QUANTIFYING THE QGP

Derek Everett (on behalf of JS SIMS WG)
The Ohio State University

OVERVIEW

- Big questions in heavy ion physics
- Bayesian parameter estimation
- Summary of hybrid model
- Estimating the properties of Quark Gluon Plasma (QGP)
- Quantifying observables sensitivity
- Bayesian model selection for QGP

BIG QUESTIONS

Is the system produced in heavy ion collisions ‘strongly-coupled’?

What is the structure of nuclei probed at high energies?

What are the dynamics of partons in a QCD medium?

BIG QUESTIONS (QUANTITATIVE)

Is the system produced in heavy ion collisions ‘strongly-coupled’?

→ what is the shear viscosity?

What is the structure of nuclei probed at high energies?

→ what scales characterize the initial energy?

What are the dynamics of partons in a QCD medium?

→ what is the transverse momentum diffusion?

*and how sure are we?

BAYESIAN PARAMETER ESTIMATION

Procedure to estimate probability distributions of model parameters, given ingredients:

1. A theory/model (viscous hydro hybrid model)
2. Assumption for model and exp. error (multivariate normal)
3. Our prior belief about the parameter's probability distribution
4. Measurements (data from Large Hadron Collider (LHC) and Relativistic Heavy Ion Collider (RHIC))

BAYESIAN PARAMETER ESTIMATION

…producing a **posterior:**

Posterior : the joint probability distribution of all model parameters

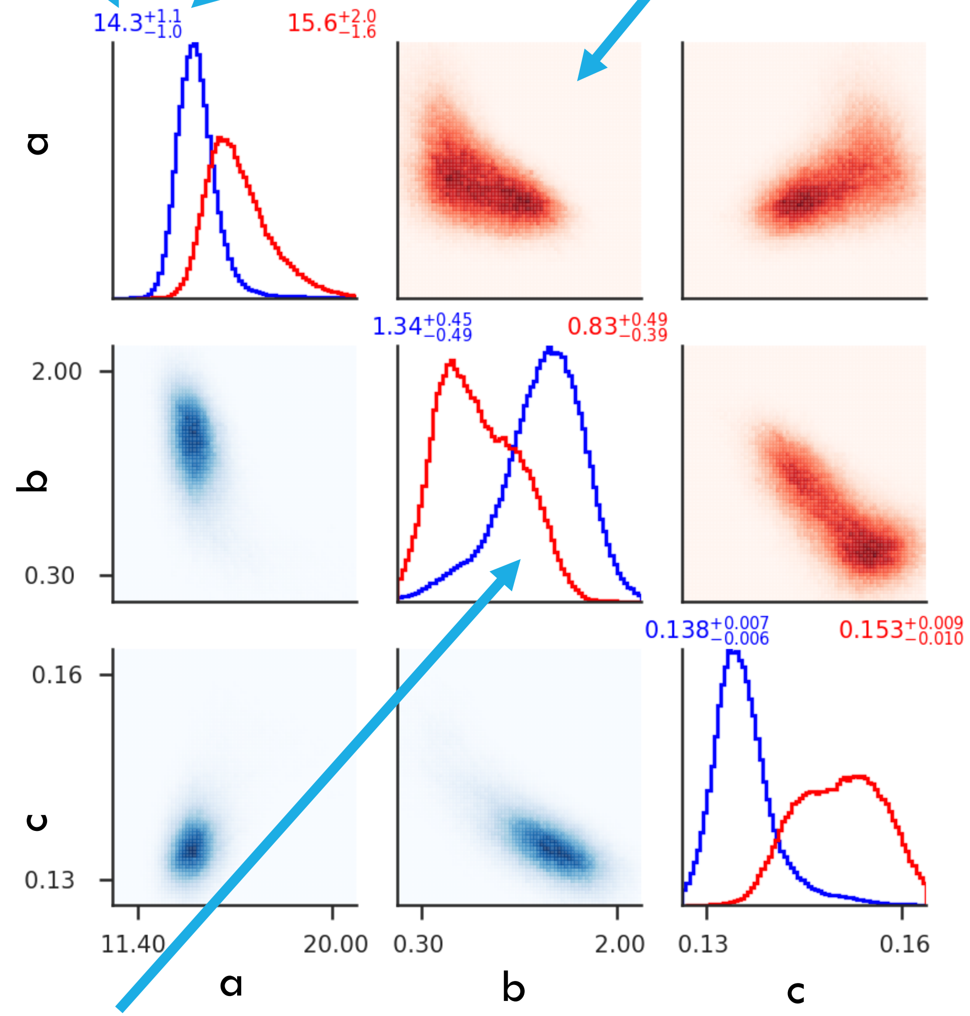
Note : For 3+ model parameters, visualization is hard…

We often resort to plotting 'corner plots' (example at right)

Median

90% Confidence Interval

Off-diagonal : joint posterior of pair, having marginalized over rest.

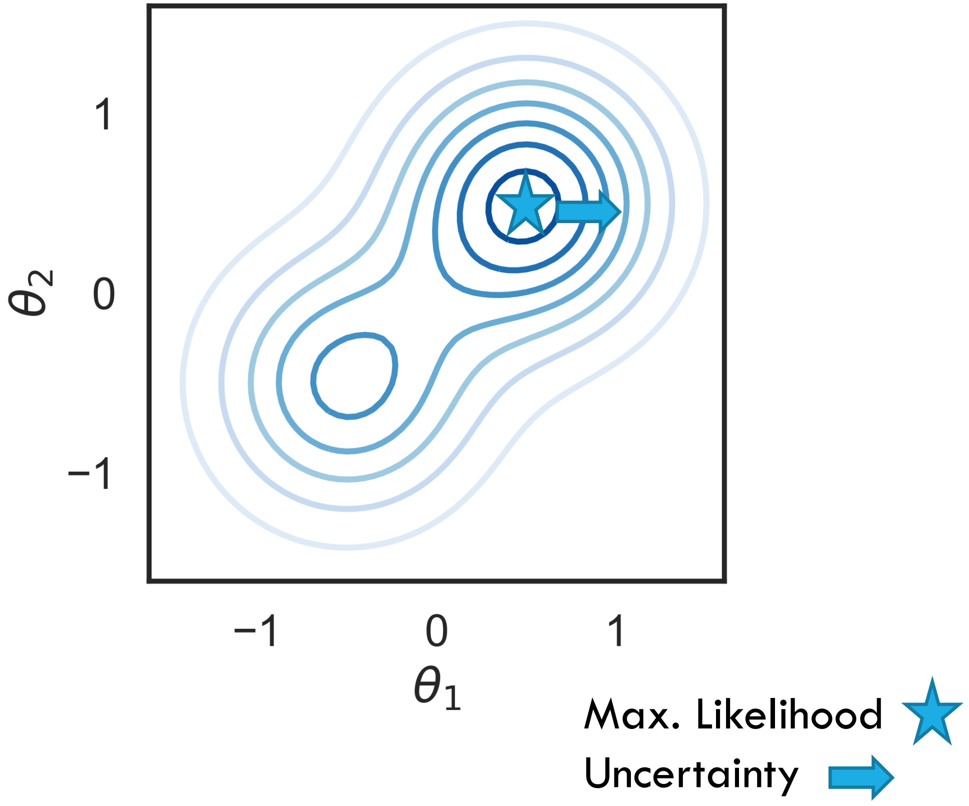


Diagonal : posterior of single parameter, having marginalized over rest.

Blue and Red are <u>different models</u>

WHY BAYES IS BETTER

Model w/bimodal likelihood



The max. likelihood and uncertainty don't reveal all useful info:

1. θ_1 and θ_2 are correlated
2. The distribution is bi-modal

Most distributions (except Gaussians) can not be understood with two numbers

Also : we can handle `nuisance parameters' straightforwardly (we will come back to this later)

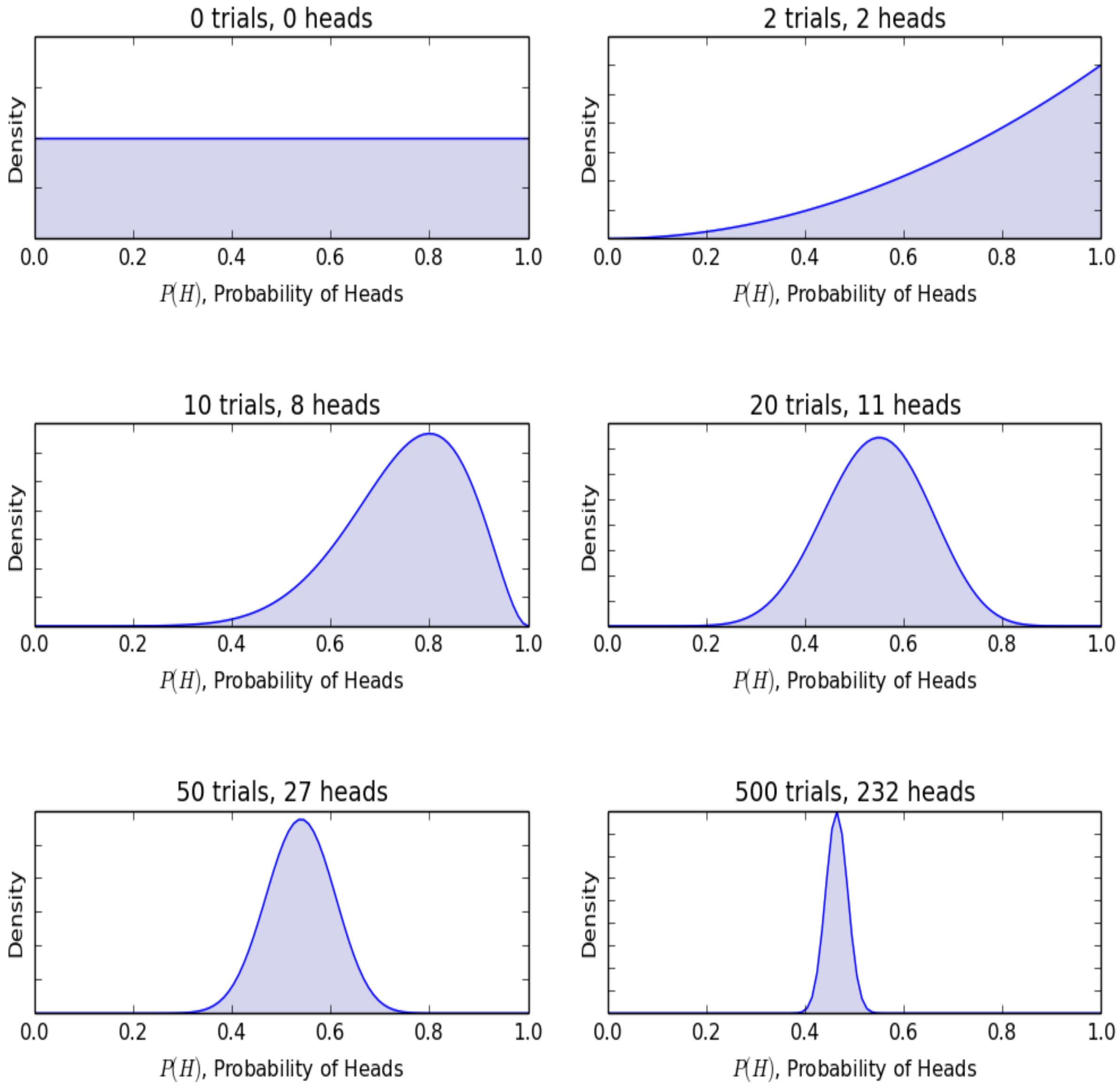
BAYES' THEOREM

Suppose we have a model with a parameter θ…

Bayes' THM : $p(\theta|D) \sim p(D|\theta)p(\theta)$

- $p(\theta)$: our ***prior*** belief for parameter θ (before we see data D)
- $p(D|\theta)$: ***likelihood*** we would see data D given the parameter θ
- $p(\theta|D)$: our ***posterior*** for parameter θ, given the data D

Example: Flipping a coin



https://www.analyticsvidhya.com/blog/2016/06/bayesian-statistics-beginners-simple-english/

