Parton Energy Loss in a Hard-soft Factorized Approach

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Outline

- Hard-soft factorized model of parton energy loss
- Numerical implementation of the factorized model
- Parton energy loss at small coupling
- Parton energy loss at larger coupling
- Application: in-medium parton energy cascade

Hard-soft factorized model of parton energy loss

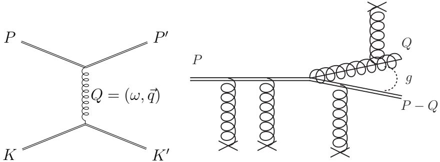
J. Ghiglieri, G. Moore, D. Teaney, JHEP03 (2016) 095

Weakly-coupled effective kinetic approach

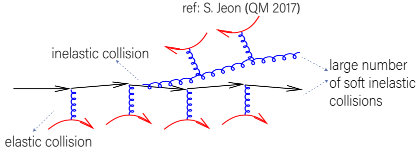
- Perturbative parton-medium interaction
- Dynamics of quasiparticles are described by transport equations
- Energy gain and loss are naturally included

Leading-order realizations (e.g. MARTINI):

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f^a(\mathbf{p}, \mathbf{x}, t) = -\mathcal{C}_a^{2\leftrightarrow 2}[f] - \mathcal{C}_a^{1\leftrightarrow 2}[f]$$



Hard-soft factorization of energy loss



Interactions with the medium:

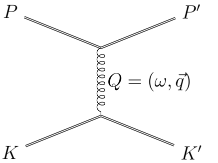
- Large number of soft interactions
- Rare hard scatterings

Parton energy loss factorized as **hard interactions + diffusion process**

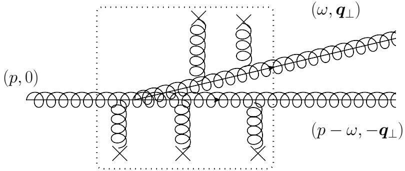
Benefits of factorized transport model

- Systematically factorized soft and hard parton-plasma interactions
- Efficient and flexible stochastic description of soft interactions
- Diffusion process does not rely on the quasiparticle assumption
- Can be extended to next-to-leading order

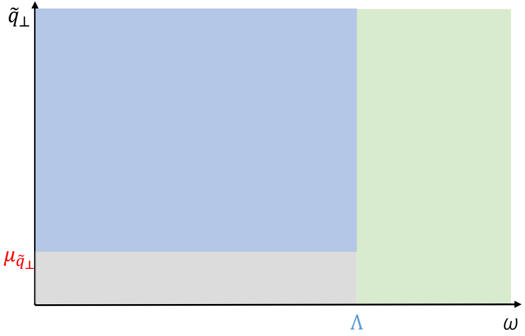
Hard-soft factorization of energy loss



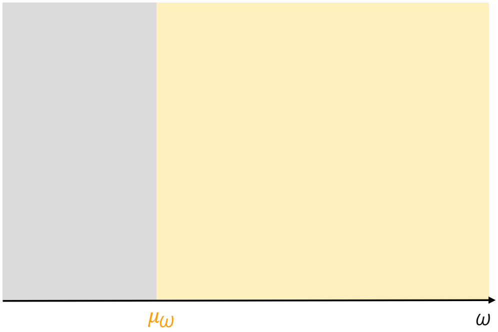
$$\tilde{q}_\perp \equiv \sqrt{q^2 - \omega^2}$$



Elastic interactions:



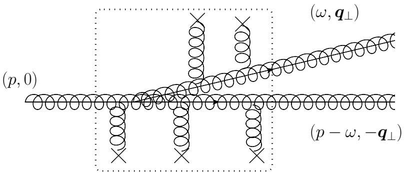
Inelastic interactions:



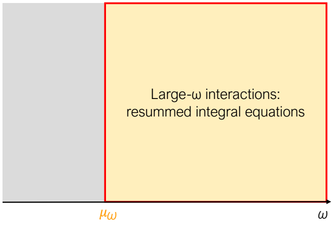
Hard $1 \leftrightarrow 2$ Interactions - Large ω Interactions

Multiple soft interactions with the plasma induce the collinear radiation of a parton of energy ω.

Soft interactions are resummed to account for LPM effect.

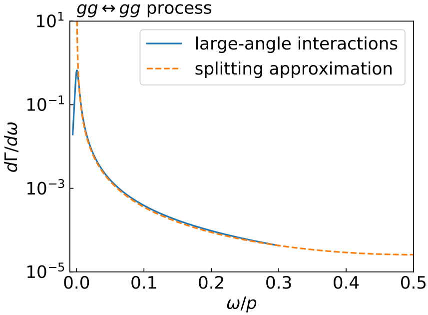


- $\omega > \mu_\omega$, $\mu_\omega \lesssim T$
- Described with emission rates (obtained from AMY integral equations)
- Leading order calculation

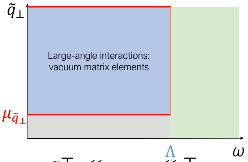


Hard $2 \leftrightarrow 2$ Interactions

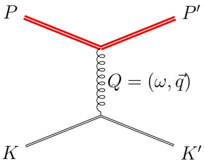
- Leading order vacuum pQCD matrix elements
- Neglect $\mathcal{O}(\frac{T}{p})$



