Helicity and local parity violation

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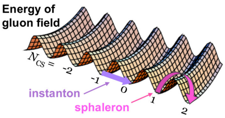
In collaboration with F. Becattini, A. Palermo and G. Prokhorov
based on [Phys. Lett. B (2021) 822, 136706, 2009.13449]

Motivations

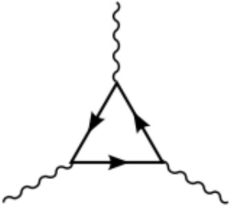
Relativistic heavy ion collisions allow to look for

- Quantum effects on fluids (Polarization and thermal vorticity)
- Local parity violation in QCD (via the Chiral Magnetic Effect)

Measurable effects of axial chemical potential?



In QCD plasma, sphalerons are abundant and induce the quark chirality non-conservation



$$\partial_\mu \widehat{j}^\mu_{\mathrm{A}}(x) = -\frac{g^2 N_f}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

CME

$$\mathbf{j} = \frac{\mu_{\mathrm{A}}}{2\pi}\mathbf{B}$$

Observables:

- Charge separation
- CME & Helicity [Finch and Murray, Phys.Rev.C 96 (2017)]

[K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys.Rev.D 78 (2008)]

[Shi et al., PRL (2020)] [D. E. Kharzeev and J. Liao, nature rev. phys. (2020)]

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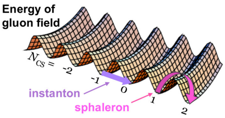

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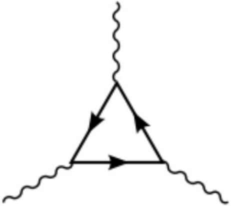


CME

$$\mathbf{j} = \frac{\mu_A}{2\pi}\mathbf{B}$$

Observables:

- Charge separation
- CME & Helicity [Finch and

Proposal

Use Lambda helicity correlations to reveal local parity violation, a probe which is independent of the Elctro-Magnetic field

[F.Becattini, MB, A. Palermo, G. Prokhorov Phys. Lett. B (2021) 822, 136706]

Mean spin vector with axial imbalance at local thermodynamic equilibrium

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \frac{1}{Z_{\rm LE}} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_A \widehat{j}^\mu_{\rm A} \right) \right]$$

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p) + S^\mu_\chi(p)$$

$$S^\mu_\varpi(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p\, n_F(1-n_F)\partial_\rho\beta_\sigma}{\int_\Sigma {\rm d}\Sigma_\lambda p^\lambda n_F} \leftarrow \text{ Thermal vorticity}$$

$$S^\mu_\xi(p) = \frac{1}{8m}\epsilon^{\mu\nu\tau\sigma} \frac{p_\tau p^\rho}{\varepsilon} \frac{\int_\Sigma {\rm d}\Sigma \cdot p\, n_F(1-n_F)\hat{t}_\nu(\partial_\sigma\beta_\rho + \partial_\rho\beta_\sigma)}{\int_\Sigma {\rm d}\Sigma \cdot p\, n_F} \leftarrow \text{ Thermal shear}$$

$$S^\mu_\chi(p) \simeq \frac{g_h}{2} \frac{\int_\Sigma {\rm d}\Sigma \cdot p\, \zeta_{\rm A} n_{\rm F}\left(1-n_{\rm F}\right)}{\int_\Sigma {\rm d}\Sigma \cdot p\, n_{\rm F}} \frac{\varepsilon p^\mu - m^2 \hat{t}^\mu}{m\varepsilon} \leftarrow \text{ Axial imbalance}$$

for spin polarization of massless fermions induced by $\zeta_{\rm A}$ see

[Y. C. Liu, K. Mameda and X. G. Huang, Chin.Phys.C 44 (2020) 2002.03753]

[S. Shi, C. Gale and S. Jeon, Phys. Rev. C 103 (2021) 2008.08618]

[J. H. Gao, Phys. Rev. D 104 (2021) 2105.08293]

Mean spin vector with axial imbalance at local thermodynamic equilibrium

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \frac{1}{Z_{\rm LE}} \exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_A \widehat{j}^\mu_{\rm A}\right)\right]$$