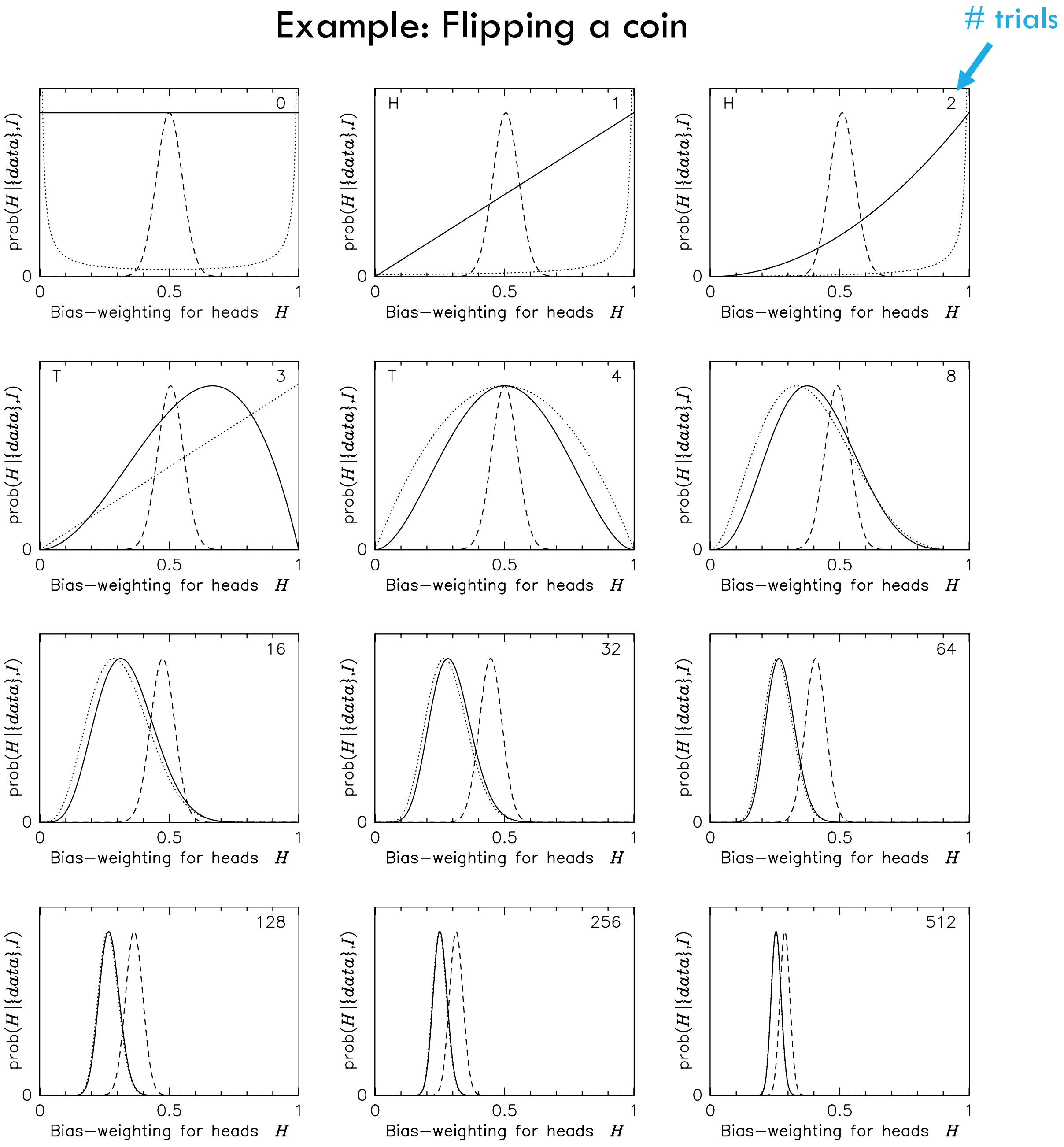
CHOOSING THE PRIOR

The prior is important, especially when we don't have 'enough data'

Three different priors (solid, dashed, dotted)

With ~8 trials, dashed posterior is ~same as dashed prior ('Returning the prior')

Example: Flipping a coin



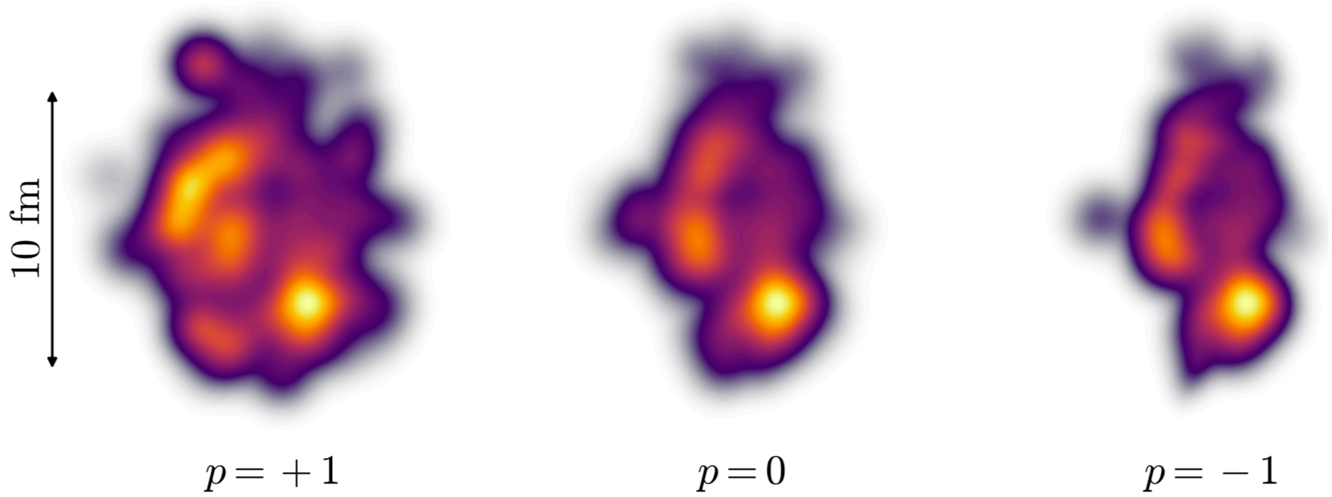
Bayesian Data Analysis (Sivia) p.18

Description of Hybrid Model

INITIAL ENERGY DEPOSITION (TRENTO)

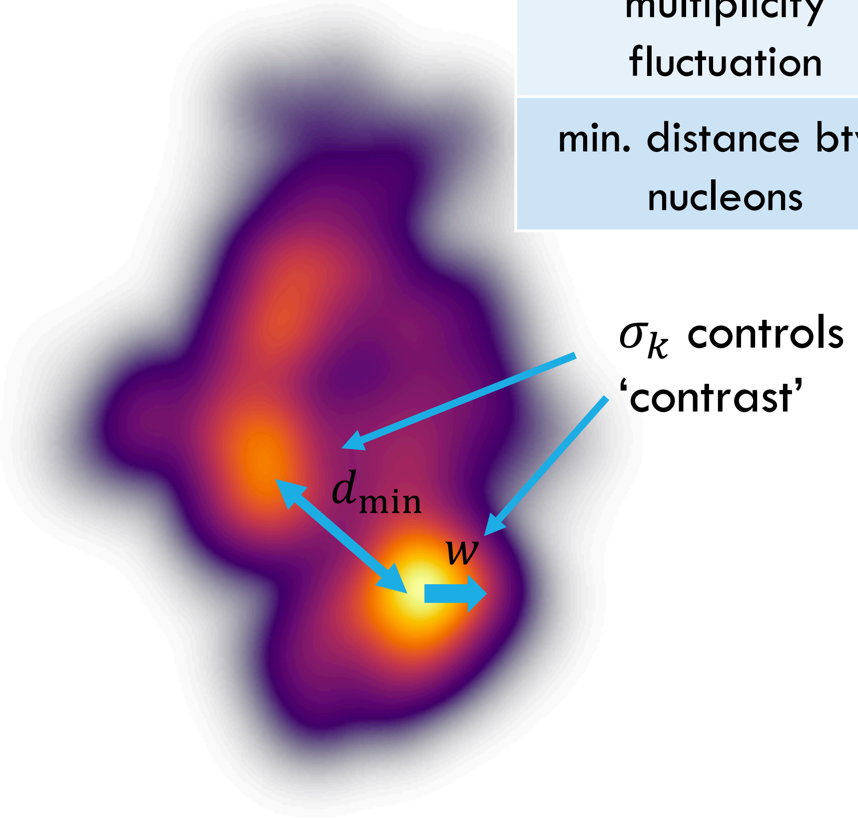
Parameter	Symbol
reduced thickness	p
nucleon width	w
energy normalization	N
multiplicity fluctuation	σ_k
min. distance btw. nucleons	$d_{\min}$

Parameterization for energy deposition at $\tau = 0^+$



Pb-Pb @ 2.76 TeV $w = 0.4$fm

arXiv:1904.08290v1



PRE-HYDRO (FREE-STREAMING)

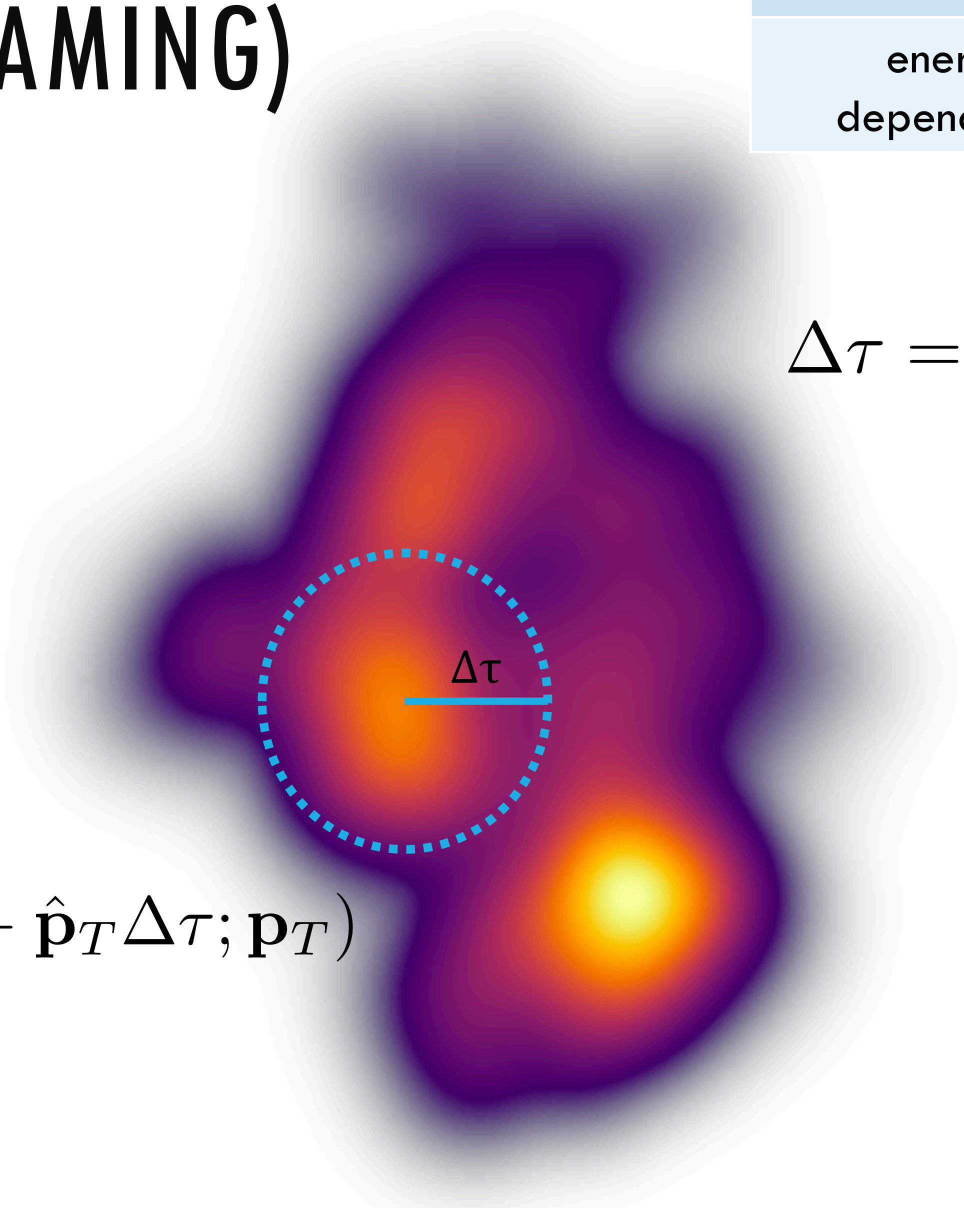
Parameter	Symbol
ref. proper time	τ_R
energy dependence	α

Free-stream massless particles:

$$f(t, \mathbf{x}; \mathbf{p}) = f(t_0, \mathbf{x} - \mathbf{v}\Delta t; \mathbf{p})$$

$$\Delta\tau = \tau_R \left(\frac{\langle \epsilon \rangle}{\epsilon_R} \right)^{\alpha}$$

Take initial momentum-distribution isotropic in transverse plane



$$T^{\mu\nu}(\tau_h) = \frac{\tau_0}{\tau_h} \int \frac{d\phi}{2\pi} \hat{p}^{\mu} \hat{p}^{\nu} T^{\tau\tau}(\tau_0, \mathbf{x}_T - \hat{\mathbf{p}}_T \Delta\tau; \mathbf{p}_T)$$

VISCOUS HYDRO (MUSIC)

Energy-momentum conservation $\nabla_\mu T^{\mu\nu} = 0$

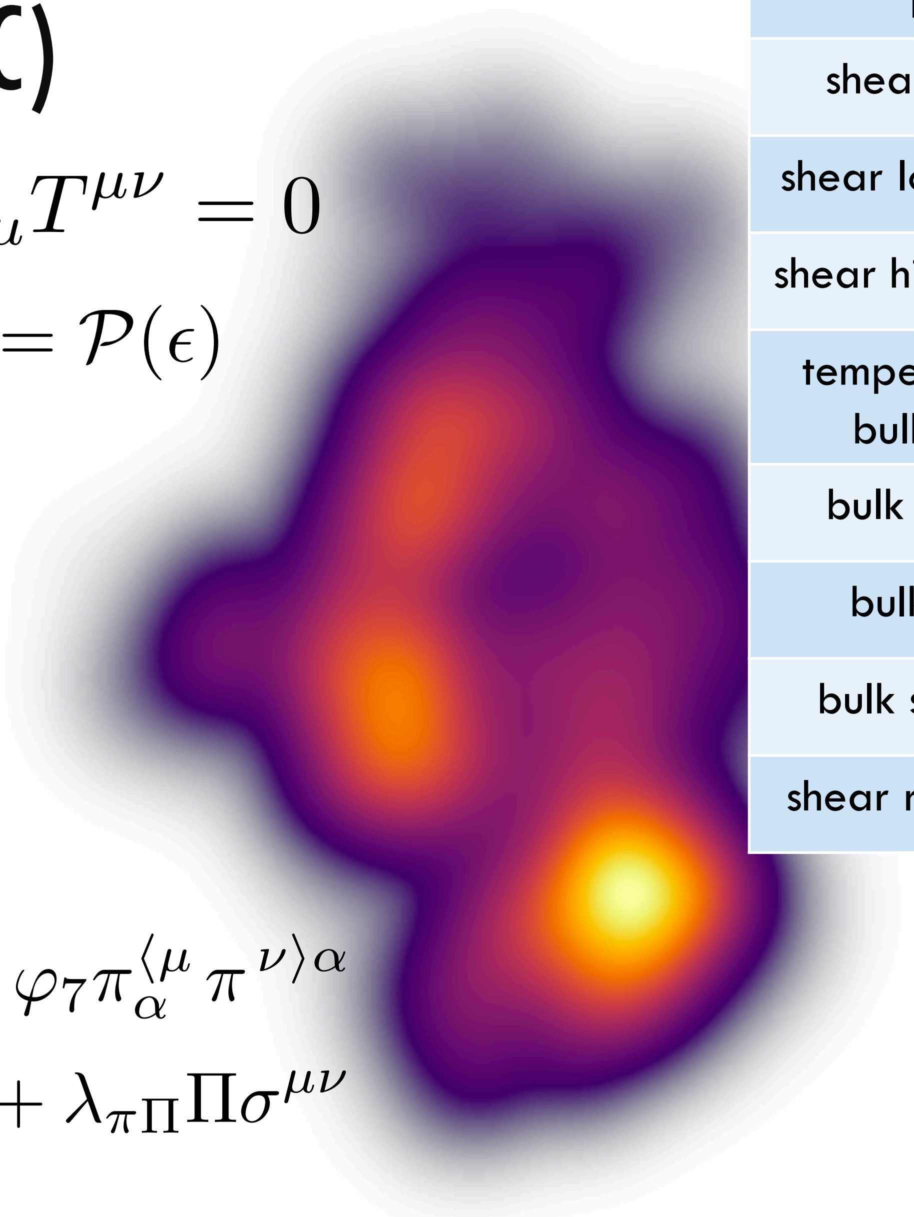
Eqn. of state matches lattice and hadron resonance gas $\mathcal{P} = \mathcal{P}(\epsilon)$

and relaxation eqns…

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

$$\tau_\pi = b_\pi \frac{\eta}{sT}$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}$$



