Self-correlations in fluctuation studies of heavy ion collisions

Aritra De

19 March 2020

UNIVERSITY OF MINNESOTA
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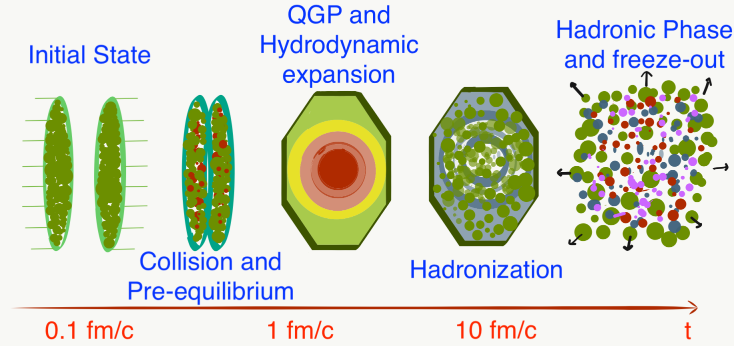
3rd JETSCAPE Winter Workshop 2020

A. De, C. Plumberg and J. I. Kapusta, arXiv:2003.04878 [nucl-th]

A very short introduction to Quark Gluon Plasma

- If we subject the protons and neutrons residing inside the nucleus to sufficiently high temperature, the quarks and gluons get deconfined and form a thermalized state called quark-gluon plasma (QGP).
- QGP is believed to have existed during the early universe.



Electric charge diffusion in QGP

- Hydrodynamics involves study of conservation equations for energy-momentum, electric charge, baryon charge etc.
- $J_Q^\mu = nu^\mu + \Delta J^\mu + I^\mu$
  n is the proper charge density, ΔJ^μ is viscous correction and I^μ is the stochastic noise.
- Consider the case of white noise and ordinary diffusion equation.
- $\left(\frac{\partial}{\partial t} - D\nabla^2\right) n = 0$
- The 2-point function of white noise (Fluctuation-Dissipation Theorem) [**White noise**]

$$\langle I^\mu(x_1) I^\nu(x_2)\rangle = 2\sigma_Q T\, h^{\mu\nu} \delta^4(x_1 - x_2)$$

White noise and Colored noise

- The ordinary diffusion equation leads to an infinite speed of signal propagation.
- $\left(\frac{\partial}{\partial t} - D\nabla^2 + \tau_Q \frac{\partial^2}{\partial t^2}\right) n = 0$. This is called the Catteneo equation.
- Finite signal propagation speed of $v_Q = \sqrt{\frac{D}{\tau_Q}}$.
- Now the noise function becomes [**Colored noise**]

$$\langle I^i(x_1) I^j(x_2)\rangle = \frac{\sigma_Q T}{\tau_Q} \delta^3(\vec{x}_1 - \vec{x}_2) e^{-\frac{|t_1 - t_2|}{\tau_Q}} \delta_{ij}$$

Stochastic Differential Equations

- Bjorken coordinates $\tau = \sqrt{t^2 - z^2}$ and $\xi = \tanh^{-1}(\frac{z}{t})$ in 1+1D.
- The noise part of the current is

$$I^0 = s(\tau) f(\xi, \tau) \sinh \xi \qquad I^3 = s(\tau) f(\xi, \tau) \cosh \xi$$

  f is dimensionless and is the fluctuating part.
- We use the quantity $X = \tau \delta n$ for two-point correlation in δn.
- The charge conservation equation is $\partial_\mu J^\mu_Q = 0$ where $J^\mu_Q = nu^\mu + \Delta J^\mu + I^\mu$.

$$\boxed{\left(\frac{\tau}{D\chi T} + \tau_Q \frac{\partial}{\partial \tau}\left(\frac{\tau}{D\chi T}\right)\right)\frac{\partial X}{\partial \tau} + \frac{\tau_Q \tau}{D\chi T}\frac{\partial^2 X}{\partial \tau^2} - \frac{1}{\tau \chi T}\frac{\partial^2 X}{\partial \xi^2} + \left(\frac{\tau s}{D\chi T} + \tau_Q \frac{\partial}{\partial \tau}\left(\frac{\tau s}{D\chi T}\right)\right)\frac{\partial f}{\partial \xi} + \frac{\tau_Q \tau s}{D\chi T}\frac{\partial^2 f}{\partial \xi \partial \tau} = 0}$$

