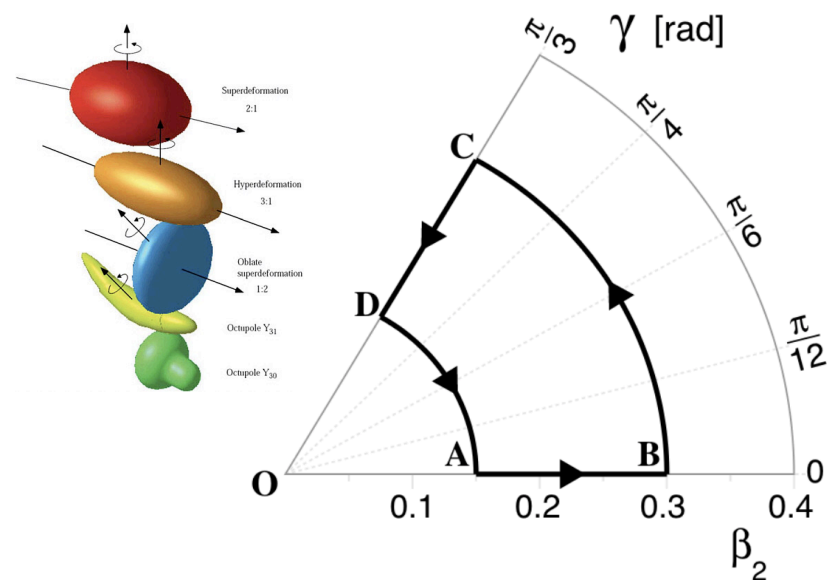


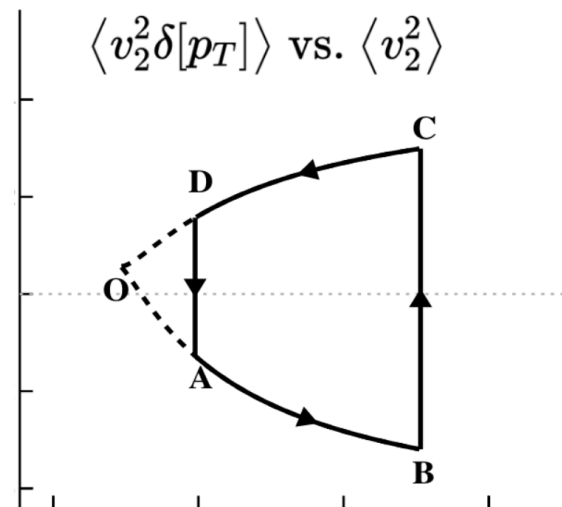
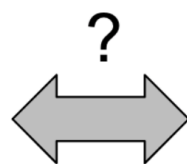
# Effects of nuclear structure in isobar collisions

Jiangyong Jia

Nuclear structure

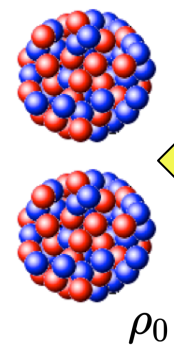


High-energy collisions



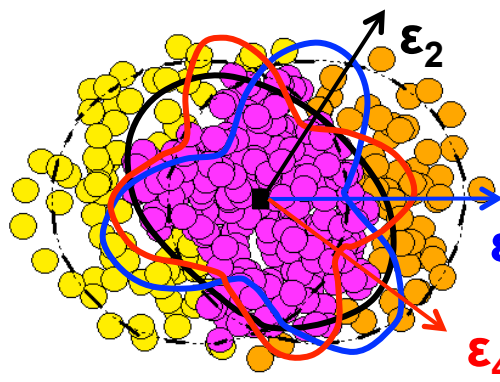
# Hydrodynamic response to initial state

Nuclear Structure



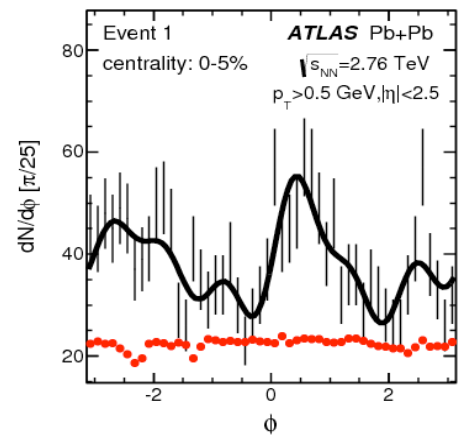
Imaging?  
↔

Initial State



Hydro-response  
↔  
Space-time dynamics

Final Particle flow



$$1 + e^{(r - R_0(1 + \sum_n \beta_n Y_n^0(\theta, \phi)))/a_0}$$

- $\beta_2 \rightarrow$  Quadrupole deformation
- $\beta_3 \rightarrow$  Octupole deformation
- $a_0 \rightarrow$  Surface diffuseness
- $R_0 \rightarrow$  Nuclear size

Initial Size      Initial Shape

$$R_{\perp} \propto \langle r_{\perp}^2 \rangle, \quad \epsilon_n \propto \langle r_{\perp}^n e^{in\phi} \rangle$$

??

$R_0$      $a_0$      $\beta_n$

Radial Flow      Harmonic Flow

$$\frac{d^2 N}{d\phi dp_T} = N(p_T) \left( \sum_n V_n e^{-in\phi} \right)$$

arXiv:1206.1905

High energy: approx. linear response in each event:

$$\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_{\perp}}{R_{\perp}} \quad V_n \propto \epsilon_n$$

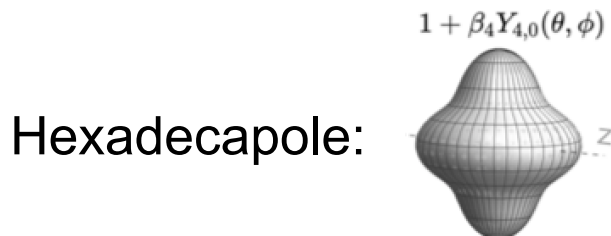
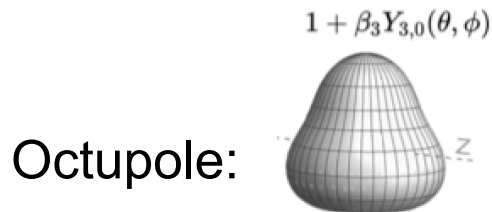
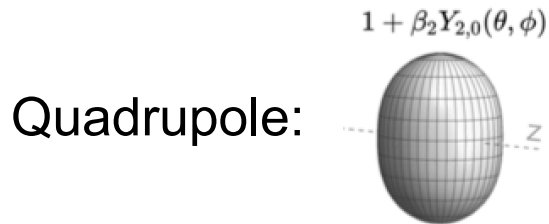
Previous works: PRC34.185, PRC61.021903, PRC61.034905, PRC.80.054903, nucl-th/0411054, 0712.0088, 1409.8375, 1507.03910, 1609.01949, 1711.08499, 2007.00780.2103.05595. H. Stoecker, W. Geiner, BA Li, E. Shuryak, PBM, U. Heinz, P. Philip, N.Xu, Q. Shou, P. Sorensen, F. Videbaek, A. Tang, P. Dasgupta, R. Chatterjee, D. Krivastava, F.Wang, H. Xu, Jaki. M. Luzum, P.Carzon...

# Shape of nuclei

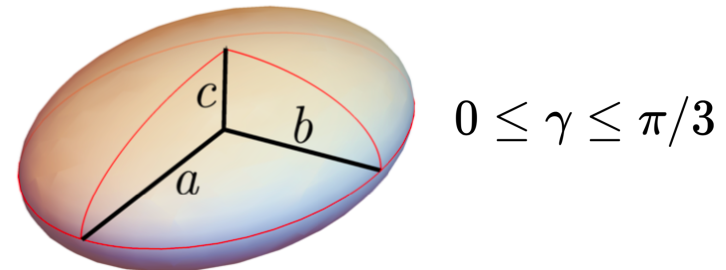
Most ground state stable nuclei are deformed

$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{(r-R(\theta,\phi))/a_0}}$$

$$R(\theta, \phi) = R_0 \left( 1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^3 \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^4 \alpha_{4,m} Y_{4,m} \right)$$



Triaxial spheroid:  $a \neq b \neq c$ .



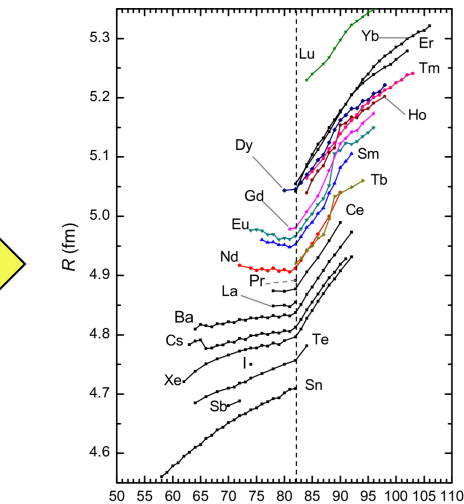
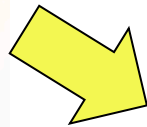
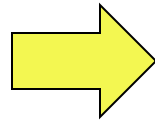
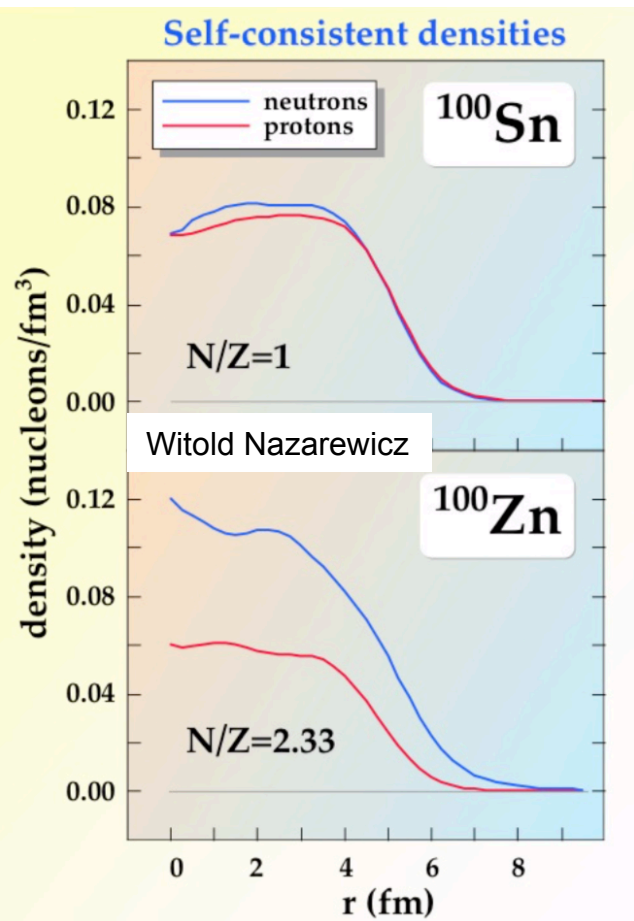
Prolate:  $a=b < c \rightarrow \beta_2, \gamma=0$

