# Relaxation Time Approximation for Relativistic Quantum Systems

*Pengfei Zhuang*
*Physics Department, Tsinghua University*

- *Kadanoff-Baym Equations with Spin*
- *Spin Distribution in Equilibrium*
- *Spin Distribution in Near Equilibrium*
- *Damping and Polarization Rates in NJL*

*based on the 2 papers:*

1) *Equilibrium spin distribution from detailed balance,*
   *Ziyue Wang, Xingyu Guo and PZ, Eur. Phys. J. C81, 799(2021)*
2) *Relaxation time approximation for relativistic quantum systems,*
   *Ziyue Wang and PZ, ArXiv: 2105.00915*

# Relaxation Time Approximation (RTA)

*Classical RTA:* *[J.Anderson and H.Witting, Physica 74, 466(1974)]*

$$p^\mu \partial_\mu f = -p^\mu u_\mu \frac{\delta f}{\tau}, \qquad \delta f = f - f_{eq}$$

*Quantum RTA:*

$$(\gamma^\mu p_\mu - m)W + \frac{i\hbar}{2}\gamma^\mu \partial_\mu W = -\frac{i\hbar}{2}\gamma^\mu u_\mu \frac{\delta W}{\tau}, \qquad \delta W = W - W_{eq}$$

*A single time scale to control the relaxation process for all the components.*

*Motivations:*

*1) A multi-component kinetic theory needs more time scales to control different degrees of freedom.*

*2) Calculating the relaxation times in Kadanoff-Baym formalism.*

# Kadanoff-Baym Equations

*Collision terms:*

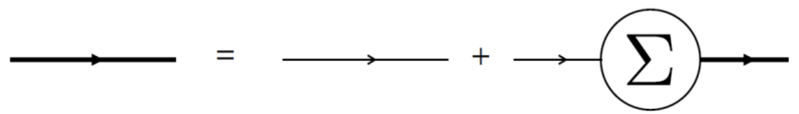
*J.Y.Chen, D.T.Son, M.A.Stephanov, Collisions in chiral kinetic theory, Phys. Rev. Lett. 115, 021601(2015).*
***D.L.Yang, K.Hattori, Y.Hidaka, Effective quantum kinetic theory …… with collisional effects, JHEP20, 070(2020).***
*N.Weickgenannt, E.Speranza, X.L.Sheng, Q.Wang, D.H.Rischke, Generating spin polarization ……, arXiv:2005.01506.*
*S.Carignano, C.Manuel, J.M.Torres-Rincon, Chiral kinetic theory …… collision terms, Phys. Rev. D102, 016003(2020).*
*S.Li, H.U.Yee, Quantum kinetic Theory of spin polarization …… QCD: Leading Log, Phys. Rev. D100, 056022(2019).*
*D. Hou, S. Lin, Polarization of chiral fermions in vortical fluid, arXiv:2008.03862.*
*……*

*Lesser and Greater propagators defined on Schwinger-Keldysh time contour:*

$$\tilde{S}^{<}_{\alpha\beta}(x+y/2, x-y/2) = \left\langle \bar{\psi}_\beta(x-y/2)\psi_\alpha(x-y/2)\right\rangle$$

$$\tilde{S}^{>}_{\alpha\beta}(x+y/2, x-y/2) = \left\langle \psi_\alpha(x-y/2)\bar{\psi}_\beta(x-y/2)\right\rangle$$