- For the case $\tau_Q = 0$ (white noise), this equation becomes

$$\frac{\partial X}{\partial \tau} - \frac{D}{\tau^2}\frac{\partial^2 X}{\partial \xi^2} + s\frac{\partial f}{\partial \xi} = 0$$

Setting up the problem

The Objective : Calculation of effect of hydrodynamic fluctuations on the final particle correlations in the detectors.

The Procedure : Simulating the stochastic hydro equations to calculate the charge fluctuations and calculating the charge balance functions.

Discrete lattice with noise

- These equations are coupled stochastic differential equations (SDEs).
- First we set up white noise on a discrete lattice.

$$f \sim \mathcal{N}\left(0, \frac{1}{\sqrt{\text{grid-size}}}\right) \Rightarrow \langle f_i f_{i'} \rangle = \frac{1}{\text{grid-size}} \delta_{ii'}$$

- Figuring out derivative of white noise on a lattice.

$$\langle f(x_i) f'(x_{i'}) \rangle = \frac{\delta_{i+1,i'} - \delta_{i,i'}}{\Delta x^2}$$

$$\langle f'(x_i) f'(x_{i'}) \rangle = -\frac{\delta_{i,i'+1} + \delta_{i,i'-1} - 2\delta_{i,i'}}{\Delta x^3}$$

- Figuring out integration of white noise on a lattice.

$$\left\langle \int_{z_i}^{z} dz' f(z') \int_{z_i}^{z} dz'' f(z'') \right\rangle = (z - z_i)$$

Analytical solution to the white noise SDE

The white noise SDE is

$$\frac{\partial X}{\partial \tau} - \frac{D}{\tau^2}\frac{\partial^2 X}{\partial \xi^2} + s\frac{\partial f}{\partial \xi} = 0$$

The two-point correlation for the noise fluctuation which is a solution of this SDE in our discretized system turns out to be (Kapusta,Plumberg, 2018)

$$\langle \delta n(\xi_1, \tau_2) n(\xi_2, \tau_2) \rangle = \frac{\chi_Q(\tau_2) T_f}{A\tau_2} \left[\delta(\xi_1 - \xi_2) - \frac{1}{\sqrt{\pi w^2}} e^{-\frac{(\xi_1 - \xi_2)^2}{w^2}} \right]$$

where $w^2 = 8D_Q(\frac{1}{\tau_i} - \frac{1}{\tau_f})$ and A is the transverse area of the Bjorken system. In our discrete lattice, what we expect is:

$$\langle \delta n(\xi_1, \tau_2) \delta n(\xi_2, \tau_2) \rangle = \frac{\chi_Q(\tau_2) T_f}{A\tau_2} \left[\frac{1}{\Delta \xi} \delta_{\xi_1 \xi_2} - \frac{1}{\sqrt{\pi w^2}} e^{-\frac{(\xi_1 - \xi_2)^2}{w^2}} \right]$$

