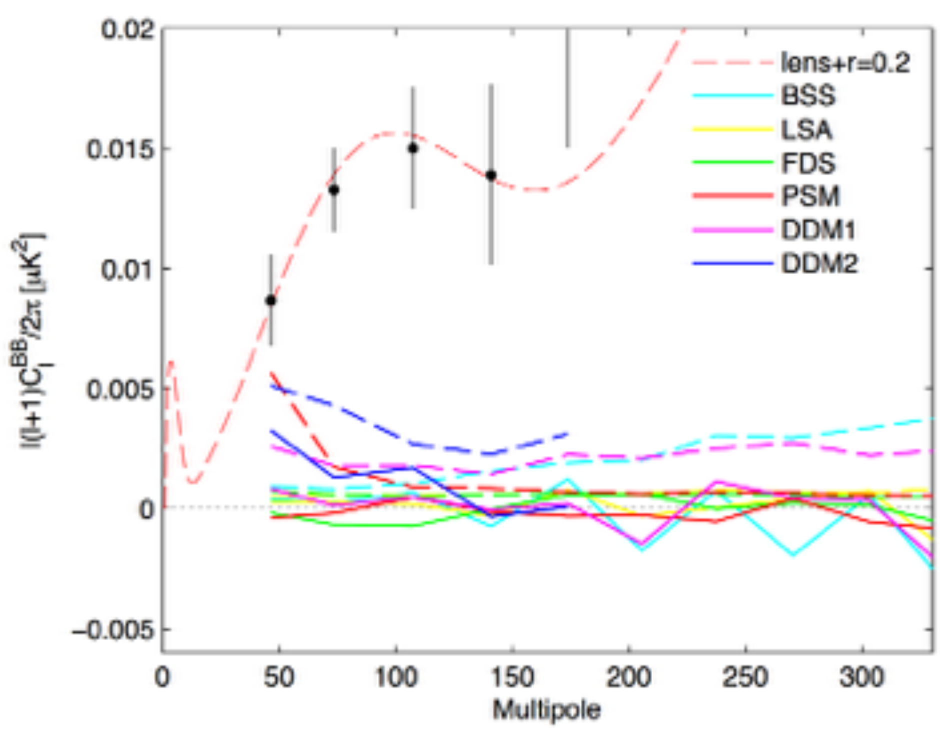
BICEP2 paper argues:

- Frequency dependence of signal is consistent with CMB (blackbody), inconsistent at $\sim 2\sigma$ with dust (ν^2).

(Statistical power of this test is low since most data is at 150 GHz)

- Amplitude of signal is too large to be consistent with wide range of dust models



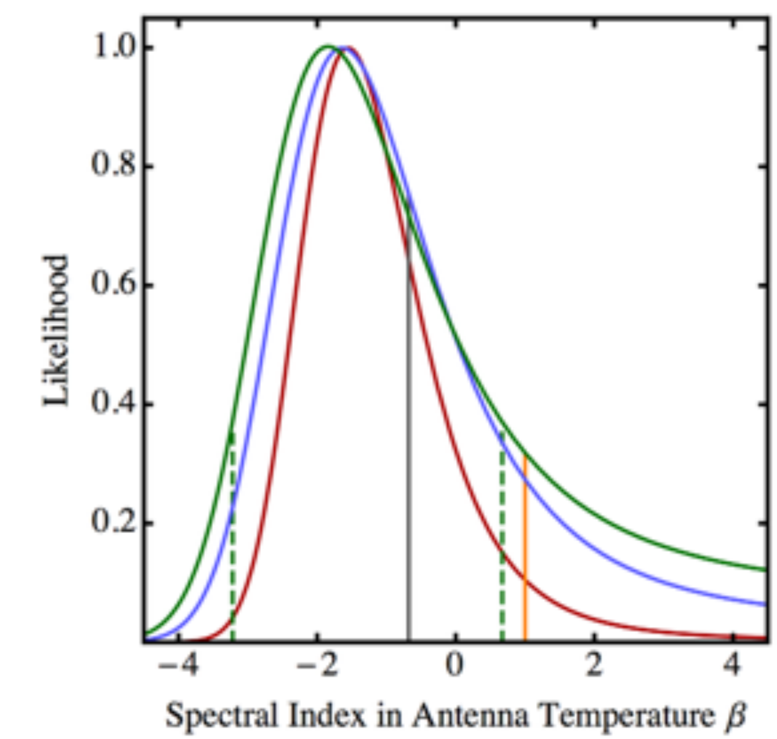


BICEP2 collaboration

BICEP2: gravity waves or dust?