Oblate:  $a < b=c \rightarrow \beta_2, \gamma=\pi/3$  or  $-\beta_2, \gamma=0$

# Radial structure of nuclei

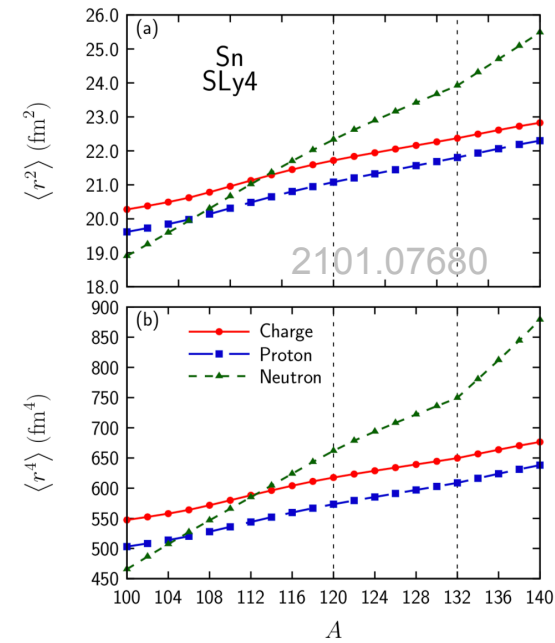
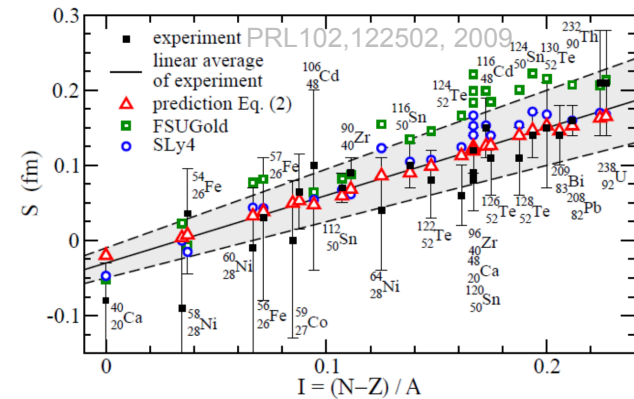
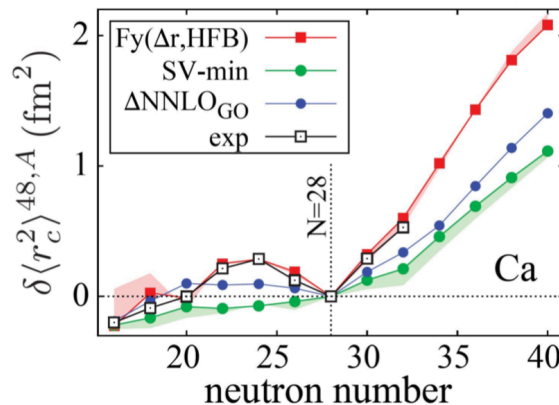
$$\rho = \frac{\rho_0}{1 + e^{(r - R_0(1 + \sum_n \beta_n Y_n^0(\theta, \phi)))/a_0}}$$

$$\Delta r_{np} = R_n - R_p$$



## Higher radial moments

1911.00699 2101.00320



# Isobar collisions as precision tool

- Unique running mode of RHIC and STAR to minimize the detector systematics
  - 0.4% precision is achieved in ratio of many observables between two isobar systems  $\rightarrow$  precision imaging tool

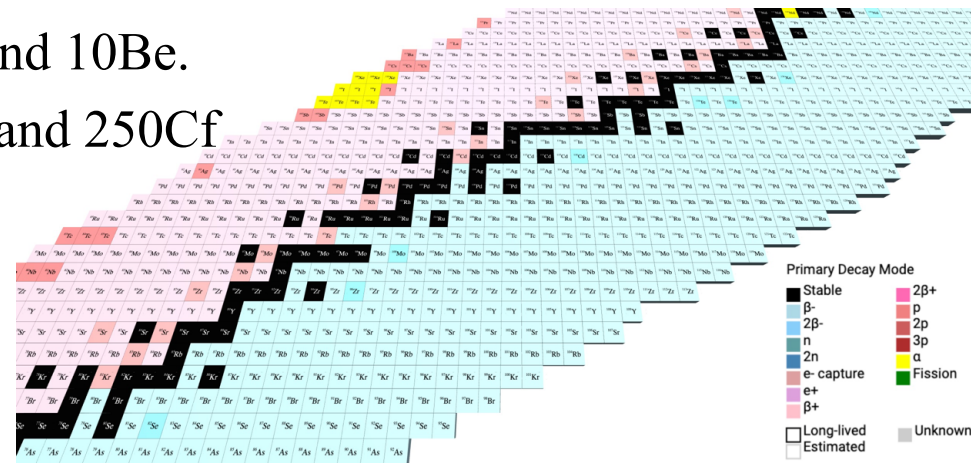
A key question for any  
HI observable  $\bigcirc$

$$\frac{O_{X+X}}{O_{Y+Y}} \stackrel{?}{=} 1 \quad {}^A X + {}^A X \text{ vs } {}^A Y + {}^A Y$$

Deviation from 1 must have its origin in the nuclear structure, which is reflected by the initial state and then survives the final state. A precision tool to study initial state and final state responses

- Many such pairs of isobars in the nuclear chart.
  - Small system isobar such as  ${}^{10}\text{B}$  and  ${}^{10}\text{Be}$ .
  - Large system isobar up to  ${}^{250}\text{Cm}$  and  ${}^{250}\text{Cf}$

Can help many heavy ion  
physics questions



# Parametric forms: nuclear shape

2109.00604

- $\epsilon_n$  has the form 
$$\epsilon_n = \underbrace{\epsilon_{n;0}}_{\text{undeformed}} + \sum_{m=2}^4 \underbrace{p_{n;m}(\Omega_1, \Omega_2)}_{\text{phase factor}} \beta_m + \mathcal{O}(\beta^2)$$
- $R_\perp^2 = \langle x^2 \rangle + \langle y^2 \rangle$  has the form 
$$\delta d_\perp / d_\perp = \delta_d + \sum_{m=2}^4 p_{0;m}(\Omega_1, \Omega_2) \beta_m + \mathcal{O}(\beta^2)$$
  
 $d_\perp \equiv 1/R_\perp$

- Two particle correlation

$$\langle \epsilon_n^2 \rangle \approx \langle \epsilon_{n;0}^2 \rangle + \sum_{m,m'} \langle p_{n;m} p_{n;m'}^* \rangle \beta_m \beta_{m'} \quad \left\langle \left( \frac{\delta d_\perp}{d_\perp} \right)^2 \right\rangle \approx \langle \delta_d^2 \rangle + \sum_{m,m'} \langle p_{0;m} p_{0;m'} \rangle \beta_m \beta_{m'}$$