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p) + S^\mu_\chi(p)$$

$\zeta_{\rm A}$ sign fluctuates on an event by event bais
Average over multiple events

$$\langle\langle S^\mu(p)\rangle\rangle = \langle\langle S^\mu_\varpi(p)\rangle\rangle + \langle\langle S^\mu_\xi(p)\rangle\rangle + \cancel{\langle\langle S^\mu_\chi(p)\rangle\rangle}$$

$$\langle\langle \zeta_{\rm A}\rangle\rangle = 0 \qquad \langle\langle \zeta_{\rm A}^2\rangle\rangle \neq 0$$

Polarization formula for a fermion

Given the density matrix $\widehat{\rho}$

- Wigner function

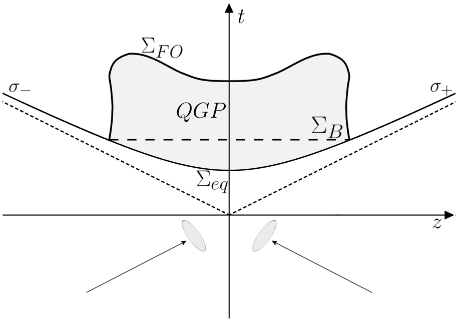
$$W_+(x,p)_{AB} = \theta(p^0)\theta(p^2)\int \frac{\mathrm{d}^4 y}{(2\pi)^4} \mathrm{e}^{-\mathrm{i}p\cdot y}\ \mathrm{tr}\left[\widehat{\rho} : \bar{\Psi}_B(x+y/2)\Psi_A(x-y/2) :\right]$$

- Spin vector

$$S^\mu(p) = \frac{1}{2}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\,\mathrm{tr}\left[\gamma^\mu\gamma^5 W_+(x,p)\right]}{\int_\Sigma \mathrm{d}\Sigma\cdot p\,\mathrm{tr}\left[W_+(x,p)\right]}$$

In a nuclear collision Σ is the freeze-out hypersurface

Local thermodynamic equilibrium



$$\beta^\nu = \frac{u^\nu}{T}$$

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\int_{\Sigma_{\rm eq}} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_{\rm A}\widehat{j}^\mu_{\rm A} \right) \right]$$

$$\int_{\Sigma_{\rm eq}} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_A\widehat{j}^\mu_{\rm A} \right) = \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_A\widehat{j}^\mu_{\rm A} \right) + \int_\Omega {\rm d}\Omega \left(\widehat{T}^{\mu\nu}\partial_\mu\beta_\nu - \widehat{j}^\mu_{\rm A}\partial_\mu\zeta_A - \zeta_A\partial_\mu\widehat{j}^\mu_{\rm A} \right)$$

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \frac{1}{Z_{\rm LE}} \exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_A\widehat{j}^\mu_{\rm A} \right) \right]$$

[Becattini, MB, Grossi, Particles 2 (2019) 2, 197-207; 1902.01089]

Hydrodynamic limit

At Local therm. eq. and by neglecting dissipative terms:

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z_{\mathrm{LE}}} \exp\left[- \int_\Sigma \mathrm{d}\Sigma_\mu(y) \left(\widehat{T}^{\mu\nu}(y) \beta_\nu(y) - \zeta_A(y) \widehat{j}^\mu_{\mathrm{A}}(y) \right) \right]$$

$$W(x,p) = \mathrm{tr}\left[\widehat{\rho}\, \widehat{W}(x,p) \right]$$

Slowly varying $\beta \Rightarrow$ Taylor expansion

$$\beta_\nu(y) = \beta_\nu(x) + \underbrace{\frac{1}{2}\left[\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x)\right]}_{\text{Thermal vorticity } \varpi}(y-x)^\mu + \underbrace{\frac{1}{2}\left[\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x)\right]}_{\text{Thermal shear } \xi}(y-x)^\mu + \cdots$$

$$\widehat{\rho}_{\mathrm{LE}} \simeq \frac{1}{Z_{\mathrm{LE}}} \exp\left[-\beta(x)\cdot\widehat{P} + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x + \int_\Sigma \mathrm{d}\Sigma_\rho \zeta_A \widehat{j}^\rho_A \right]$$