Large-angle interactions:

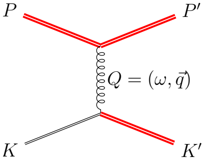


$gT \ll \mu_{\tilde{q}_\perp} \ll T$

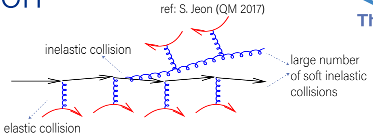


Splitting approximation:

$3T \ll \Lambda \ll p$



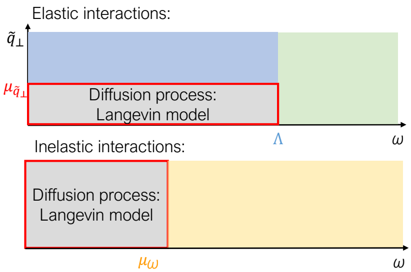
Identity **Preserving** Soft Interactions - Diffusion



Number and identity preserving soft collisions are described stochastically with drag and diffusion.

$$\mathcal{C}^{\mathrm{diff}}[f] = -\frac{\partial}{\partial p^i}\left[\eta_D(p)p^i f(p)\right] - \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\left(\hat{p}^i\hat{p}^j\hat{q}_L(p) + \frac{1}{2}\left(\delta^{ij} - \hat{p}^i\hat{p}^j\right)\hat{q}(p)\right)f(p)\right]$$

- Include both elastic and inelastic soft interactions
- Transport coefficients: pQCD
- Treated with Langevin model

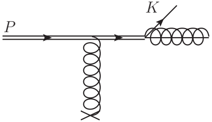


Identity **Non-preserving** Soft Interactions - Conversion

- Change parton identity by conversion rate
- Suppressed by T/p

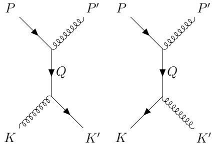
Inelastic conversion processes

q ↔ gq, g ↔ q$\bar{\text{q}}$



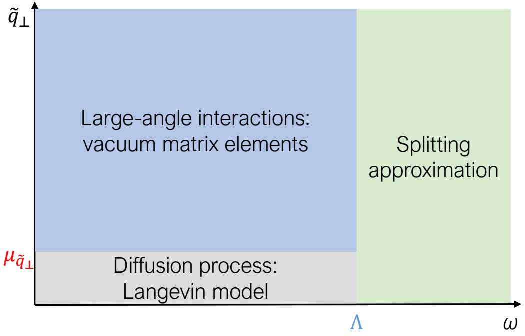
Elastic conversion processes

Soft fermion exchange w/ medium

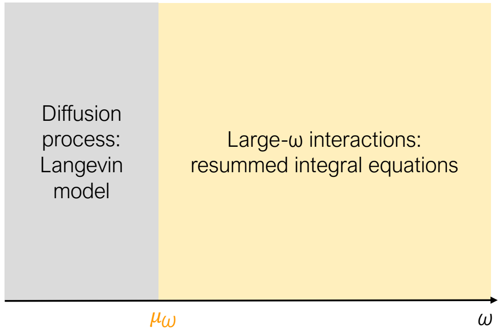


Summary of the Treatments to Different Processes

Elastic interactions:



Inelastic interactions:



$$\mathcal{C}^{2\leftrightarrow 2} + \mathcal{C}^{1\leftrightarrow 2} = \mathcal{C}^{\text{large-angle}}(\mu_{\tilde{q}_\perp}, \Lambda) + \mathcal{C}^{\text{split}}(\Lambda) + \mathcal{C}^{\text{large-}\omega}(\mu_\omega) + \mathcal{C}_a^{\text{diff}}(\mu_{\tilde{q}_\perp}, \mu_\omega)$$

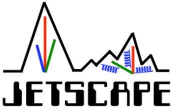
If the division of the energy loss model is valid, energy loss should be **independent on the cutoffs** (in a reasonable range).

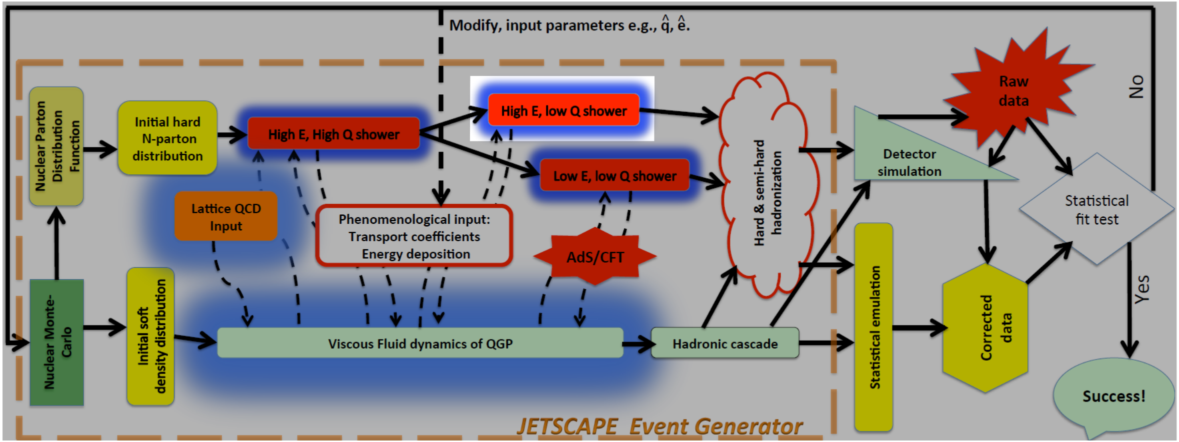
Numerical implementation of the factorized model