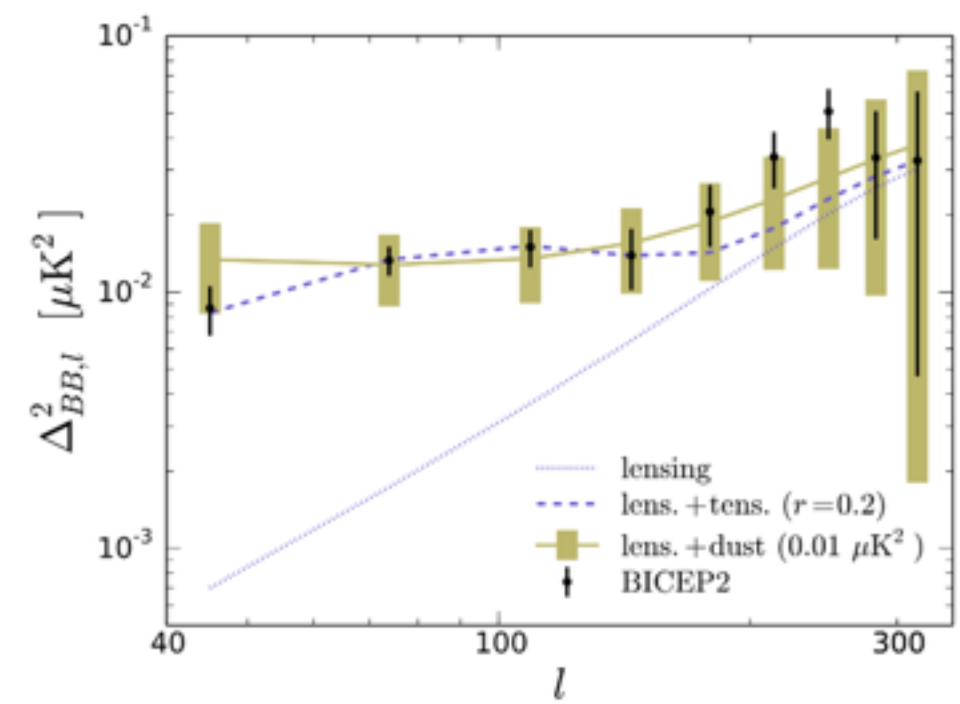
More recent work has argued that:

- Frequency dependence is consistent with dust, when dust sample variance and CMB lensing are accounted for



Flauger, Hill & Spergel, 1405.7351

- BICEP2 points can be fit by lensed CMB + dust model assuming ~10% polarization fraction, and this level of polarization is consistent with our current knowledge of foregrounds



Mortonson & Seljak 1405.5857

BICEP2: gravity waves or dust?

This controversy will be resolved soon, by some combination of:

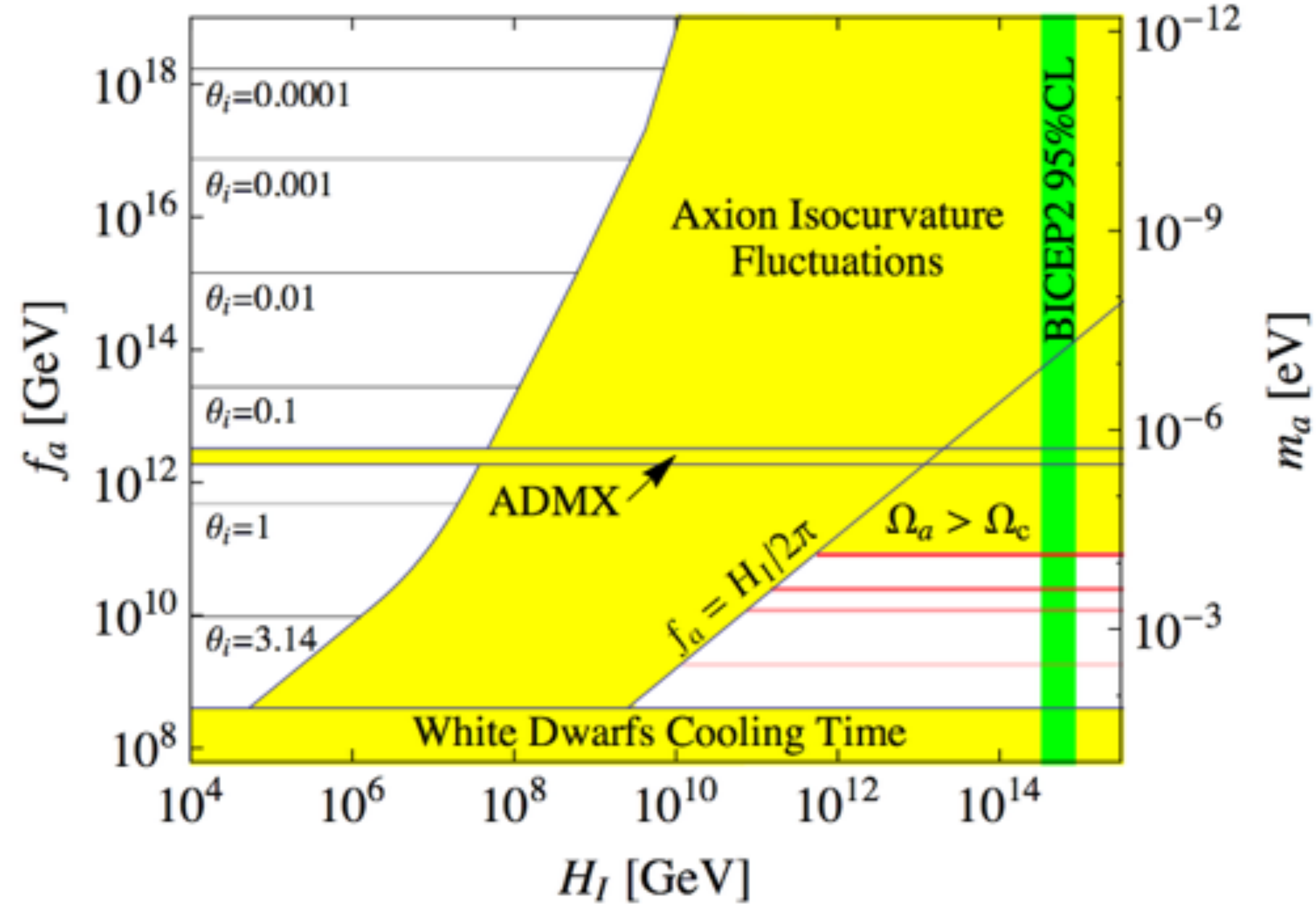
- Planck measurements at 353 GHz CMB polarization in the BICEP2 field
- Planck measurement of r (combining many frequencies)
- More 100 GHz data from Keck (BICEP2 successor)
- Data from other experiments? (SPTpol, ACTpol, Polarbear, ABS, others?)