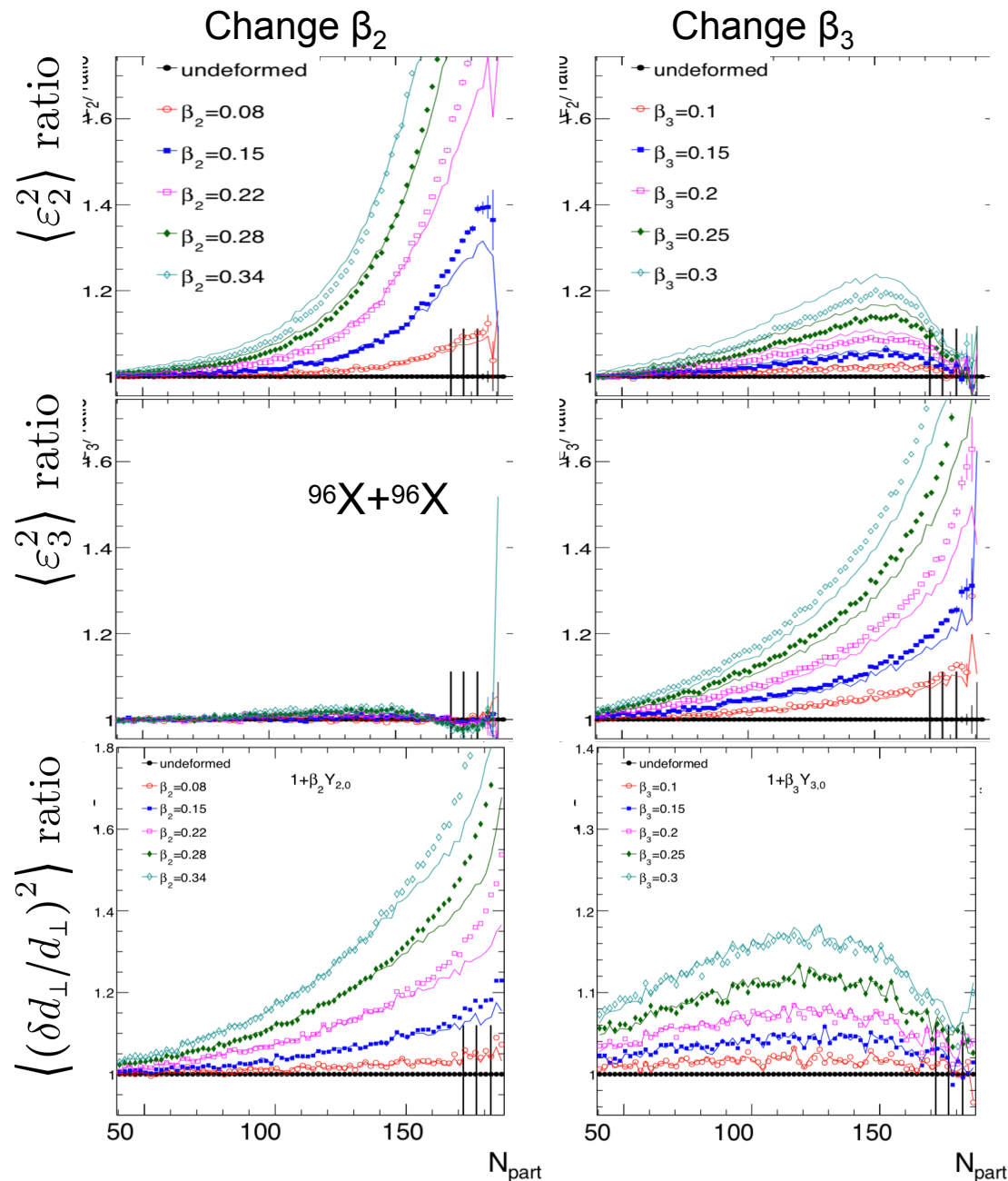
- In reality for medium size nucleus and ignore  $\beta_4$ .

$$\langle \epsilon_2^2 \rangle = a'_2 + b'_2 \beta_2^2 + b'_{2,3} \beta_3^2 \quad \langle \epsilon_3^2 \rangle = a'_3 + b'_3 \beta_3^2$$

$$\left\langle \left( \delta d_\perp / d_\perp \right)^2 \right\rangle = a'_0 + b'_0 \beta_2^2 + b'_{0,3} \beta_3^2$$

- Again linear response to relate to final state: 
$$v_n \propto \epsilon_n \frac{\delta[p_T]}{[p_T]} \propto \frac{\delta d_\perp}{d_\perp}$$

# Parametric dependence



medium size system:

$$\varepsilon_2^2 = a'_2 + b'_2 \beta_2^2 + b'_{2,3} \beta_3^2$$

$$\varepsilon_3^2 = a'_3 + b'_3 \beta_3^2$$

$$(\delta d_\perp / d_\perp)^2 = a'_0 + b'_0 \beta_2^2 + b'_{0,3} \beta_3^2$$



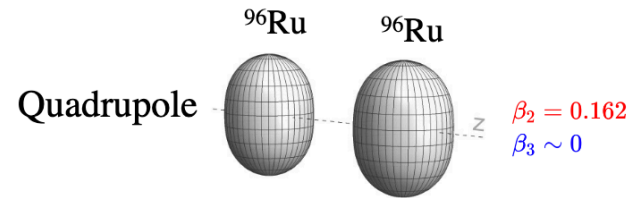
$$v_2^2 = a_2 + b_2 \beta_2^2 + b_{2,3} \beta_3^2$$

$$v_3^2 = a_3 + b_3 \beta_3^2$$

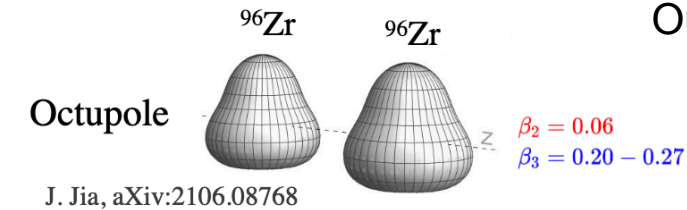
$$(\delta p_T / p_T)^2 = a_0 + b_0 \beta_2^2 + b_{0,3} \beta_3^2$$

# Expectation of some ratios in isobar

H. Xu, et.al. 2103.05595



$^{96}\text{Ru}$		$^{96}\text{Zr}$	
$R$	$a$	$R$	$a$
5.065	0.485	4.961	0.544



Our choice:

Species	$\beta_2$	$\beta_3$	$a_0$	$R$
Ru+Ru	0.162	0.00	0.46	5.09
Zr+Zr	0.06	0.20	0.52	5.02

$$\Delta R = +0.07 \text{ fm}$$

$$\Delta a = -0.06 \text{ fm}$$

Heavy ion expectation:

$$\frac{v_{n,\text{Ru}}^2}{v_{n,\text{Zr}}^2} \approx 1 + \frac{b_{n,2}}{a_n} (\beta_{2,\text{Ru}}^2 - \beta_{2,\text{Zr}}^2) + \frac{b_{n,3}}{a_n} (\beta_{3,\text{Ru}}^2 - \beta_{3,\text{Zr}}^2)$$

In general:

$$\frac{v_{n,\text{Ru}}}{v_{n,\text{Zr}}} \approx 1 + c_1 \Delta \beta_2^2 + c_2 \Delta \beta_3^2 + c_3 \Delta a + c_4 \Delta R \quad c_1 = \frac{b_{n,2}}{2a_n} \quad c_2 = \frac{b_{n,3}}{2a_n}$$

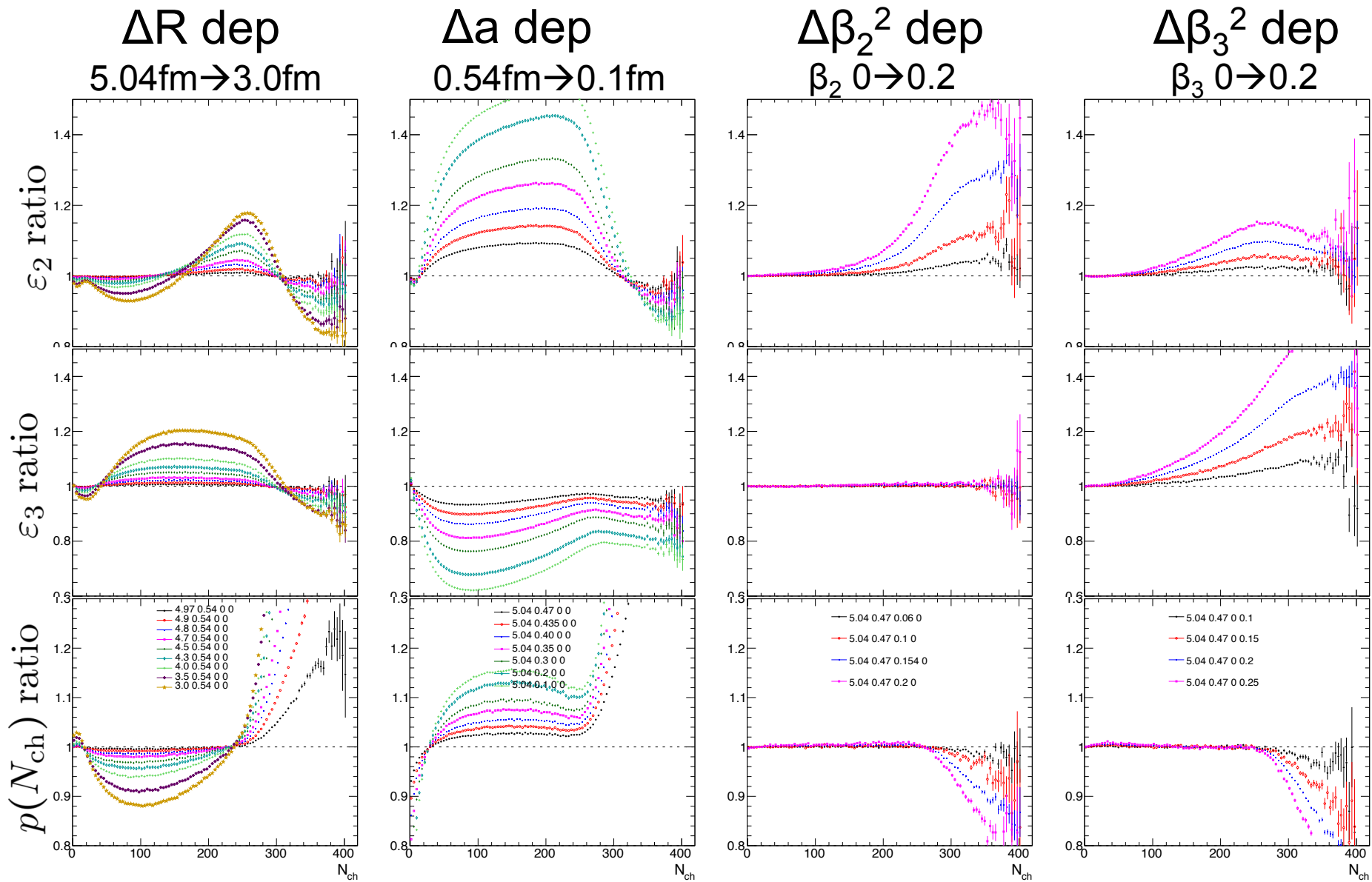
Similarly:

$$\frac{(\delta p_{\text{T}}/p_{\text{T}})_{\text{Ru}}^2}{(\delta p_{\text{T}}/p_{\text{T}})_{\text{Zr}}^2} \approx 1 + c_1 \Delta \beta_2^2 + c_2 \Delta \beta_3^2 + c_3 \Delta a + c_4 \Delta R$$