*Wigner transformation of the Dyson-Schwinger equation:*




*Kadanoff-Baym transport and constraint equations:*

$$\left\{(\gamma^\mu p_\mu - m), S^<\right\} + \frac{i\hbar}{2}\left[\gamma^\mu, \partial_\mu S^<\right] = \frac{i\hbar}{2} C(\Sigma, S)$$

$$\left[(\gamma^\mu p_\mu - m), S^<\right] + \frac{i\hbar}{2}\left\{\gamma^\mu, \partial_\mu S^<\right\} = \frac{i\hbar}{2} D(\Sigma, S)$$

$$S(x,p) = \int d^4y\, e^{\frac{i}{\hbar}p\cdot y} S(x+y/2, x-y/2)$$

$$W(x,p) = S^<(x,p)$$

# Spin Decomposition

*Complete and orthogonal matrices in spin space:*

$$\{I, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}/2\}$$

$\rightarrow scalar, pseudoscalar, vector, axial - vector\ and\ tensor\ components\ of\ S\ and\ \Sigma$:

$$S^< = \{F, P, V_\mu, A_\mu, S_{\mu\nu}\}$$

$$S^> = \{\bar{F}, \bar{P}, \bar{V}_\mu, \bar{A}_\mu, \bar{S}_{\mu\nu}\}$$

$$\Sigma^< = \{\Sigma_F, \Sigma_P, \Sigma_V^\mu, \Sigma_A^\mu, \Sigma_S^{\mu\nu}\}$$

$$\Sigma^> = \{\bar{\Sigma}_F, \bar{\Sigma}_P, \bar{\Sigma}_V^\mu, \bar{\Sigma}_A^\mu, \bar{\Sigma}_S^{\mu\nu}\}$$

● *2 Kadanoff-Baym equations for the Wigner function* $W = S^<$
→ *16 transport + 16 constraint equations for 16 spin components* $F, P, V_\mu, A_\mu, S_{\mu\nu}$

# *Classical Transport Equations*

*$\hbar$ −expansion for $S$ and $\Sigma$:*

$$F = F^{(0)} + \hbar F^{(1)} + \hbar^2 F^{(2)} + \cdots$$
$$\Sigma_F = \Sigma_F^{(0)} + \hbar \Sigma_F^{(1)} + \hbar^2 \Sigma_F^{(2)} + \cdots$$
$$\cdots\cdots$$

*Classical limit:*

*Constraint equations → only 4 independent spin components:*

*particle number $V_0$ and spin density $\vec{A}$ → covariant $V_\mu, A_\mu$*

*under constraints $(p^\mu \partial^\nu + p^\nu \partial^\mu) V_\mu = 0$ and $p^\mu A_\mu = 0$*

$$\begin{aligned} p^\nu \partial_\nu V_\mu^{(0)} =& - m \widehat{\Sigma_F^{(0)} V_\mu^{(0)}} - p^\nu \widehat{\Sigma_{V\nu}^{(0)} V_\mu^{(0)}} + p_\mu \widehat{\Sigma_A^{(0)\nu} A_\nu^{(0)}} \\ & - \frac{m}{2} \epsilon_{\alpha\beta\lambda\mu} \widehat{\Sigma_S^{(0)\alpha\beta} A^{(0)\lambda}} + \frac{p_\beta}{m} \epsilon_{\mu\nu\alpha\lambda} p^\nu \widehat{\Sigma_S^{(0)\alpha\beta} A^{(0)\lambda}}, \\ p^\nu \partial_\nu A_\mu^{(0)} =& - m \widehat{\Sigma_F^{(0)} A_\mu^{(0)}} - p^\nu \widehat{\Sigma_{V\nu}^{(0)} A_\mu^{(0)}} + p_\mu \widehat{\Sigma_{A\nu}^{(0)} V^{(0)\nu}} \\ & - \frac{m}{2} \epsilon_{\alpha\beta\lambda\mu} \widehat{\Sigma_S^{(0)\alpha\beta} V^{(0)\lambda}} - \widehat{\Sigma_{A\mu}^{(0)} p^\nu V_\nu^{(0)}}, \end{aligned}$$