Parameter	Symbol
temperature of kink	T_η
shear at kink	$(\eta/s)_{\mathrm{kink}}$
shear low-T slope	a_{low}
shear high-T slope	a_{high}
temperature of bulk peak	T_ζ
bulk at peak	$(\zeta/s)_{\mathrm{max}}$
bulk width	w_ζ
bulk skewness	λ
shear relax. time	b_π

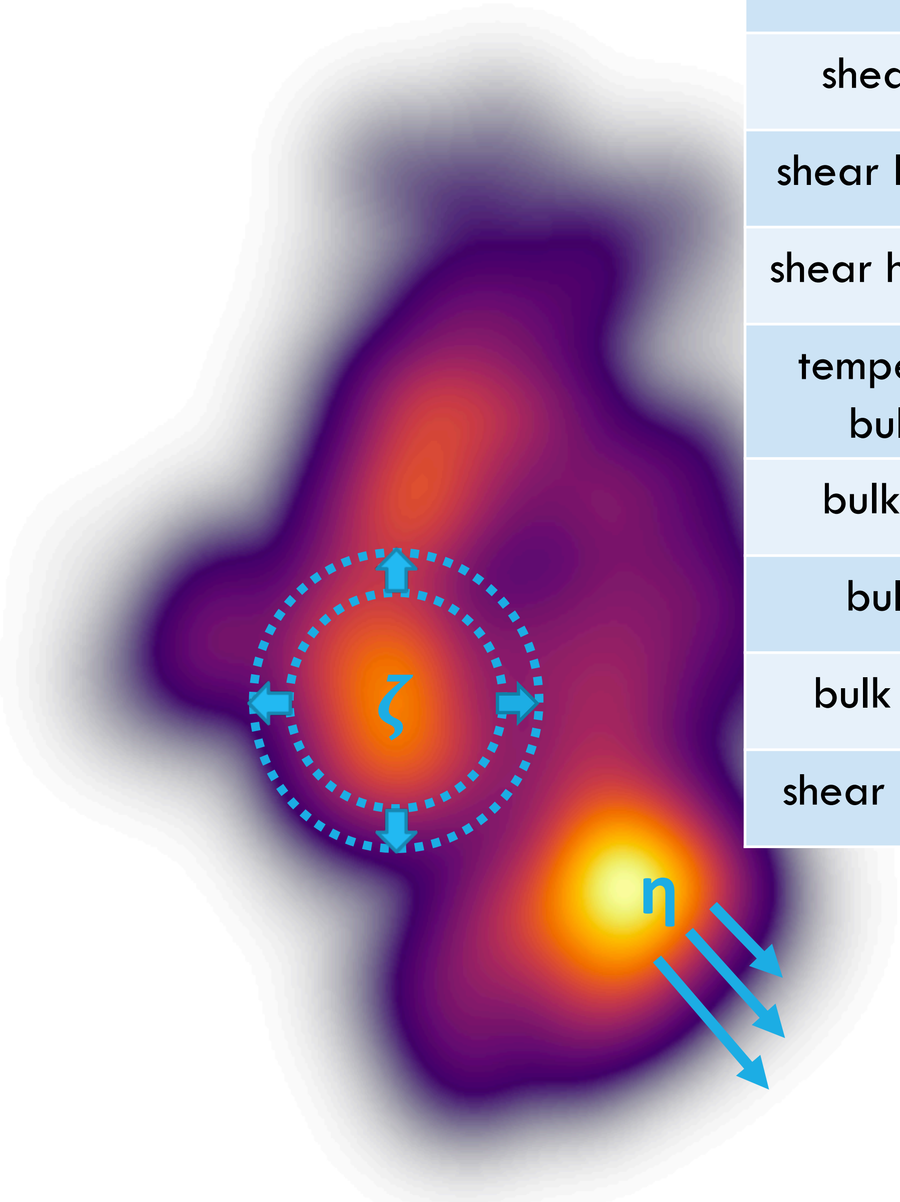
VISCOUS HYDRO

The viscosity of QGP:

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \ldots$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \ldots$$

Quantify transport properties : shear and bulk viscosities



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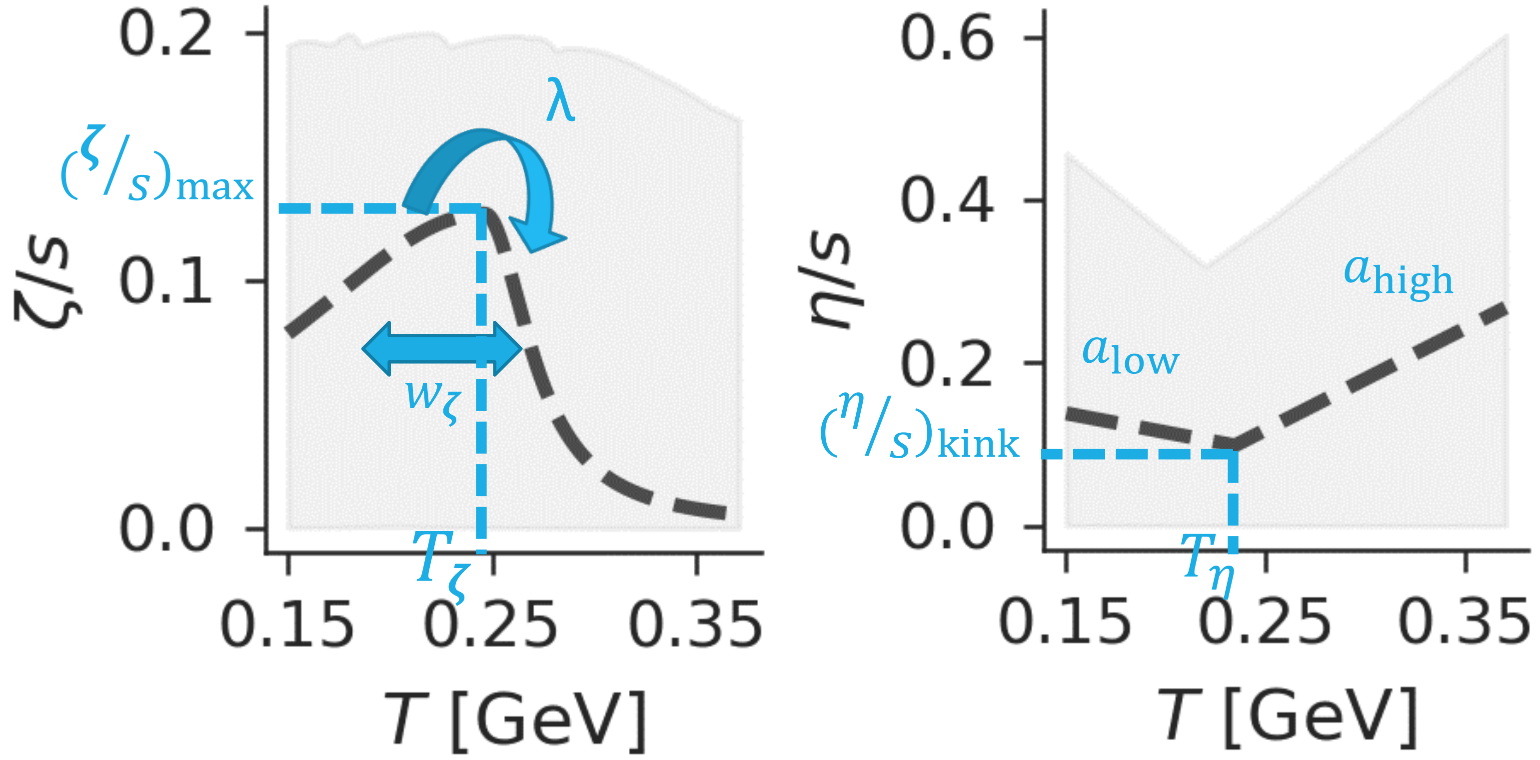
VISCOUS HYDRO

Viscosity parameterizations:



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PARTICLIZATION

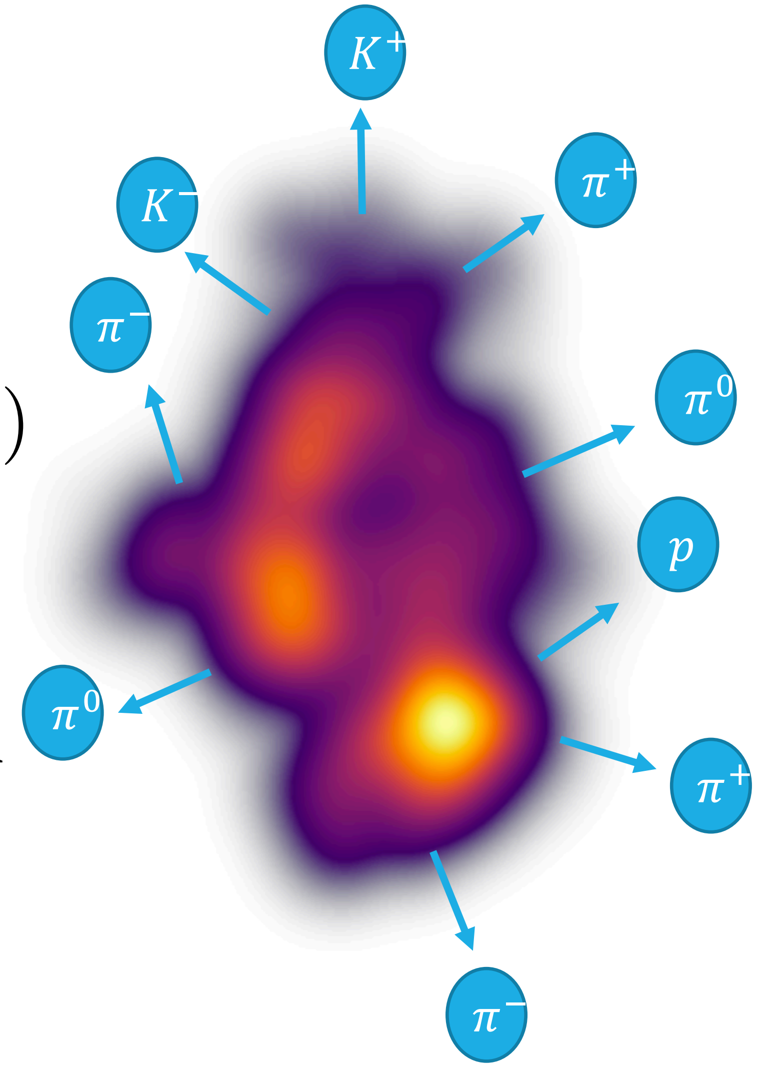
$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int_\Sigma d^3\sigma_\mu p^\mu f_i(x;p)$$

Out-of-equil. fluid $f_i \neq f_{i,\mathrm{eq}}(T(x), u^\mu(x))$

Must apply ansatz/models for $\delta f(x;p)$:

1. Expansion of $\delta f(x;p)$ in momenta
2. Relaxation time approx. Boltzmann eqn
3. `Modified equilibrium'

Choice affects parameter estimates!



