Francesco Becattini,
University of Florence and INFN

Theory advances in spin physics

OUTLINE

- Motivations and a short theory summary
- Spin tensor and spin hydrodynamics
- Spin-thermal shear coupling and isothermal local equilibrium
- What is the spin good for?

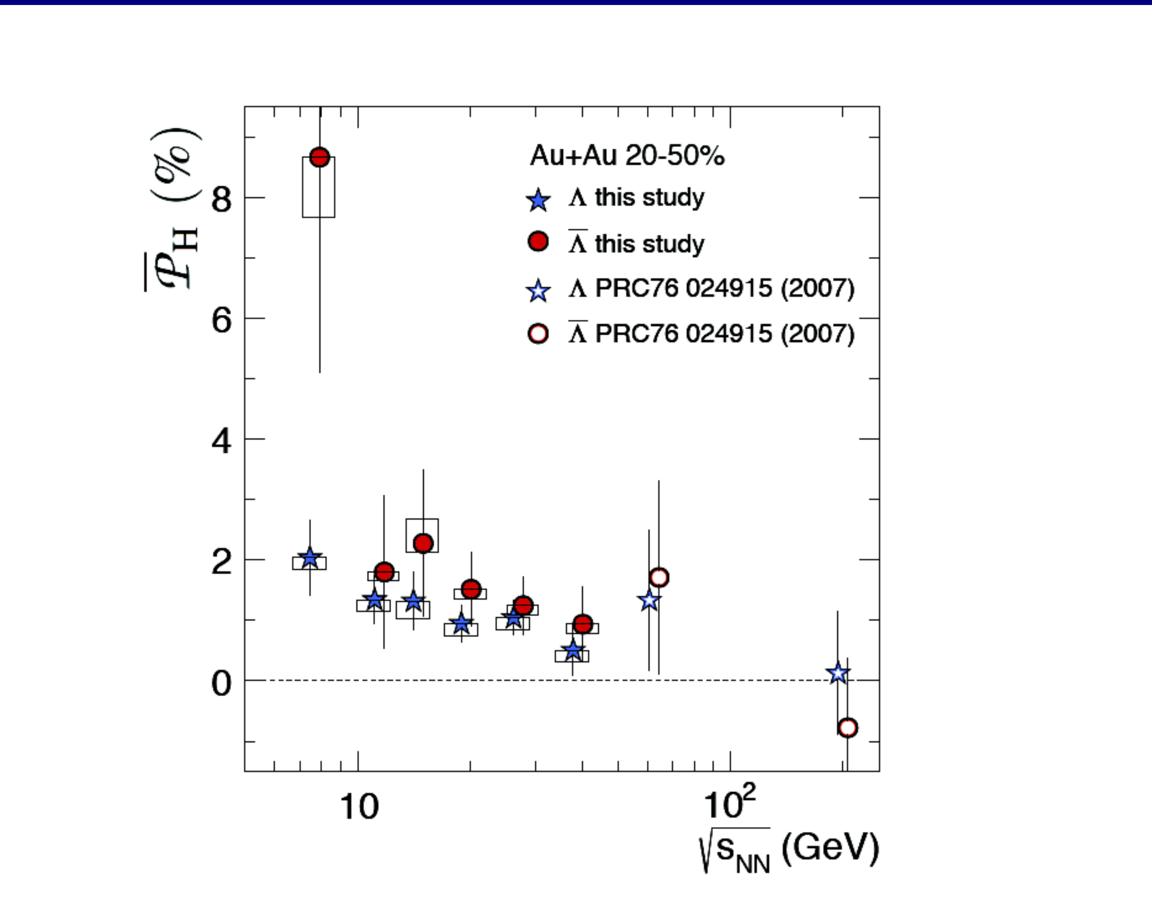
Chirality 2021, Nov 2nd, Stonybrook

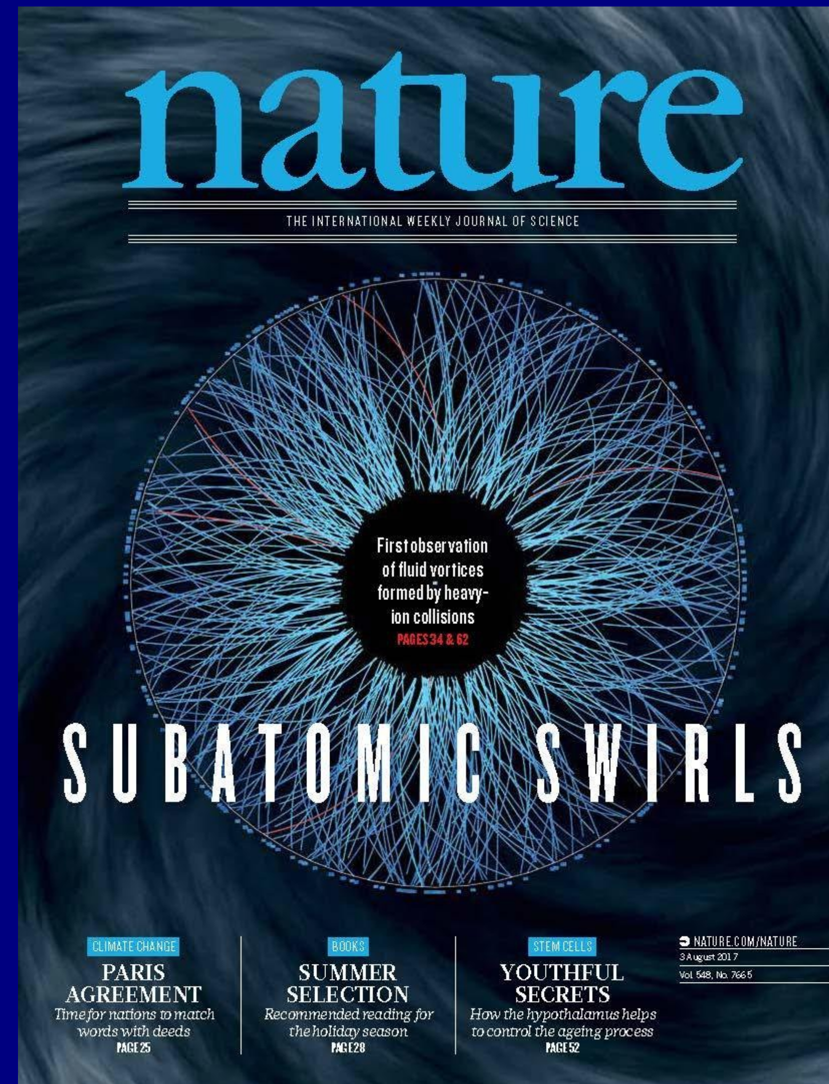
Disclaimer

- The last two years have seen an intense and quick theoretical development of spin physics in heavy ion collisions, also in the theory sector
- This is a personal overview, far from being comprehensive
- Several approaches and languages: I shall use the language of QFT and operators

Spin in heavy ion physics: prologue

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017

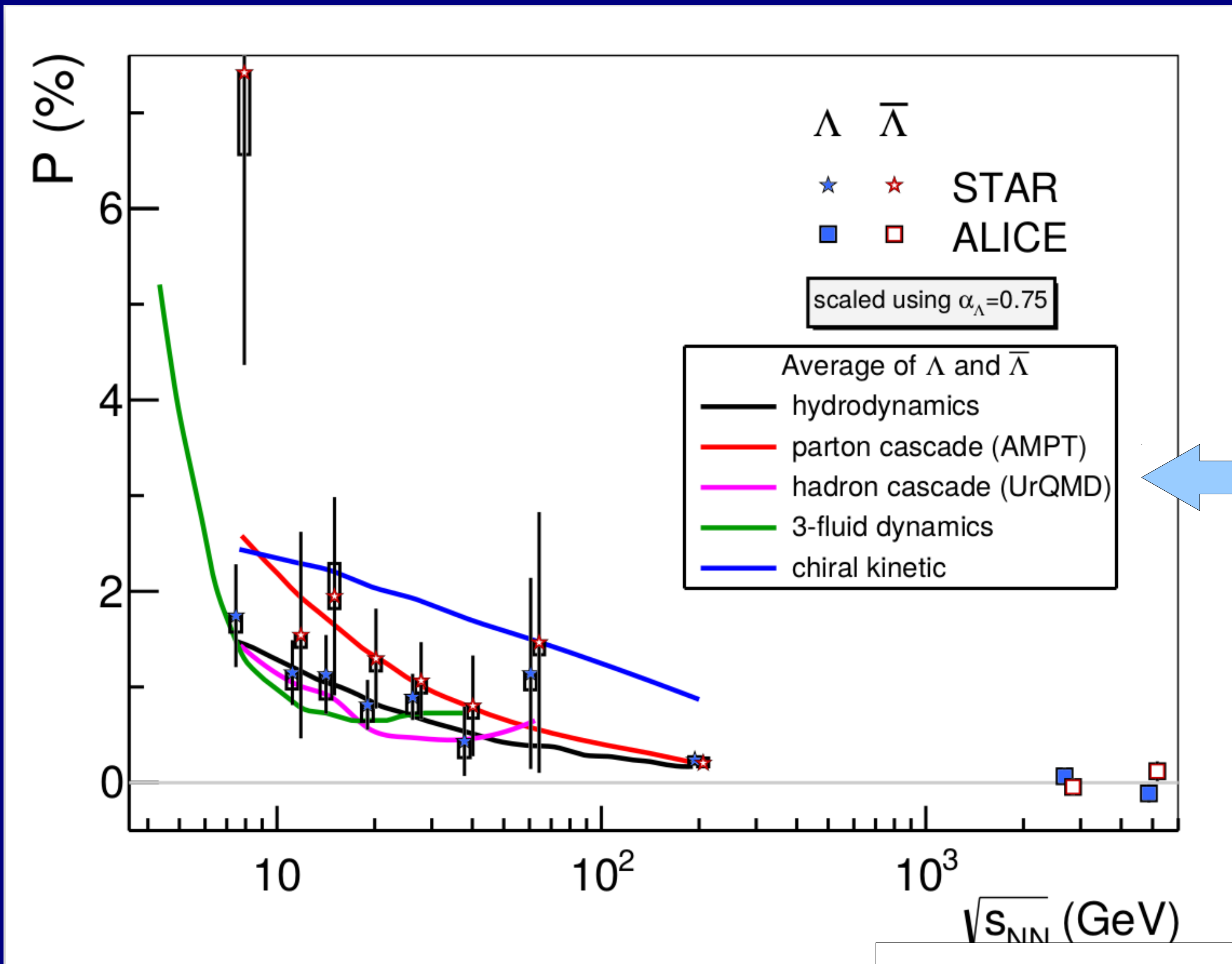




Particle and antiparticle have the same polarization sign.
This shows that the phenomenon cannot be driven
by a mean field (such as EM) whose coupling is *C-odd.*
Definitely favours the thermodynamic (equipartition) interpretation

Data-model comparison 2020

F. B., M. Lisa, Polarization and vorticity in the QGP, Ann. Rev. Part, Nucl. Sc. 70, 395 (2020)

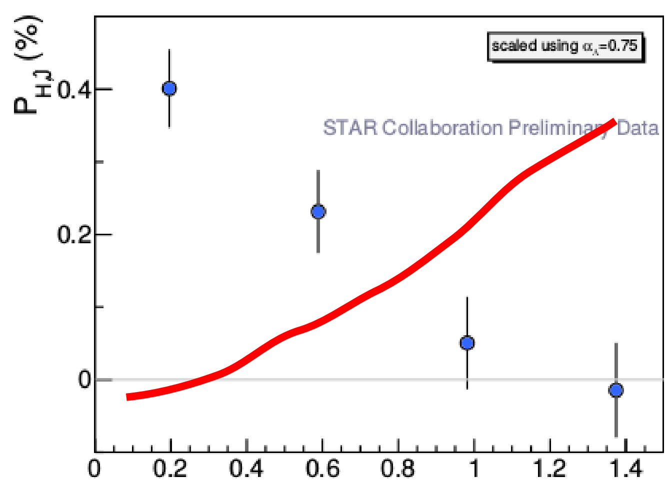


Different models of the collision, same formula for polarization:

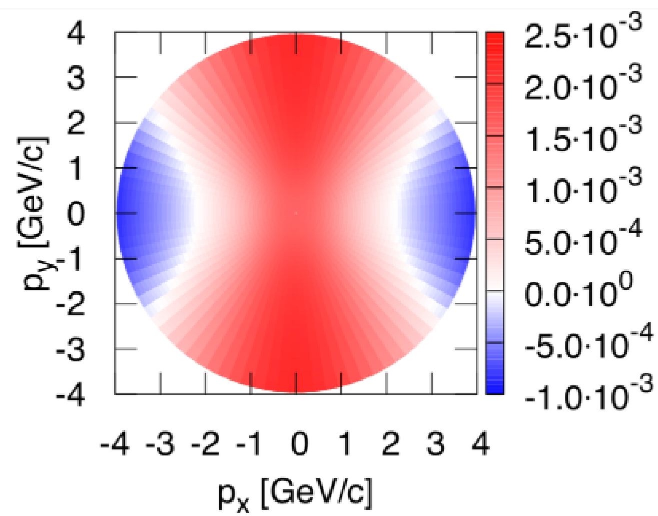
$$S^{\mu}(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int_{\Sigma} \mathrm{d}\Sigma_{\tau}p^{\tau}n_F(1-n_F)\partial_{\nu}\beta_{\rho}}{\int_{\Sigma} \mathrm{d}\Sigma_{\tau}p^{\tau}\ n_F}$$

Puzzles: momentum dependence of polarization

a strong motivation for theoretical investigation!

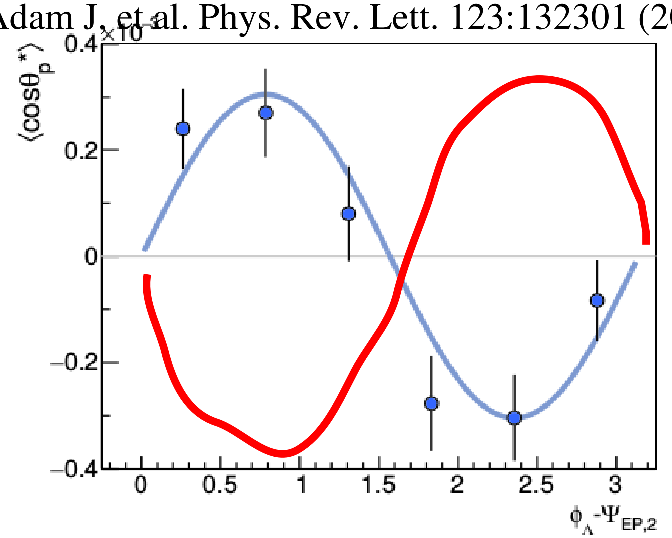


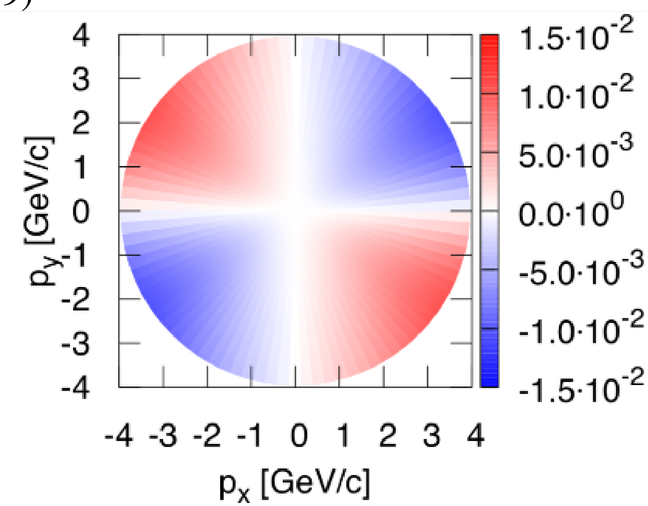
Niida T. Nucl. Phys. A982:511 (2019)



Theory prediction

Adam J, et al. Phys. Rev. Lett. 123:132301 (2019)





Not the effect of decays:

F.B., I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, Phys.Rev.C 95 (2017) 5, 054902.

X. L. Xia, H. Li, X.G. Huang and H. Z. Huang, Phys. Rev. C 100 (2019), 014913

F. B., G. Cao and E. Speranza, Eur. Phys. J. C 79 (2019) 741

Polarization of fermions in a relativistic fluid: basic theory tools

The covariant Wigner function of the free Dirac field (of the effective hadronic fields):

F. B., arXiv:2004.04050, Springer Lecture Notes in Physics 987 (2021)

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y\ \mathrm{e}^{-ik\cdot y}\langle :\Psi_A(x-y/2)\overline{\Psi}_B(x+y/2):\rangle$$
$$= \frac{1}{(2\pi)^4}\int \mathrm{d}^4y\ \mathrm{e}^{-ik\cdot y}\langle :\overline{\Psi}_B(x+y/2)\Psi_A(x-y/2):\rangle$$

$$\langle \widehat{X}\rangle = \mathrm{tr}(\widehat{\rho}\widehat{X})$$

It allows to calculate the spin density matrix for spin ½:

$$\Theta(p)_{rs} = \frac{\int \mathrm{d}\Sigma_\mu p^\mu \bar{u}_r(p) W_+(x,p) u_s(p)}{\sum_t \int \mathrm{d}\Sigma_\mu p^\mu \bar{u}_t(p) W_+(x,p) u_t(p)}$$

And the mean spin vector in these three equivalent forms:

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}_4(\gamma^\mu\gamma^5 W_+(x,p))}{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}_4 W_+(x,p)}$$

$$S^\mu(p) = -\frac{1}{2m}\epsilon^{\mu\beta\gamma\delta}p_\delta\frac{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4(\Sigma_{\beta\gamma}W_+(x,p))}{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4 W_+(x,p)}$$

$$S^\mu(p) = -\frac{1}{4}\epsilon^{\mu\beta\gamma\delta}p_\delta\frac{\int \mathrm{d}\Sigma_\lambda \mathrm{tr}_4(\{\gamma^\lambda,\Sigma_{\beta\gamma}\}W_+(x,p))}{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4 W_+(x,p)}$$

Density operator of a quantum relativistic fluid

Needed to calculate the Wigner function!

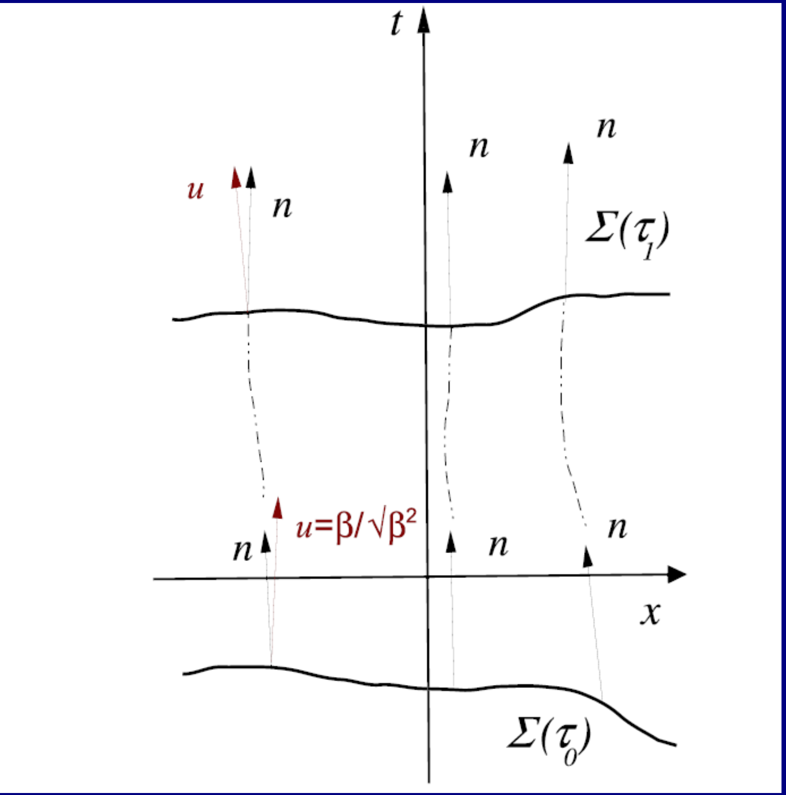
$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

General covariant
Local thermodynamic
Equilibrium density operator

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right)\right]$$

$$\beta = \frac{1}{T}u$$

$$\zeta = \frac{\mu}{T}$$



The operator is obtained by maximizing the entropy

$$S = -\mathrm{tr}(\widehat{\rho}\log\widehat{\rho})$$

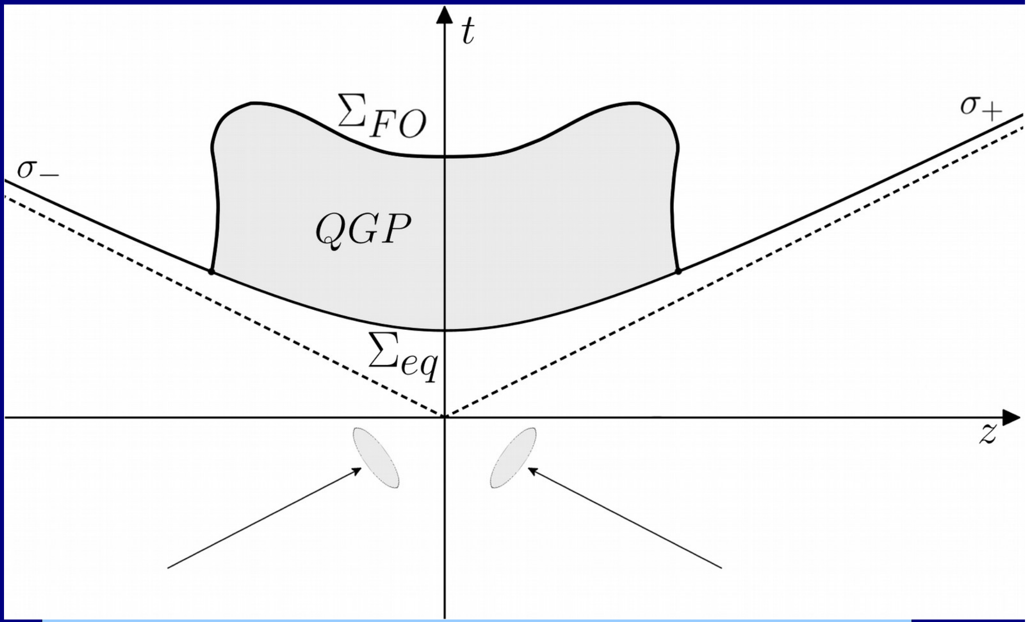
with the constraints of fixed energy-momentum density

The actual density operator

The above density operatot is "time" dependent, cannot be the actual one!

In the Zubarev's theory, this is the LTE at some initial "time":

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right].$$



NOTE: T_B stands for the symmetrized Belinfante stress-energy tensor

With the Gauss theorem

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) + \int_\Theta \mathrm{d}\Theta \left(\widehat{T}_B^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right],$$

Local equilibrium, non-dissipative terms

Dissipative terms

Local thermodynamic equilibrium approximation

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \tfrac{1}{Z} \exp\left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \, \widehat{T}^{\mu\nu}_B \beta_\nu - \zeta \widehat{j}^\mu \right]$$

$$W(x,k) \simeq W(x,k)_{\rm LE} = \frac{1}{Z} {\rm Tr} \left(\exp\left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y) \widehat{T}^{\mu\nu}_B(y) \beta_\nu(y) - \zeta(y) \widehat{j}^\mu(y) \right] \widehat{W}(x,k) \right)$$

Separation of scales: hydro functions β and ζ are slowly varying.