- *$V_\mu - A_\mu$ coupling*
- *local interaction*

# Quantum Transport Equations

*At order $\mathcal{O}(\hbar)$:*

*Constraint equations → still 4 independent spin components: $V_\mu, A_\mu$*

$$
\begin{aligned}
p^\nu \partial_\nu V_\mu^{(1)} = & - m\widehat{\Sigma_F^{(0)} V_\mu^{(1)}} - p^\nu \widehat{\Sigma_{V\nu}^{(0)} V_\mu^{(1)}} + p_\mu \widehat{\Sigma_A^{(0)\nu} A_\nu^{(1)}} + \frac{p_\nu}{m}\epsilon_{\rho\sigma\alpha\mu} p^\rho \widehat{\Sigma_S^{(0)\alpha\nu} A^{(1)\sigma}} - \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma_S^{(0)\sigma\nu} A^{(1)\lambda}} \\
& - m\widehat{\Sigma_F^{(1)} V_\mu^{(0)}} - p^\nu \widehat{\Sigma_{V\nu}^{(1)} V_\mu^{(0)}} + p_\mu \widehat{\Sigma_A^{(1)\nu} A_\nu^{(0)}} + \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda} p^\beta \widehat{\Sigma_S^{(1)\alpha\nu} A^{(0)\lambda}} - \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma_S^{(1)\sigma\nu} A^{(0)\lambda}} \\
& - \frac{1}{2m} p^\nu [\widehat{\Sigma_{S\mu\nu}^{(0)} (p^\alpha V_\alpha^{(0)})}]_{\text{P.B.}} + \frac{m}{2}[\widehat{\Sigma_{S\mu\nu}^{(0)} V^{(0)\nu}}]_{\text{P.B.}} - \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} p^\nu [\widehat{\Sigma_A^{(0)\alpha} V^{(0)\beta}}]_{\text{P.B.}} \\
& - \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha \Sigma_V^{(0)\nu}) A^{(0)\beta}} + \frac{1}{2m} p_\mu \widehat{(\nabla^\nu \Sigma_P^{(0)}) A_\nu^{(0)}} + \frac{1}{2m}\widehat{(p^\nu \nabla_\nu \Sigma_P^{(0)}) A_\mu^{(0)}} - \frac{1}{2m} p^\nu \epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha \Sigma_F^{(0)}) A^{(0)\beta}} \\
& + \frac{1}{2m} p_\nu \widehat{\Sigma_{S\alpha\mu}^{(0)} \nabla^{[\alpha} V^{(0)\nu]}} - \frac{1}{2m} p_\nu \widehat{\Sigma_S^{\alpha\nu(0)} \nabla_{[\alpha} V_{\mu]}^{(0)}} - \frac{1}{2}\epsilon_{\beta\nu\lambda\mu}\widehat{\Sigma_A^{(0)\beta} \nabla^\nu V^{(0)\lambda}}
\end{aligned}
$$