In the meantime, best to take an agnostic approach?

In this talk, I'll consider both scenarios, where BICEP2 either does or does not hold up

Example: axion dark matter

Can dark matter be the QCD axion?



Visinelli & Gondolo, 1403.4594

If BICEP2 does not hold up: two regions of parameter space are allowed; axion mass can be between 10^{-12} and 10^{-2} eV.

If BICEP2 does hold up: only a small region of parameter space survives; axion mass is

$$m_a = (7 \times 10^{-5}\ \mathrm{eV})(\alpha + 1)^{6/7}$$

where α represents conversion rate of axion topological defects to axions; computable in principle with simulations

"Local" non-Gaussianity

In single-field slow roll inflation, the curvature perturbation ζ is Gaussian to an extremely good approximation (Maldacena 2002)

A simple non-Gaussian model: "local model"

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL}^{\rm loc} \zeta_G(\mathbf{x})^2 + \cdots$$

where ζ_G is a Gaussian field and $f_{NL}^{\rm loc}$ is a free parameter.

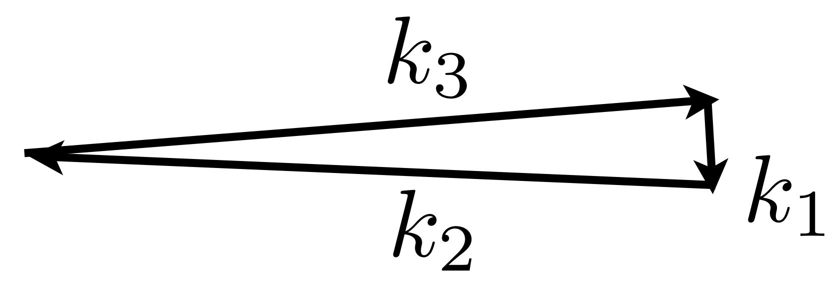
Occurs generically in multifield inflation, when fields other than the inflaton contribute to the curvature perturbation ζ

e.g. modulated reheating: spectator field σ controls decay rate of inflaton

e.g. "curvaton": spectator field decays to SM particles and gives dominant contribution to ζ

“Local” non-Gaussianity

$f_{NL}^{\rm loc}$ generates a nonzero three-point function $\langle \zeta_{k1}\, \zeta_{k2}\, \zeta_{k3} \rangle$, which turns out to have highest signal-to-noise in “squeezed” triangles