Expand the β and ζ fields from the point *x* where the Wigner operator is to be evaluated

$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda \beta_\nu(x) (y-x)^\lambda + \ldots$$

$$\int_\Sigma {\rm d}\Sigma_\mu T^{\mu\nu}_B(y) \beta_\nu(x) = \beta_\nu(x) \int_\Sigma {\rm d}\Sigma_\mu T^{\mu\nu}_B(y) = \beta_\nu(x) \widehat{P}^\nu$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp\left[-\beta_\mu(x) \widehat{P}^\mu - \frac{1}{2}(\partial_\mu \beta_\nu(x) - \partial_\nu \beta_\mu(x)) \widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu \beta_\nu(x) + \partial_\nu \beta_\mu(x)) \widehat{Q}^{\mu\nu}_x + \ldots \right]$$

$$S^\mu(p) = \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma {\rm d}\Sigma_\tau p^\tau n_F (1-n_F) \partial_\nu \beta_\rho}{\int_\Sigma {\rm d}\Sigma_\tau p^\tau \, n_F}$$

Neglected
by prejudice

Spin and dissipative corrections

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right) + \int_\Theta \mathrm{d}\Theta \left(\widehat{T}_B^{\mu\nu}\nabla_\mu\beta_\nu - \widehat{j}^\mu\nabla_\mu\zeta\right)\right],$$

GOAL: to determine the dissipative corrections to the Wigner function:

$$S^\mu(p) = -\frac{1}{4}\epsilon^{\mu\beta\gamma\delta}p_\delta \frac{\int \mathrm{d}\Sigma_\lambda \mathrm{tr}_4(\{\gamma^\lambda, \Sigma_{\beta\gamma}\}W_+(x,p))}{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4 W_+(x,p)}$$

Beware the difference between the dissipative corrections to the spin polarization vector of final hadrons and the dissipative corrections to the spin tensor in spin hydrodynamics!

First formula connecting dissipative terms of the stress-energy tensor to spin vector (for massless fermions)

S. Shi, C. Gale, S. Jeon, Phys. Rev. C 103 (2021) 4, 044906

$$\begin{aligned} S^\mu(p) = \frac{1}{2m_H}&\left\{\left[\int_\Sigma f_{V,0}\right] + \int_\Sigma f_{V,0}(1-f_{V,0})(\lambda_\nu \nu^\alpha p_\alpha + \lambda_\pi \pi^{\alpha\beta}p_\alpha p_\beta)\right\}^{-1} \\ &\times\left\{\left[-\frac{\hbar}{4}\epsilon^{\mu\nu\rho\sigma}\int_\Sigma p_\nu \varpi_{\rho\sigma} f_{V,0}(1-f_{V,0})\right] + \int_\Sigma p^\mu f_{V,0}(1-f_{V,0})\frac{\mu_A}{T}\right. \\ &\left. + \int_\Sigma p^\mu f_{V,0}(1-f_{V,0})\left(\frac{\lambda_\nu}{2}\nu_A^\alpha p_\alpha + \frac{\lambda_\nu^+ - \lambda_\nu^-}{2}\nu^\alpha p_\alpha + \frac{\lambda_\pi^+ - \lambda_\pi^-}{2}\pi^{\alpha\beta}p_\alpha p_\beta\right)\right\} + \mathcal{O}(\hbar^2), \end{aligned}$$

Spin tensor and spin hydrodynamics

Pseudo-gauge transformations with a superpotential $\widehat{\Phi}$ in flat spacetime

F. W. Hehl, Rept. Math. Phys. 9 (1976) 55

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\Phi}^{\alpha,\mu\nu} - \widehat{\Phi}^{\mu,\alpha\nu} - \widehat{\Phi}^{\nu,\alpha\mu}\right)$$
$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

They leave the conservation equations and spacial integrals (=generators, or total energy, momentum and angular momentum operators) invariant

Belinfante pseudogauge $\mathcal{S}^{\lambda,\mu\nu} = 0$

Heavy ion Physics: our beloved stress-energy tensor $T^{\mu\nu}(x) = \mathrm{Tr}(\widehat{\rho}\widehat{T}^{\mu\nu}(x))$ *is not objective up to quantum terms. It plays the same role as of a vector potential in electrodynamics. What is objective are final particle distributions which <u>should</u> be invariant under pseudo-gauge transformations*

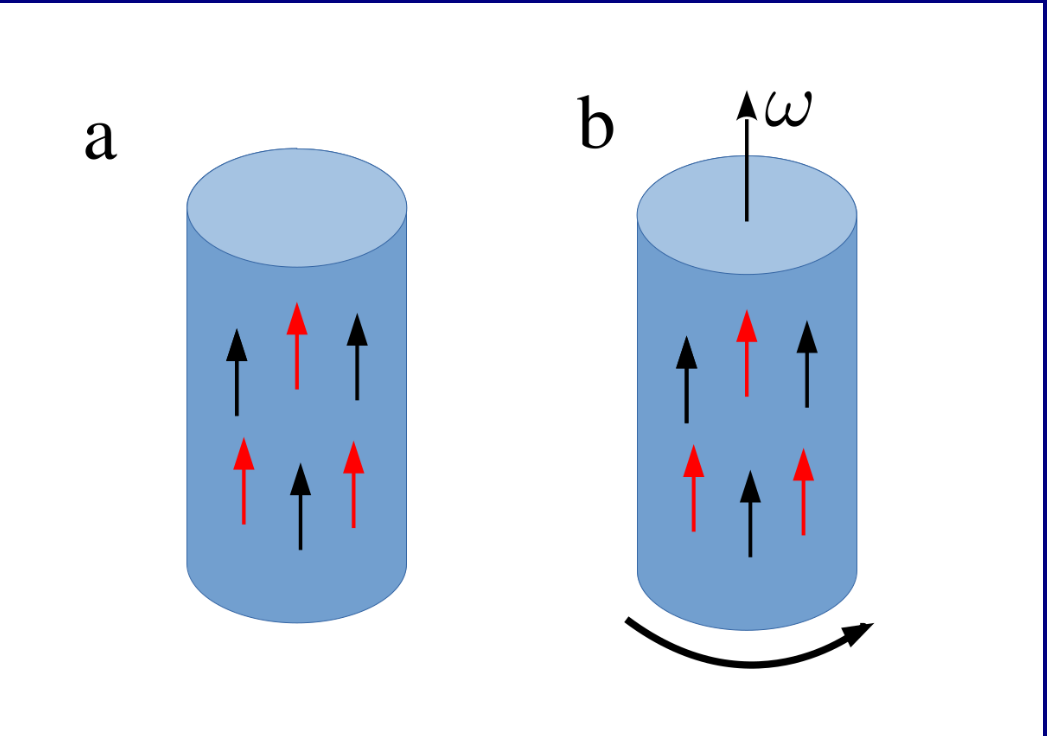
Quantum states MAY depend on the pseudo-gauge

F. B. Nucl.Phys.A 1005 (2021) 121833

$|p,\sigma\rangle$ Free particle physical states depend on the generators P and J of the Poincare' group and are independent of the pseudo-gauge, i.e. of $(T\ ,\mathcal{S})$ and yet:

$$\sum C(p_1,\sigma_1,p_2,\sigma_2,\ldots)_{(T,\mathcal{S})}|p_1,\sigma_1\rangle|p_2,\sigma_2\rangle\ldots$$

In general, a density operator $\widehat{\rho}(\widehat{T},\widehat{\mathcal{S}})$ may not be pseudo-gauge invariant.
In this case, measurements would break the pseudo-gauge invariance



C-invariant relativistic matter

a) with u=(1,0,0,0) T = constant and particles-antiparticles polarized in the same direction without thermal vorticity. Impossible with Belinfante pseudogauge.

b) Belinfante pseudo-gauge: only non-zero thermal vorticity can sustain such a configuration

Local equilibrium pseudo-gauge dependence analysis

Start from Belinfante pseudo-gauge:

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}_B \beta_\nu - \zeta \, \widehat{j}^\mu \right) \right],$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}_C \beta_\nu - \frac{1}{2}\varpi_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu}_C - \frac{1}{2}\xi_{\lambda\nu}\left(\widehat{\mathcal{S}}^{\lambda,\mu\nu}_C + \widehat{\mathcal{S}}^{\nu,\mu\lambda}_C \right) - \zeta \widehat{j}^\mu \right) \right],$$

$$\varpi_{\lambda\nu} = \frac{1}{2}(\nabla_\nu \beta_\lambda - \nabla_\lambda \beta_\nu) \qquad \xi_{\lambda\nu} = \frac{1}{2}(\nabla_\nu \beta_\lambda + \nabla_\lambda \beta_\nu)$$

This operator is non-invariant under a pseudo-gauge transformation!
ONLY at global equilibrium it is

If the spin tensor is non-zero (non-Belinfante) angular momentum constraints must be additionally implemented

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta \widehat{j}^\mu \right) \right].$$

$\Omega_{\lambda\nu} \equiv$ spin potential

Confirmed in the analysis by K. Fukushima, Shi Pu, Phys. Lett. B 817 (2021) 136346.

Hydrodynamics with a spin tensor

W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709

$$\partial_\mu j^\mu = 0 \qquad \partial_\mu T^{\mu\nu} = 0 \qquad \partial_\lambda \mathcal{S}^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

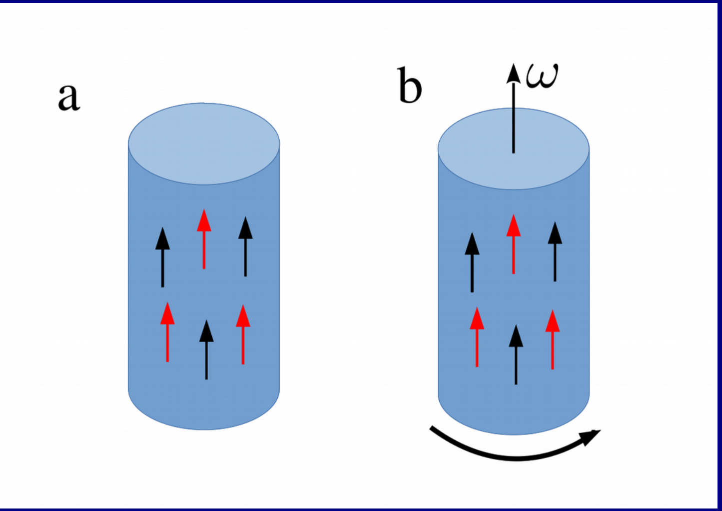
$$j^\mu = j^\mu(\beta,\zeta,\Omega), \qquad T^{\mu\nu} = T^{\mu\nu}(\beta,\zeta,\Omega), \qquad \mathcal{S}^{\lambda,\mu\nu} = \mathcal{S}^{\lambda,\mu\nu}(\beta,\zeta,\Omega).$$

Spin hydrodynamics is necessary if the spin relaxation time scale is much longer than the time scale of e.g. kinetic equilibration

F. B., W. Florkowski, E. Speranza, Phys. Lett. B 789 (2019) 419

A detailed and insightful analysis of time scales and Spin-hydro regimes in:

M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov, H.U. Yee, arXiv 2107.14321



Is it all relevant to heavy ion collisions? Strictly speaking, the prepared initial state are two colliding nuclei, which is not pseudo-gauge dependent. However, it can be relevant in an effective description!