d

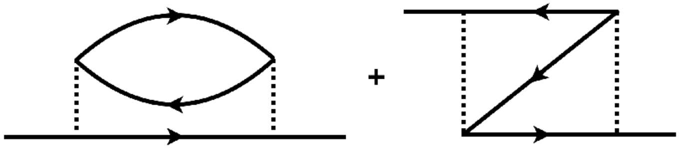
$$
\begin{aligned}
p^\nu \partial_\nu A_\mu^{(1)} = & - m\widehat{\Sigma_F^{(0)} A_\mu^{(1)}} - p^\nu \widehat{\Sigma_{V\nu}^{(0)} A_\mu^{(1)}} - p^\nu \widehat{\Sigma_{A\mu}^{(0)} V_\nu^{(1)}} - \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma_S^{(0)\alpha\beta} V^{(1)\lambda}} + p_\mu \widehat{\Sigma_{A\nu}^{(0)} V^{(1)\nu}} \\
& - m\widehat{\Sigma_F^{(1)} A_\mu^{(0)}} - p^\nu \widehat{\Sigma_{V\nu}^{(1)} A_\mu^{(0)}} - p^\nu \widehat{\Sigma_{A\mu}^{(1)} V_\nu^{(0)}} - \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma_S^{(1)\alpha\beta} V^{(0)\lambda}} + p_\mu \widehat{\Sigma_{A\nu}^{(1)} V^{(0)\nu}} \\
& + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\widehat{(\nabla^\sigma \Sigma_V^{(0)\nu}) V^{(0)\rho}} + \frac{m}{2}[\widehat{\Sigma_P^{(0)} V_\mu^{(0)}}]_{\text{P.B.}} - \frac{1}{2m} p_\mu [\widehat{\Sigma_P^{(0)} (p^\nu V_\nu^{(0)})}]_{\text{P.B.}} - \frac{1}{2}\epsilon_{\mu\sigma\nu\rho}\nabla^\sigma \widehat{\Sigma_A^{(0)\nu} A^{(0)\rho}} \\
& - \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}[\widehat{\Sigma_A^{(0)\nu} (p^\alpha A^{(0)\beta})}]_{\text{P.B.}} + \frac{m}{2}[\widehat{\Sigma_{S\mu\nu}^{(0)} A^{(0)\nu}}]_{\text{P.B.}} - \frac{1}{2m} p_\mu [\widehat{\Sigma_{S\rho\nu}^{(0)} (p^\rho A^{(0)\nu})}]_{\text{P.B.}} \\
& + \frac{1}{2m} p_\sigma \nabla^\sigma (\widehat{\Sigma_{S\mu\nu}^{(0)} A^{(0)\nu}}) - \frac{1}{2m} p^\nu \nabla^\sigma (\widehat{\Sigma_{S\mu\nu}^{(0)} A_\sigma^{(0)}}) - \frac{1}{2m} p_\mu \nabla^\sigma (\widehat{\Sigma_{S\sigma\nu}^{(0)} A^{(0)\nu}}) + \frac{1}{2m} p^\nu \nabla^\sigma (\widehat{\Sigma_{S\sigma\nu}^{(0)} A_\mu^{(0)}}).
\end{aligned}
$$

- *local and non-local (derivative terms) interactions*

# Equilibrium Distribution from Detailed Balance (I)

*NJL model, a specific self-energy Σ:*

$$\mathcal{L} = \bar{\psi}(i\hbar\partial\!\!\!/ - m)\psi + G(\bar{\psi}\psi)^2$$



*Classical limit:*

$$V_\mu = p_\mu f_V \delta(p^2 - m^2)$$
$$A_\mu = m a_\mu f_A \delta(p^2 - m^2)$$
$$collision\ terms \sim \left[f_V(p_1)\bar{f}_V(p)\bar{f}_V(p_2)f_V(p_3) - f_V(p)\bar{f}_V(p_1)f_V(p_2)\bar{f}_V(p_3)\right]$$

*Detailed balance principle:*

*lose and gain terms cancel to each other in equilibrium*

$\rightarrow$

Fermi-Dirac distribution $f_V(p^\mu \beta_\mu)$, $\quad \beta_\mu = u_\mu / T$

$$f_A = 0$$
$$P, S_{\mu\nu}, \Sigma_p, \Sigma_A^\mu, \Sigma_S^{\mu\nu} = 0$$

- *If the system is not initially polarized, spin density keeps zero in classical limit.*

# Equilibrium Distribution from Detailed Balance (II)

*Quantum transport equation in equilibrium:*

$$p^\nu \partial_\nu A^{(1)}_\mu = -m\widehat{\Sigma^{(0)}_F A^{(1)}_\mu} - p_\nu \widehat{\Sigma^{(0)\nu}_V A^{(1)}_\mu} - p_\nu \widehat{\Sigma^{(1)}_{A\mu} V^{(0)\nu}} + p_\mu \widehat{\Sigma^{(1)}_{A\nu} V^{(0)\nu}} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}(\nabla^\sigma \widehat{\Sigma^{(0)\nu}_V) V^{(0)\rho}} - \frac{m}{2}\epsilon_{\rho\nu\lambda\mu}\widehat{\Sigma^{(1)\rho\nu}_S V^{(0)\lambda}}.$$