Solution to the white noise SDE

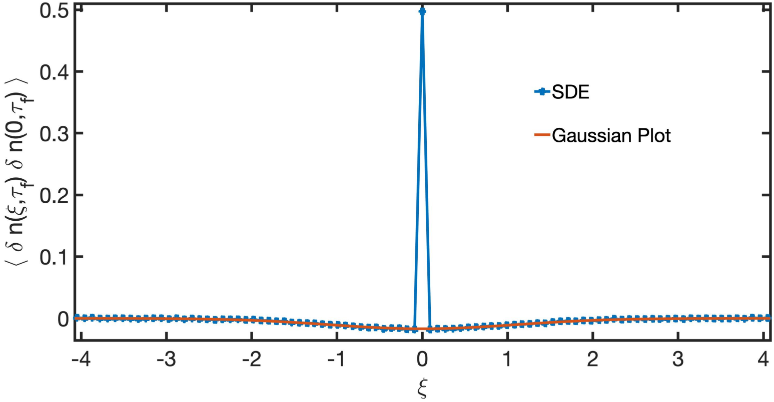
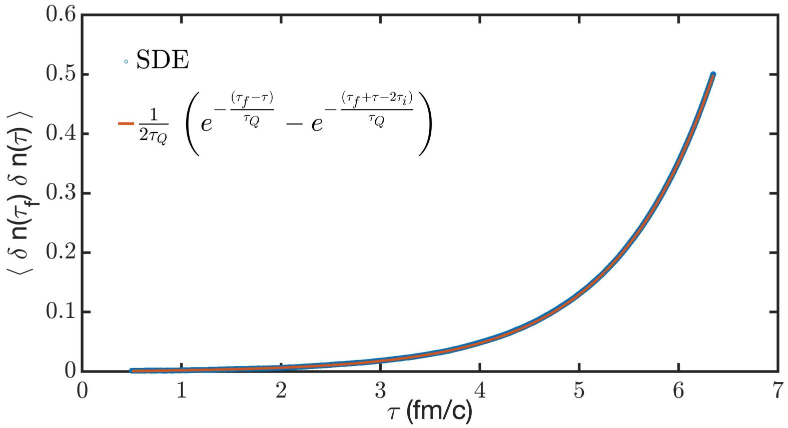


Figure: $\langle \delta n(\xi_1, \tau_2) \delta n(\xi_2, \tau_2) \rangle = \frac{\chi_Q(\tau_2) T_f}{A \tau_2} \left[\frac{1}{\Delta \xi} \delta_{\xi_1 \xi_2} - \frac{1}{\sqrt{\pi w^2}} e^{-\frac{(\xi_1 - \xi_2)^2}{w^2}} \right]$ for one million random events.

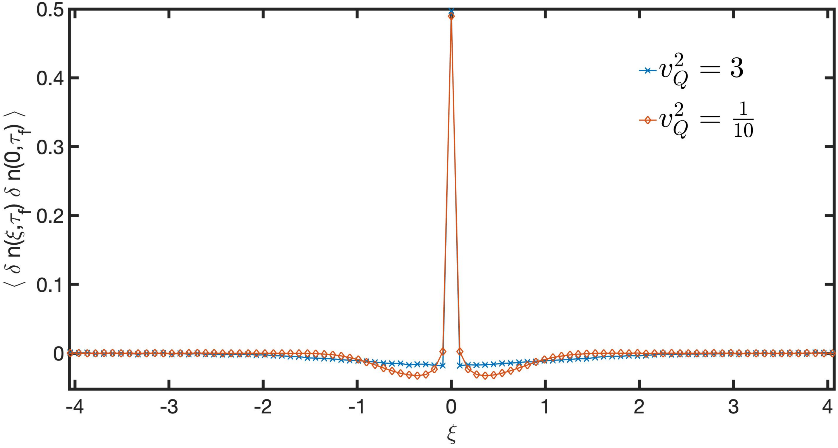
Colored noise

- We solve the Langevin equation. $f + \tau_Q \frac{\partial f}{\partial \tau} = \zeta$