$\widehat{P}$ is the total four-momentum

Linear response theory

In nuclear collisions $\zeta_{\rm A}$ is supposed to be small

$$\mathrm{e}^{\widehat{A}+\widehat{B}} = \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z\, \mathrm{e}^{z\widehat{A}}\, \widehat{B}\, \mathrm{e}^{-z\widehat{A}}\, \mathrm{e}^{\widehat{A}} + \cdots\,,$$

$$\widehat{A} = -\beta(x)\cdot\widehat{P}, \quad \widehat{B} = \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x + \int_\Sigma \mathrm{d}\Sigma_\rho(y)\zeta_A(y)\widehat{j}^\rho_A(y).$$

Wigner function

$$\langle\widehat{W}_+(x,p)\rangle_{\rm LE} \simeq \langle\widehat{W}_+(x,p)\rangle_{\beta(x)} + \Delta W_+(x,p)$$

$$\Delta W_+(x,p) = \int_\Sigma \mathrm{d}\Sigma_\rho\zeta_A\int_0^1 \mathrm{d}z\langle\widehat{W}_+(x,p)\widehat{j}^\rho_A(y+\mathrm{i}z\beta)\rangle_{c,\beta(x)}$$

$$\langle\widehat{O}\rangle_{\beta(x)} = \frac{1}{Z}\mathrm{tr}\left[\exp[-\beta(x)\cdot\widehat{P}]\,\widehat{O}\right] \qquad \langle\widehat{O}_1\widehat{O}_2\rangle_c \equiv \langle\widehat{O}_1\widehat{O}_2\rangle - \langle\widehat{O}_1\rangle\langle\widehat{O}_2\rangle$$

Hadronic axial current

- $\widehat{j}^{\mu}_{\mathrm{A}}$ is the color singlet axial current
- Decompose it on the multi-hadronic Hilbert space

[S. Weinberg, The Quantum theory of fields. Vol. 1]

$$\widehat{j}^{\mu}_{A}(x) = \sum_{\substack{N=0\\M=0}}^{\infty} \sum_{\substack{j_1,\dots,j_N\\k_1,\dots,k_M}} \int \frac{\mathrm{d}^3 q_1'}{2\varepsilon_1'} \cdots \int \frac{\mathrm{d}^3 q_N'}{2\varepsilon_N'} \int \frac{\mathrm{d}^3 q_1}{2\varepsilon_1} \cdots \int \frac{\mathrm{d}^3 q_M}{2\varepsilon_M}$$

$$\widehat{a}^{\dagger}_{j_1}(q_1') \cdots \widehat{a}^{\dagger}_{j_N}(q_N') \widehat{a}_{k_1}(q_1) \cdots \widehat{a}_{k_M}(q_M) J^{\mu}(q', q, x)^{j_1,\dots,j_N,k_1,\dots,k_M}$$

- $\langle \widehat{W}^{h}_{+} \widehat{j}^{\rho}_{A} \rangle_{c,\beta} \to$ contribution from the same species h
- predominant contribution: $N = M = 1$, $j_1 = k_1 = h$

$$J^{\mu}(q', q, x)^{hh} = \langle 0 | \widehat{a}_{h,\sigma'}(q') \widehat{j}^{\mu}_{A}(x) \widehat{a}^{\dagger}_{h,\sigma}(q) | 0 \rangle = \langle q', \sigma' | \widehat{j}^{\mu}_{A}(x) | q, \sigma \rangle$$

$$= \frac{\mathrm{e}^{it\cdot x}}{(2\pi)^3} \bar{u}_{\sigma'}(q') \left[G_{A1}(t^2) \gamma^{\mu} \gamma^5 + \frac{t^{\mu}}{2m_h} G_{A2}(t^2) \gamma^5 \right] u_{\sigma}(q)$$

$$t = q' - q$$

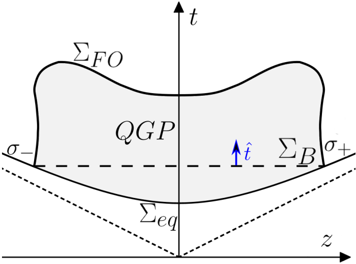
Form factors G_{A1} and G_{A2} depend on the flavour-space transformation properties of the axial current