Putschke, et al., arXiv:1902.05934 (2019).
Dai, T., Bass, S. A., Paquet, J-F., & Teaney, D., arXiv:1901.07022.

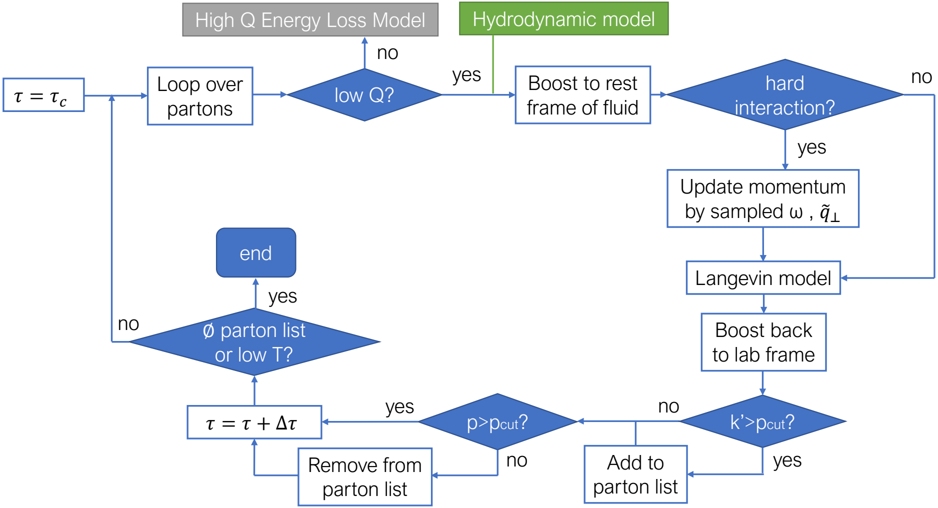
Numerical Implementation

Reformulated energy loss model is **added as a separate external module** in a modified version of the public JETSCAPE2.0 code.



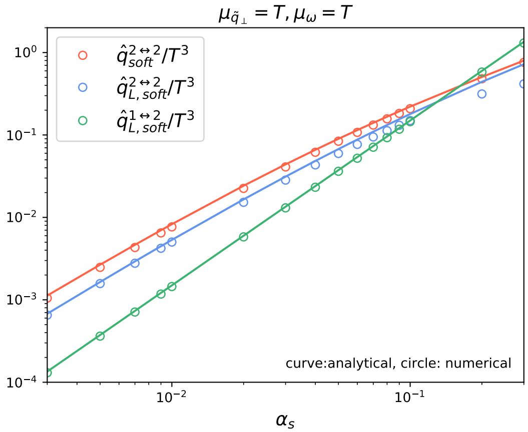


Numerical Implementation



pcut : We only track hard partons with p > p_{cut}

Numerical vs analytical diffusion coefficients - coupling



Analytical diffusion coefficients:

$$\hat{q}^{2\to 2}_{\rm soft} = \frac{g^2 C_R T m_D^2}{4\pi} \ln\left[1 + \left(\frac{\mu_{\tilde{q}_\perp}}{m_D}\right)^2\right]$$

$$\hat{q}^{2\to 2}_{\rm L,soft} = \frac{g^2 C_R T M_\infty^2}{4\pi} \ln\left[1 + \left(\frac{\mu_{\tilde{q}_\perp}}{m_\infty}\right)^2\right]$$

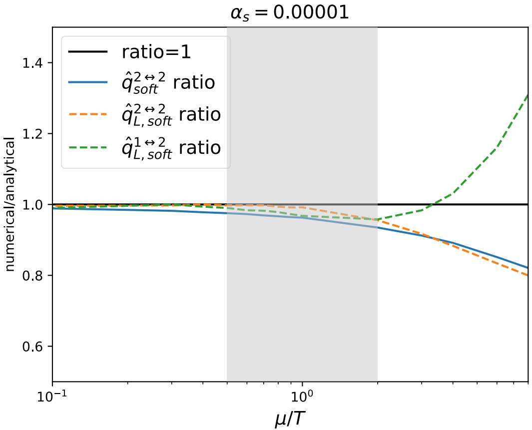
$$\hat{q}^{1\to 2}_{\rm L,soft} = \frac{(2-\ln 2) g^4 C_R C_A T^2 \mu_\omega}{4\pi^3}$$