Spin tensor dissipative hydrodynamics: recent advances

K. Hattori et al., Phys.Lett.B 795 (2019) 100;

R. Singh and R. Ryblewski, Acta Phys.Polon.B 51 (2020) 1537;

S. Bhadury et al,, Eur.Phys.J.ST 230 (2021) 3, 655

D. She et al,, arXiv: 2105.04060;

S. Bhadury et al., Phys.Rev.D 103 (2021) 1, 014030;

S. Shi, C. Gale, S. Jeon, Phys. Rev. C 103 (2021) 4, 044906;

M. Hongo et al., arXiv 2107.14321

Which is the right spin tensor after all?

In principle, there are infinitely many choices. Only in curved space-time theories you can pick one, like in Poincarè gauge theory (canonical spin tensor)

$$\mathcal{S}^{\lambda,\mu\nu} = \frac{1}{2}\bar{\Psi}\{\gamma^\lambda, \Sigma^{\mu\nu}\}\Psi$$

F. W. Hehl and Y. N. Obukhov, arXiv:1909.01791

Other arguments favour the GLW tensor

$$S^{\lambda,\mu\nu}_{\rm GLW} = \frac{\hbar}{4}\int {\rm d}^4k\, {\rm tr}_4\left[\left(\{\sigma^{\mu\nu},\gamma^\lambda\} + \frac{2i}{m}\left(\gamma^{[\mu}k^{\nu]}\gamma^\lambda - \gamma^\lambda\gamma^{[\mu}k^{\nu]}\right)\right)\mathcal{W}(x,k)\right].$$

W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709

Spin vector in spin hydrodynamics: leading order

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta \widehat{j}^\mu \right) \right].$$

M. Buzzegoli, arXiv 2109.12084 for the canonical spin tensor $\mathcal{S}^{\lambda,\mu\nu} = \frac{1}{2}\bar{\Psi}\{\gamma^\lambda, \Sigma^{\mu\nu}\}\Psi$

$$S^\mu_{\rm C}(k) \simeq S^\mu_\varpi(k) + S^\mu_\xi(k) + \Delta^{\rm C}_\Theta S^\mu(k).$$

$$\Delta^{\rm C}_\Theta S^\mu(k) = \frac{\epsilon^{\lambda\rho\sigma\tau}\hat{t}_\lambda(k^\mu k_\tau - \eta^\mu_\tau m^2)}{m\varepsilon_k} \times \frac{\int_\Sigma {\rm d}\Sigma(x)\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,(\varpi_{\rho\sigma} - \Omega_{\rho\sigma})}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}}. \quad (41)$$

Y. C. Liu, X. G. Huang, arXiv 2109.15034 for the canonical spin tensor

$$\mathcal{A}^\mu = -8\pi\hbar\delta(p^2-m^2)n_F(1-n_F)\left\{ \frac{1}{4}\epsilon^{\mu\rho\sigma\nu}p_\rho\omega_{\sigma\nu} + \Sigma^{\mu\nu}_{\hat{t}}\left[(\xi^{\nu\lambda} + \Delta\omega^{\nu\lambda})p^\sigma + \partial_\nu\alpha \right] \right\},$$

Relativistic kinetic theory with spin

Trying to solve the dynamical Wigner equation for interacting fermions without the introduction of the density operator

$$\left[\gamma\cdot\left(p+i\,\frac{\hbar}{2}\partial\right)-m\right]W_{\alpha\beta}=\hbar\,\mathcal{C}_{\alpha\beta}\ ,$$

Technique: semiclassical expansion in *h*
Recent advances in this field: see talks by P. Zhuang, N. Weickgenannt, X. G. Huang,…

EQUILIBRIUM PROBLEM

Equilibrium form of the Wigner function is usually an *ansatz*

The exact form of the equilibrium solution for free fermions at all orders has been recently derived: see A. Palermo's talk:

$$W\left(x,k\right)=\frac{1}{(2\pi)^3}\int\frac{\mathrm{d}^3\mathrm{p}}{2\varepsilon}\sum_{n=1}^{\infty}(-1)^{n+1}\mathrm{e}^{-n\widetilde{\beta}_n\cdot p}\times \left[S\left(\Lambda\right)^n\left(m+\not{p}\right)\delta^4\left(k-\frac{\Lambda^n p+p}{2}\right)+\left(m-\not{p}\right)S\left(\Lambda\right)^{-n}\delta^4\left(k+\frac{\Lambda^n p+p}{2}\right)\right],$$

Back to local equilibrium: spin-thermal shear coupling

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp\left[-\beta_\mu(x) \widehat{P}^\mu - \frac{1}{2}(\partial_\mu \beta_\nu(x) - \partial_\nu \beta_\mu(x)) \widehat{J}_x^{\mu\nu} - \frac{1}{2}(\partial_\mu \beta_\nu(x) + \partial_\nu \beta_\mu(x)) \widehat{Q}_x^{\mu\nu} + \ldots \right]$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x) \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x) \widehat{J}_x^{\mu\nu} - \frac{1}{2}\xi_{\mu\nu}(x) \widehat{Q}_x^{\mu\nu} + \ldots]$$

$$\widehat{J}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

Angular-momentum boost operators

$$\widehat{Q}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

Quadrupole-like operators

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

Thermal vorticity
Adimensional in natural units

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$$

Thermal shear
Adimensional in natural units

At global equilibrium the thermal shear vanishes because of the Killing equation

Surprise: thermal shear does contribute!

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma {\rm d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma {\rm d}\Sigma_\tau p^\tau \; n_F}$$

$$n_F = ({\rm e}^{\beta\cdot p - \xi} + 1)^{-1}$$

NON-dissipative effect

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma {\rm d}\Sigma\cdot p \; n_F(1-n_F)\hat{t}_\nu\xi_{\sigma\rho}}{\int_\Sigma {\rm d}\Sigma\cdot p \; n_F},$$

F. B., M. Buzzegoli, A. Palermo, Phys. Lett. B 820 (2021) 136519

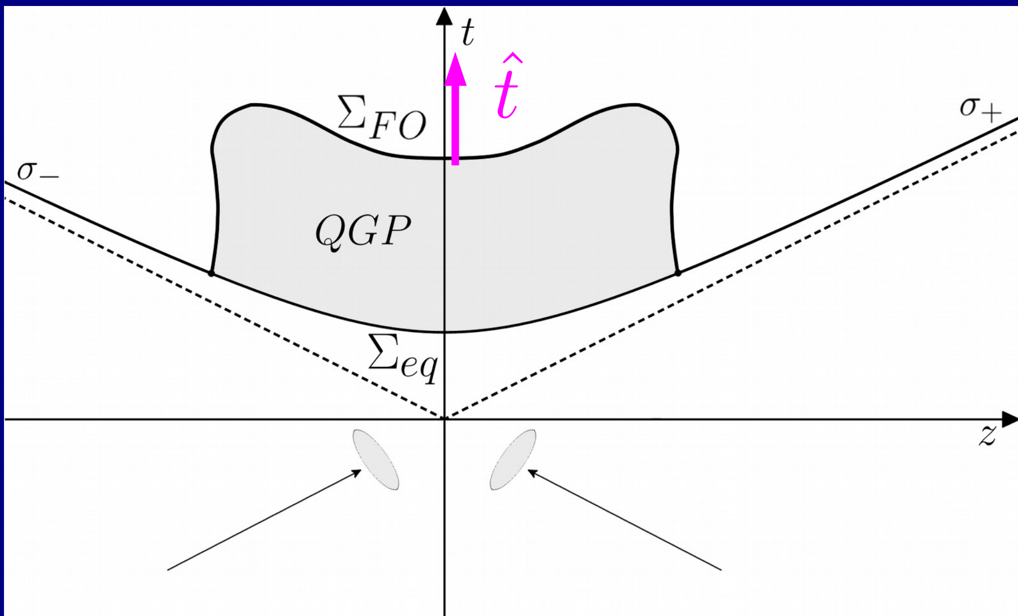
Same (though not precisely the same) formula obtained by Liu and Yin with a different method:

S. Liu, Y. Yin, JHEP 07 (2021) 188 See Yi Yin's talk

The additional local equilibrium term has been confirmed in more analyses:

C. Yi, S. Pu, D. L. Yang, 2106.00238
Y. C. Liu, X. G. Huang, arXiv 2109.15034

Why do we have a dependence on Σ ?



The thermal shear term depends on the correlator:

$$\langle \widehat{Q}_x^{\mu\nu} \widehat{W}(x,p) \rangle$$

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int_{\Sigma_{FO}} \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

The divergence of the integrand of $J^{\mu\nu}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and J is thus a tensor operator:

$$\widehat{\Lambda} \widehat{J}_x^{\mu\nu} \widehat{\Lambda}^{-1} = \Lambda^{-1\mu}_{\alpha} \Lambda^{-1\nu}_{\beta} \widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{\mu\nu}$ does not vanish, therefore it does depend on the integration hypersurface and Q is NOT a tensor operator

$$\widehat{\Lambda} \widehat{Q}^{\mu\nu} \widehat{\Lambda}^{-1} \neq \Lambda^{-1\mu}_{\alpha} \Lambda^{-1\nu}_{\beta} \widehat{Q}^{\alpha\beta}$$

Application to relativistic heavy ion collisions

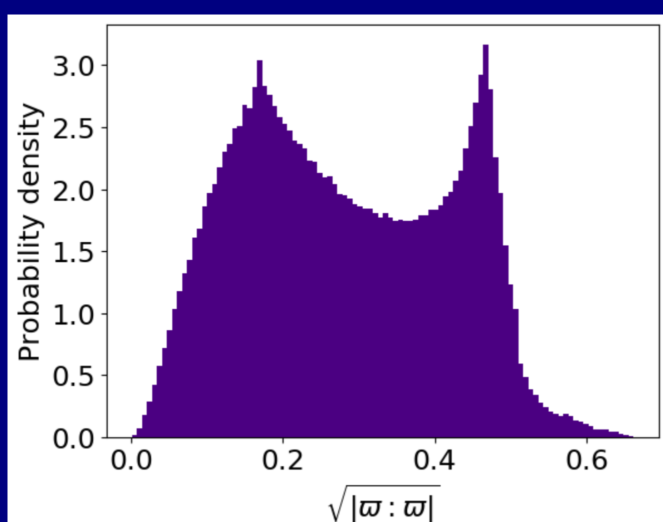
F. B., M. Buzzegoli, A. Palermo, G. Inghirami and I. Karpenko, arXiv:2103.14621, maybe appearing in PRL soon

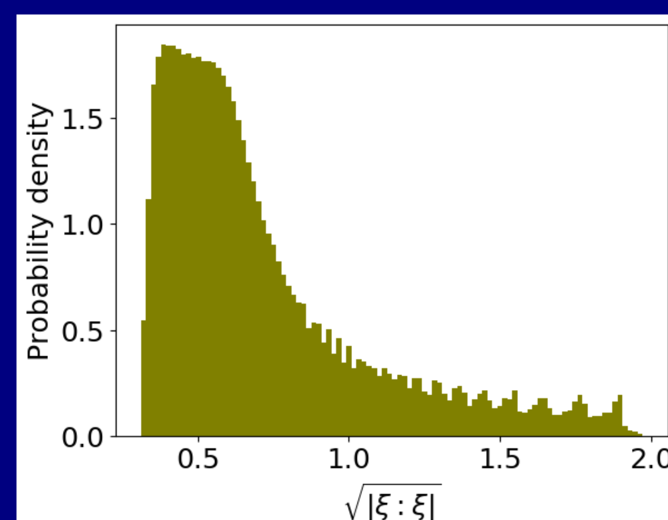
$$S^\mu = S^\mu_\varpi + S^\mu_\xi$$

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F (1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