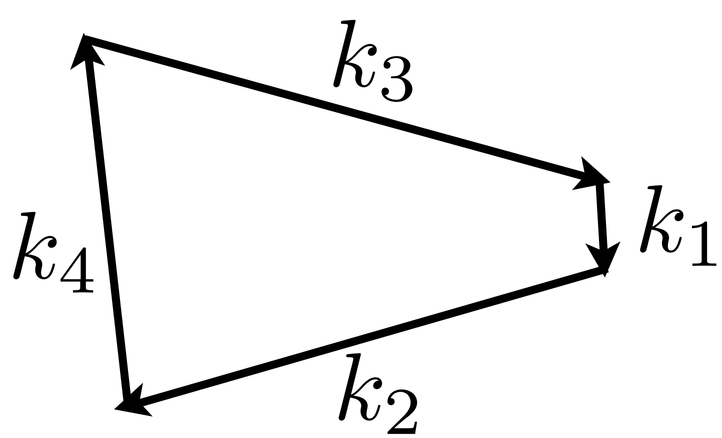
Planck: $f_{NL}^{\rm local} = 2.7 \pm 5.8$ (consistent with Gaussian)

“Local” non-Gaussianity

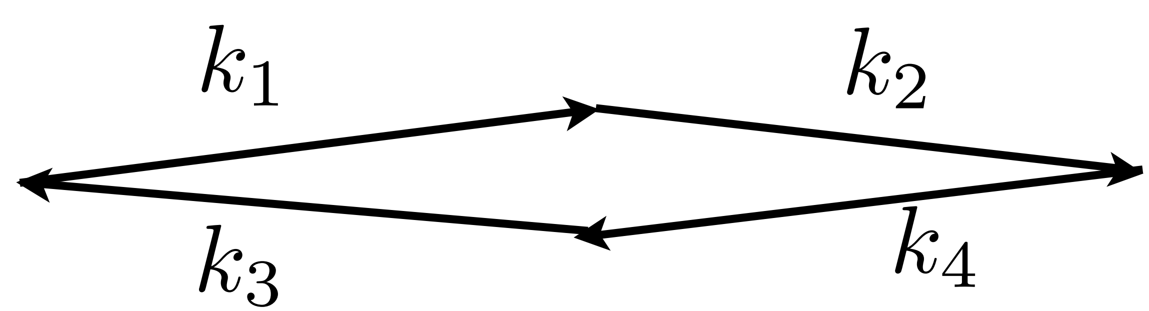
The local model can be generalized to include cubic terms or multiple non-Gaussian source fields:

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL}^{\mathrm{loc}} \zeta_G(\mathbf{x})^2 + \frac{9}{25} g_{NL} \zeta_G(\mathbf{x})^3$$
$$+ \zeta'_G(\mathbf{x}) + \frac{3}{5} f'_{NL} \zeta'_G(\mathbf{x})^2 + \cdots$$

This gives rise to 4-point signals…



g_{NL} (cubic terms)



τ_{NL} (multiple sources)

"Local" non-Gaussianity

If BICEP2 holds up: $f_{NL}^{\rm loc}$ is still possible, but more artificial and must be accompanied by four-point signal τ_{NL}

Reason: the "standard" relation r=16ε assumes that only the inflaton contributes to ζ. If other fields contribute then it generalizes to r=16εQ, where Q is the fraction (in power) contributed by the inflaton.