Final result

$$S^\mu_\chi(p) \simeq \frac{g_h}{2}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\,\zeta_\mathrm{A} n_\mathrm{F}\,(1-n_\mathrm{F})}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_\mathrm{F}}\frac{\varepsilon p^\mu - m^2\hat{t}^\mu}{m\varepsilon}$$

$$g_h = G_{\mathrm{A}1}(0)$$

$$\mathbf{S}_0 = \mathbf{S} - \frac{\mathbf{p}}{\varepsilon(\varepsilon+m)}\mathbf{S}\cdot\mathbf{p}, \quad \hat{\mathbf{p}} = \frac{\mathbf{p}}{|\mathbf{p}|}$$



In the rest frame of the hadron

$$\mathbf{S}_{0,\chi} = \frac{g_h}{2}\frac{|\mathbf{p}|}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\,\zeta_A n_\mathrm{F}\,(1-n_\mathrm{F})}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_\mathrm{F}}\hat{\mathbf{p}} \equiv h_\chi(\mathbf{p})\hat{\mathbf{p}}$$

Helicity

$$\text{Helicity:}\quad h(\mathbf{p}) = \mathbf{S}_0(\mathbf{p}) \cdot \hat{\mathbf{p}}$$

Induced by axial imbalance

$$h_\chi(\mathbf{p}) = \frac{g_h}{2}\frac{|\mathbf{p}|}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \zeta_A n_{\mathrm{F}}\,(1-n_{\mathrm{F}})}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_{\mathrm{F}}}$$

A special case:

- ζ_{A} is almost constant
- $(1-n_{\mathrm{F}}) \simeq 1$

$$\mathbf{S}_{0,\chi} \simeq \frac{g_h}{2}\frac{|\mathbf{p}|}{\varepsilon}\zeta_A\,\hat{\mathbf{p}} \qquad h_\chi \simeq \frac{g_h}{2}\frac{|\mathbf{p}|}{\varepsilon}\zeta_{\mathrm{A}}$$

Search for local parity violation

Signature of axial imbalance

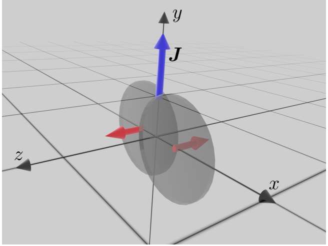
$$\text{Linear effect:} \quad \mathbf{S}_{0,\chi} = h_\chi(\mathbf{p})\,\hat{\mathbf{p}} \qquad h_\chi(\mathbf{p}) \propto \zeta_{\mathrm{A}}$$

Problem: Over many events, the axial chemical potential averages to zero: $\langle\langle \zeta_{\mathrm{A}} \rangle\rangle = 0$

Search for parity breaking terms

- Average of the square: $\langle\langle \zeta_{\mathrm{A}}^2 \rangle\rangle \neq 0$
- Look for parity breaking terms in the helicity

Helicity and Parity breaking



Symmetries: Parity P and Rotation $\mathsf{R}_J(\pi)$
$\rightarrow$ also sym. of freeze-out surface
P in momentum space: $\mathbf{p} \rightarrow -\mathbf{p}$

No axial charge

$\widehat{\rho}$ is P invariant, then
Helicity is a pseudoscalar h_{P}

$$h(-\mathbf{p}) = -h(\mathbf{p})$$

Axial charge

$\widehat{\rho}$ is not P invariant, then
Helicity has a scalar part: h_{S}

$$h_{\mathrm{S}}(-\mathbf{p}) = h_{\mathrm{S}}(\mathbf{p})$$

$h_\chi(\mathbf{p})$ is a scalar

$$h = h_{\mathrm{P}} + h_{\mathrm{S}}$$

Model independent analysis

Consider particles emitted at midrapidity,
i.e. transverse momentum ($p_z = 0$): $\mathbf{p} \to (p_T,\ \phi)$