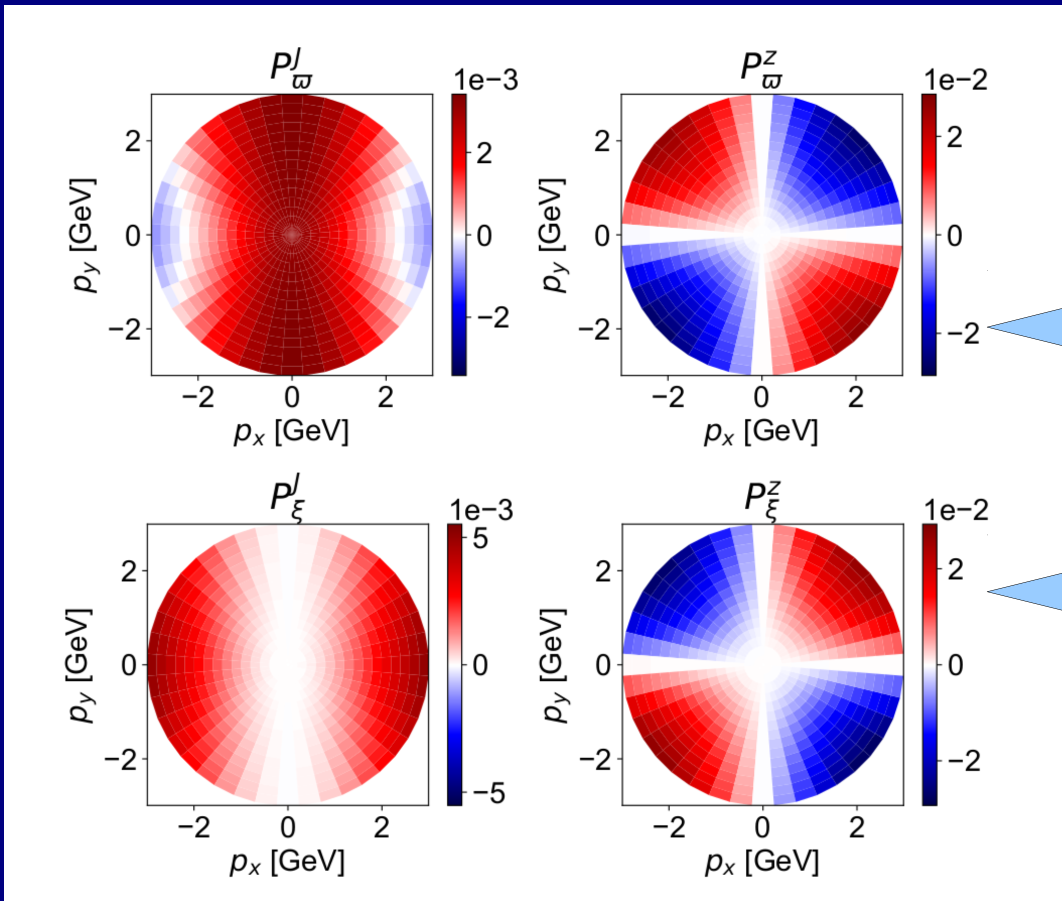
$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\rho\sigma\tau} \frac{p_\tau p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F (1-n_F)\hat{t}_\rho \xi_{\sigma\lambda}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

Is linear response theory adequate?





New calculations

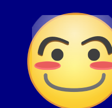


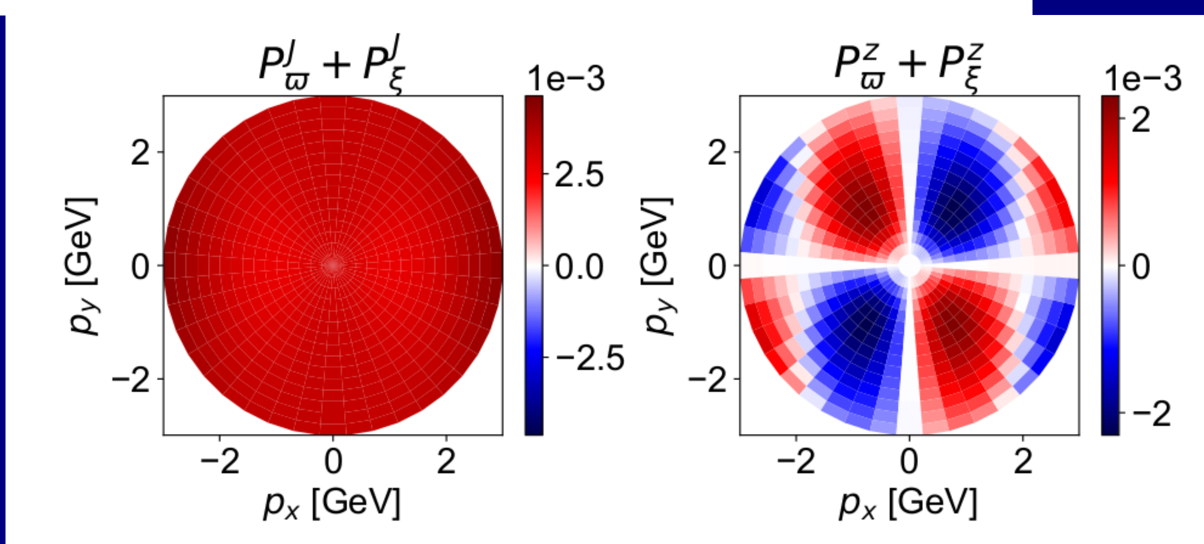
Based on the hydrodynamic code VHLLE (author I. Karpenko) tuned to reproduce Au-Au momentum spectra at RHIC top energy.
Similar output with ECHO-QGP (main author G. Inghirami).

Thermal vorticity

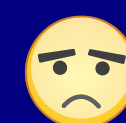
Thermal shear

Right pattern!





SUMMING UP



Not sufficient to restore the agreement between data and model

Calculations fully consistent with:
B. Fu, S. Liu, L. Pang, H. Song and Y. Yin, Phys. Rev. Lett. 127 (2021) 14, 142301

Isothermal local equilibrium

The most appropriate setting for relativistic heavy ion collisions at very high energy!

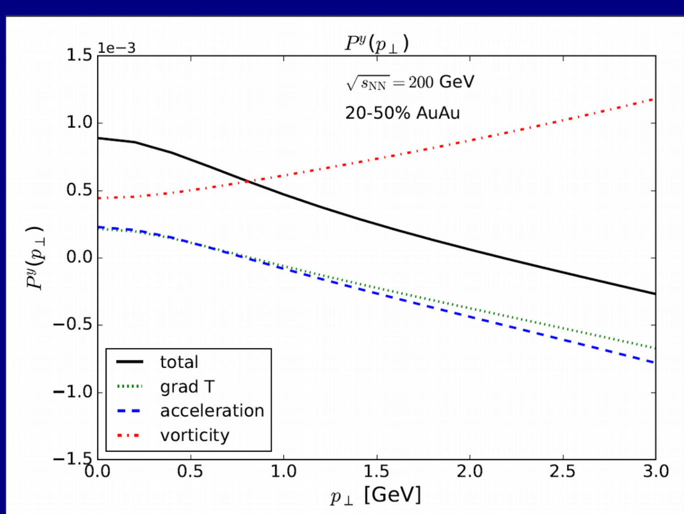
Both thermal shear and thermal vorticity include temperature gradients

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$$

$$\beta^\mu = (1/T)u^\mu$$

Thermal gradients do contribute to the polarization



$$\mathbf{S}^* \propto \frac{\hbar}{KT^2}\mathbf{u} \times \nabla T + \frac{\hbar}{KT}(\boldsymbol{\omega} - \boldsymbol{\omega} \cdot \mathbf{v}\mathbf{u}/c^2) + \frac{\hbar}{KT}\mathbf{A} \times \mathbf{u}/c^2$$

I. Karpenko, Nucl. Phys. A 982 (2019), 519-522, talk given at QM 2018

Is it the best thing to do?

The formulae of the spin vector are based on a Taylor expansion of the density operator

$$W(x,k)_{\rm LE} = \frac{1}{Z}{\rm Tr}\left(\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y)\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y)\right]\widehat{W}(x,k)\right)$$

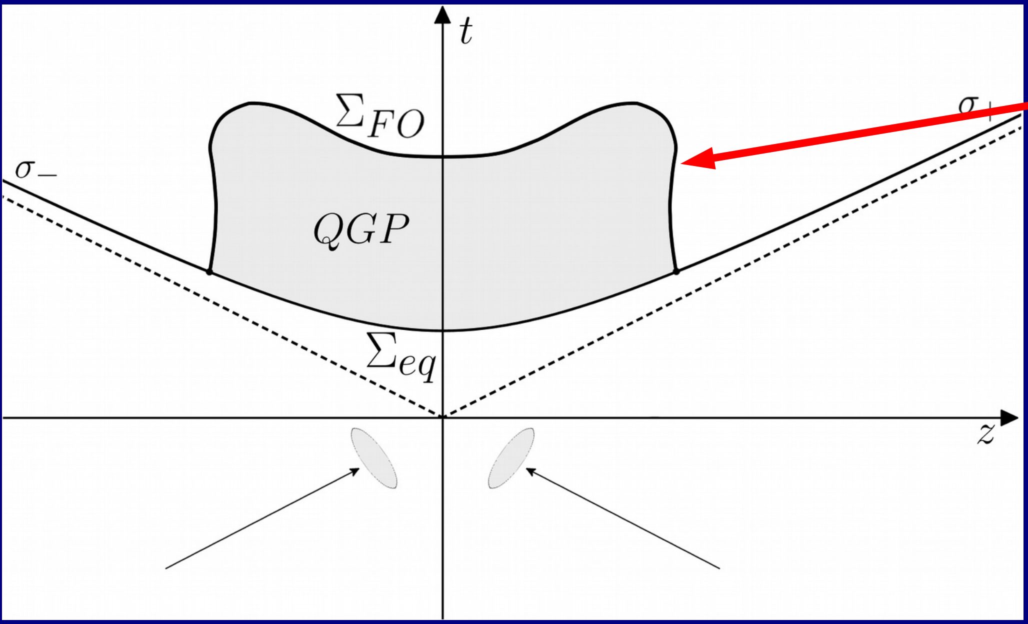
$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \dots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}^{\mu\nu}_x + \dots]$$

This is generally correct, but it is an approximation after all.

Can we find a better approximation for a special case?