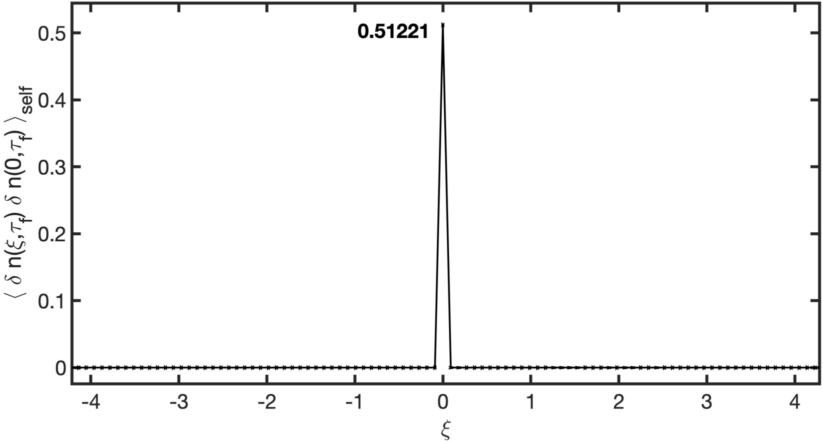
Solution of Catteneo noise SDE

$$\left(\frac{\tau}{D\chi T}+\tau_Q\frac{\partial}{\partial\tau}\left(\frac{\tau}{D\chi T}\right)\right)\frac{\partial X}{\partial\tau}+\frac{\tau_Q\tau}{D\chi T}\frac{\partial^2 X}{\partial\tau^2}-\frac{1}{\tau\chi T}\frac{\partial^2 X}{\partial\xi^2}+\left(\frac{\tau s}{D\chi T}+\tau_Q\frac{\partial}{\partial\tau}\left(\frac{\tau s}{D\chi T}\right)\right)\frac{\partial f}{\partial\xi}+\frac{\tau_Q\tau s}{D\chi T}\frac{\partial^2 f}{\partial\xi\partial\tau}=0$$



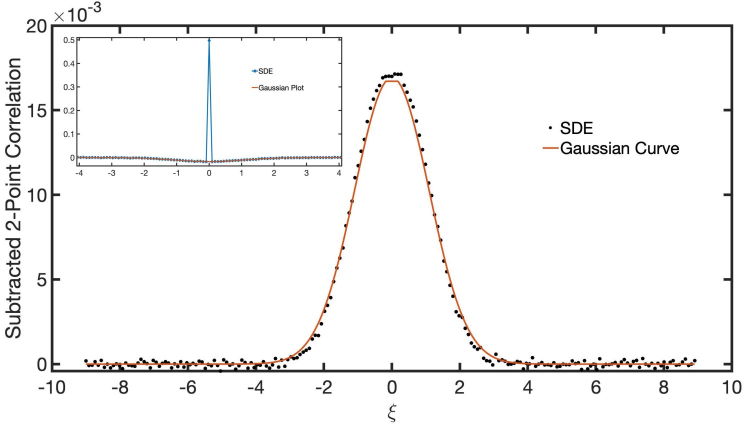
Self-correlations for white noise

The Self-correlations are trivial for the white noise case. It is a Dirac-delta function with the expected amplitude $\frac{\chi_f T_f}{\tau_f \Delta\xi} = 0.5144\ \mathrm{MeV}^3\,\mathrm{fm}^{-3}$.

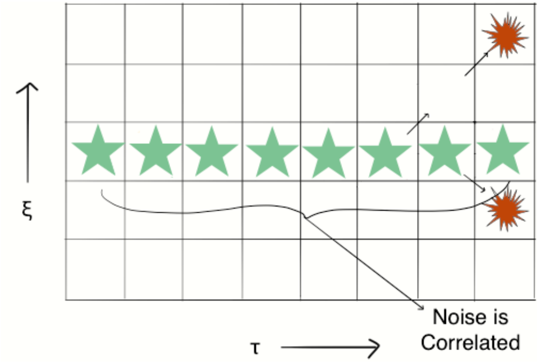


Subtracted correlation function for white noise

$$\langle \delta n(\xi_1, \tau_2) \delta n(\xi_2, \tau_2) \rangle = \frac{\chi_Q(\tau_2) T_f}{A\tau_2} \left[\frac{1}{\Delta\xi} \delta_{\xi_1 \xi_2} - \frac{1}{\sqrt{\pi w^2}} e^{-\frac{(\xi_1 - \xi_2)^2}{w^2}} \right]$$

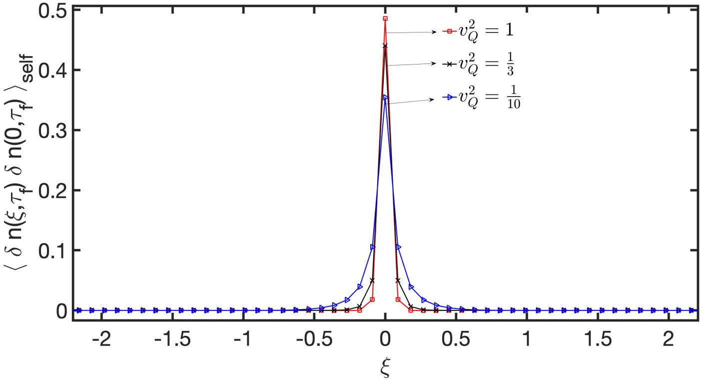


Definition of self-correlation



The definition of self-correlation can be interpreted as the correlation of a charge fluctuation generated in ξ_1 at final time τ_f with another charge fluctuation generated at the same ξ_1 but at a previous time and hence has traveled to a different ξ_2 at τ_f .