PARTICLIZATION MODELS

1. Grad : expand $\delta f(x;p)$ in momenta

$$\delta f = f_{\mathrm{eq}}\bar{f}_{\mathrm{eq}}c_{\mu\nu}p^{\nu}p^{\nu}$$

2. Chapman-Enskog (C.E.) RTA : Boltzmann EQN

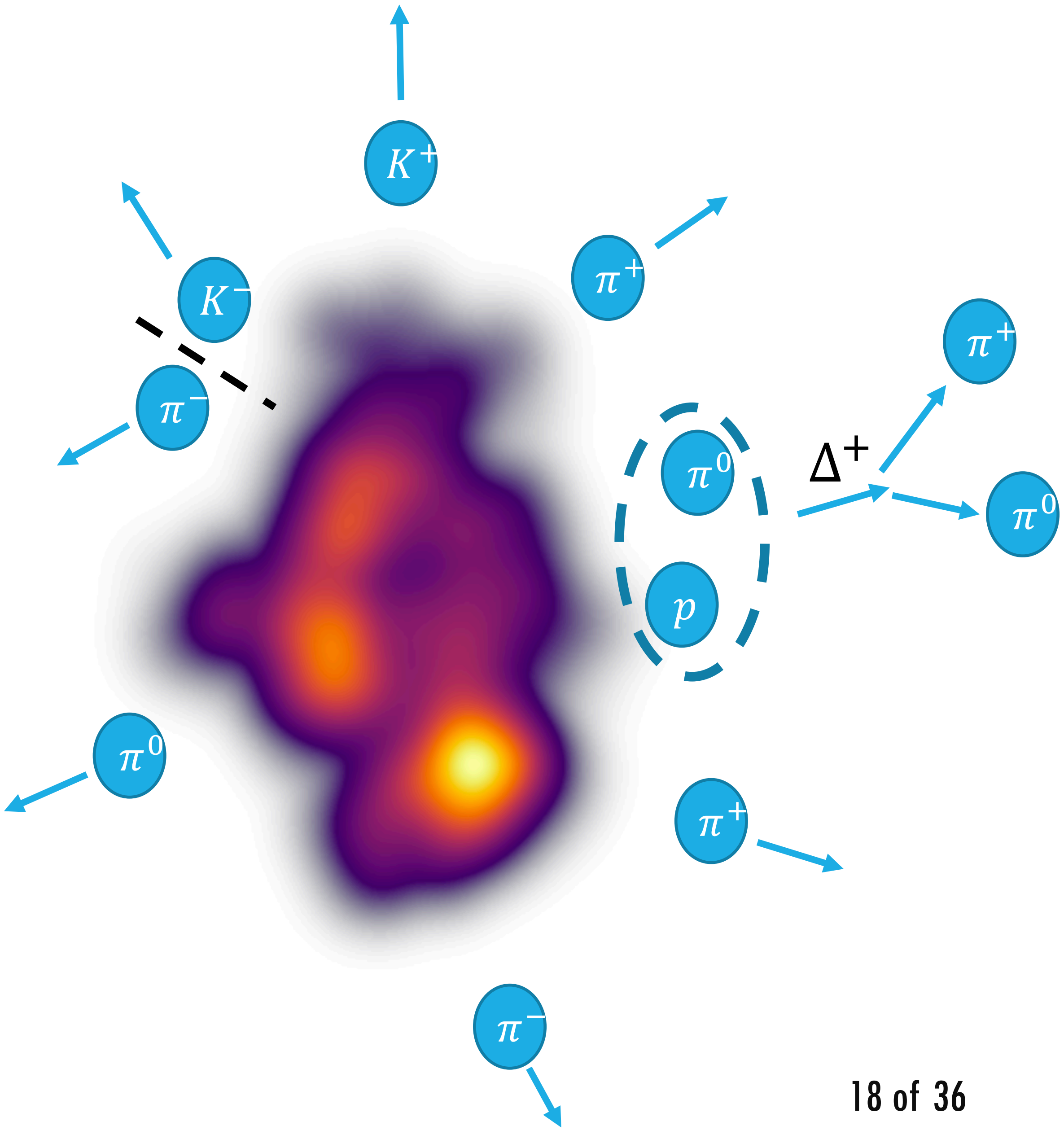
$$p^{\mu}\partial_{\mu}f = -\frac{u\cdot p}{\tau_r}(f - f_{\mathrm{eq}}) \qquad \delta f = -\frac{\tau_R}{u\cdot p}p^{\mu}\partial_{\mu}f_{\mathrm{eq}} + \mathcal{O}\left(\partial^2\right)$$

3. Pratt-Bernhard (P.B.) : 'Modified Equilibrium'

$$f = \frac{\mathcal{Z}_{\Pi}}{\det\Lambda}g\left[\exp\left(\frac{|\mathbf{p}'|^2 + m^2}{T}\right) + \Theta\right]^{-1} \qquad \Lambda_{ij} \equiv (1 + \lambda_{\Pi})\,\delta_{ij} + \frac{\pi_{ij}}{2\beta_{\pi}}$$

HADRONIC PHASE (SMASH)

- Hadrons scatter, form resonances, decay
- Lattice EoS matched to EoS of SMASH hadrons s.t. energy, pressure, … continuous at particlization
- No parameters varied

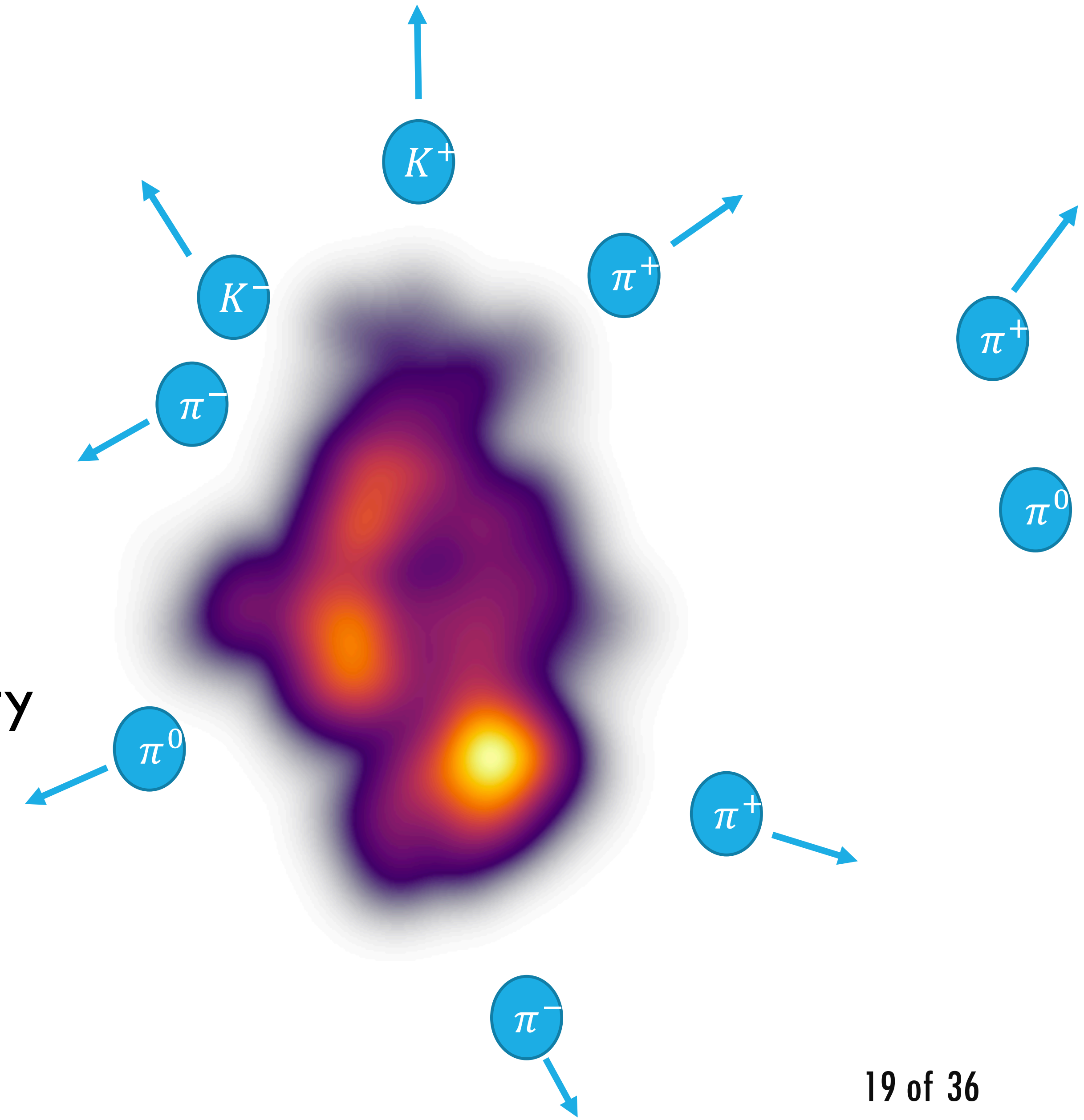


OBSERVABLES

LHC Pb-Pb 2.76 TeV	RHIC Au-Au 200 GeV
dN_i/dy	dN_i/dy
$< p_T >_i$	$< p_T >_i$
$dN_{ch}/d\eta$	
$v_n\{2\}$	$v_n\{2\}$
$dE_T/d\eta$	
$\frac{\delta p_T}{< p_T >}$	
$i \in \{\pi, K, p\}$ $n \in \{2,3,4\}$	$i \in \{\pi, K\}$ $n \in \{2,3\}$

p_T-integrated observables

Same centrality bins as experiments



Bayesian Parameter Estimation

CHOOSING OUR PRIORS (<u>THEY MATTER</u>)

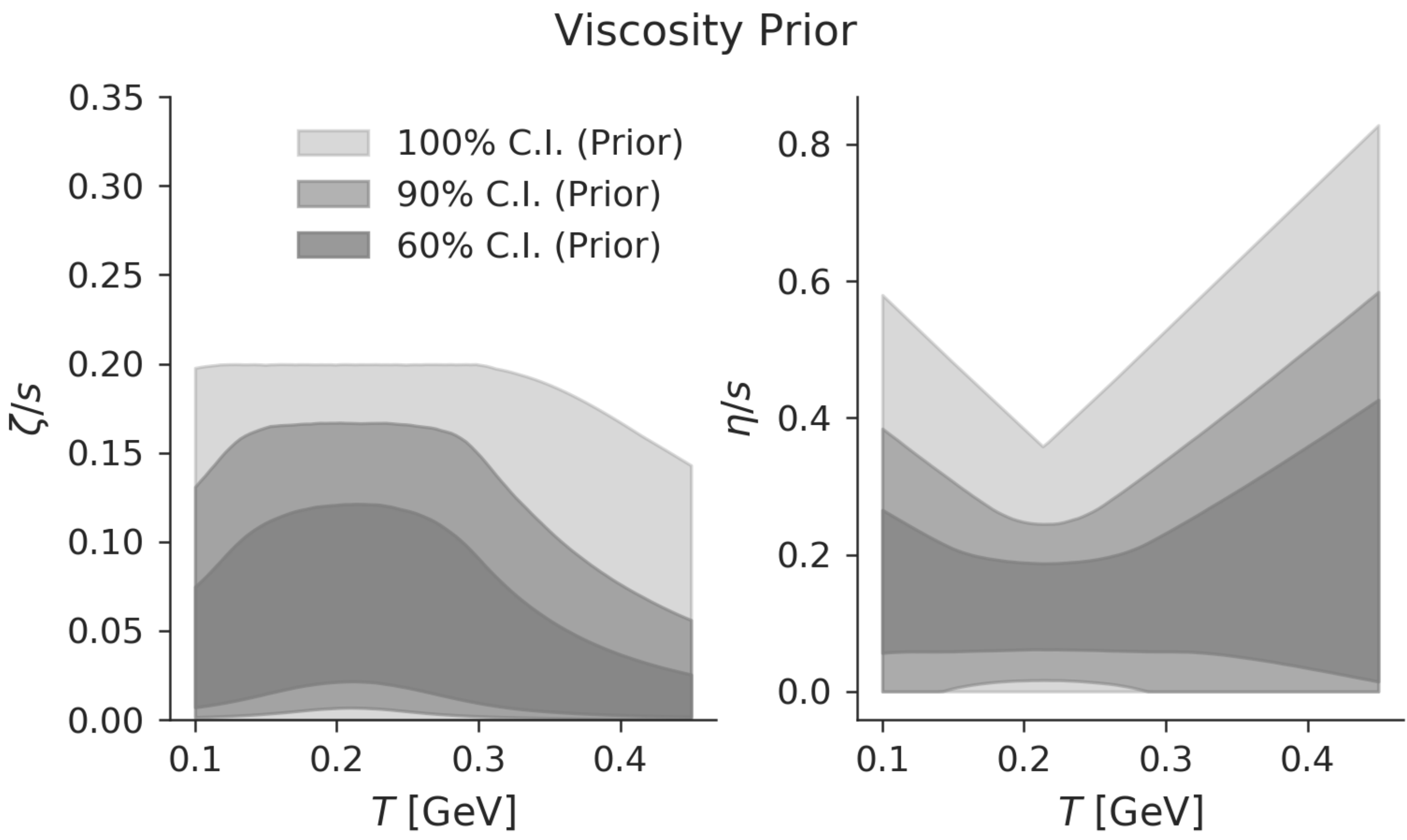
$$p(\theta_i) = \begin{cases} \frac{1}{\theta_{\max} - \theta_{\min}} & \theta \in [\theta_{\max}, \theta_{\min}] \\ 0 & \text{else} \end{cases}$$