Isothermal hadronization at very high energy



At high energy, Σ_{FO} expected to be T= constant!

$$\beta^\mu = (1/T)u^\mu$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu}\beta_\nu\right] = \frac{1}{Z}\exp\left[-\frac{1}{T}\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu}u_\nu\right]$$

NOW u (and just u) can be expanded!

$$u_\nu(y) = u_\nu(x) + \partial_\lambda u_\nu(x)(y-x)^\lambda + \ldots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{2T}(\partial_\mu u_\nu(x) - \partial_\nu u_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2T}(\partial_\mu u_\nu(x) + \partial_\nu u_\mu(x))\widehat{Q}^{\mu\nu}_x + \ldots]$$

Spin mean vector at leading order with isothermal local equilibrium (ILE)

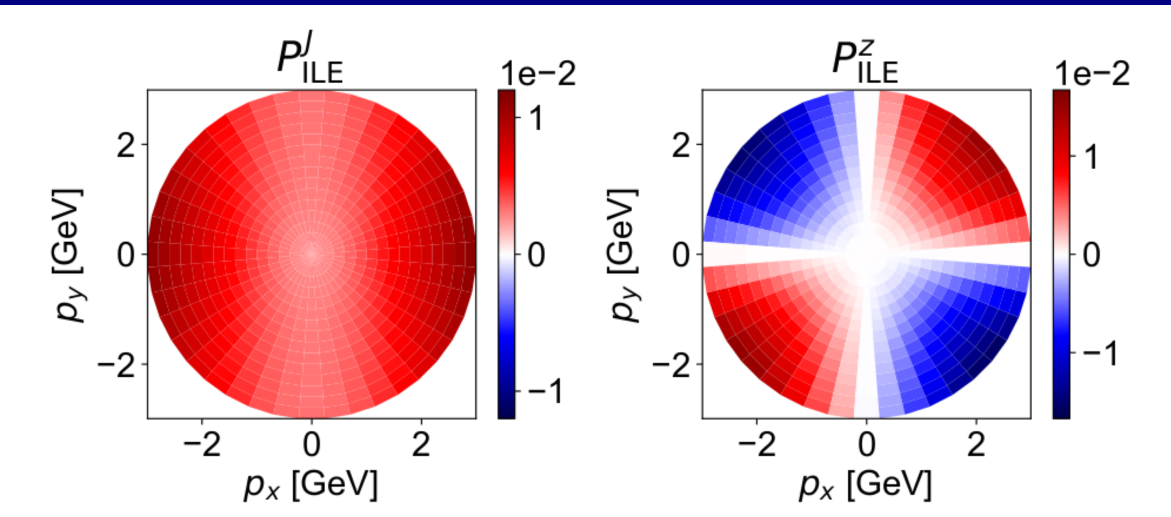
Readily found by replacing the gradients of β with those of *u*

$$S^\mu_{\rm ILE}(p) = \qquad (\\ - \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F (1 - n_F) \left[\omega_{\rho\sigma} + 2 \, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma} \right]}{8 m T_{\rm dec} \int_\Sigma {\rm d}\Sigma \cdot p \, n_F}$$

$$\omega_{\rho\sigma} = \frac{1}{2} \left(\partial_\sigma u_\rho - \partial_\rho u_\sigma \right)$$

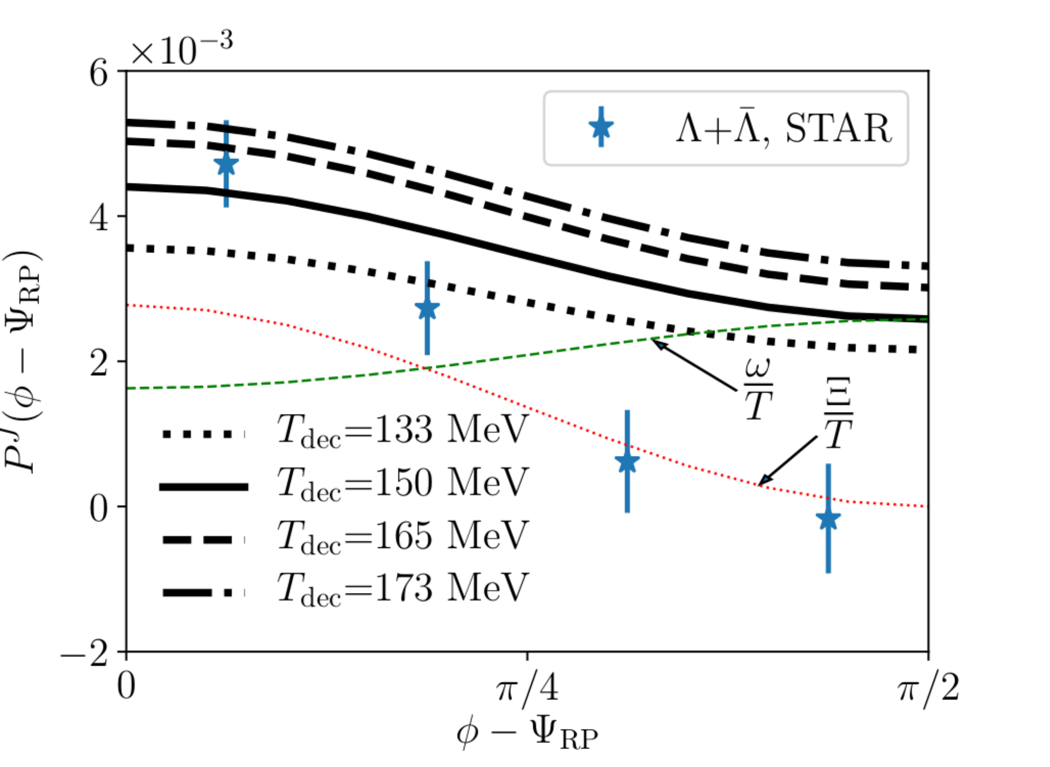
$$\Xi_{\rho\sigma} = \frac{1}{2} \left(\partial_\sigma u_\rho + \partial_\rho u_\sigma \right)$$

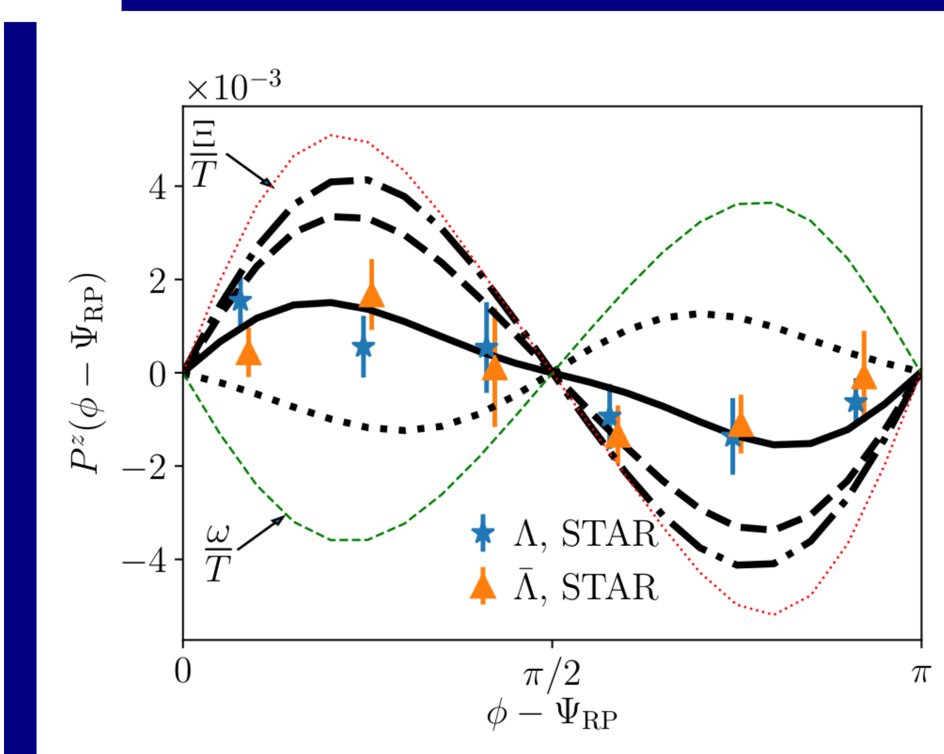
Isothermal local equilibrium: results



Apply the new formula (for primary hadrons)

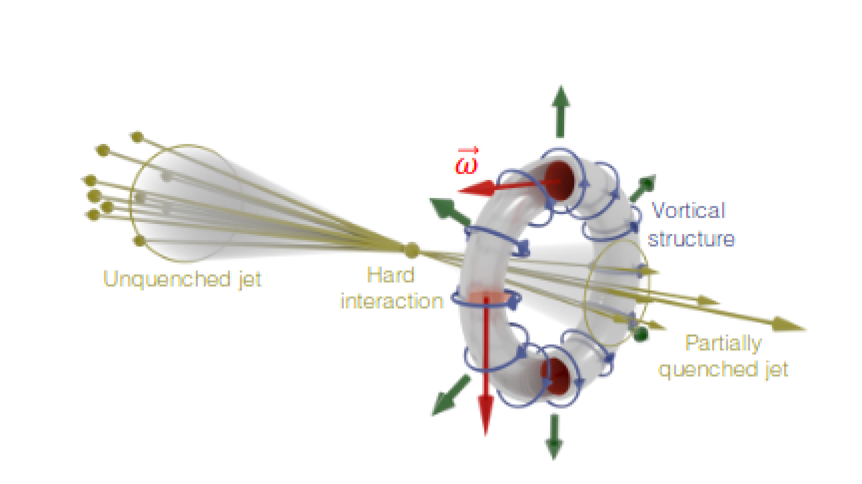
$$S^\mu_{\rm ILE}(p) = - \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F (1-n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma} \right]}{8 m T_{\rm dec} \int_\Sigma {\rm d}\Sigma \cdot p \, n_F}$$





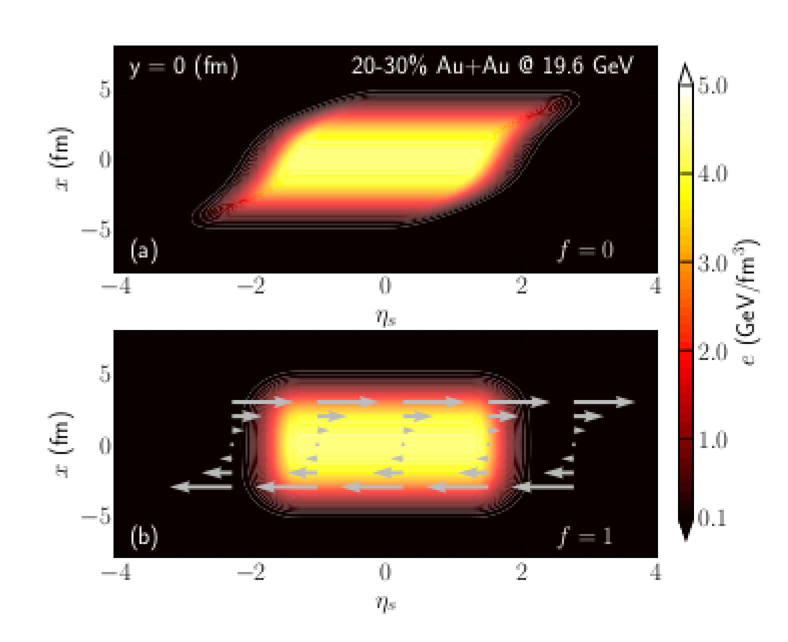
More polarization phenomenology

Many new phenomenological predictions both at high and low energy
(L. Bravina et al., A. Lei et al., B. Fu et al., Yu. Ivanov, Y. Guo et al., X. G. Dei et al.,)



W. Serenone, J. Barbon, D. Chinellato, M. Lisa, C. Shen, J. Takahashi, G. Torrieri, Phys.Lett.B 820 (2021) 136500

Use of Λ polarization to detect vortices induced by the jet energy loss



S. Ryu, V. Jupic, C. Shen, arXiv: 2106.08125

Study of the effect of longitudinal flow velocity on Λ polarization with 3+1D viscous hydro code

What is the spin good for?