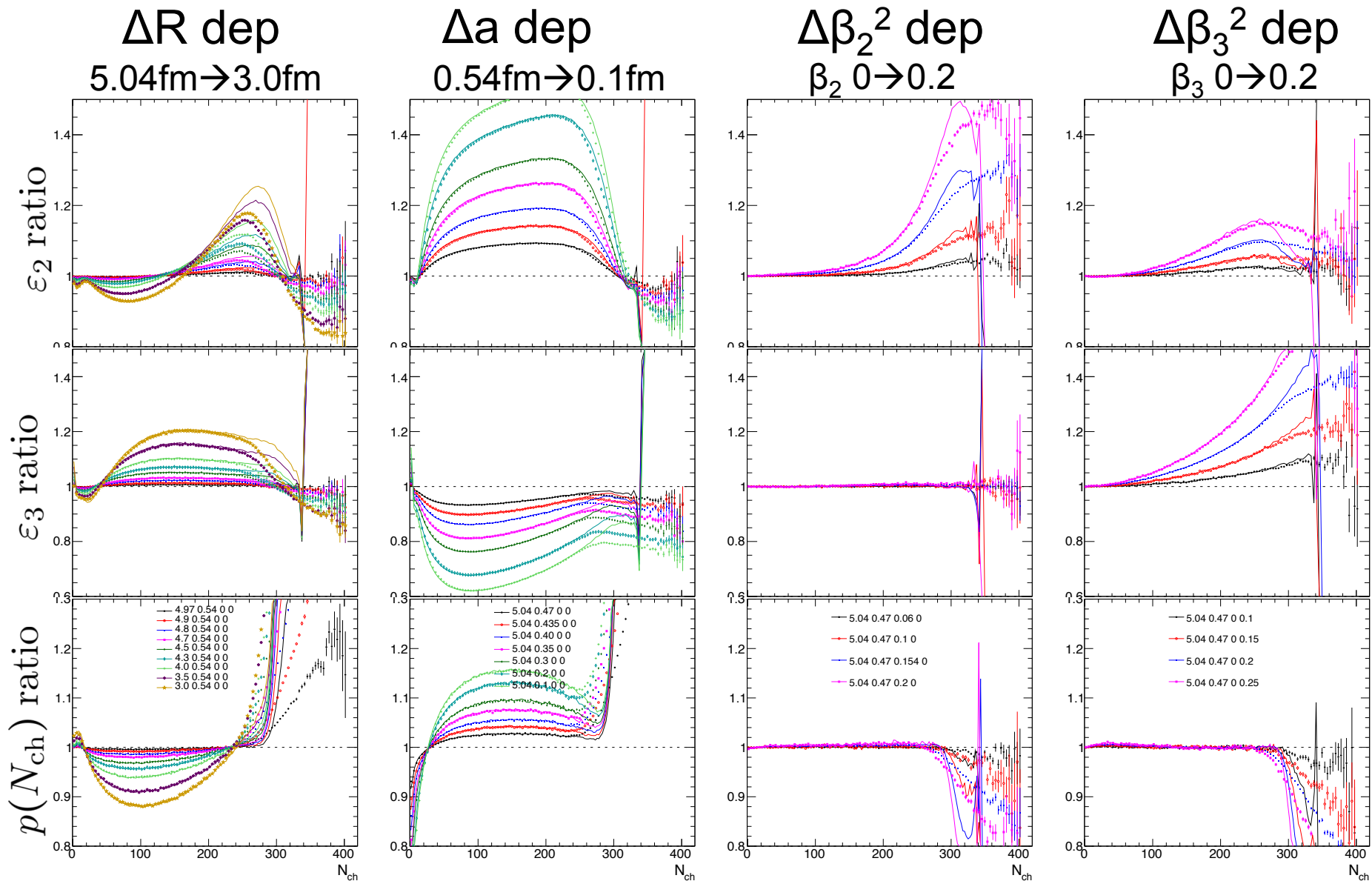
$$\frac{p(N_{\text{ch}})_{\text{Ru}}}{p(N_{\text{ch}})_{\text{Zr}}} \approx 1 + c_1 \Delta \beta_2^2 + c_2 \Delta \beta_3^2 + c_3 \Delta a + c_4 \Delta R$$