A uniform prior is **not** <u>'uninformed'</u>

Our theoretical bias is included in the shape, magnitude…

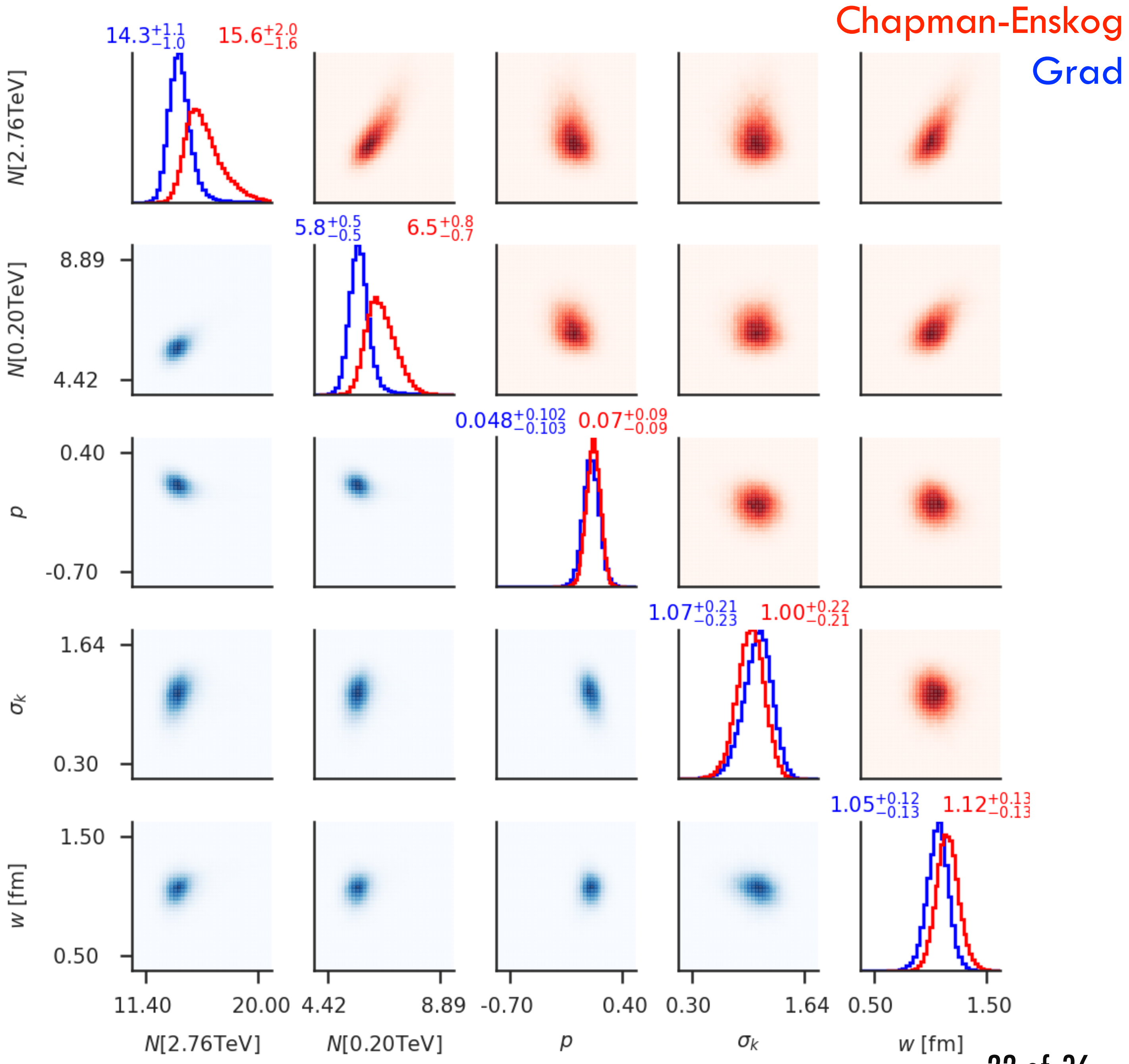
Our prior should <u>not</u> be informed by the hadronic data we will use!

*more general than previous works, w/ room for future generalization



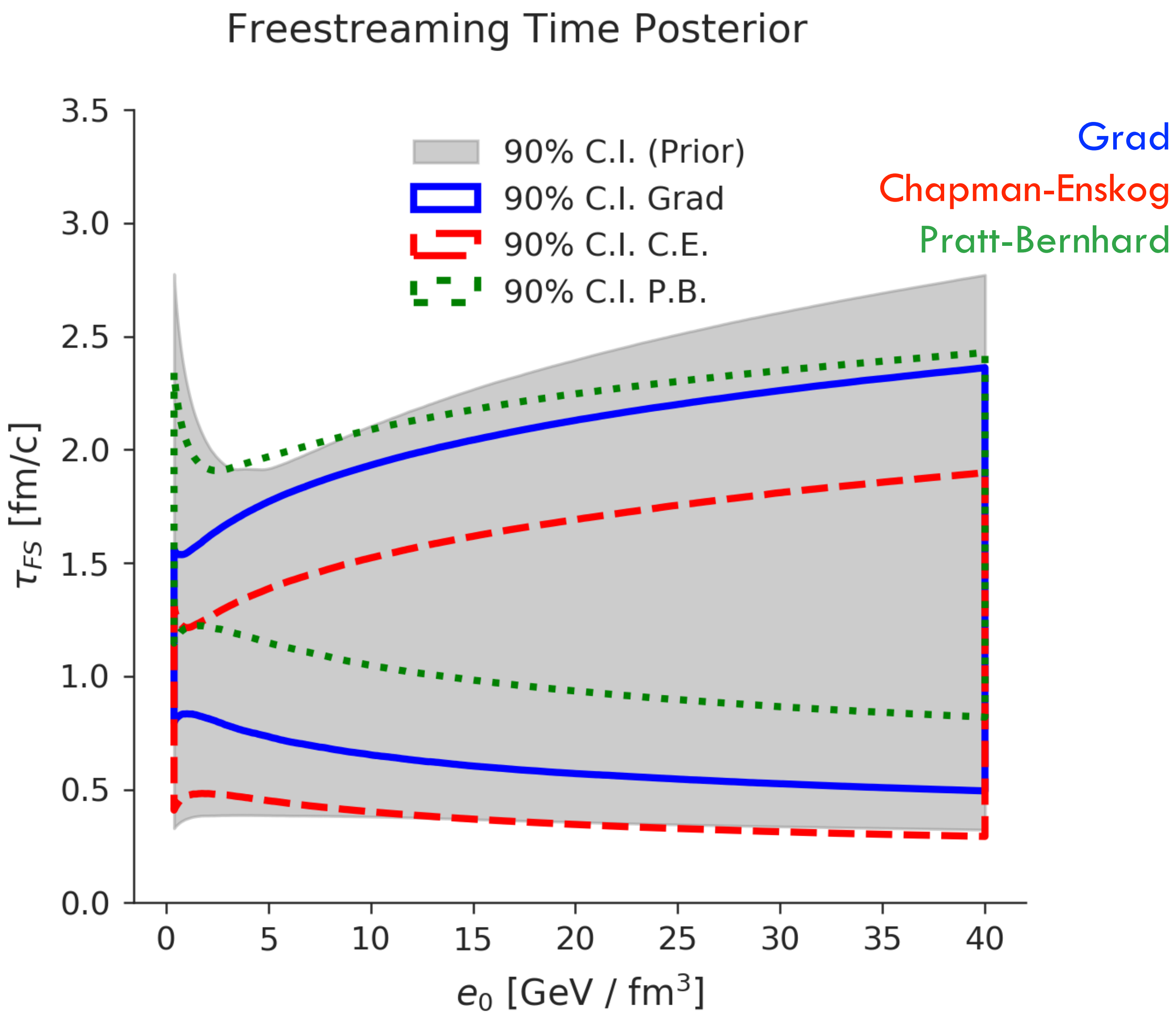
QUANTIFYING THE QGP INITIAL STATE

- Estimates w/ both LHC and RHIC data
- Parameters well constrained by data
- Reduced-thickness, fluctuation and width robust under viscous correction model (Grad/Chapman-Enskog)



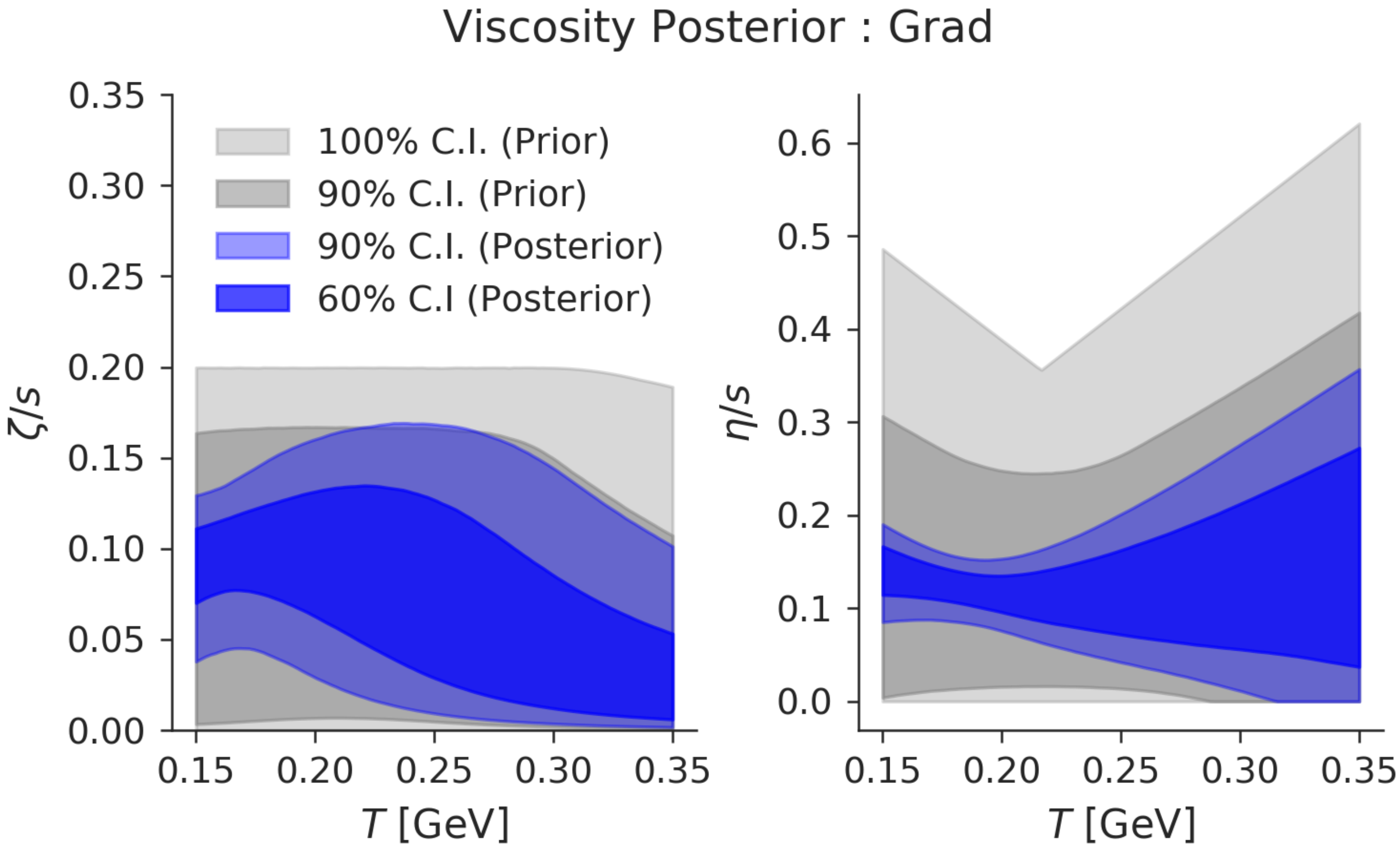
QUANTIFYING THE QGP INITIAL STATE

- No strong energy dependence (opposed to theory expectation)
- Estimate highly dependent on viscous correction model



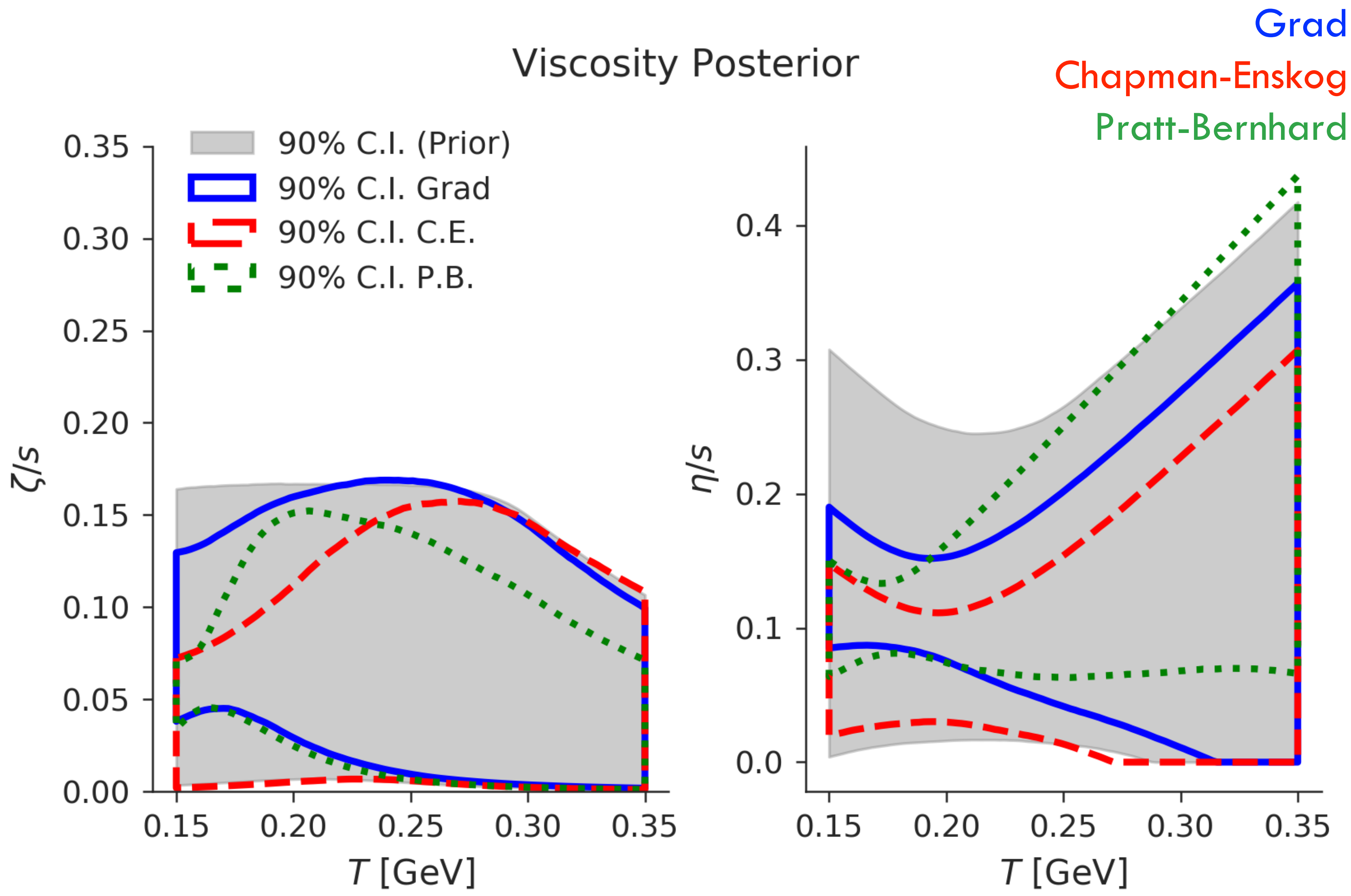
QUANTIFYING THE QGP COUPLING STRENGTH

- Estimates w/ both LHC and RHIC data
- Better constraint near switching temperature
- `Returning the prior' at high temperature, for bulk viscosity!