If r=0.2, and we need ε small, then Q can't be << 1

=> the inflaton contributes in addition to non-Gaussian sources

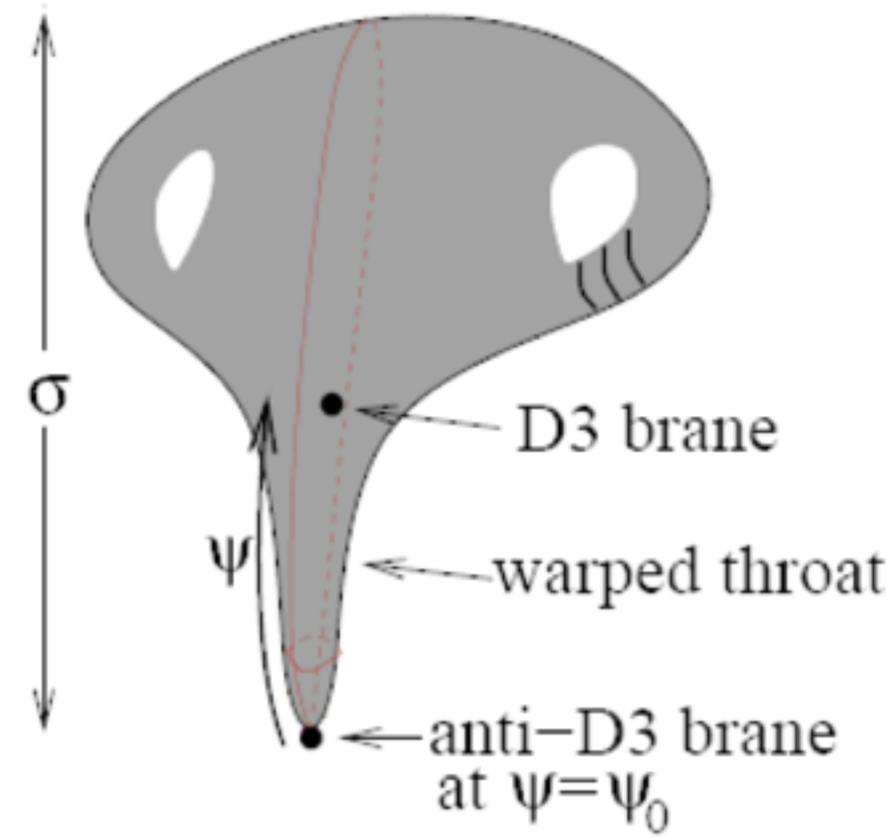
=> some level of tuning needed

=> if $f_{NL}^{\rm loc}$ is nonzero, must have at least two sources (inflaton + a non-Gaussian source), so expect τ_{NL}

“Non-local” NG: DBI inflation

String-motivated model of inflation (Alishahiha, Silverstein & Tong)

$$\mathcal{L} = -\frac{1}{g_s}\left(\frac{\sqrt{1+f(\phi)(\partial\phi)^2}}{f(\phi)} + V(\phi)\right)$$



After a suitable change of variables, the effective action can be approximated as a massless scalar with a $\dot{\sigma}^3$ interaction

$$S = \frac{1}{2}\int d\tau\, d^3x\, a(\tau)^2\left[\left(\frac{\partial\sigma}{\partial\tau}\right)^2 - (\partial_i\sigma)^2\right] + f\,a(\tau)\left(\frac{\partial\sigma}{\partial\tau}\right)^3$$

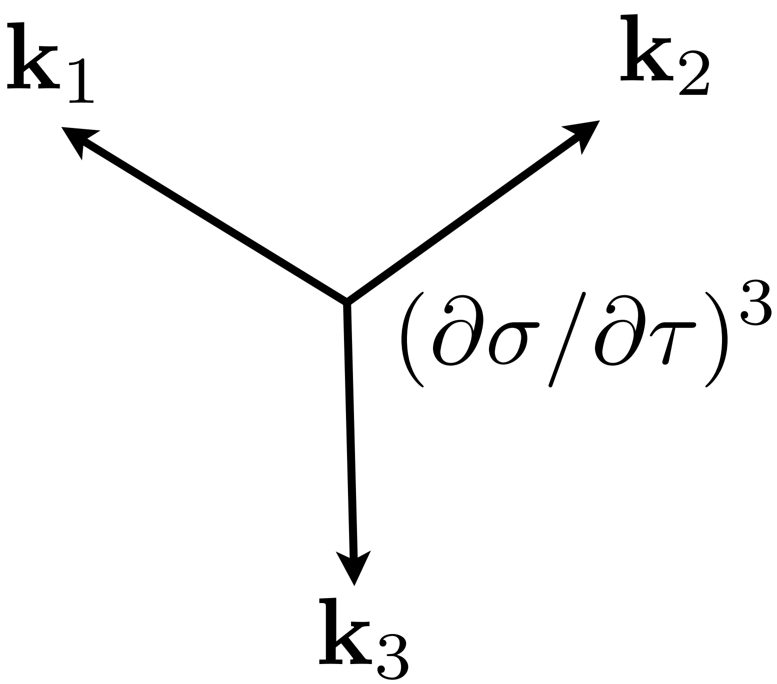
small coupling constant

“Non-local” NG: DBI inflation

DBI example:

$$S = \frac{1}{2}\int d\tau\, d^3x\, a(\tau)^2 \left[\left(\frac{\partial\sigma}{\partial\tau}\right)^2 - (\partial_i\sigma)^2\right] + f\, a(\tau)\left(\frac{\partial\sigma}{\partial\tau}\right)^3$$

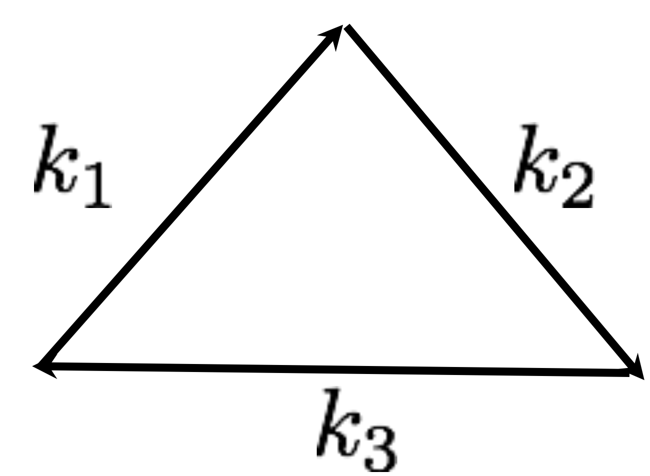
To first order in f, non-Gaussianity shows up in the 3-point function