*Detailed balance principle →*

*equilibrium spin distribution*

$$A^{eq}_\mu(x,p) = -\frac{\hbar}{4}\epsilon_{\mu\nu\rho\sigma}p^\nu \omega^{\rho\sigma} n'_F(x,p)\delta(p^2-m^2)$$

*Killing condition in global equilibrium*

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0$$

- *Equilibrium spin distribution is generated through thermal vorticity.*
- *Consistent with non-local collisions (N.Weickgenannt, E.Speranza, X.L.Sheng, Q.Wang, D.H.Rischke, arXiv:2005.01506) and relaxation from spin chemical potential to thermal vorticity in global equilibrium (W.Florkowski, B.Friman, A.Jaiswal, E.Speranza, PRC97, 041901(2018), F.Becattini, W.Florkowski, E.Speranza, PLB789, 419(2019), F.Becattini, PRL108, 244502(2012), K.Hattori, M.Hongo, X.Huang, M.Matsuo, H.Taya, PLB795, 100(2019).).*
- *While the relaxation depends on the interaction, the equilibrium distributions themselves should be model independent.*

# Collision Terms in Near Equilibrium

*Quantum transport equations to the first order in ħ:*

$$F = F^{(0)} + \hbar F^{(1)}$$

$$A_\mu = A_\mu^{(0)} + \hbar A_\mu^{(1)}$$

$$p^\nu \partial_\nu F = -m\overline{\Sigma_F F} - p_\nu \overline{\Sigma_V^\nu F} + \hbar\Big[m\overline{\Sigma_A^\nu A_\nu} - \frac{p^\alpha p_\nu}{m}\overline{\Sigma_A^\nu A_\alpha} - \frac{1}{2}\epsilon_{\alpha\nu\sigma\rho}p^\nu\overline{\Sigma_S^{\sigma\rho}A^\alpha}$$
$$- \frac{1}{2m}\epsilon_{\alpha\nu\rho\sigma}p^\rho\overline{(\partial^\sigma\Sigma_V^\nu)A^\alpha} + \frac{1}{2}\overline{(\partial_\mu\Sigma_P)A^\mu}\Big] + \mathcal{O}(\hbar^2),$$

$$p^\nu \partial_\nu A_\mu = -m\overline{\Sigma_F A_\mu} - p_\nu \overline{\Sigma_V^\nu A_\mu} - \hbar\Big[m\overline{\Sigma_{A\mu}F} - \frac{p_\mu p_\nu}{m}\overline{\Sigma_A^\nu F} - \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}p^\nu\overline{\Sigma_S^{\sigma\rho}F}$$
$$- \frac{1}{2m}\epsilon_{\mu\nu\rho\sigma}p^\rho\overline{(\partial^\sigma\Sigma_V^\nu)F} + \frac{1}{2}\overline{(\partial_\mu\Sigma_P)F}\Big] + \mathcal{O}(\hbar^2)$$

*Equilibrium distributions:*

$$F^{eq}(x,p) = m n_F(x,p)\delta(p^2 - m^2),$$

$$A_\mu^{eq}(x,p) = -\frac{\hbar}{4}\epsilon_{\mu\nu\rho\sigma}p^\nu\omega^{\rho\sigma}n_F'(x,p)\delta(p^2-m^2),$$

$$\text{Collision term } C(\Sigma, W) = \begin{cases} 0 & \text{in equilibrium} \\ C'(\Sigma_{eq}, W_{eq})\delta W & \text{in near equilibrium} \end{cases}$$

# Relaxation Equations (I)

*Relaxation equations for $F$ and $A_\mu$:*

$$p^\nu \partial_\nu \begin{pmatrix} F \\ A_\mu \end{pmatrix} = -\begin{pmatrix} r & 0 \\ r_\mu & r \end{pmatrix} \begin{pmatrix} \delta F \\ \delta A_\mu \end{pmatrix}$$