$h = h_{\rm P} + h_{\rm S}$
From rotational symmetry $\phi \to \pi - \phi$
and reflection properties $\phi \to \pi + \phi$:

$$h_P(p_T, \phi) = \sum_k P_k(p_T) \sin[(2k+1)\phi]$$

$$h_S(p_T, \phi) = \sum_k S_k(p_T) \cos[2k\phi]$$

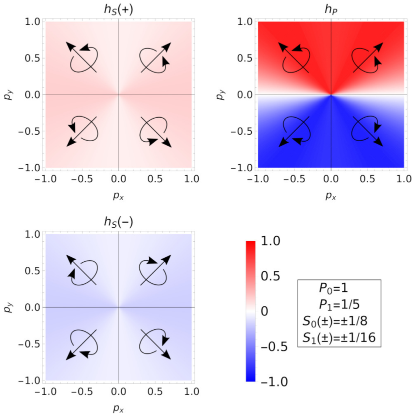


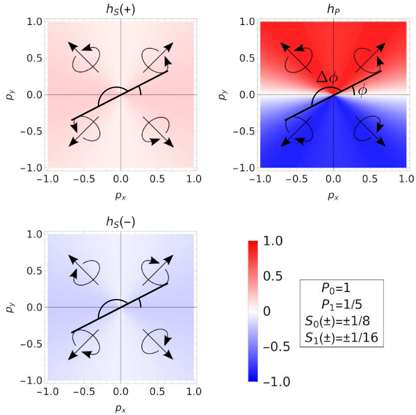
Figure is for illustration, not actual predictions

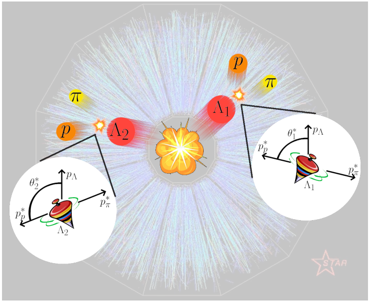
Local parity violation $S_k(p_T) \neq 0$
Global parity conservation $\langle\langle S_k(p_T)\rangle\rangle = 0$

Helicity-helicity correlator

$\langle h_1 h_2(\Delta\phi)\rangle =$ correlator between two hyperons emitted in the same event with angles ϕ and $\phi + \Delta\phi$

$$\langle h_1 h_2(\Delta\phi)\rangle = \frac{1}{N}\int \mathrm{d}^2\mathbf{p}_{T1}\mathrm{d}^2\mathbf{p}_{T2}\, n(\mathbf{p}_{T1},\mathbf{p}_{T2})\,\delta(\phi_2-\phi_1-\Delta\phi)\times h_1(\mathbf{p}_{T1})h_2(\mathbf{p}_{T2})$$





Local parity violation → Positive correlation at large angles

E.g. at $\Delta\phi = \pi$ same sign of h_1 and h_2

Helicity-helicity correlator

From leading harmonics $\rightarrow \bar{S}_0, \bar{P}_0 =$ transverse momentum average

$$\langle h_1 h_2(\Delta\phi)\rangle \simeq \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi \left(\bar{S}_0^2 + \bar{P}_0^2 \sin^2\phi \cos\Delta\phi\right) = \bar{S}_0^2 + \frac{1}{2}\bar{P}_0^2 \cos\Delta\phi$$

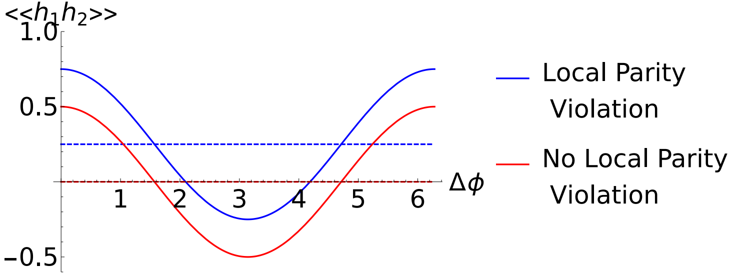


Figure is for illustration, not actual predictions

Signature of local parity violation

A constant term in $\langle\langle h_1 h_2(\Delta\phi)\rangle\rangle$

Conclusions

- Spin as a probe of axial chemical potential
- Local parity violation in relativistic nuclear collisions can be detected by measuring polarization of e.g. Λ hyperons
- Search for parity breaking terms in the helicity azimuthal dependence
- Independent of magnetic field (Complementary to CME)