$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \propto f\int_{-\infty}^{0} d\tau\, \frac{\tau^2 e^{(k_1+k_2+k_3)\tau}}{k_1k_2k_3}$$

$$= \frac{2f}{k_1k_2k_3(k_1+k_2+k_3)^3}$$

Signal-to-noise comes from equilateral triangles



EFT of inflation

π = Goldstone boson of spontaneously broken time translations

1-1 correspondence between operators in S_π and f_{NL}-like parameters
(Degree-N operator shows up in N-point CMB correlation function)

$$S_\pi = \int d^4x \sqrt{-g}(-\dot{H} M_{\rm pl}^2) \left[\frac{\dot{\pi}^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2} \right.$$

$$+ \frac{A}{c_s^2} \dot{\pi}^3 + \frac{1 - c_s^2}{c_s^2} \frac{\dot{\pi}(\partial_i \pi)^2}{a^2}$$

Equilateral+orthogonal 3-point functions
(Senatore, KMS & Zaldarriaga 2009)

$$+ B \pi_{ttt}^3 + C \pi_{ttt} \pi_{ijk}^2 + \cdots$$

Higher-derivative 3-point functions
(Behbahani, Mirbabayi, Senatore & KMS to appear)

$$+ D \dot{\pi}^4 + E \dot{\pi}^2 (\partial_i \pi)^2 + F (\partial_i \pi)^2 (\partial_j \pi)^2 + \cdots$$

4-point functions
(Senatore & Zaldarriaga 2009)

$$\left. + \frac{1}{2} \dot{\sigma}^2 - \frac{1}{2} (\partial_i \sigma)^2 - \frac{m^2}{2} \sigma^2 + \rho \dot{\pi} \sigma + G \sigma^3 \right]$$

Quasi single-field inflation
(Chen & Wang 2009,
Baumann & Green 2011)

Equilateral and orthogonal shapes

$$S = \int d^4x \, \sqrt{-g} \, (-\dot{H} M_{\rm Pl}^2) \times \left[\frac{\dot{\pi}^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2} + \frac{A}{c_s^2} \dot{\pi}^3 + \frac{1 - c_s^2}{c_s^2} \frac{\dot{\pi} (\partial_i \pi)^2}{a^2} \right]$$

The 3-point functions of the operators $\dot{\pi}^3, \; \dot{\pi}(\partial_i \pi)^2$ are highly correlated but not identical. Therefore, a "generic" linear combination gives the equilateral 3-point function, but there is a specific linear combination which gives a new "orthogonal" 3-point function.

Following this logic one can define 3-point observables $f_{NL}^{\rm equil}, \; f_{NL}^{\rm orthog}$ (Senatore, KMS & Zaldarriaga 2009)

Planck: $f_{NL}^{\rm equil} = -42 \pm 75 \qquad f_{NL}^{\rm orthog} = -25 \pm 39$

Equilateral and orthogonal shapes

Interesting fact: the coefficient of the operator $\dot{\pi}(\partial_i \pi)^2$ is determined by the sound speed c_s of the fluctuations