*Damping and polarization rates are calculated at equilibrium:*

$$r = m\Sigma_F + p_\nu \Sigma_V^\nu,$$

$$r_\mu = m\hat{\Sigma}_{A\mu} - \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}p^\nu \hat{\Sigma}_S^{\sigma\rho} - \frac{p_\mu p_\nu}{m}\hat{\Sigma}_A^\nu - \frac{\hbar}{2m}\epsilon_{\mu\nu\rho\sigma}p^\rho \partial^\sigma \hat{\Sigma}_V^\nu + \frac{1}{2}\partial_\mu \widehat{\Sigma}_P$$

- *all the degrees of freedom share a same damping rate*
- $\Sigma_F, \Sigma_V \neq 0$ *and* $\Sigma_p, \Sigma_A^\mu, \Sigma_S^{\mu\nu} = 0$ *in classical limit*

→ *damping rate at order $\mathcal{O}(\hbar^0)$ and polarization rate at order $\mathcal{O}(\hbar^1)$,*

$$r \gg r_A$$

*Damping time and polarization time:*

$$\tau = \frac{1}{r}, \qquad \tau_A = \frac{1}{r_A}$$

$$\tau \ll \tau_A$$

- *Spin polarization still happens when all the degrees of freedom reach equilibrium.*

# Relaxation Equations (II)

*Relaxation equations for $f_V$ and $f_A$:*

$$F = mf_V\delta(p^2 - m^2)$$
$$A_\mu = ma_\mu f_A\delta(p^2 - m^2)$$
$$p^\mu\partial_\mu \begin{pmatrix} f_V \\ f_A \\ \boldsymbol{a} \end{pmatrix} = -\begin{pmatrix} r & 0 & 0 \\ r_A & r & 0 \\ \boldsymbol{r}_a & 0 & r \end{pmatrix} \begin{pmatrix} \delta f_V \\ \delta f_A \\ \delta\boldsymbol{a} \end{pmatrix}$$

*damping and polarization rates:*

$$r_A = (\boldsymbol{p}\cdot\boldsymbol{a})(\boldsymbol{p}\cdot\boldsymbol{r})/p_0^2 - \boldsymbol{a}\cdot\boldsymbol{r},$$
$$\boldsymbol{r}_a = \left[\boldsymbol{a}^2\boldsymbol{r} + \left((\boldsymbol{p}\cdot\boldsymbol{a})(\boldsymbol{p}\cdot\boldsymbol{r})/p_0^2 - \boldsymbol{a}\cdot\boldsymbol{r}\right)\boldsymbol{a}\right]/f_A,$$

*Relaxation equations for $f_\pm = (f_V \pm f_A)/2$:*

$$p^\mu\partial_\mu f_+ = -(r + r_A/2)\delta f_+ - r_A/2\delta f_-,$$
$$p^\mu\partial_\mu f_- = -(r - r_A/2)\delta f_- + r_A/2\delta f_+.$$

- *different damping times: $\tau_\pm = 1/(r \pm r_A)$*
- *spin flipping between the two kinds of fermions: $r_A$*

# Damping and Polarization Rates in NJL

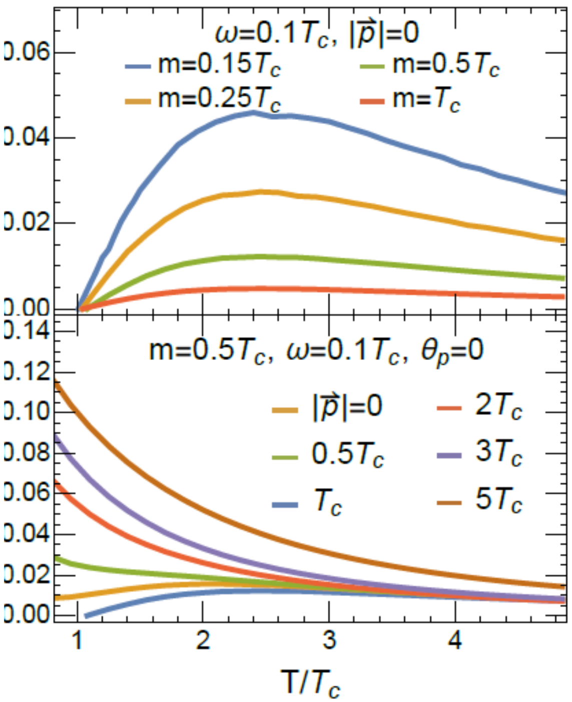
## Damping and polarization rates in chiral restored phase in NJL:

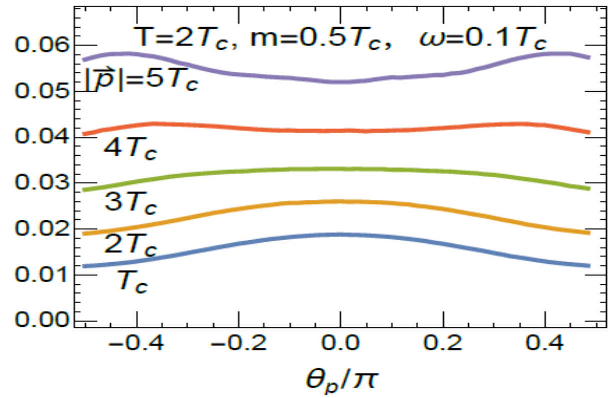
$$r(p) = G^2 \int dP \left(m^2 + p_2 \cdot p_3\right)\left(m^2 + p \cdot p_1\right)\left(\bar{f}_1 f_2 \bar{f}_3 + f_1 \bar{f}_2 f_3\right)\delta_1\delta_2\delta_3,$$

$$\begin{aligned} r_\mu(p) = G^2/2 \int dP m \left(m^2 + p_2 \cdot p_3\right)\delta_1\delta_2\delta_3 \\ \times\big[\left(m^2 + p \cdot p_1\right)\tilde{\omega}_{\mu\nu} p_1^\nu f_1' \left(\bar{f}_2 f_3 - f_2 \bar{f}_3\right) - \left(p_\mu + p_{1\mu}\right) p^\nu p_1^\rho \tilde{\omega}_{\nu\rho} f_1' \left(\bar{f}_2 f_3 - f_2 \bar{f}_3\right) \\ + \epsilon_{\mu\nu\alpha\beta} p^\nu p_1^\beta \left(\partial^\alpha f_1\right)\left(\bar{f}_2 f_3 - f_2 \bar{f}_3\right) + \epsilon_{\mu\nu\alpha\beta} p^\nu p_1^\beta \partial^\alpha \left(\bar{f}_1 f_2 \bar{f}_3 + f_1 \bar{f}_2 f_3\right)\big],\end{aligned}$$

*Killing condition leads to $r_\mu \sim \omega_{th}$*

$$\frac{\tau}{\tau_A} = \frac{r_A}{r}$$




- *light (high momentum) fermions are easily polarized*
- *polarization is almost direction independent*

# Summary

- *Equilibrium distributions from detailed balance*

$$F^{eq}(x,p) = m n_F(x,p)\delta(p^2 - m^2),$$
$$A_\mu^{eq}(x,p) = -\frac{\hbar}{4}\epsilon_{\mu\nu\rho\sigma}p^\nu\omega^{\rho\sigma}n'_F(x,p)\delta(p^2 - m^2),$$