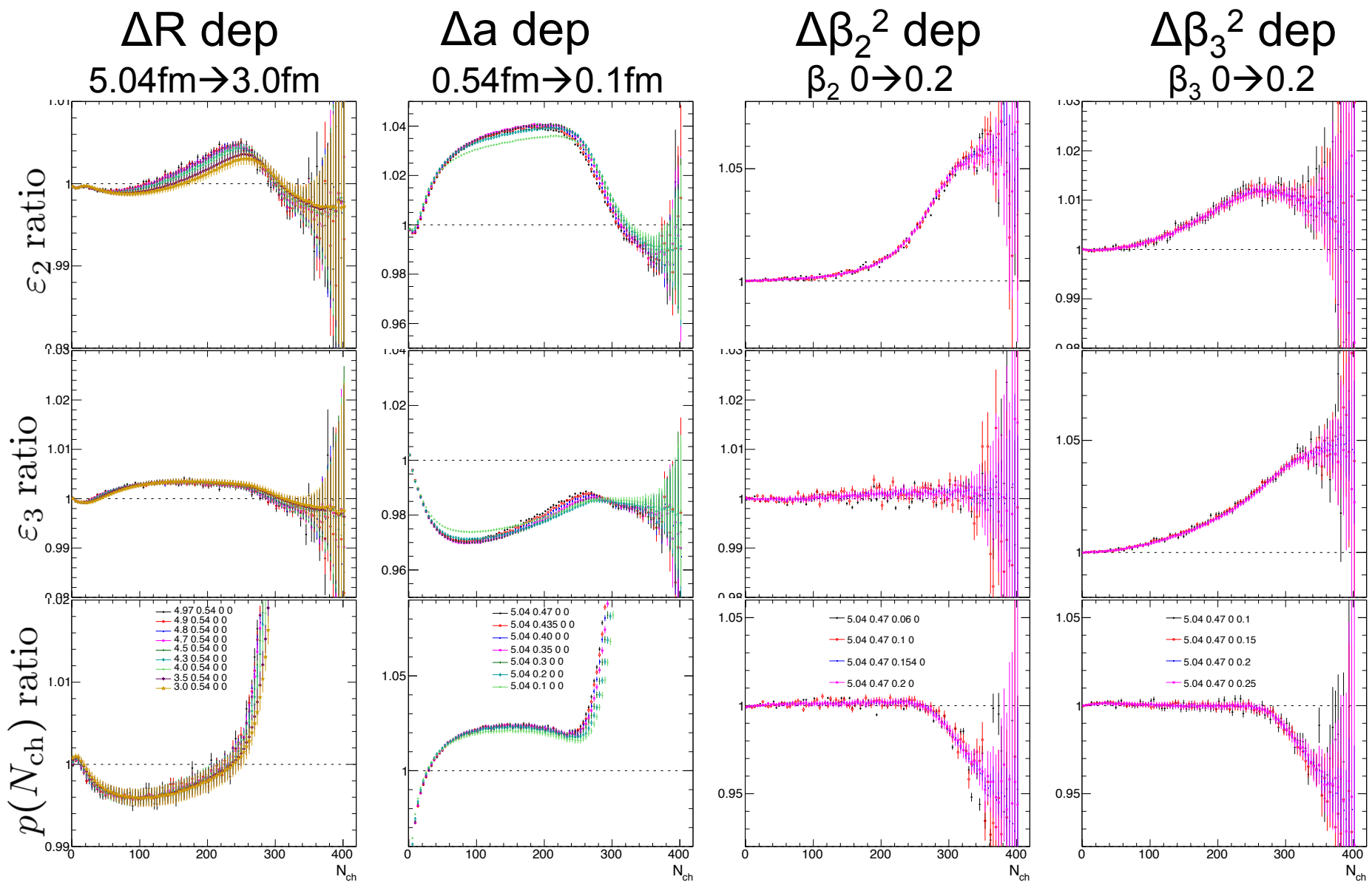
# Glauber results: $N_{ch}$ dep



# Glauber results: $N_{ch}$ vs $N_{qp}$ dep



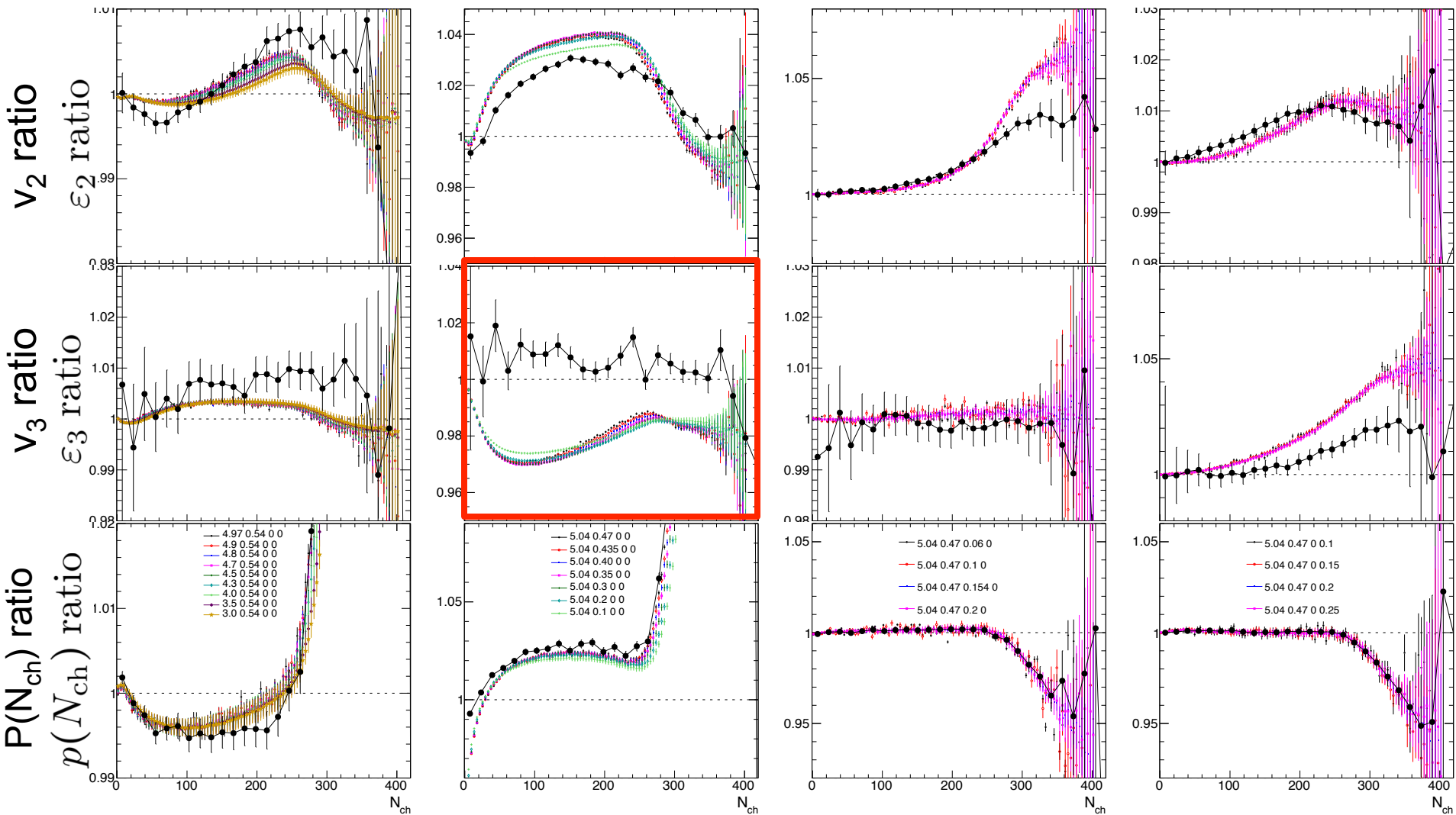
# Glauber results scaled



Verifies the relation:  $1 + c_1 \Delta\beta_2^2 + c_2 \Delta\beta_3^2 + c_3 \Delta a + c_4 \Delta R$