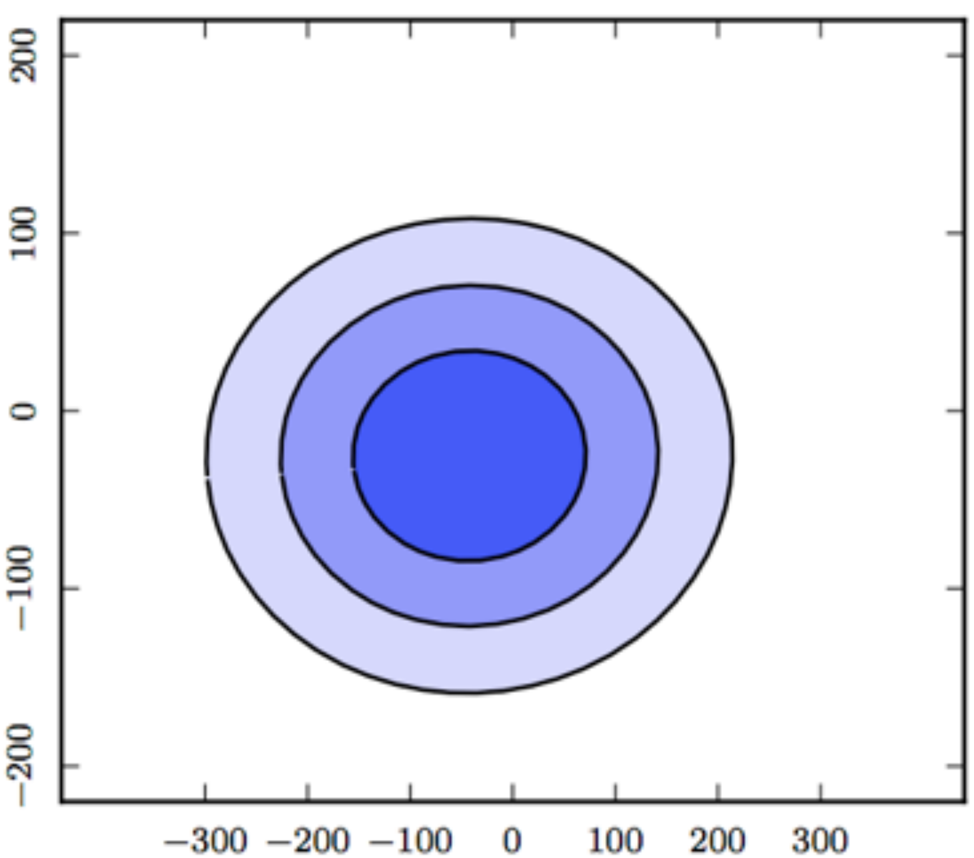
$$S = \int d^4x \sqrt{-g}\,(-\dot{H} M_{\rm Pl}^2) \times \left[\frac{\dot{\pi}^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2} + \frac{A}{c_s^2}\dot{\pi}^3 + \frac{1-c_s^2}{c_s^2}\frac{\dot{\pi}(\partial_i \pi)^2}{a^2}\right]$$

so we can interpret the measurement of $f_{NL}^{\rm equil},\ f_{NL}^{\rm orthog}$ as a measurement of c_s and the "nuisance" parameter A

Equilateral and orthogonal shapes

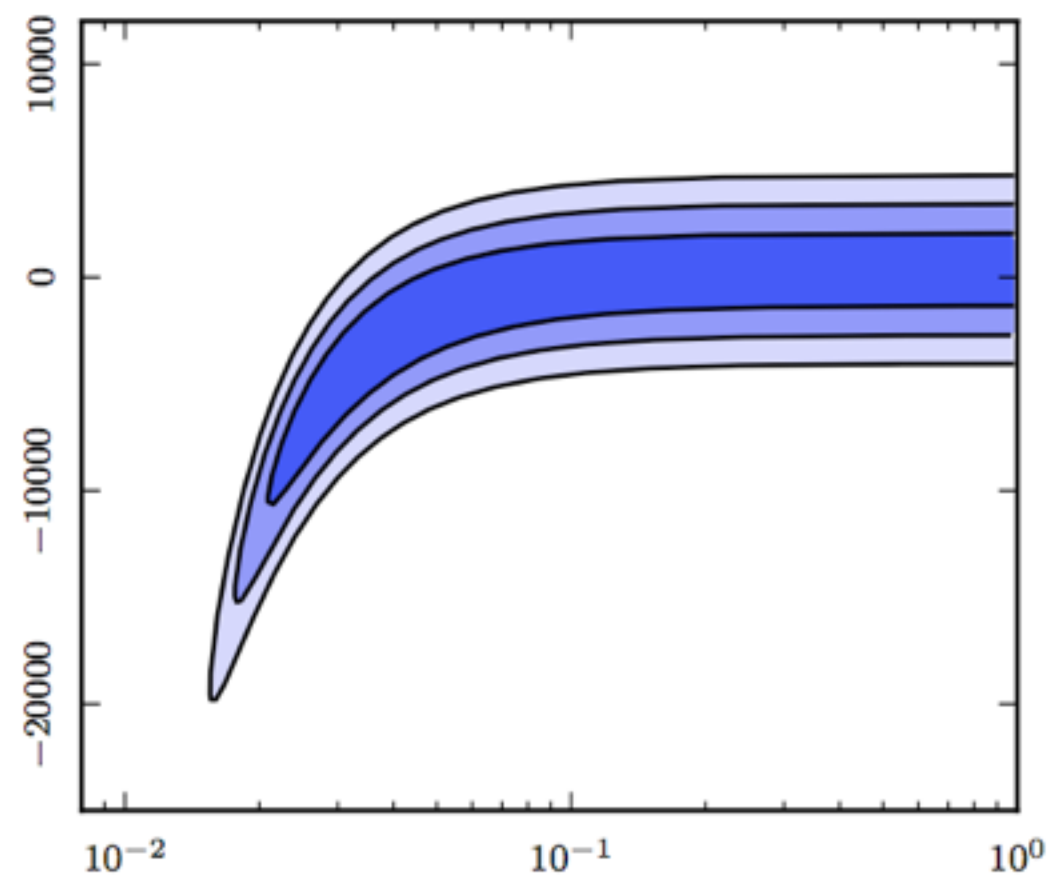
$$S=\int d^4x\,\sqrt{-g}\,(-\dot{H}M_{\rm Pl}^2)\times\left[\frac{\dot\pi^2}{c_s^2}-\frac{(\partial_i\pi)^2}{a^2}+\frac{A}{c_s^2}\dot\pi^3+\frac{1-c_s^2}{c_s^2}\frac{\dot\pi(\partial_i\pi)^2}{a^2}\right]$$