QUANTIFYING THE QGP COUPLING STRENGTH

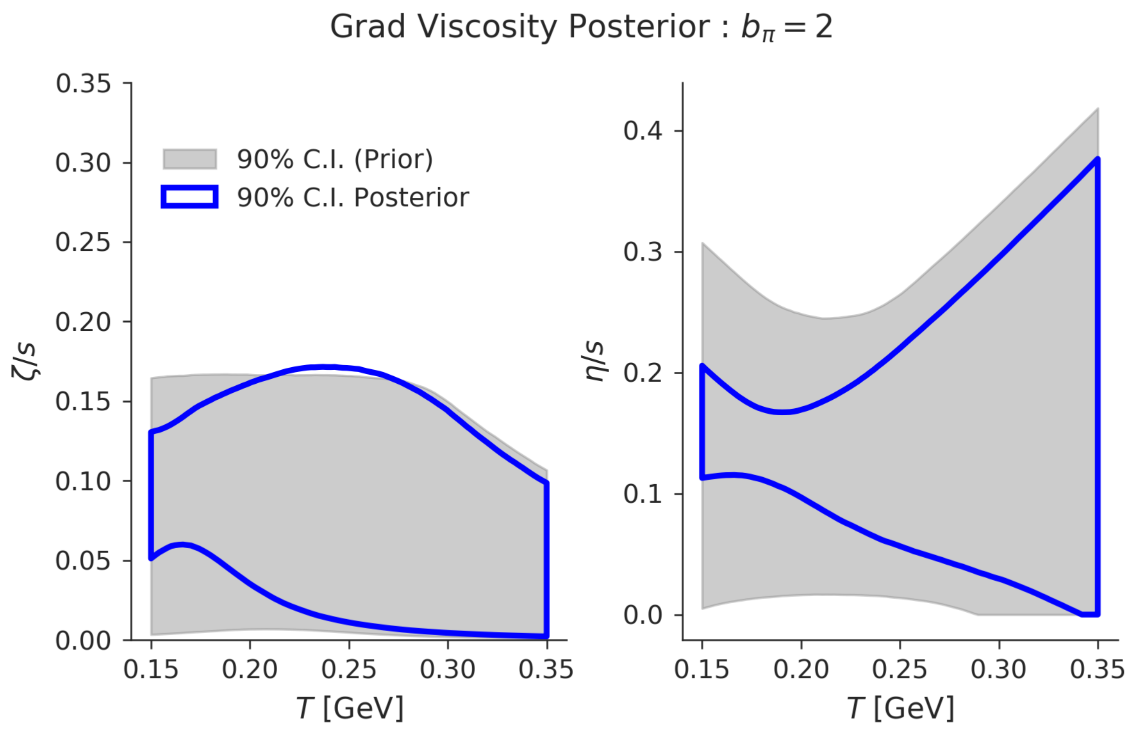
- Estimates w/ both LHC and RHIC data
- Better constraint near switching temperature
- Viscosity estimates strongly depend on viscous correction.
- `Returning the prior' at high temperature, for bulk viscosity!



QUANTIFYING THE 2ND ORDER TRANSPORT COEFF.

Consider fixing the shear relaxation time to smaller value…

Posterior for viscosity plotted at right

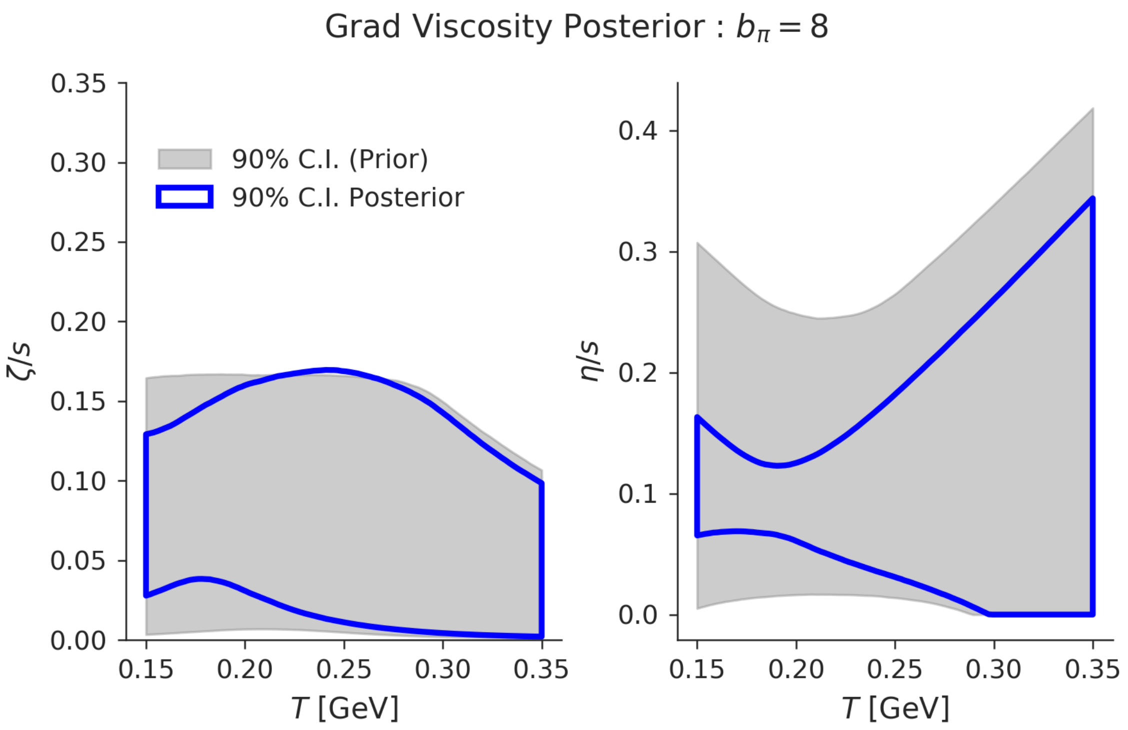


QUANTIFYING THE 2ND ORDER TRANSPORT COEFF.

Now, fix shear relax. time to larger value…

Posterior for viscosity plotted at right

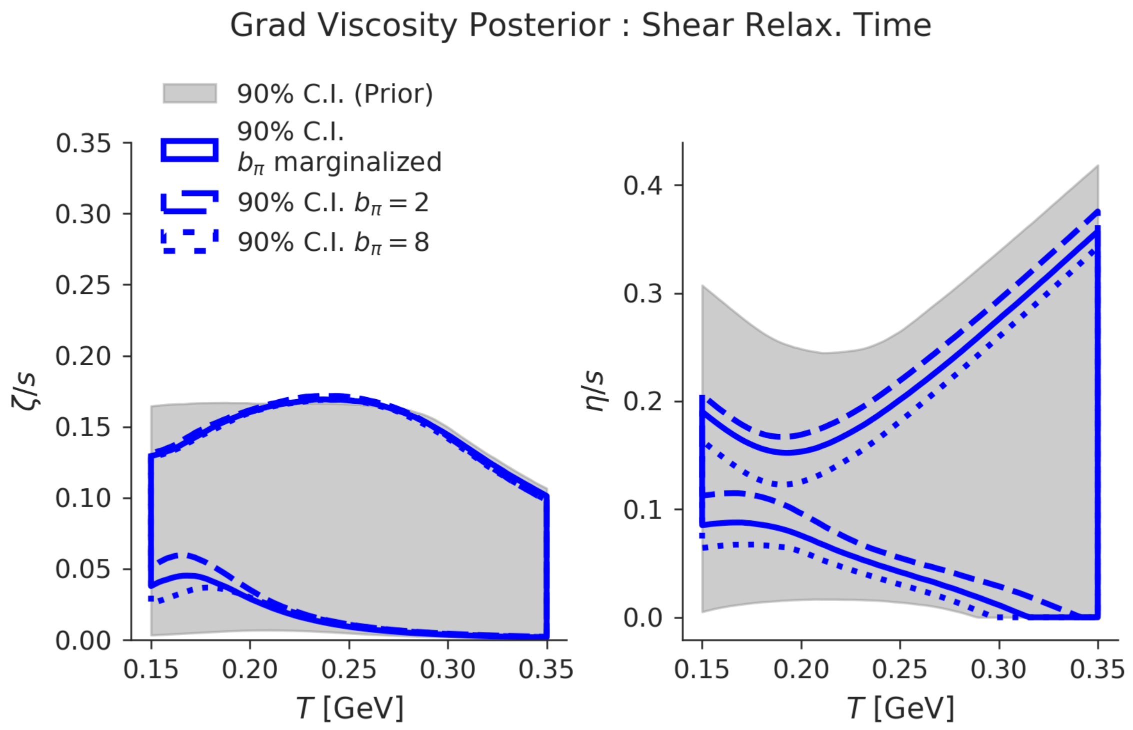
Increasing either shear relax. time or shear viscosity have similar effect on observables (reducing flows, etc…)



QUANTIFYING THE 2ND ORDER TRANSPORT COEFF.

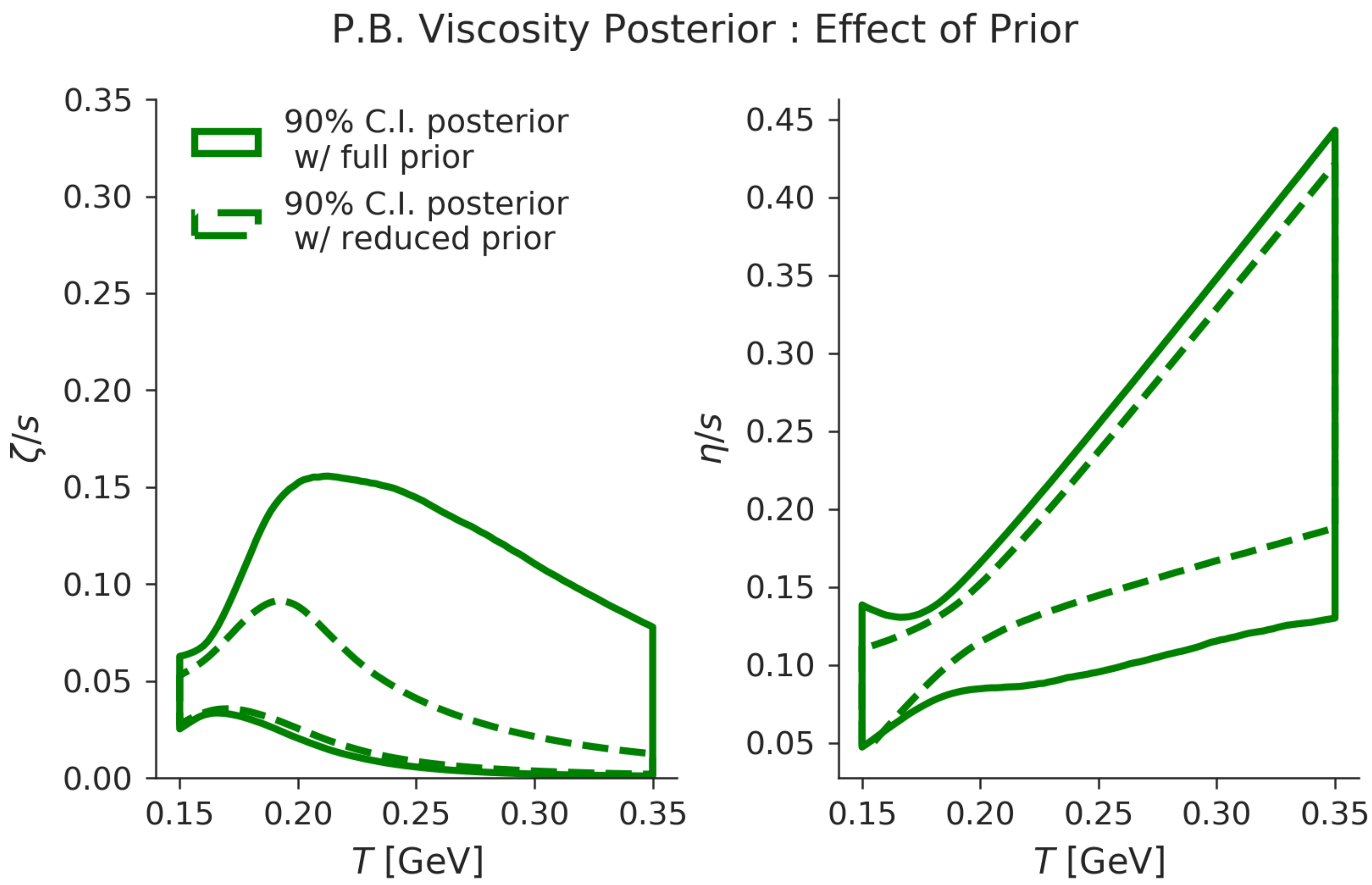
Remember, we can marginalize of 'nuisance parameters'

Marginalizing over shear relaxation time gives more robust estimation of shear viscosity



OUR PRIORS MATTER

- We said our prior affects our posterior…
- Demonstration : replace our prior by a tighter one, similar to previous works*
- Estimate viscosity posteriors using LHC Pb-Pb 2.76 TeV data with each prior



*Nat. Phys. **15**, 1113–1117 (2019)

Which observables are sensitive?