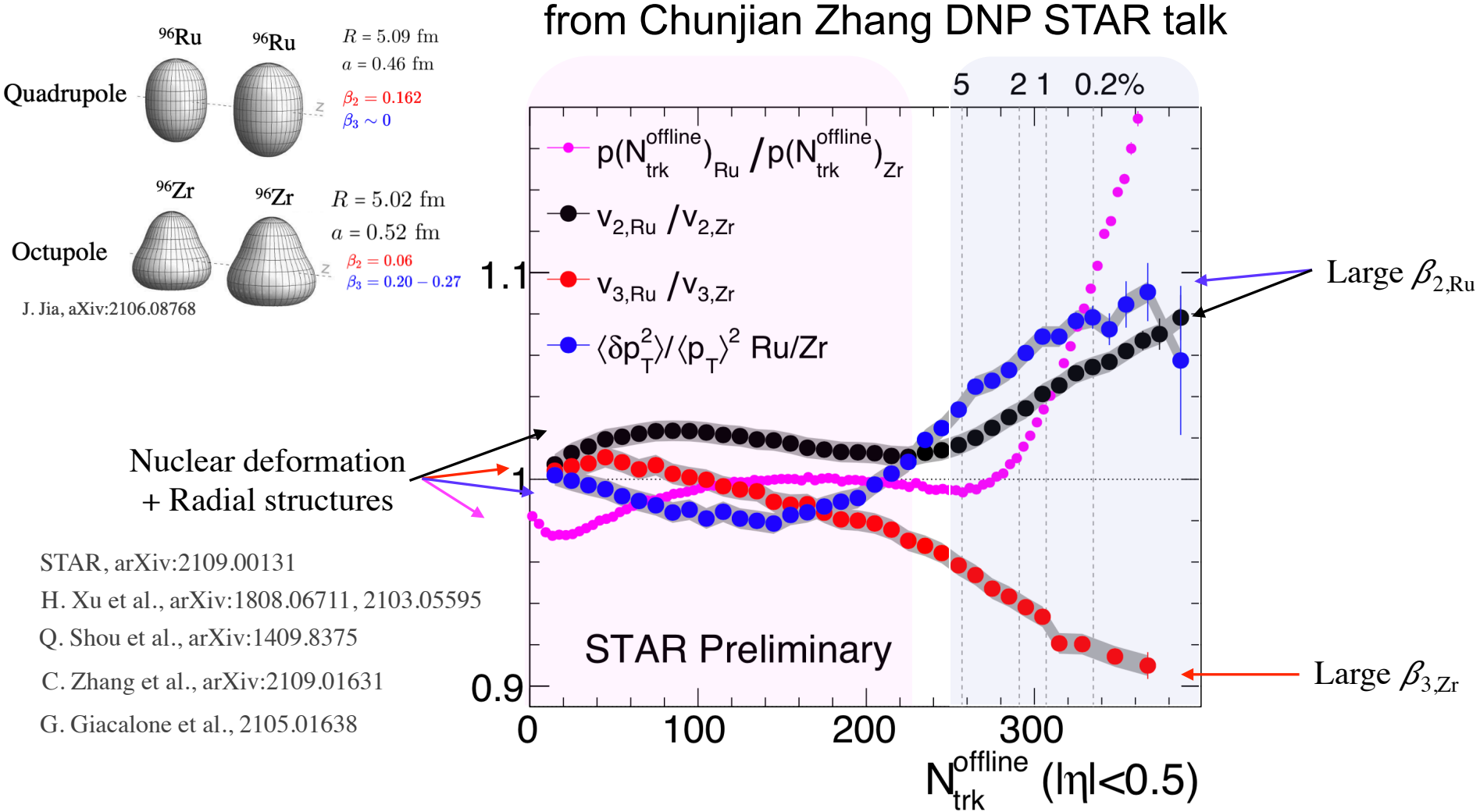
# Compare with AMPT final state results

Final state ratio follow similar trends as data. Except one



Note: Glauber model implemented in AMPT can be improved

# Compare with isobar data



STAR, arXiv:2109.00131

H. Xu et al., arXiv:1808.06711, 2103.05595

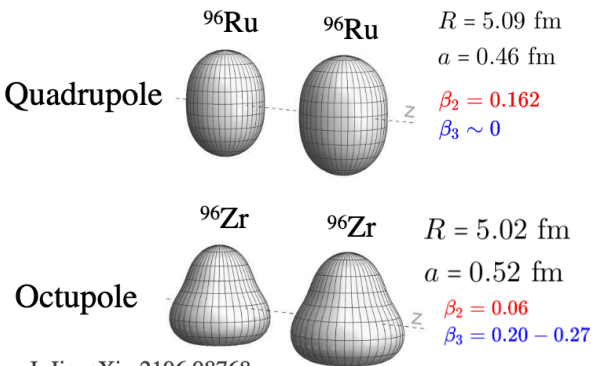
Q. Shou et al., arXiv:1409.8375

C. Zhang et al., arXiv:2109.01631

G. Giacalone et al., 2105.01638

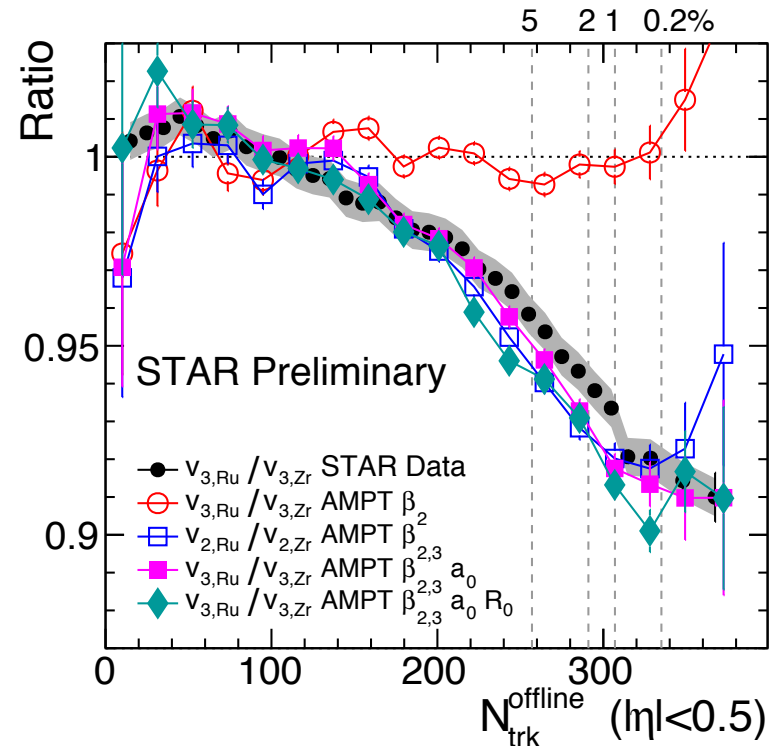
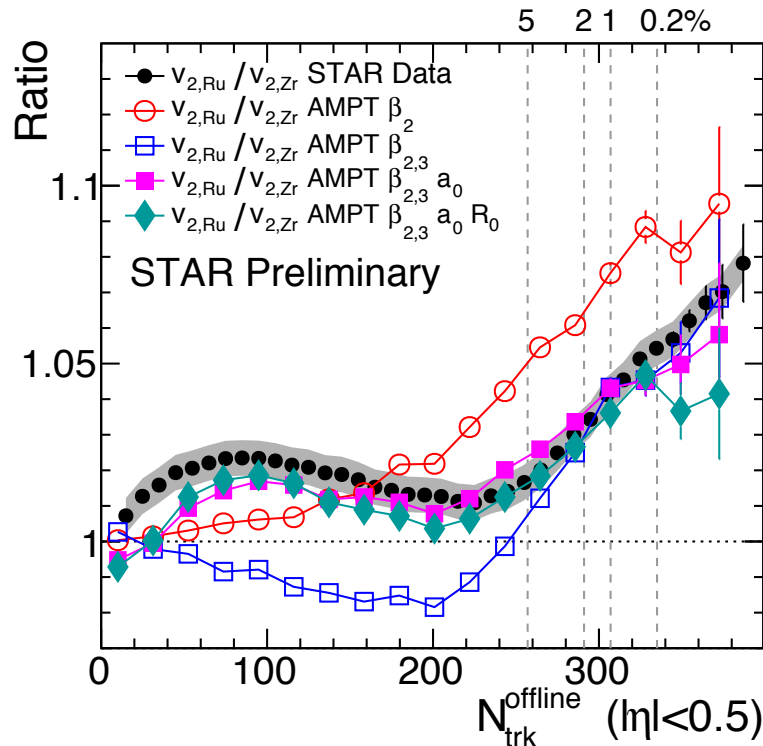
Use these ratios to probe shape and radial structure of nuclei.

# Nuclear structure via $v_n$ -ratio



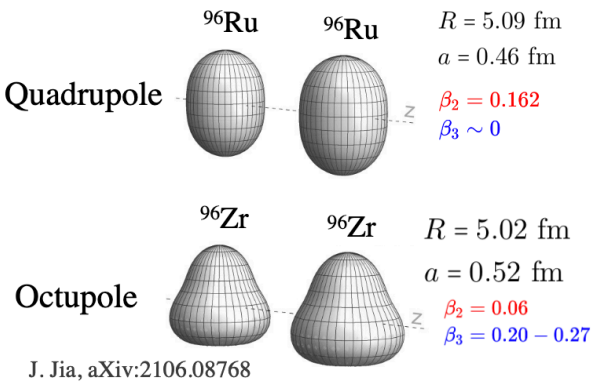
J. Jia, aXiv:2106.08768

- $\beta_{2\text{Ru}} \sim 0.16$  increase  $v_2$ , no influence on  $v_3$  ratio
- $\beta_{3\text{Zr}} \sim 0.2$  decrease  $v_2$  in mid-central, decrease  $v_3$  ratio
- diffu.  $\Delta a_0 = -0.06 \text{ fm}$  increase  $v_2$  mid-central, no influe. on  $v_3$ .
  - Similar study by Haojie et.al.
- Radius  $\Delta R_0 = 0.07 \text{ fm}$  only slightly affects  $v_2$  and  $v_3$  ratio.

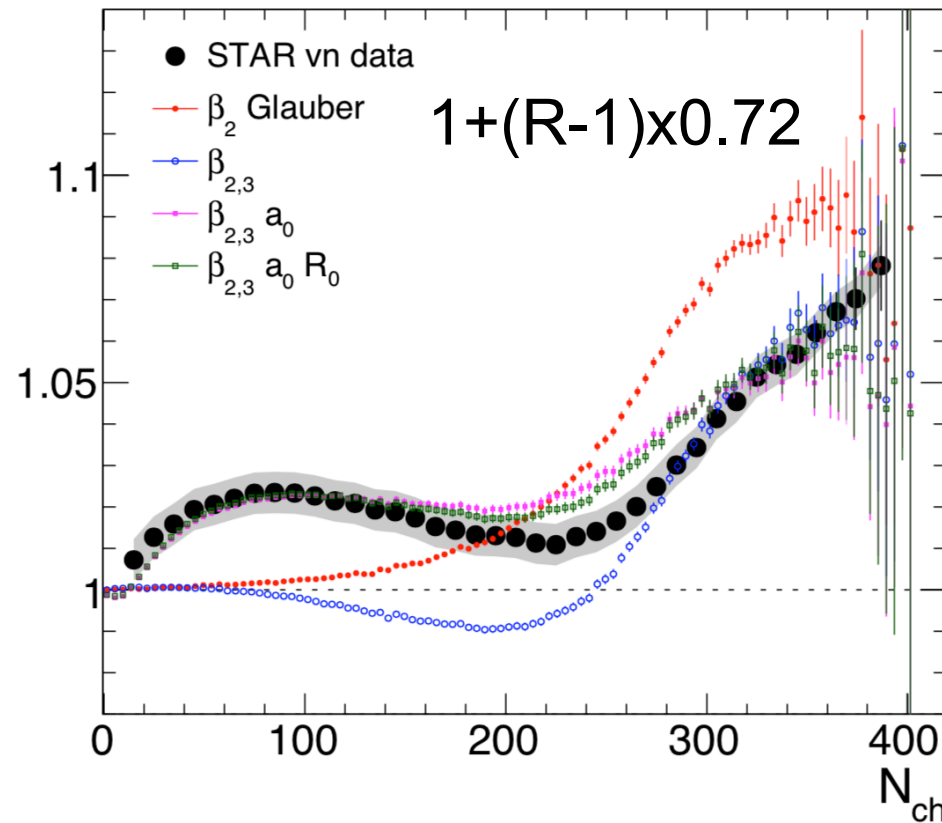


Simultaneously constrain these parameters using different  $N_{\text{ch}}$  regions

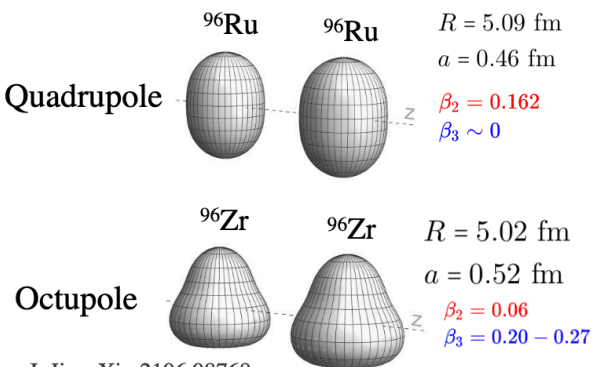
# Nuclear structure via $v_n$ -ratio



- $\beta_{2\text{Ru}} \sim 0.16$  increase  $v_2$ , no influence on  $v_3$  ratio
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  - Similar study by Haojie et.al.
- Radius  $\Delta R_0 = 0.07 \text{ fm}$  only slightly affects  $v_2$  and  $v_3$  ratio.
- Trends are reproduced by Glauber model with rescaling signal down by 0.72.

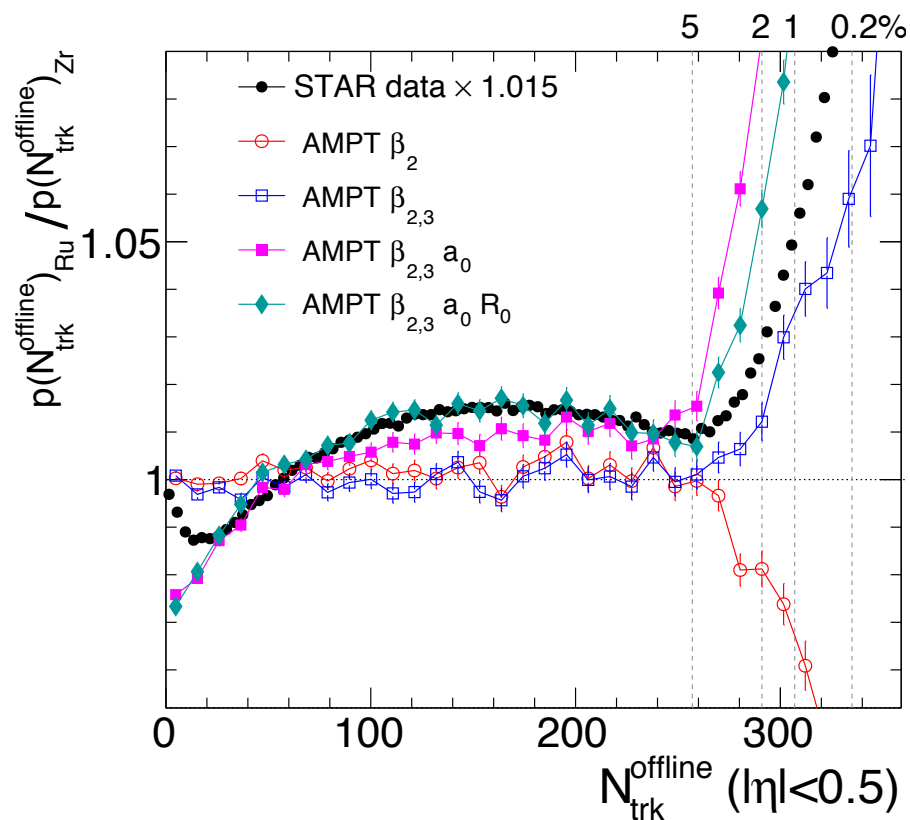


# Nuclear structure via $p(N_{ch})$ -ratio



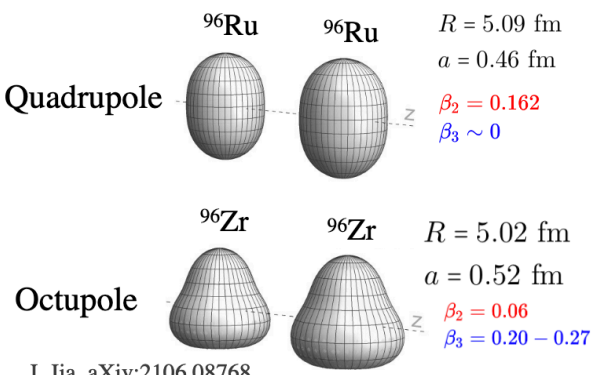
- $\beta_{2\text{Ru}} \sim 0.16$  decrease ratio, increase after considering  $\beta_{3\text{Zr}} \sim 0.2$
- The bump structure in non-central region mostly sensitive to differences in surface diffuseness  $\Delta a_0$  and radius  $\Delta R_0$