$f_{NL}^{\rm orthog}$



$f_{NL}^{\rm equil}$

A/c_s^2



c_s

Planck collaboration

Equilateral and orthogonal shapes

If the BICEP2 result holds up, what are the implications?

1. Lower bound on sound speed cs is suddenly much better!

   $r = 16\epsilon c_s$ => if r=0.2 and we need small ε to get inflation,
   then c_s cannot be too small

2. Can get $f_{NL}^{\rm equil}$ but not $f_{NL}^{\rm orthog}$!

$$S = \int d^4x \sqrt{-g}\, (-\dot{H} M_{\rm Pl}^2) \times \left[\frac{\dot{\pi}^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2} + \frac{A}{c_s^2}\dot{\pi}^3 + \underbrace{\frac{1-c_s^2}{c_s^2}}_{\text{small}} \frac{\dot{\pi}(\partial_i\pi)^2}{a^2} \right]$$

Non-Gaussianity: current status

Blue = constraint reported in recent Planck papers

Red = no constraint reported

3-point

local $\left\{ f_{NL}^{\text{local}} \right.$

nonlocal $\left\{ \begin{array}{l} f_{NL}^{\text{equil}}, f_{NL}^{\text{orthog}} \\ \text{Quasi-single field inflation} \\ \text{Higher-derivative models} \end{array} \right.$

4-point

local $\left\{ \begin{array}{l} \tau_{NL} \\ g_{NL} \end{array} \right.$

nonlocal $\left\{ \begin{array}{l} \text{Operators} \\ \quad (\dot{\pi}^4, \dot{\pi}^2(\partial\pi)^2, (\partial\pi)^4) \\ \text{Quasi-single field inflation} \end{array} \right.$

In all cases, no deviation from Gaussian statistics is found, but not all cases have been analyzed

Conclusion: what's next?

Most immediate priority is to reach a conclusion on whether BICEP2 is seeing gravity waves or dust emission. Need more data but this will happen very soon (months!)

If BICEP2 holds up, there are many implications:

- inflation confirmed and alternatives ruled out
- energy scale of inflation known (GUT-scale)
- many parameter spaces dramatically reduced (axion DM, primordial NG)
- next frontier will be precision measurements of the gravity wave B-mode, to get as much information as we can about $V(\phi)$ (e.g. $m^2\phi^2$ or something else?)

Conclusion: what's next?

In parallel, we'll continue the program of "parametrizing all possible surprises" and shrinking error bars. (curvature, neutrino mass, dark energy equation of state, primordial NG, etc.)

A medium-term goal: measure the neutrino mass, or more properly the sum of neutrino masses $\sum m_\nu$, guaranteed to be > 0.06 eV

Conclusion: what's next?

Looking further ahead, we're a long way from saturating ultimate limits on cosmological parameters, essentially by mode-counting:

CMB: 2d field, l_{max}=2000
$\Rightarrow$ (4 x 10^6 modes)

Large-scale structure: 3d field, k_{max}=0.5 Mpc^{-1}, z_{max}=1.0
$\Rightarrow$ (2 x 10^8 modes)

Ultimate futuristic scenario: large-scale structure at high redshift, measured via 21-cm line. In this case there is essentially no theoretical limit to the number of modes which might ultimately be measured (number of modes larger than the Jeans scale is 10^{18})

Conclusion: what's next?

Future experiments will need to solve many hard problems (hardware, modeling, and statistical). It's easy to be pessimistic in the face of hard problems. However, historical evidence suggests a perspective of cautious optimism…

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- Sunyaev: "I did not think that the acoustic oscillation would ever be observed"

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- Peebles: "I did not continue (with computation of CMB anisotropy), in part because I had trouble imagining that such tiny disturbances could be observed"
- Sunyaev: "I did not think that the acoustic oscillation would ever be observed"
- Mukhanov: "I thought it would take 1000 years to detect the logarithmic dependence of the power spectrum"