Spin as a tool to reveal local parity violation without the mediation of the EM field

See M. Buzzegoli's talk

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma_{\rm eq}} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta_A \widehat{j}^\mu_A \right) \right] \quad \zeta_A = \frac{\mu_A}{T}$$

$$S^\mu_{tot}(p) = S^\mu_{\rm hyd}(p) + S^\mu_\chi(p)$$

What we have seen before

$$S^\mu_\chi(p) = \frac{g_h}{2} \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, \zeta_A n_F \left(1 - n_F\right)}{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F} \frac{\varepsilon p^\mu - m^2 \hat{t}^\mu}{m\varepsilon}$$

Spin vector induced by the axial chemical potential

S. Shi, C. Gale, S. Jeon, Phys. Rev. C 103 (2021) 4, 044906 (massless particles);

J. H. Gao, arXiv: 2105.08293; Y. C. Liu et al., Chin. Phys.C 44 (2020) 9, 094101

Summary and outlook

Spin physics in heavy ion collisions is a rapidly evolving and exciting field

Many theoretical advances over the past two years of confinement

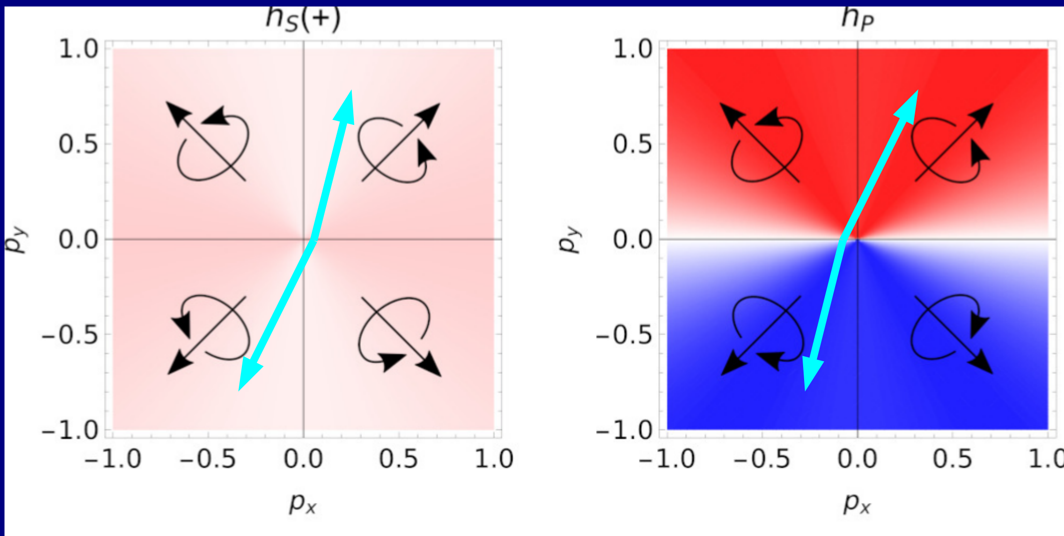
Spin is a tool to investigate fundamental physics in the QGP
and beyond

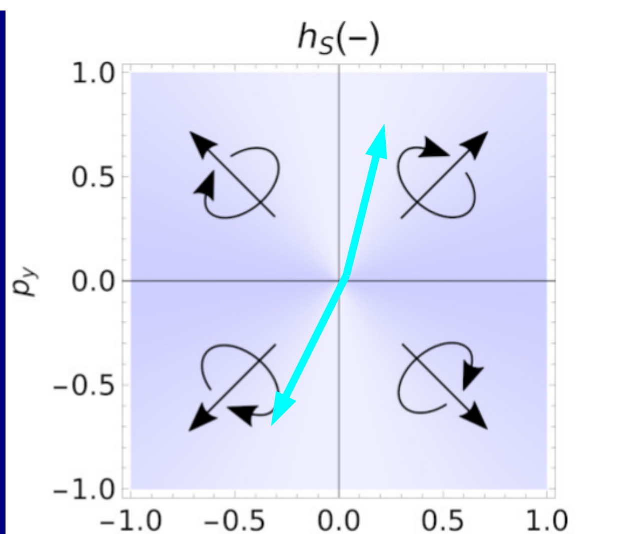
Post-doctoral position available in Florence, deadline Nov. 9Th

https://sites.google.com/unifi.it/francescobecattiniwebpage/relativistic-and-nuclear-matter/people-and-jobs

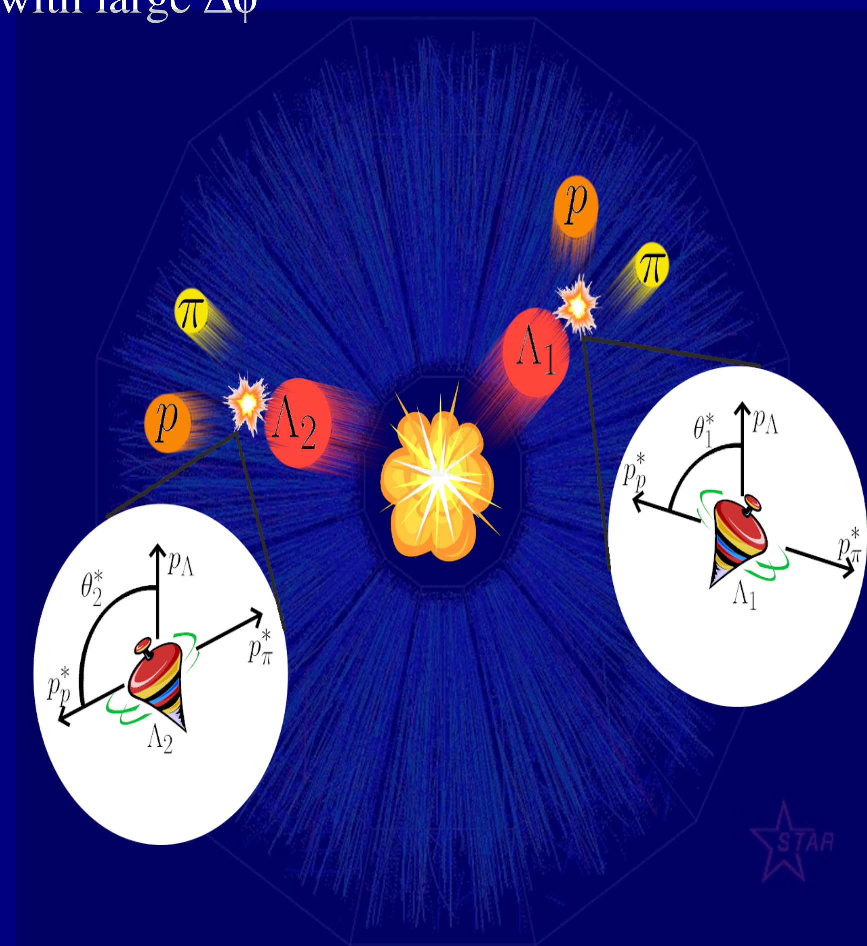
Azimuthal analysis of helicity and helicity correlations

Helicity can be measured by projecting the p momentum in the Λ rest frame onto the momentum of the Λ in the QGP frame





If local parity violation is there, there should appear an anomalous positive correlation between the proton-Λ angles of Λ pairs with large Δϕ



No coupling to EM field required!
Completely independent of the CME

What is this new term?

Does it have a non-relativistic limit?
Let us decompose it

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma\left(\frac{1}{T}\right)u_\rho + \frac{1}{2}\partial_\rho\left(\frac{1}{T}\right)u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

A is the acceleration field

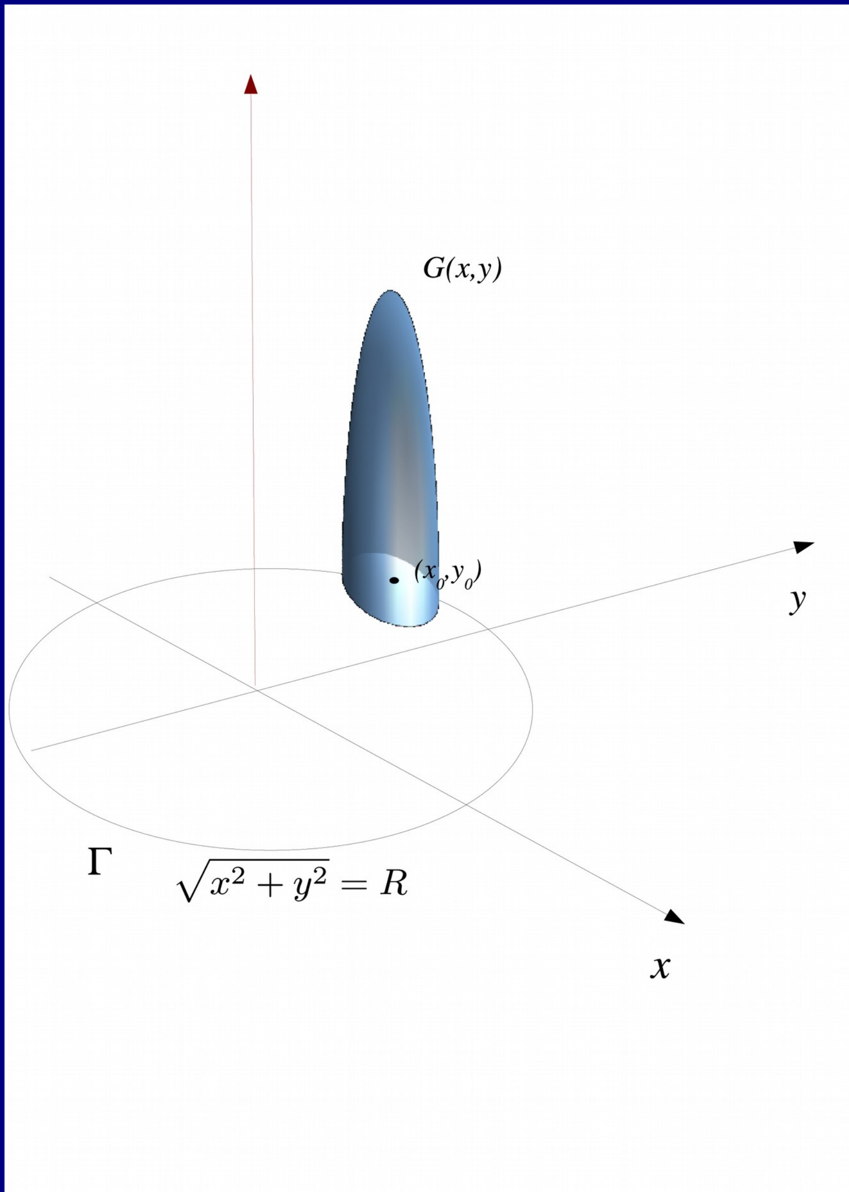
$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

All terms are relativistic (they vanish in the infinite *c* limit) EXCEPT grad T terms, which give rise to:

$$\mathbf{S}_\xi = \frac{1}{8}\mathbf{v} \times \frac{\int \mathrm{d}^3\mathrm{x}\, n_F(1-n_F)\nabla\left(\frac{1}{T}\right)}{\int \mathrm{d}^3\mathrm{x}\, n_F}$$

There is an equal contribution in the NR limit from thermal vorticity

Understand the point: a simple example



Task: approximate the integral

$$W = \int_\Gamma \mathrm{e}^{\sqrt{x^2+y^2}} G(x,y)ds$$

where *G(x,y)* is a peaked function around the point (x_0,y_0) on the circle.

Since G is peaked, one can Taylor expand the exponent about (x_0,y_0)

$$\begin{aligned} W &\simeq \mathrm{e}^{\sqrt{x_0^2+y_0^2}} \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s \\ &= \mathrm{e}^R \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s \\ &= \mathrm{e}^R \int_\Gamma \mathrm{e}^{\nabla r|_{(x_0,y_0)}\cdot(\mathbf{x}-\mathbf{x}_0)} G(x,y)\mathrm{d}s \end{aligned}$$

But it is just pointless if we integrate over the circle!

$$W = \mathrm{e}^R \int_\Gamma G(x,y)\mathrm{d}s$$

In the previous example, the Taylor expansion at first order introduces an undesired term:

$$\mathrm{W} = \mathrm{e}^{R} \int_{\Gamma} G(x,y)\mathrm{d}s \qquad\qquad W \simeq \mathrm{e}^{R} \int_{\Gamma} \mathrm{e}^{\nabla r|_{(x_0,y_0)}\cdot(\mathbf{x}-\mathbf{x}_0)} G(x,y)\mathrm{d}s$$

exact — With gradient of r expansion

which is proportional to the gradient of the constant quantity on the circle, perpendicular to the integration line. This term does not vanish in the integration!