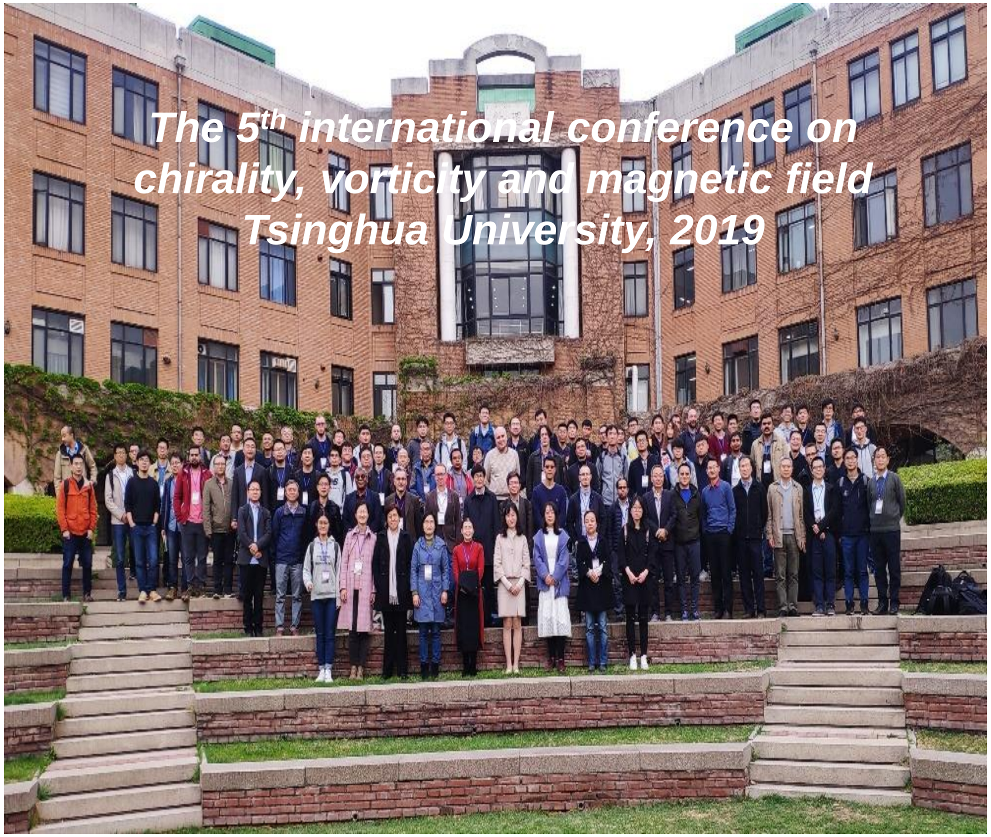
- *Damping and polarization process in near equilibrium state*

$$p^\nu\partial_\nu\begin{pmatrix} F \\ A_\mu \end{pmatrix} = -\begin{pmatrix} r & 0 \\ r_\mu & r \end{pmatrix}\begin{pmatrix} \delta F \\ \delta A_\mu \end{pmatrix}$$

$$r = m\Sigma_F + p_\nu\Sigma_V^\nu,$$
$$r_\mu = m\hat{\Sigma}_{A\mu} - \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}p^\nu\hat{\Sigma}_S^{\sigma\rho} - \frac{p_\mu p_\nu}{m}\hat{\Sigma}_A^\nu - \frac{\hbar}{2m}\epsilon_{\mu\nu\rho\sigma}p^\rho\partial^\sigma\hat{\Sigma}_V^\nu + \frac{1}{2}\partial_\mu\widehat{\Sigma}_P$$

- *Calculations in QED and QCD are needed.*

***Thank You !***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Tsinghua University, 2019

## Total angular momentum conservation

*Question:*
*Initial orbital angular momentum is partially transferred to spin angular momentum, is the total angular momentum conservation guaranteed?*

*Total angular momentum tensor:*

$$M_{\rho,\mu\nu} = x_\mu T_{\varrho\nu} - x_\upsilon T_{\rho\mu} + \hbar S_{\rho,\mu\nu}$$

*Energy-momentum tensor:*

$$T_{\mu\nu} = \int d^4p V_\mu p_\nu$$

*Spin angular momentum tensor:*

$$S_{\mu,\nu\rho} = -\frac{1}{2}\int d^4p\, \varepsilon_{\mu\nu\rho\lambda} A^\lambda$$

*To the zeroth, first, and second order in $\hbar$,*

$$\partial^\rho M_{\rho,\mu\nu} = T_{[\mu\nu]} + \hbar \partial^\rho S_{\rho,\mu\nu} = 0$$