Self-correlations for colored noise



$$\langle \delta n(\xi_1, \tau_f) \delta n(\xi_2, \tau_f) \rangle_{\text{self}}$$

$$= \frac{\tau_f}{D_Q} \int \frac{dk}{2\pi} e^{ik(\xi_1 - \xi_2)} \int \frac{d\tau''}{\tau''} \frac{\tilde{G}(-k, \tau_f, \tau'')}{ik} N(\tau_f, \tau'')$$

Charge balance functions

- The balance function measures the difference in conditional probability of finding a particle of opposite charge versus a particle of same charge given a charged particle in a different fluid cell. The formula for calculating the balance function in our case is

$$B(\Delta y) = \frac{\langle \delta\left(\frac{(dN_+ - dN_-)}{dy_1}\right)\delta\left(\frac{(dN_+ - dN_-)}{dy_2}\right)\rangle}{\langle\frac{(dN_+ + dN_-)}{dy}\rangle} = \frac{dA\tau_f T_f}{4\pi^2}\frac{C(\Delta y)}{Q(\frac{m}{T_f})}$$

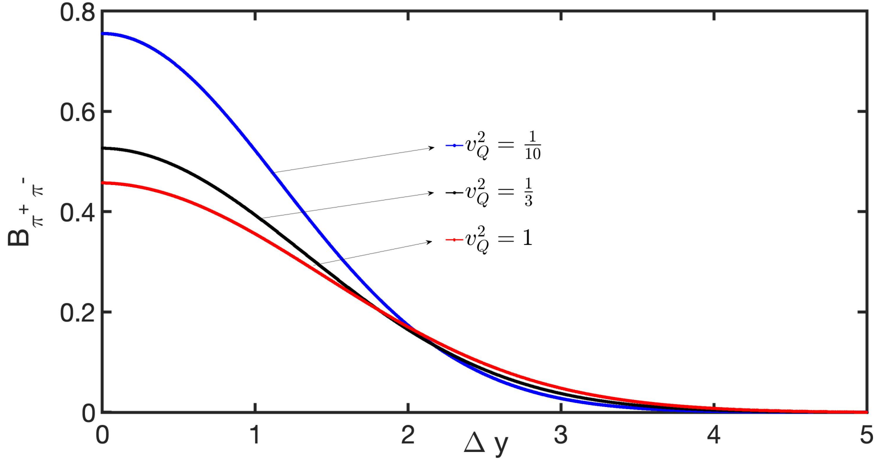
Here $B(\Delta y)$ is the balance function and $C(\Delta y)$ is the charge correlation in rapidity y

$$C(\Delta y) = \int d\xi_1 \int d\xi_2 F_n(y_1 - \xi_1) F_n(y_2 - \xi_2) C_{nn}(\xi_1 - \xi_2, \tau_f)$$

$F_n(x) = \frac{1}{\chi_Q \cosh^2 x}\Gamma(3, \frac{m}{T_f}\cosh x)$ is a thermal smearing function and $Q(\frac{m}{T_f})$ is constant for a hadron.

Charge balance functions for pions with different v_Q

The width of the balance function denotes the diffusion distance.



Summary

- Numerical simulation of the white noise and colored noise SDEs.
- Physical interpretation of the self-correlations in the case of colored noise.
- Numerical prescription for cancelling out the self-correlations in a hydro simulation in the presence of Catteneo noise. This prescription can be straightforwardly extended to the case of Gurtin-Pipkin noise.
- The pattern of balance functions for the various values of v_Q matches expectations.
- The results that we get from this exercise on a fairly simple model can also be used to benchmark results from other more involved hydrodynamic simulations in 3+1 D like MUSIC.

Thank You for listening!