Numerical diffusion coefficients:

$$\hat{q}_{\rm soft}(p) = \int_0^{\mu_{\tilde{q}_\perp}} d^2 q_\perp q_\perp^2 \frac{d\Gamma(\mathbf{p}, \mathbf{p}+\mathbf{q}, \mu_\omega)}{d^2 q_\perp}$$

$$\hat{q}_{\rm L,soft}(p) = \int_0^{\mu_\omega} dq^z (q^z)^2 \frac{d\Gamma(\mathbf{p}, \mathbf{p}+\mathbf{q}, \mu_{\tilde{q}_\perp})}{dq^z}$$

Numerical vs analytical diffusion coefficients - cutoff



Analytical diffusion coefficients assume:

$\mu_\omega \ll$ T

$\mu_{\tilde{q}_\perp} \ll$ T

For valid diffusion process:

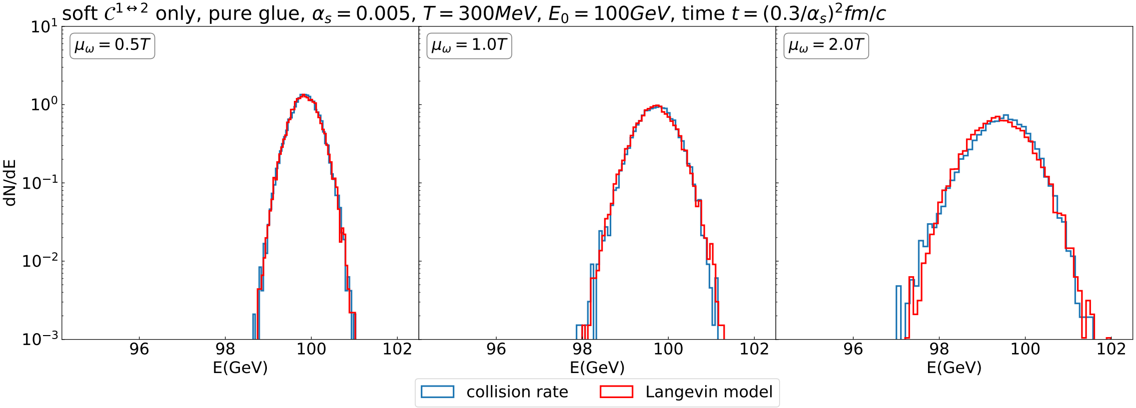
$\mu_\omega \gg$ gT

$\mu_{\tilde{q}_\perp} \gg$ gT

Parton energy loss at small coupling

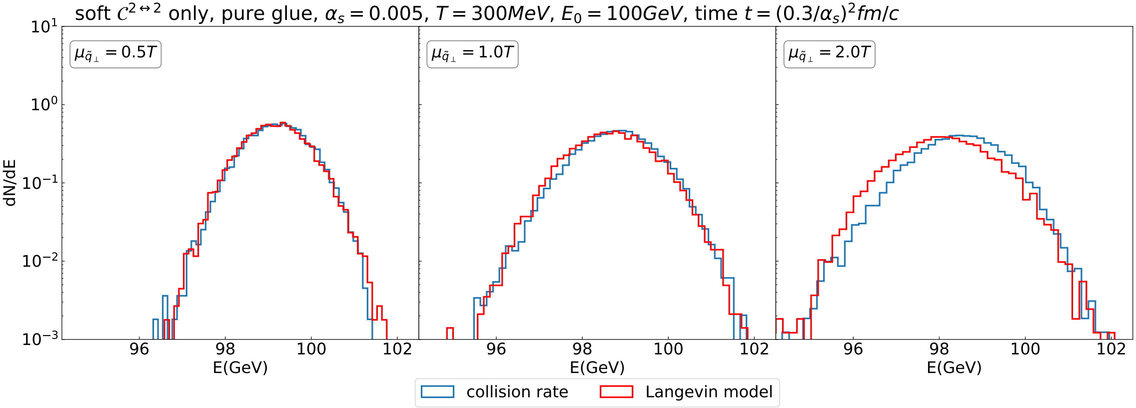
Diffusion vs collision rate - 1 $\leftrightarrow$ 2 process

energy distribution of a hard gluon propagating in a static medium when soft collisions treated with collision rate (MARTINI style) vs treated stochastically



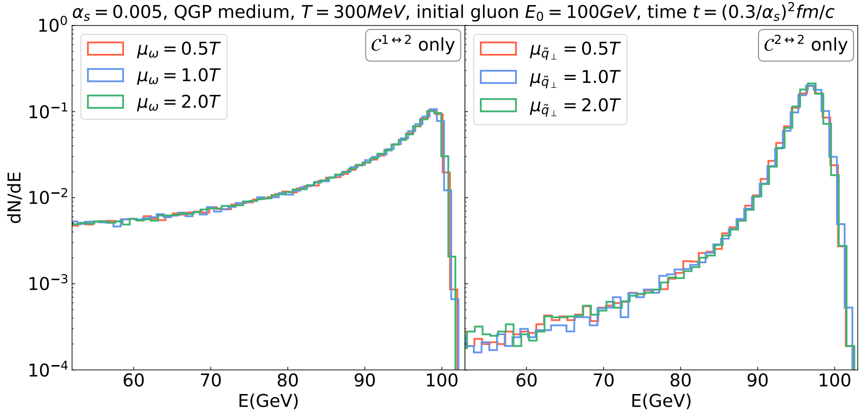
Diffusion cs collision rate - 2 $\leftrightarrow$ 2 process

energy distribution of a hard gluon propagating in a static medium when soft collisions treated with collision rate (MARTINI style) vs treated stochastically