J. Jia, aXiv:2106.08768

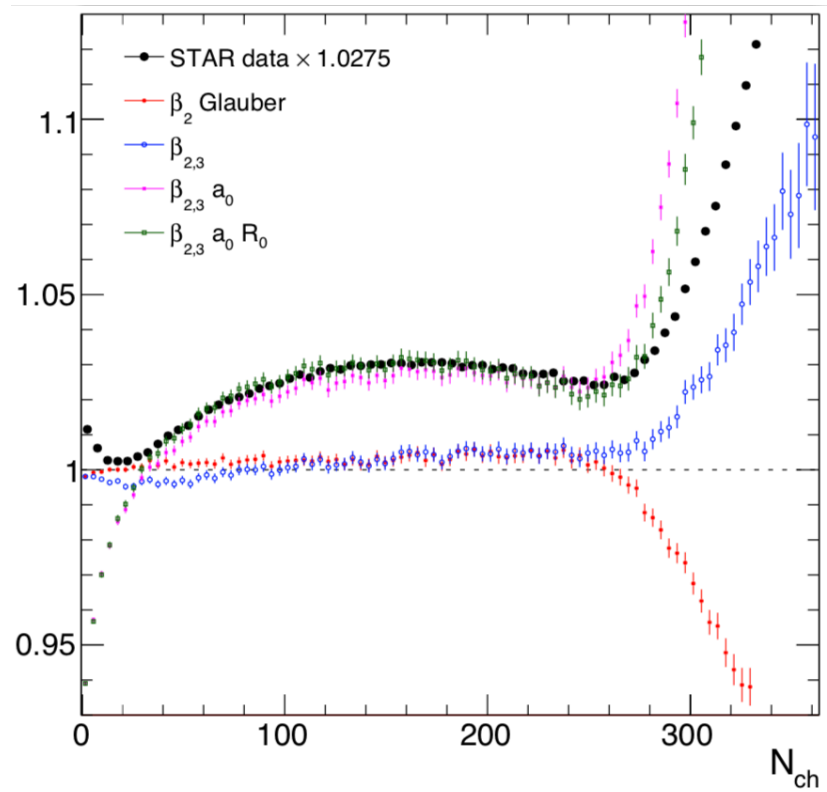
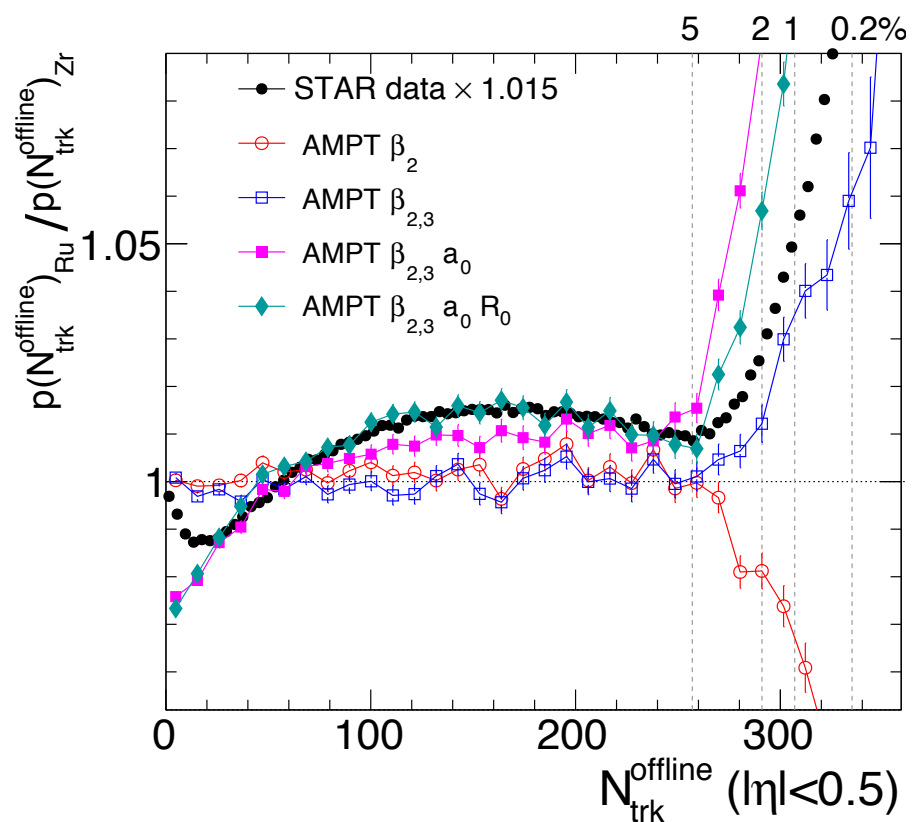




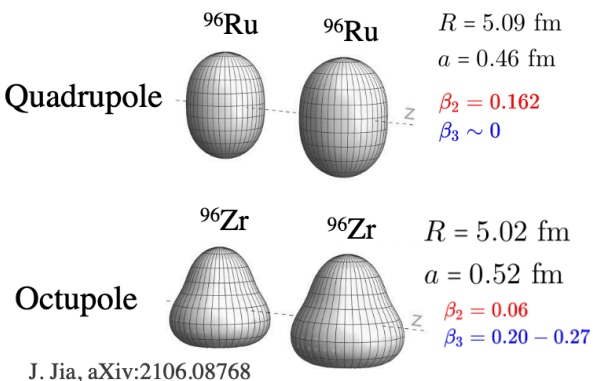
# Nuclear structure via $p(N_{ch})$ -ratio



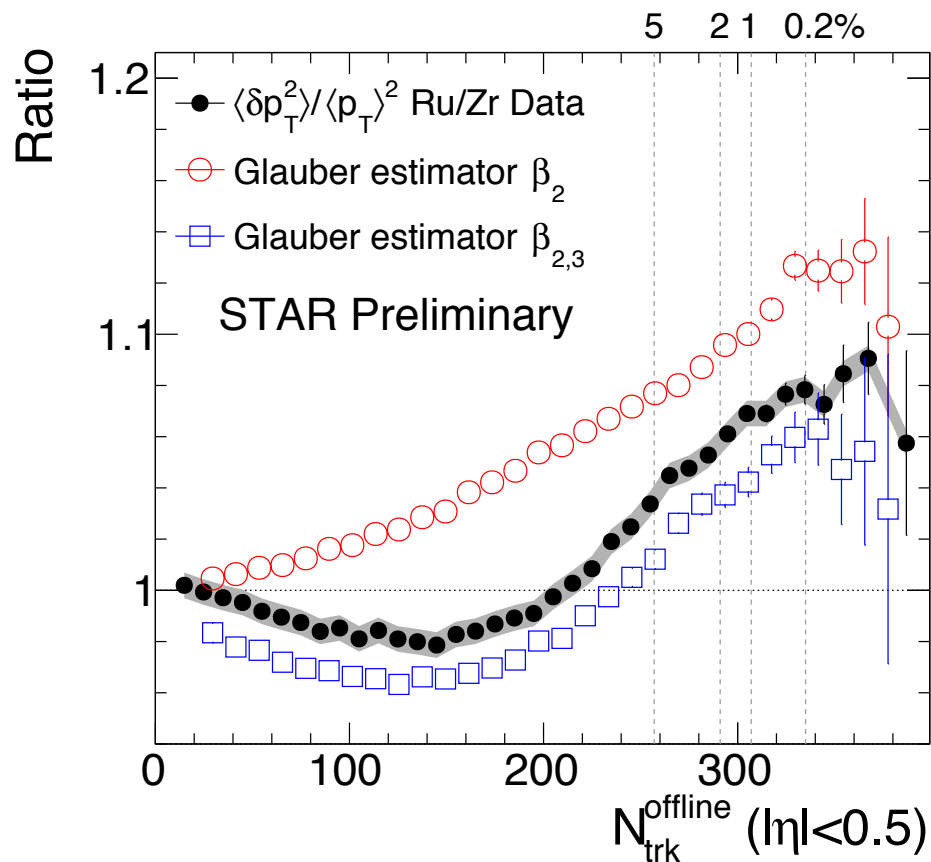
- $\beta_{2\text{Ru}} \sim 0.16$  decrease ratio, increase after considering  $\beta_{3\text{Zr}} \sim 0.2$
- The bump structure in non-central region mostly sensitive to differences in surface diffuseness  $\Delta a_0$  and radius  $\Delta R_0$
- All these trends are quantitatively reproduced by Glauber
  - Note the **normalization is sensitive to trigger efficiency!**
  - See related study by Haojie et.al.