Outlook

- Numerical simulations based on hydro codes including axial current generation

$$S^\mu_\chi(p) \simeq \frac{g_h}{2} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p \, \zeta_\mathrm{A} n_\mathrm{F} \left(1 - n_\mathrm{F}\right)}{\int_\Sigma \mathrm{d}\Sigma \cdot p \, n_\mathrm{F}} \frac{\varepsilon p^\mu - m^2 \hat{t}^\mu}{m\varepsilon}$$

Ent by event chiral hydrodynamic codes: [Shi et al, Annals Phys. 394 (2018); Shi et al, PRL 125 (2020)]

Thanks for the attention!

Backup

Estimate of μ_{A}

From Glasma models

D. Kharzeev, A. Krasnitz, and R. Venugopalan, Phys. Lett. B 545, 298 (2002), hep-ph/0109253
T. Lappi and L. McLerran, Nucl. Phys. A 772, 200 (2006), hep-ph/0602189
Y. Jiang, S. Shi, Y. Yin, and J. Liao, Chin. Phys. C 42, 011001 (2018), 1611.04586

$$\sqrt{\langle n_{\mathrm{A}}^2 \rangle} \simeq \frac{Q_s^2 (\pi \rho_{\mathrm{tube}} \tau_0) \sqrt{N_{\mathrm{Coll}}}}{16\pi^2 A_{\mathrm{overlap}}}$$

Anomalous Viscous Fluid dynamics simulations

S. Shi, Y. Jiang, E. Lilleskov, and J. Liao, Annals Phys. 394, 50 (2018), 1711.02496
S. Lin, L. Yan, and G.-R. Liang, Phys. Rev. C 98, 014903 (2018), 1802.04941
G.-R. Liang, J. Liao, S. Lin, L. Yan, and M. Li, Chin. Phys. C 44, 094103 (2020), 2004.04440

$$\lambda_5 = \frac{\int_V \sqrt{\langle n_{\mathrm{A}}^2 \rangle}}{\int_V s} \qquad n_{\mathrm{A}}^{\mathrm{initial}} \equiv \lambda_5 s$$

The simulations gives [J. Liao]:

$$\zeta_{\mathrm{A,at\ hadronization}} \sim 10^{-2}, \qquad \mu_{\mathrm{A}}^{\mathrm{Had}} \sim T^{\mathrm{Had}} \times 10^{-2} \sim 1,5\ \mathrm{MeV}$$

Wigner function

First order correction on axial imbalance

$$\langle \widehat{W}_+(x,p)\rangle_{\rm LE} \simeq \langle \widehat{W}_+(x,p)\rangle_{\beta(x)} + \int_\Sigma {\rm d}\Sigma_\rho \zeta_A \int_0^1 {\rm d}z \langle \widehat{W}_+(x,p)\widehat{j}^\rho_A(y+{\rm i}z\beta)\rangle_{c,\beta(x)}$$

using the normal mode expansion of the Dirac field and standard thermal field theory techniques

$$\begin{aligned}
\langle \widehat{W}_+(x,p)\rangle_{\beta(x)} =& \frac{m+\gamma^\mu p_\mu}{(2\pi)^3}\delta(p^2-m^2)\theta(p_0)n_{\rm F}(p) \\
\Delta W_{+ab}(x,p) =& \int_\Sigma {\rm d}\Sigma_\rho\, \zeta_A \int_0^1 \frac{{\rm d}z}{(2\pi)^6}\int \frac{{\rm d}^3k{\rm d}^3k'}{4\varepsilon_k\varepsilon_{k'}}\delta^4\left(p-\frac{k+k'}{2}\right) n_{\rm F}(k)(1-n_{\rm F}(k')) \\
& \times {\rm e}^{{\rm i}(k-k')(x-y)}{\rm e}^{z(k-k')\beta}\,\mathcal{A}^\rho(k,k')_{ab} \\
n_{\rm F}(k) =& \frac{1}{{\rm e}^{\beta(x)\cdot k}+1} \\
\mathcal{A}^\rho(k,k')_{ab} \equiv& (\not{k}'+m)\left[G_{A1}\left(t^2\right)\gamma^\rho\gamma^5 + \frac{k'^\rho-k^\rho}{2m}G_{A2}\left(t^2\right)\gamma^5\right](\not{k}+m)
\end{aligned}$$