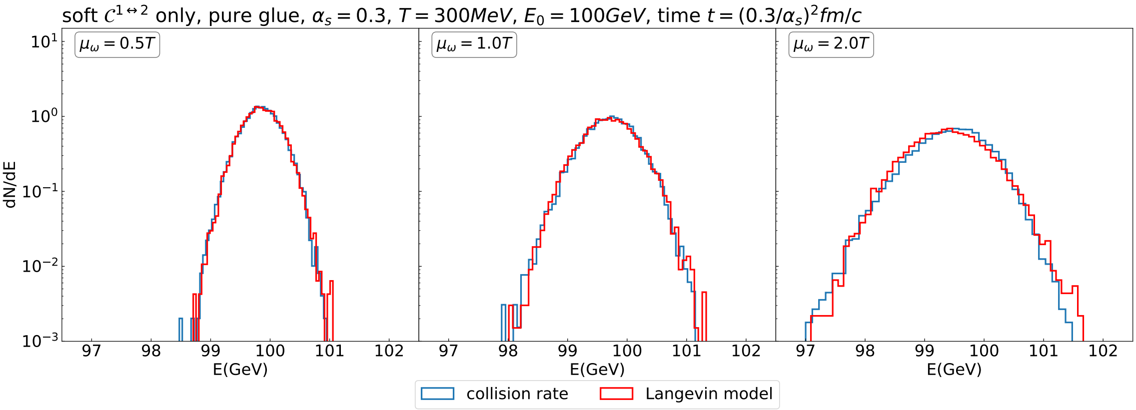
Hard-soft cutoff dependence of energy distribution

We check the dependence of the **single parton energy distribution** on the hard-soft cutoff at small coupling to validate the model.



Parton energy loss at larger coupling

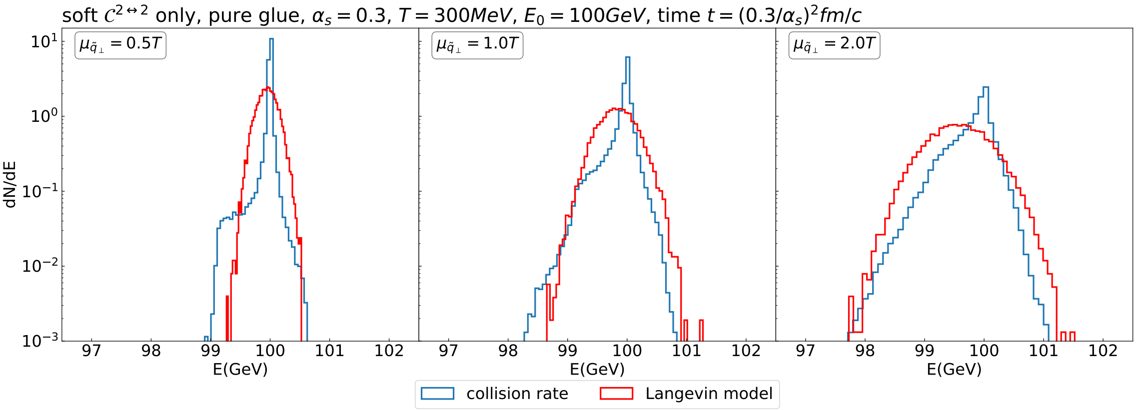
Diffusion vs collision rate - $1 \leftrightarrow 2$ process



Consistency:

- frequent soft $1 \leftrightarrow 2$ interactions

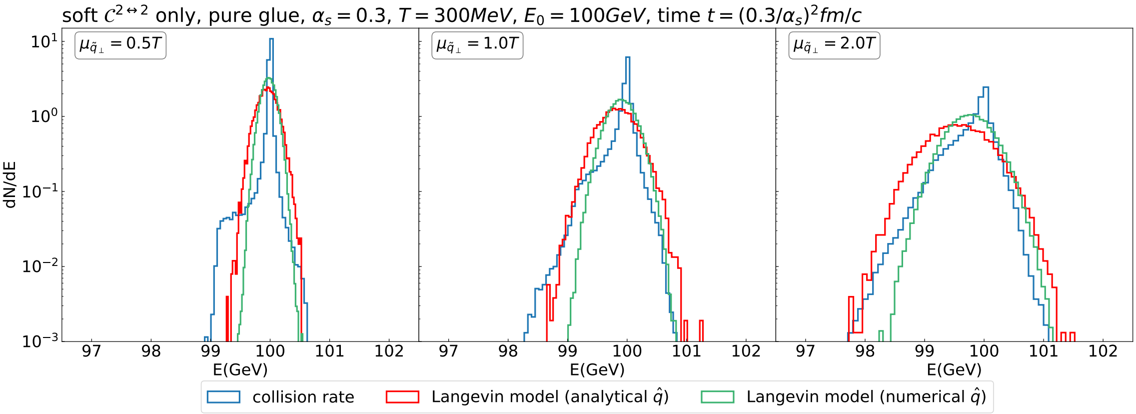
Diffusion vs collision rate - 2 $\leftrightarrow$ 2 process