Similarly, for an isothermal hadronization, the inclusion of temperature gradients results in an additional, undesirable contribution proportional to the gradient of *T*, perpendicular to Σ_{FO} :

$$\frac{1}{2}[(\partial_\mu T)u_\nu(x) - (\partial_\nu T)u_\mu(x)]\widehat{J}_x^{\mu\nu} + \frac{1}{2}[(\partial_\mu T)u_\nu(x) + (\partial_\nu T)u_\mu(x)]\widehat{Q}_x^{\mu\nu}$$

Linear response theory

$$\mathrm{e}^{\widehat{A}+\widehat{B}} = \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z\ \mathrm{e}^{z(\widehat{A}+\widehat{B})}\widehat{B}\mathrm{e}^{-z\widehat{A}}\mathrm{e}^{\widehat{A}} \simeq \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z\ \mathrm{e}^{z\widehat{A}}\widehat{B}\mathrm{e}^{-z\widehat{A}}\mathrm{e}^{\widehat{A}}$$

$$\widehat{A} = -\beta_\mu(x)\widehat{P}^\mu$$

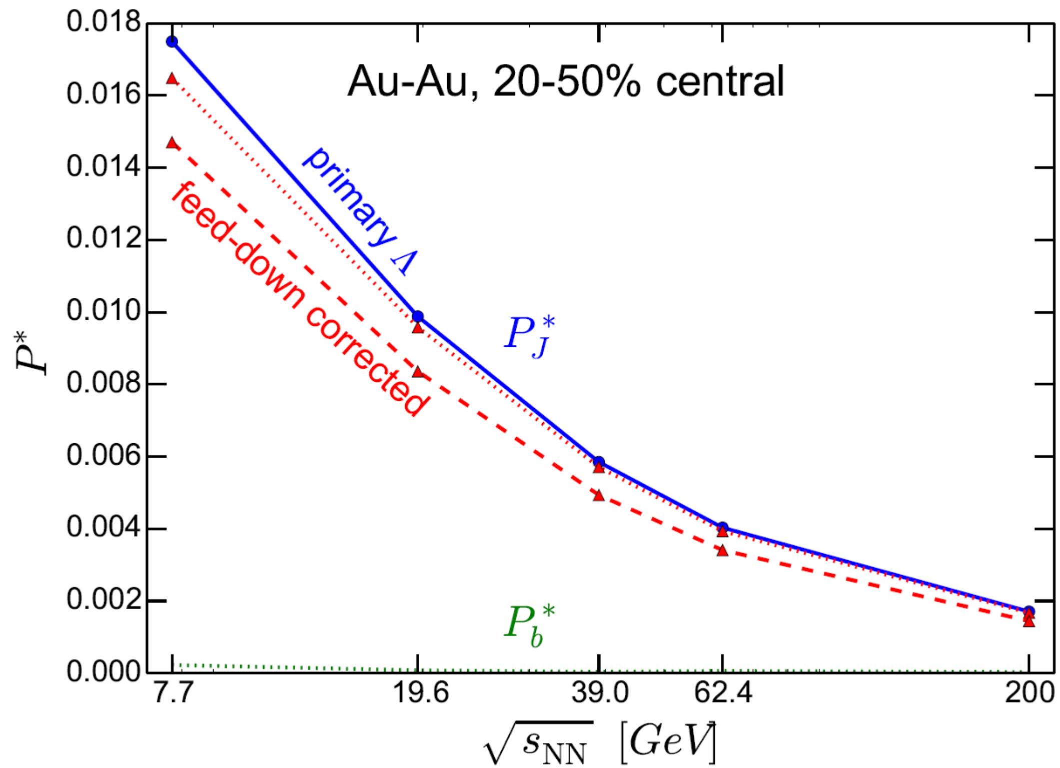
$$\widehat{B} = \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$W(x,k) \simeq \frac{1}{Z}\mathrm{Tr}(\mathrm{e}^{\widehat{A}+\widehat{B}}\widehat{W}(x,k)) \simeq \ldots$$

CORRELATORS

$$\langle \widehat{Q}^{\mu\nu}_x \widehat{W}(x,p)\rangle \qquad \langle \widehat{J}^{\mu\nu}_x \widehat{W}(x,p)\rangle$$

Agreement between hydrodynamic predictions and the data

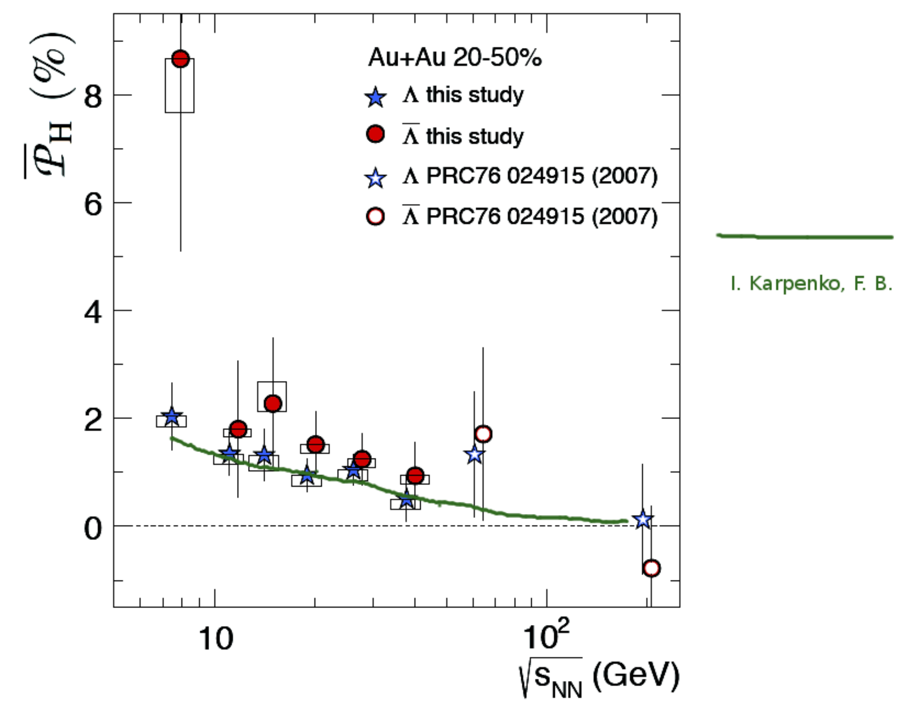


I. Karpenko and F. B., Eur. Phys. J. C 77 (2017) 213

$$S^{\mu}(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int_{\Sigma} d\Sigma_{\tau}p^{\tau}n_F(1-n_F)\partial_{\nu}\beta_{\rho}}{\int_{\Sigma} d\Sigma_{\tau}p^{\tau}\,n_F}$$

$$\mathbf{S}^*_{\text{daughter}} = C\mathbf{S}^*_{\text{parent}}$$

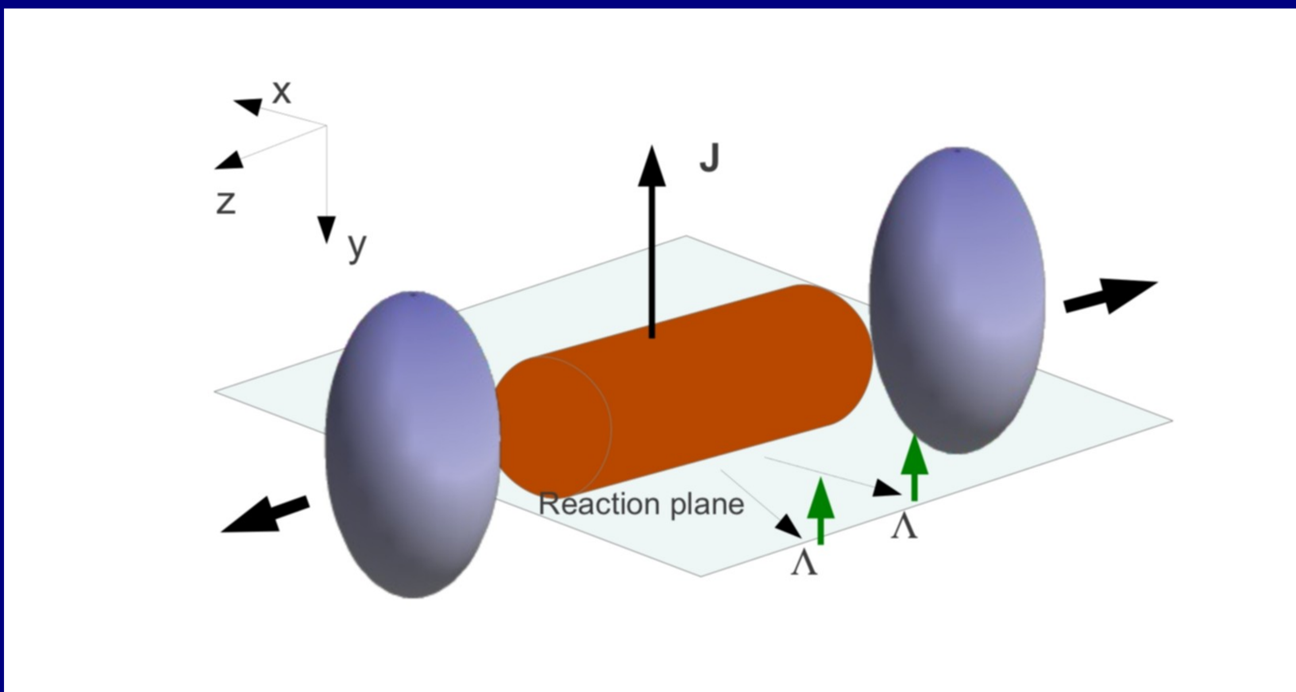
$$C = \sum_{\lambda_A,\lambda_B,\lambda'_A} T^J(\lambda_A,\lambda_B)T^J(\lambda'_A,\lambda_B)^* \sum_{n=-1}^{1} \langle\lambda'_A|\widehat{S}_{A,-n}|\lambda_A\rangle$$
$$\times \frac{c_n}{\sqrt{J(J+1)}}\langle J\lambda|J1|\lambda' n\rangle \left(\sum_{\lambda_A,\lambda_B}|T^J(\lambda_A,\lambda_B)|^2\right)^{-1}$$

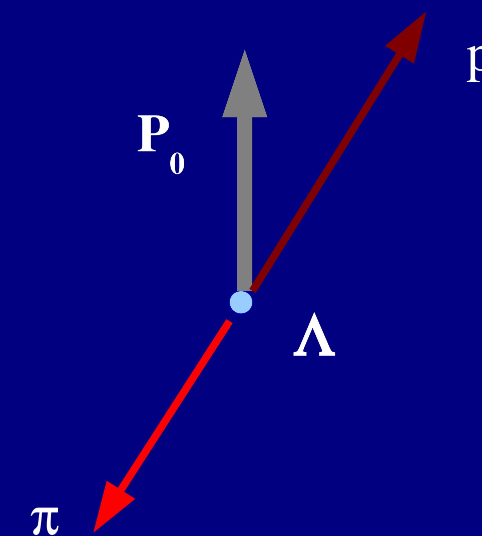


F. B., I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, Phys. Rev. C 95 054902 (2017)

How to observe it: global Λ polarization

Because of parity violation, the polarization vector of Λ can be measured in its decay Into a proton and a pion





Distribution of protons in the Λ rest frame

$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}\Omega} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_0 \cdot \hat{\mathbf{p}}^*) \qquad \mathbf{P}_0(p) = \mathbf{P}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon+m)}\mathbf{P}(p)\cdot\mathbf{p}$$

α = 0.642 → 0.75 (!) PDG 2020