Polarization

First order correction on axial imbalance

$$S^\mu_\chi(p) = \frac{1}{2}\frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \mathrm{tr}\left[\gamma^\mu\gamma^5 \Delta W_+\right]}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \mathrm{tr}\left[\langle \widehat{W}_+\rangle_\beta\right]}$$

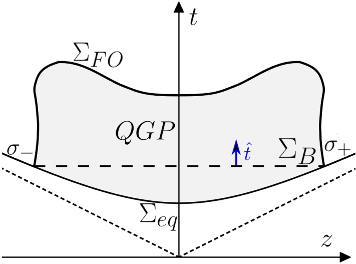
$$\begin{aligned}S^\mu_\chi(p) = &-\frac{2}{\mathcal{D}}\int_\Sigma \mathrm{d}\Sigma(x)\cdot p \int_0^1 \frac{\mathrm{d}z}{(2\pi)^6}\int\frac{\mathrm{d}^3 k}{2\epsilon_k}\int\frac{\mathrm{d}^3 k'}{2\epsilon_{k'}}\delta^4\left(p - \frac{k+k'}{2}\right)\\ &\times \int_\Sigma \mathrm{d}\Sigma_\rho(y)\ \zeta_A(y)\mathrm{e}^{\mathrm{i}(k-k')(x-y)} n_\mathrm{F}(k)(1-n_\mathrm{F}(k'))\mathrm{e}^{z(k-k')\beta}\mathcal{B}^{\mu\rho}(k,k')\end{aligned}$$

$$\mathcal{D} = \frac{4m}{(2\pi)^3}\int_\Sigma \mathrm{d}\Sigma\cdot p\ \delta(p^2-m^2)\theta(p_0)n_\mathrm{F}(p)$$

$$\begin{aligned}\mathcal{B}^{\mu\rho}(k,k') = &G_{A1}(t^2)\left[\eta^{\mu\rho}(m^2 + k\cdot k') - k^\rho k'^\mu - k^\mu k'^\rho\right]\\ &+\frac{1}{2}G_{A2}\left(t^2\right)(k'^\mu - k^\mu)(k'^\rho - k^\rho)\end{aligned}$$

Hydrodynamic approximation

- Integral in $S^\mu_\chi(p)$ decays on microscopic length scales
- ζ_A varies on longer length scales, in the hydrodynamic picture



$$\int_\Sigma \mathrm{d}\Sigma_\rho(y)\,\zeta_A(y)\mathrm{e}^{\mathrm{i}(k-k')(x-y)} \simeq \zeta_A(x)\int_\Sigma \mathrm{d}\Sigma_\rho(y)\,\mathrm{e}^{\mathrm{i}(k-k')(x-y)}$$

$$=\zeta_A(x)\int_{\sigma_\pm}\mathrm{d}\Sigma_\rho(y)\,\mathrm{e}^{\mathrm{i}(k-k')(x-y)} + \zeta_A(x)\int_{\Sigma_\mathrm{B}}\mathrm{d}\Sigma_\rho(y)\,\mathrm{e}^{\mathrm{i}(k-k')(x-y)}$$

$$-\,\mathrm{i}(k-k')_\rho\zeta_A(x)\int_{\Omega_\mathrm{B}}\mathrm{d}^4y\,\mathrm{e}^{\mathrm{i}(k-k')(x-y)}$$

$$\simeq\zeta_A(x)\hat{t}_\rho(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k'})$$

in the center-of-mass frame: $\hat{t}_\rho = \delta^0_\rho$