Discrepancy:

- not enough soft 2 $\leftrightarrow$ 2 interactions

Diffusion vs collision rate - $2 \leftrightarrow 2$ process

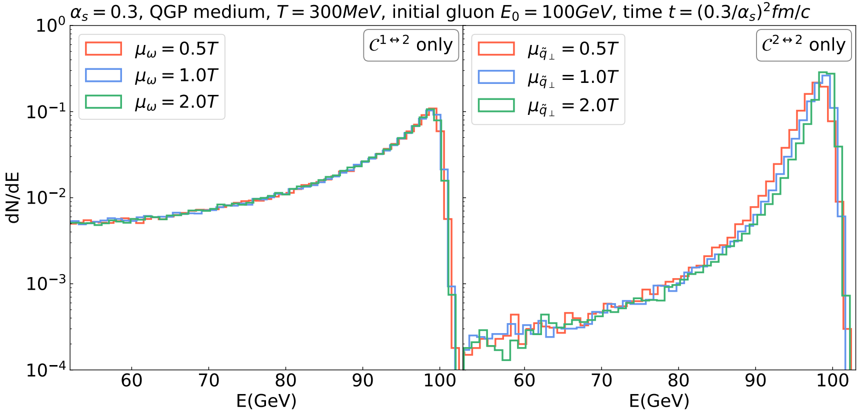


Discrepancy:

- not enough soft $2 \leftrightarrow 2$ interactions

Hard-soft cutoff dependence of energy distribution

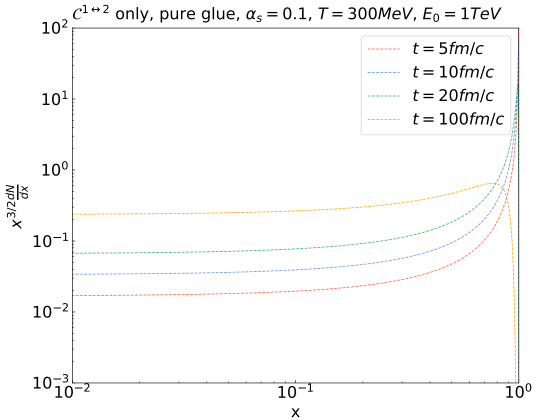
We check the dependence of the **single parton energy distribution** on the hard-soft cutoff at larger coupling to validate the model.



Application: in-medium parton energy cascade

Blaizot, Iancu, Mehtar-Tani. PRL111.5 (2013):052001.
Mehtar-Tani, Y., & Schlichting, S. (2018). JHEP, 2018(9), 144.

In-medium gluon energy cascade - analytical approximation



Assumptions:

- independent successive branchings
- approximate inelastic differential rate valid in deep LPM region

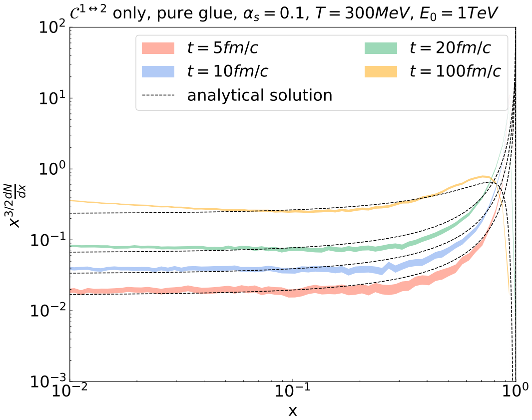
Analytical distribution of gluons (Blaizot, Iancu, Mehtar-Tani. 2013):

$$x\frac{dN}{dx} = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}e^{-\pi[\tau^2/(1-x)]}$$

where $x \equiv \omega/E_0$ and $\tau \equiv \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{E}}t$.

- In the small-x region, the power-law spectrum xdN/dx scales as $1/\sqrt{x}$.

In-medium gluon energy cascade - numerical comparison



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where $x \equiv \omega/E_0$ and $\tau \equiv \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{E}}t$.

- In the small-x region, the power-law spectrum xdN/dx scales as $1/\sqrt{x}$.
- The analytical solution is well-reproduced by the full QCD numerical model.