OBSERVABLES SENSITIVITY

Sensitivity index : how observables constrain parameters

Observable : $\hat{O}$

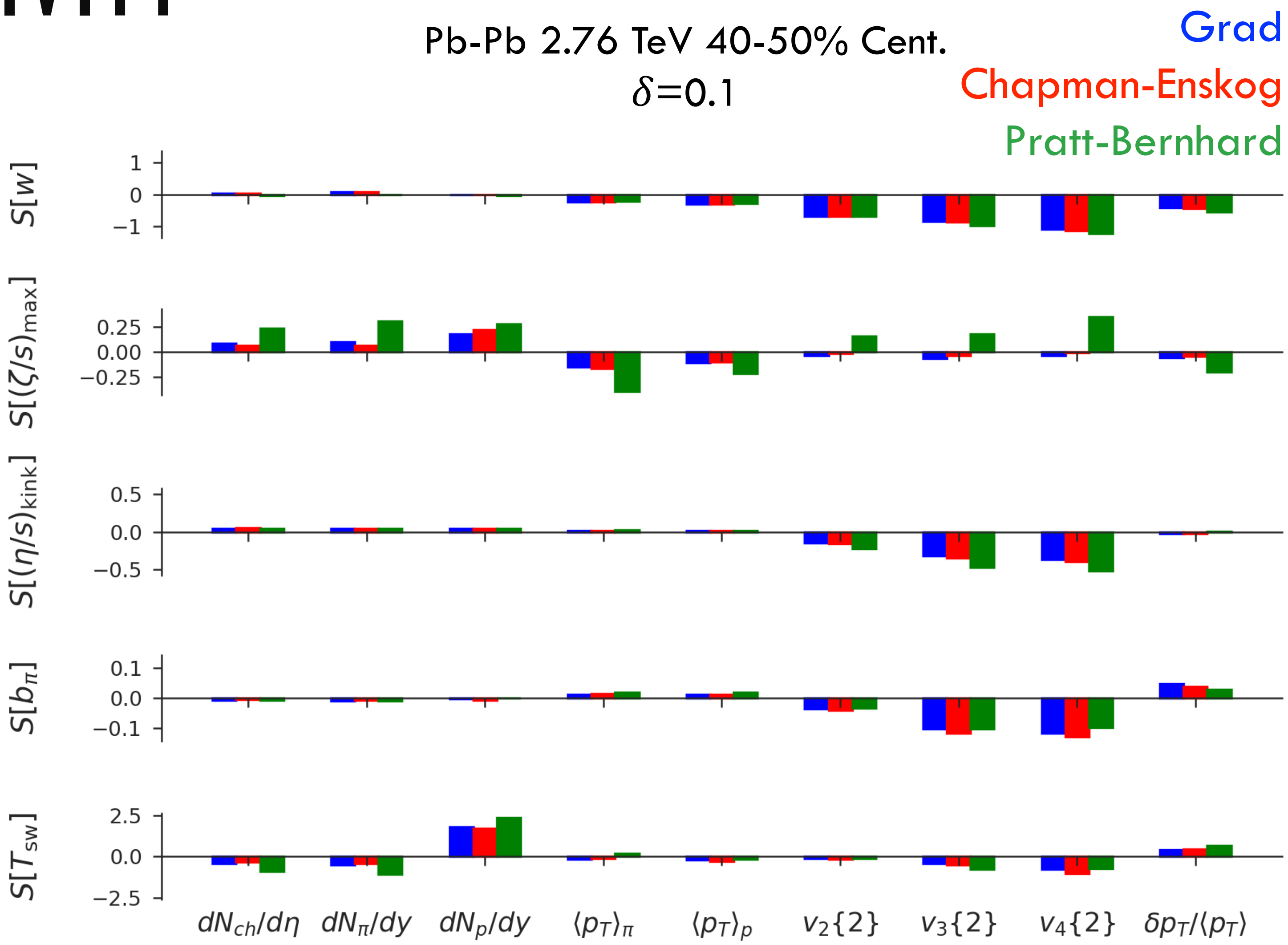
parameter : p

$$\Delta \equiv \frac{\hat{O}(p(1+\delta)) - \hat{O}(p)}{\hat{O}(p)}$$

Index : $$S[p] \equiv \Delta/\delta$$

(take δ=0.1)

Pb-Pb 2.76 TeV 40-50% Cent.
δ=0.1



Bayesian Model Selection

BAYESIAN MODEL SELECTION

- What is a good basis for choosing model A over model B, for QGP?
- Bayesian : Choose model which `fits best' with the `least number of parameters'

data theory -?

"Never let the ~~truth~~ get in the way of a good ~~story~~." – ~~Mark Twain~~

BAYESIAN MODEL SELECTION

- We have two models, A and B…
- Bayes factor is the 'odds'
  …informed by specific data $\boldsymbol{y}_{\mathrm{exp}}$

$$B_{A/B} \equiv \frac{\mathrm{prob}(\mathbf{y}_{\mathrm{exp}}|A)}{\mathrm{prob}(\mathbf{y}_{\mathrm{exp}}|B)} \underbrace{\frac{\mathrm{prob}(A)}{\mathrm{prob}(B)}}_{\text{(usually = 1)}}$$

- using sum and product rules :

$$\mathrm{prob}(\mathbf{y}_{\mathrm{exp}}|A) = \int d\mathbf{x}_A \underbrace{\mathrm{prob}(\mathbf{y}_{\mathrm{exp}}|\mathbf{x}_A, A)\mathrm{prob}(\mathbf{x}_A)}_{\text{our } \textbf{posterior}}$$

MODEL SELECTION : VISCOUS CORRECTIONS

$$B_{A/B} = \frac{\mathrm{prob}(\mathbf{y}_{\mathrm{exp}}|A)}{\mathrm{prob}(\mathbf{y}_{\mathrm{exp}}|B)}$$

- Can p_T-integrated observables constrain viscous correction models?
- Chosen data provide moderate evidence favoring Grad and Pratt-Bernhard models over Chapman-Enskog
- Chosen data do not provide evidence for Grad vs. Pratt-Bernhard

Model A	Model B	$\ln B_{A/B}$
Grad	C.E.	≈ 8
Grad	P.B	≈ 0
P.B.	C.E.	≈ 6

C.E. : Chapman-Enskog (RTA)
P.B. : Pratt-Bernhard

CONCLUSIONS

- We estimated the viscosities of QGP:
  - with 3 different viscous correction models
  - with more relaxed priors than previous works
  - using both LHC and RHIC hadronic data
- We quantified the sensitivity of observables to model parameters
- We used Bayes factors to compare viscous correction models for QGP
- *For details and more, look for upcoming paper

ACKNOWLEDGEMENTS

The JETSCAPE Collaboration

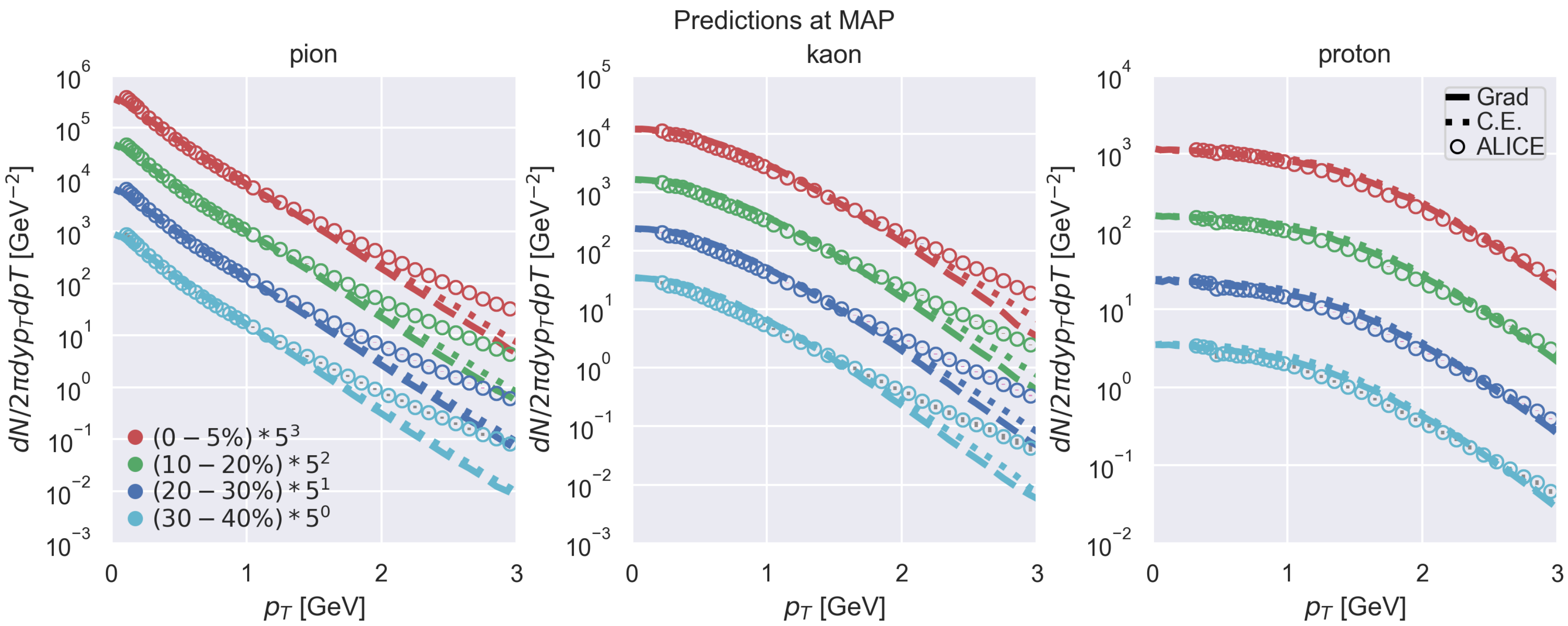
XSEDE

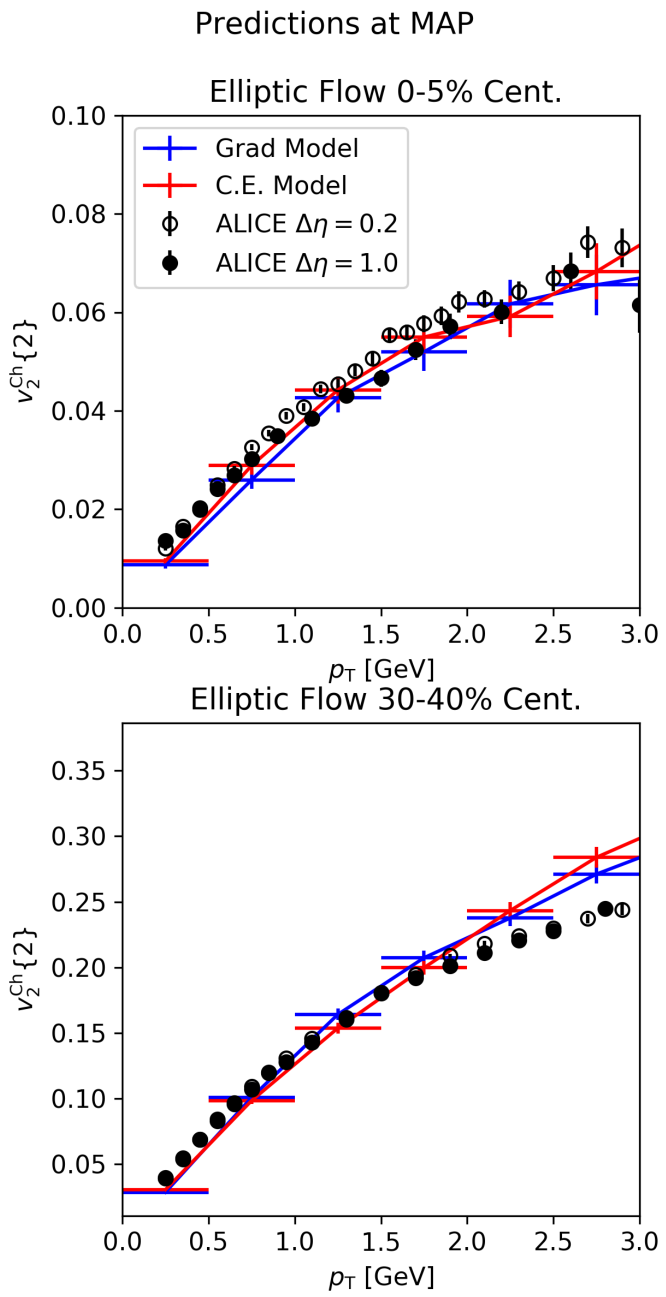
Extreme Science and Engineering Discovery Environment