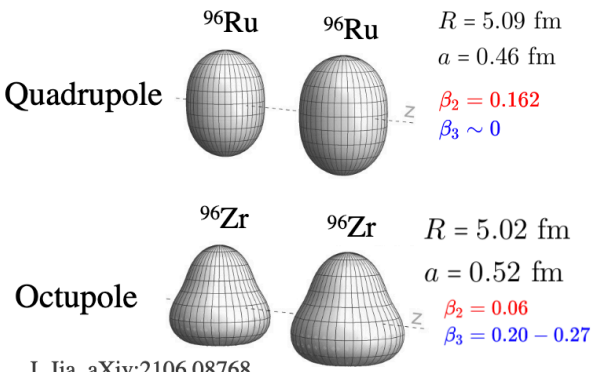
# Nuclear structure via $p_T$ fluctuations



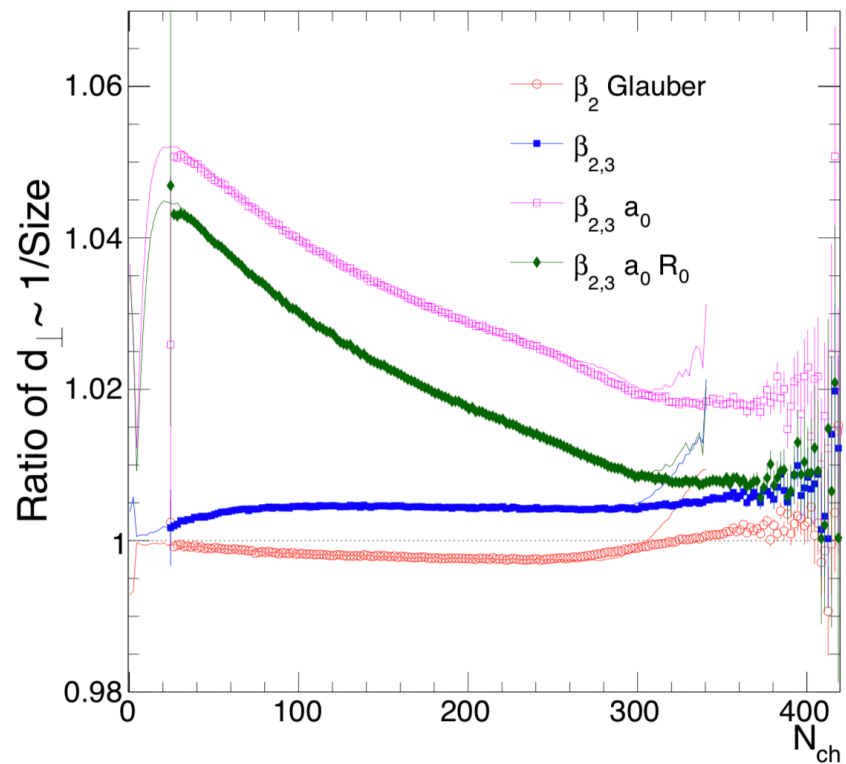
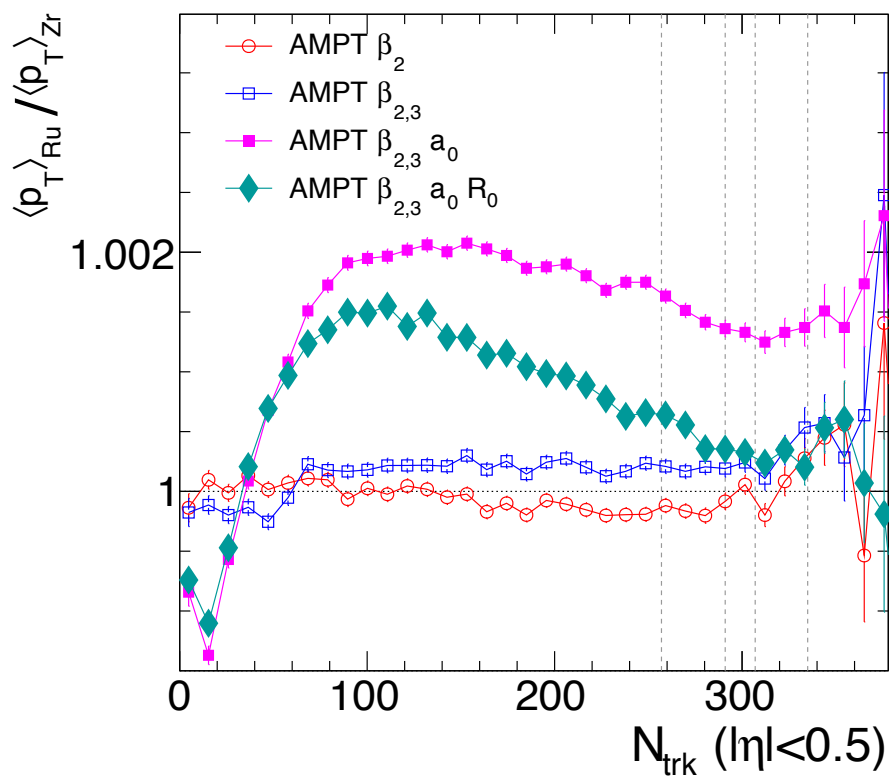
- Glauber model is used by assuming  $\frac{\delta[p_T]}{p_T} \propto \frac{\delta d_\perp}{d_\perp}$
- $\beta_{2\text{Ru}} \sim 0.16$  increase ratio while  $\beta_{3\text{Zr}} \sim 0.2$  decrease it
- AMPT has wrong  $\langle p_T \rangle$  responses (see 2109.00604), but..



# Nuclear structure via $\langle p_T \rangle$



- AMPT underestimate response by x3, so trust trends only
- $\beta_2, \beta_3$  small impact in noncentral, but some increase in UCC
- Enhancement dominated by surface diffuseness
- Radius difference leads to stronger  $N_{ch}$  dependence
- Glauber model also describe the trends..



# Outlook

- Much more..
  - Three and four-particle observables provide more constraints and more information, e.g. triaxiality.
  - Nuclear PDF effect and its centrality dependences.
  - Transport of conserved charges and effects magnetic field..
- Great opportunity for possible system scans.
  - Larger systems have better statistical sensitivity e.g.  $^{136}\text{Xe}$  vs  $^{136}\text{Ce}$
  - Small systems to disentangle geometry from initial momentum anisotropy
  - Profit from larger multiply/acceptance from LHC and also the question of energy dependence
- Model study to explore the connection to nuclear structure and potential for heavy ion physics.

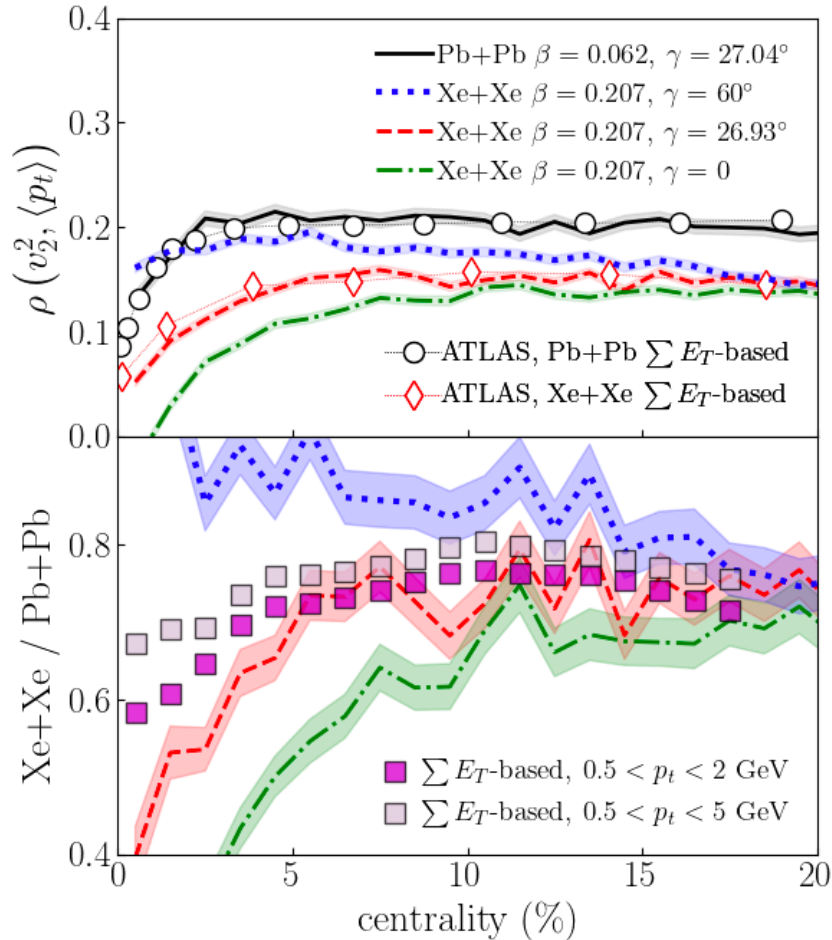
We're only limited by our imagination ...

RBRC Workshop “Physics opportunities from the RHIC Isobar run”  
**Jan 25-28, 2022.**

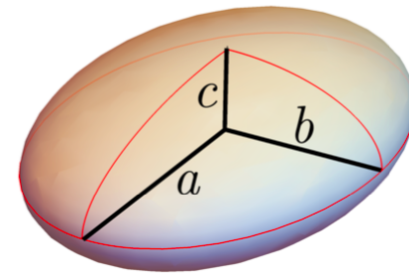
**Organizers:** Jianguo Jia (Stony Brook), Chun Shen (RBRC/Wayne State), Derek Teaney (Stony Brook), and Zhangbu Xu (BNL)

# $v_2^2$ - $p_T$ correlation at LHC

B Bally, M Bender, G Giacalone, V Somà 2108.09578



$$R(\theta, \phi) = R_0 \left( 1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] \right)$$



- Clear sensitivity to the triaxiality of  $^{129}\text{Xe}$ .