Conclusion

Energy loss model is reformulated as **hard collisions+diffusion**.

factorized interactions		identity preserving	identity non-preserving
soft interactions	small-$\tilde{q}_\perp$ elastic	diffusion processes	conversion rate
	small-ω inelastic		
hard interactions	large-angle elastic	vacuum matrix elements	
	large-ω elastic	splitting approximation	
	large-ω inelastic	AMY integral equations	

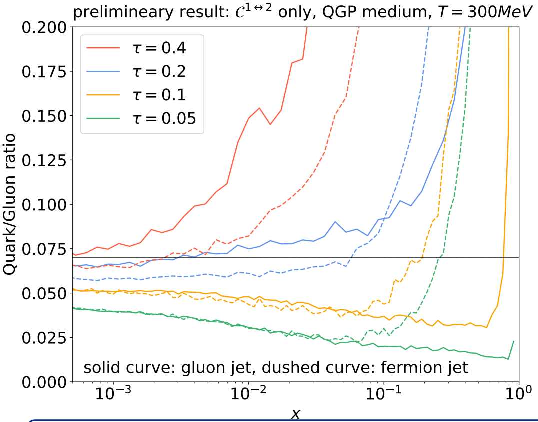
- Systematic factorization in the weakly-coupled limit
- Factorization still holds in phenomenological regime
- Factorized model is expected to be efficient and flexible

Outlook

- Make predictions with realistic hydrodynamic medium
- Apply data driven constraints on drag and diffusion coefficients
- Improve treatment of collinear emission angle
- Introduce the running coupling effect

Thanks to Jacopo Ghiglieri, Sangyong Jeon, Weiyao Ke, Chanwook Park, Yingru Xu for the useful discussion.

In-medium fermion number cascade



$$D_g \equiv x\frac{dN_g}{dx},\ D_S \equiv \sum_{i=1}^{N_F}\left(D_{q_i} + D_{\bar{q}_i}\right)$$

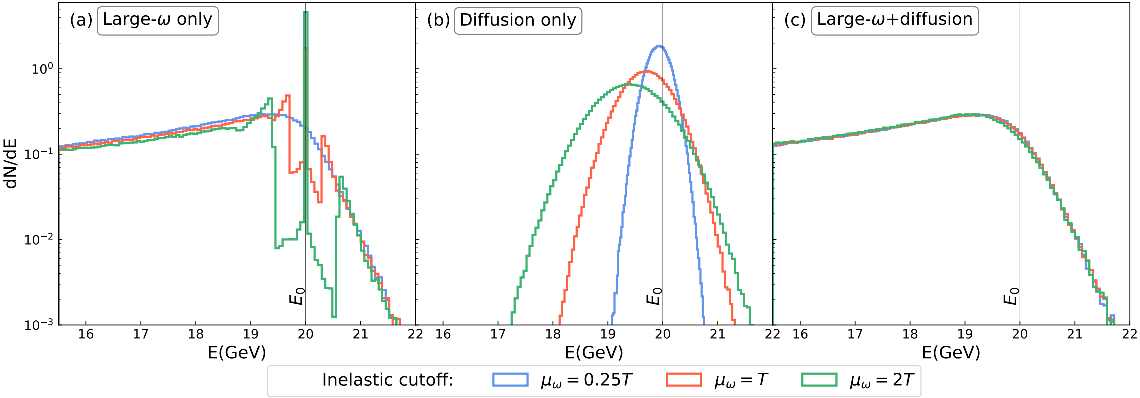
$$\frac{D_S}{2N_f D_g} = \frac{1}{2N_f}\frac{\int_0^1 dz z \mathcal{K}_{qg}(z)}{\int_0^1 dz z \mathcal{K}_{gq}(z)} \approx 0.07$$

where $\mathcal{K}_{ab}$ is the splitting function of $b \to a$.

In the small-x and large-τ region, the chemistry of fragments is determined by the balance of the $g \to q\bar{q}$ and $q \to qg$ processes.
The ratio between quark and gluon number reaches to a constant constraint.

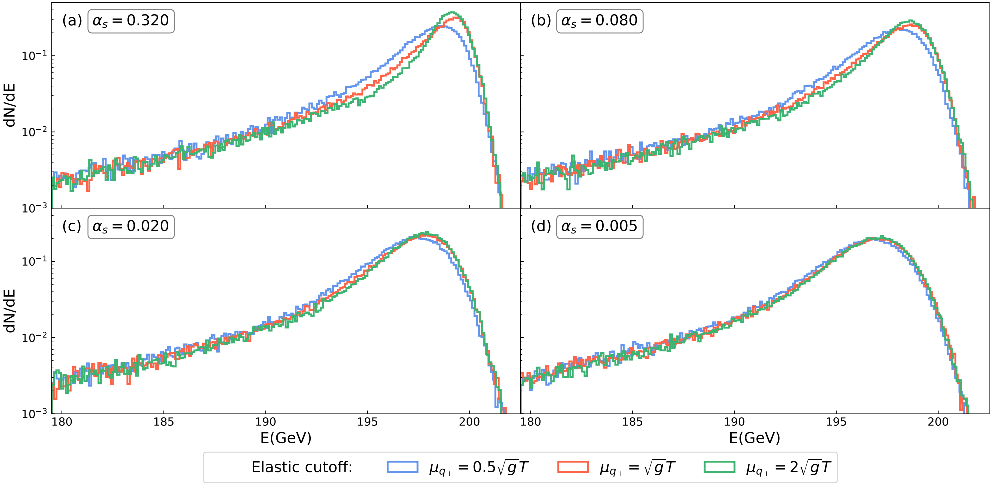
μ_ω Cutoff Independence - **Inelastic** Interactions Only