Computational resources
XSEDE & TACC

Backup

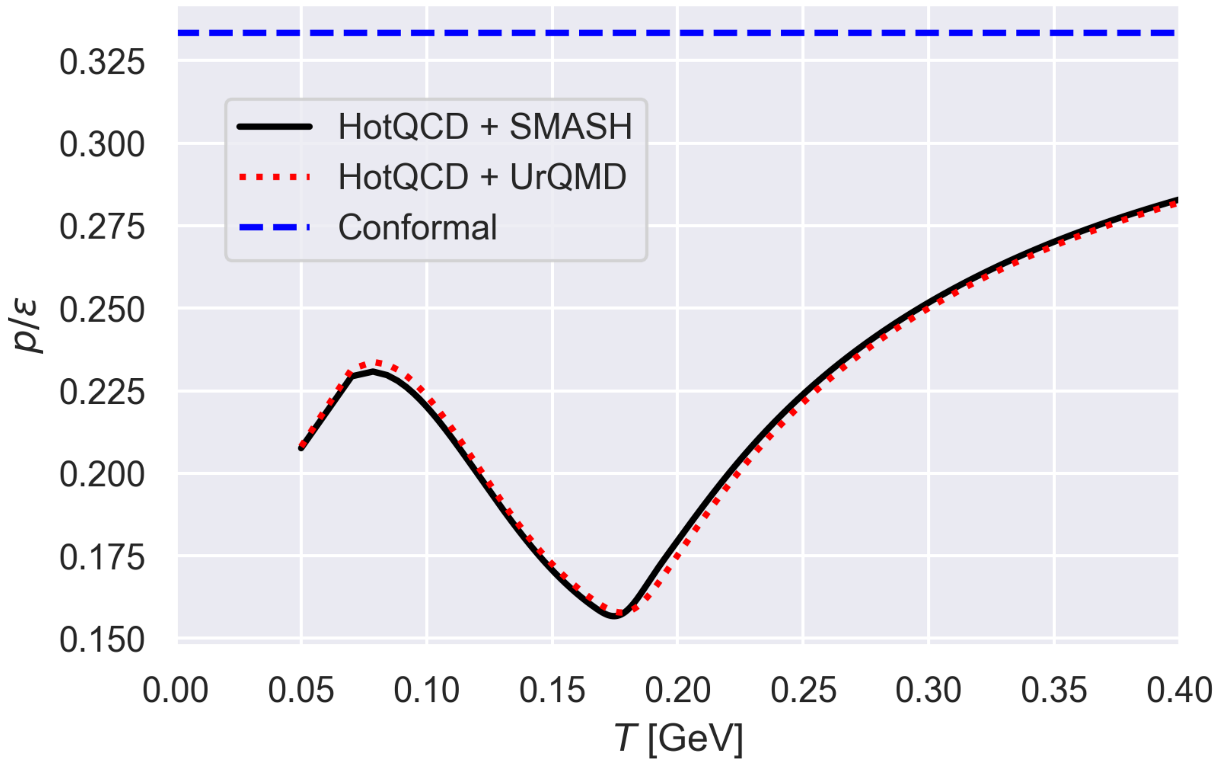
MODEL PREDICTIONS





- Spectra and elliptic flow agree well at low p_T
- Slopes too soft at high p_T
- ~Expected : We calibrated on yields, mean p_T, etc…

EQN OF STATE (EOS)



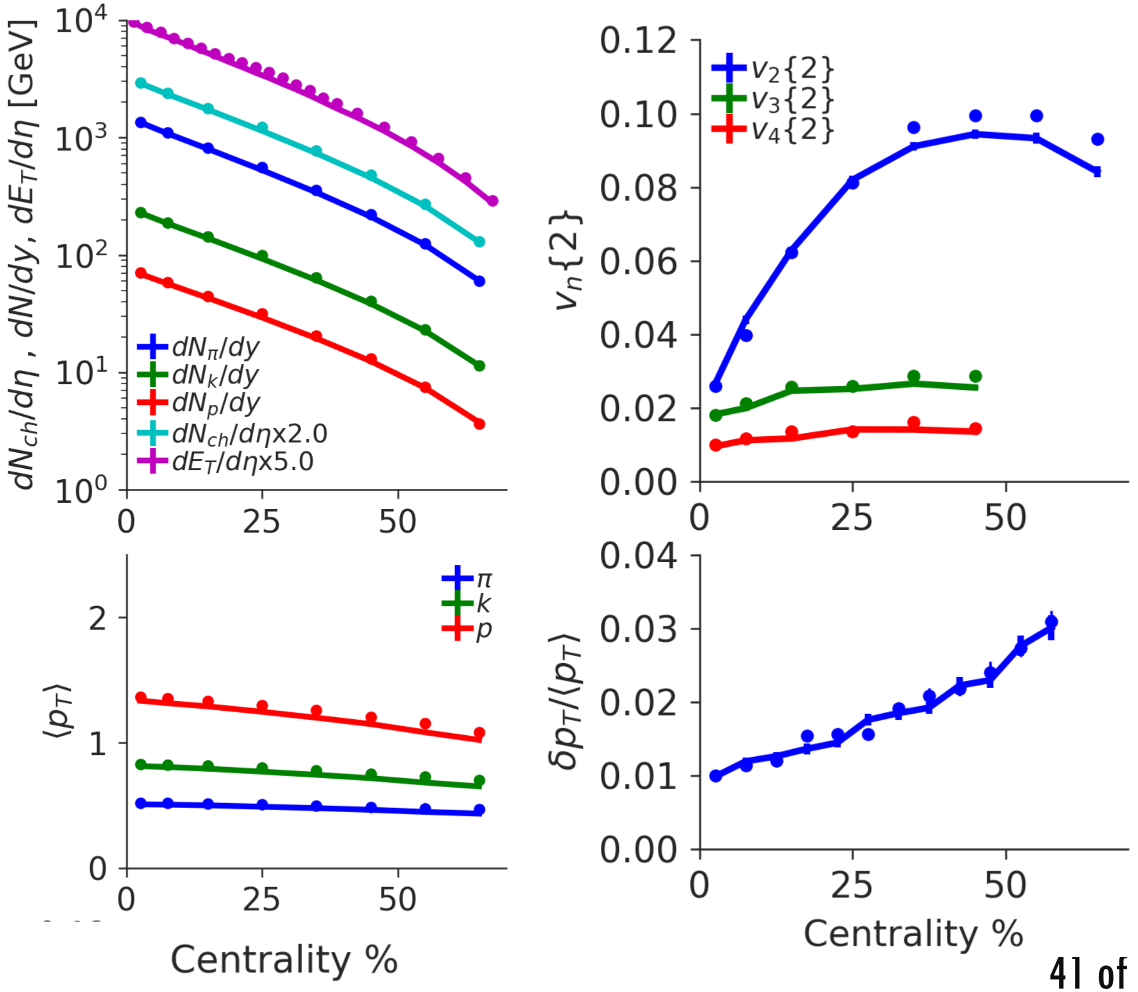
BULK RELAXATION TIME

- Check effect of doubling bulk relax. time
- Grad viscous correction model
- 5,000 fluct. events at MAP parameters

$$\tau_\Pi = b_\Pi \frac{\zeta}{\left(\frac{1}{3} - c_s^2\right)^2 (\epsilon + p)}$$

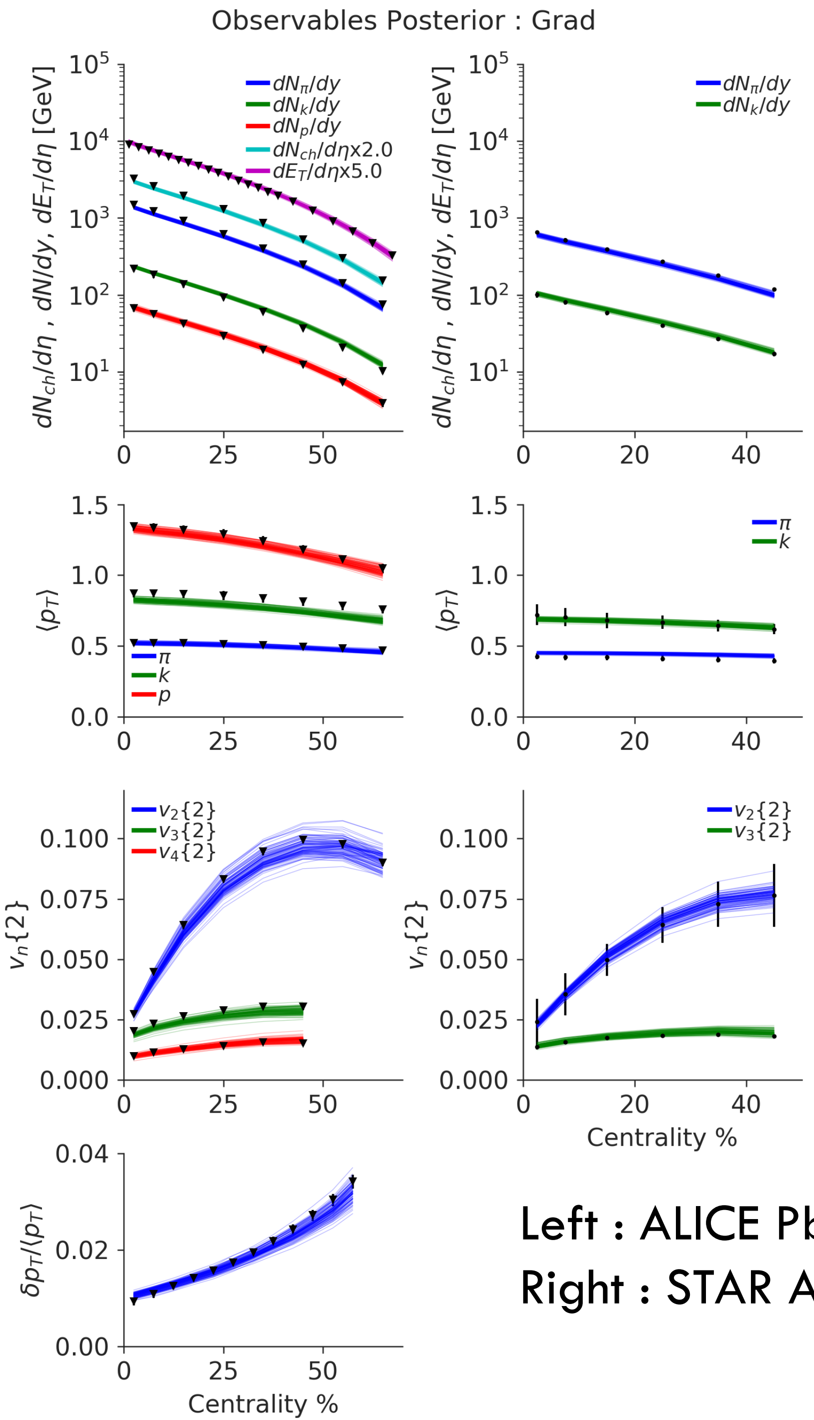
Lines : $b_\pi = 1/14.55$

Dots : $b_\pi = 2/14.55$



OBSERVABLES POSTERIOR

- 100 samples drawn from parameter posterior
- Observables predicted by Gauss. Process Emulator

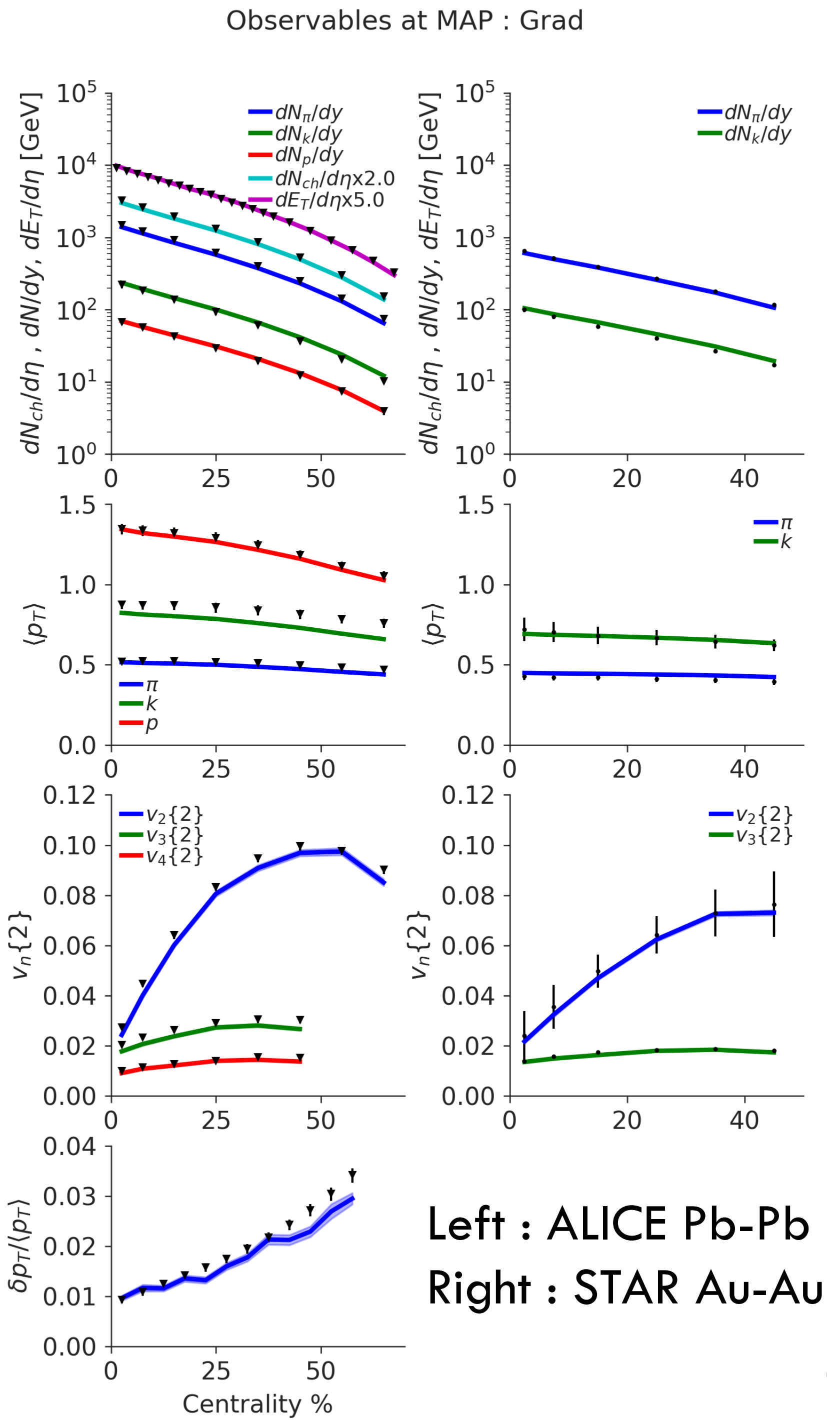


Left : ALICE Pb-Pb 2.76 TeV

Right : STAR Au-Au 200 GeV

MODEL PREDICTIONS AT MAP

- 5,000 fluct. events at max. of the posterior ’maximum a posteriori’ (MAP) predicted by the hybrid-model



Left : ALICE Pb-Pb 2.76 TeV

Right : STAR Au-Au 200 GeV