20 GeV gluon, α_s =0.3, T = 300 MeV, infinite medium, τ = 1 fm/c



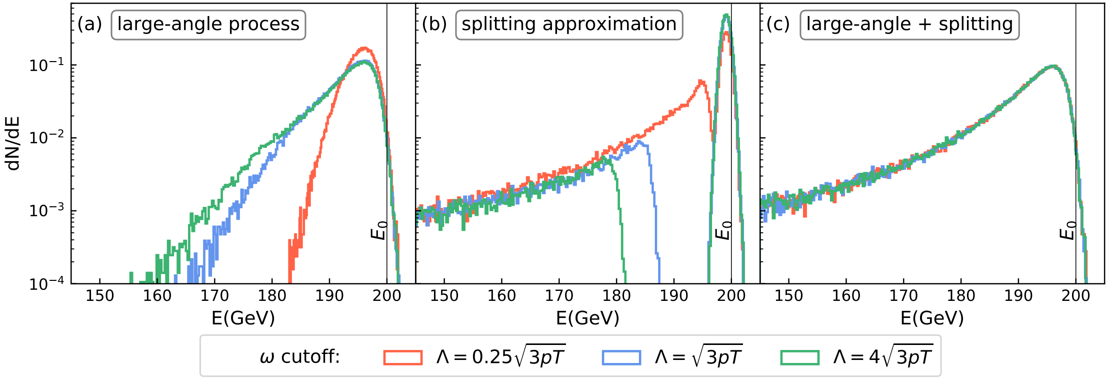
$\mu_{\tilde{q}_\perp}$ Cutoff Independence vs. α_s

200 GeV gluon, T = 300 MeV, infinite medium, $\alpha_s^2\tau = 0.3^2$fm/c, $\Lambda = \sqrt{3ET}$
Elastic interactions only:



Λ Cutoff Independence - **Elastic** Interactions Only

200 GeV gluon, α_s =0.3, T = 300 MeV, infinite medium, τ = 3fm/c, $\mu_{\tilde{q}_\perp}$ = $\mathbf{2}\sqrt{g}T$



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[3] All the plots include diffusion process.

Preliminary Comparison with MARTINI - Elastic Only

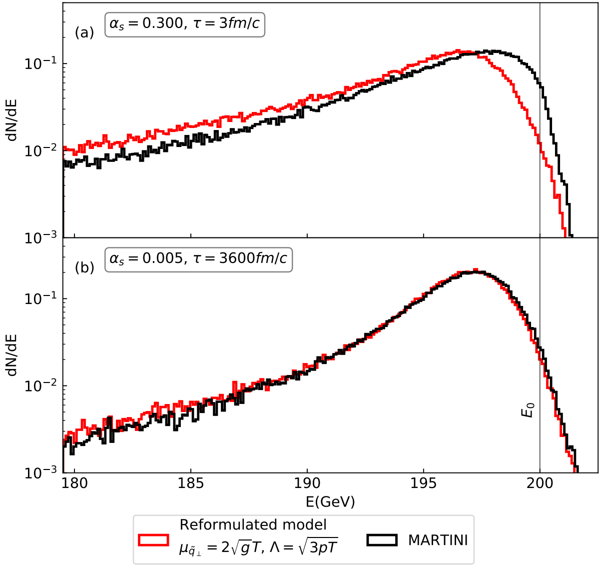
200 GeV gluon
300 MeV medium elastic interactions only

MARTINI elastic:

- screened matrix elements
- no hard / soft separation
- equivalent to reformulated model in wQGP

MARTINI inelastic:

- running coupling effect
- no hard / soft separation



Back Up - Next-to-Leading Order Calculation

$$\mathcal{C} = \mathcal{C}^{\text{large-angle}}(\mu_{\tilde{q}_\perp}, \Lambda) + \mathcal{C}^{\text{split}}(\Lambda) + \mathcal{C}^{\text{large-}\omega}(\mu_\omega) + \mathcal{C}_a^{\text{diff}}(\mu_{\tilde{q}_\perp}, \mu_\omega)$$
$$+ \delta\mathcal{C}^{\text{coll}} + \delta\mathcal{C}^{\text{diff}} + \delta\mathcal{C}^{\text{conv}} + \delta\mathcal{C}^{\text{semi-coll}}$$

Hard Thermal Loops correlators are simplified greatly when evaluated at lightlike separations.
Jet-particles propagate along the light cone: undisturbed plasma, statistical rather than dynamical

Back Up - Drag and Diffusion Coefficients

$$\hat{q} = \frac{g^2 C_R T m_D^2}{4\pi} \ln(1 + (\frac{\mu_{q_\perp}}{m_D})^2)$$

$$\hat{q}_L^{elas} = \frac{g^2 C_R T M_\infty^2}{4\pi} \ln(1 + (\frac{\mu_{\tilde{q}_\perp}}{M_\infty})^2)$$

$$\hat{q}_L^{inel} = \frac{(2-\ln 2) g^4 C_R C_A T^2 \mu_\omega}{4\pi^3}$$

$$\eta_D(p) = \frac{\hat{q}_L}{2Tp} + \frac{1}{2p^2}(\hat{q} - 2\hat{q}_L)$$

Diffusion vs collision rate - $2 \leftrightarrow 2$ process