# Spin-momentum correlation in QCD matter

1. Intro.: spin-momentum correlation and Berry curvature.
2. Signatures in heavy-ion collisions
   - Shear-induced polarization (SIP).
   - Spin Hall effect by $\mu_B$ gradient (at BESII and forward rapidity).
3. Summary and outlook.


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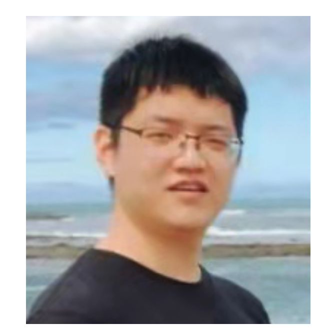
*Ref: Shuai Liu, YY, 2006.12421, PRD; 2103.09200, JHEP 21.*

*Baochi Fu, Shuai Liu, Longgang Pang, Huichao Song, YY, 2103.10403, PRL 21;*

*Baochi Fu, Longgang Pang, Huichao Song, YY, to appear;*

6th Chirality workshop, Nov. 4th, 2021


*Shuai Liu, postdoc@IMP*

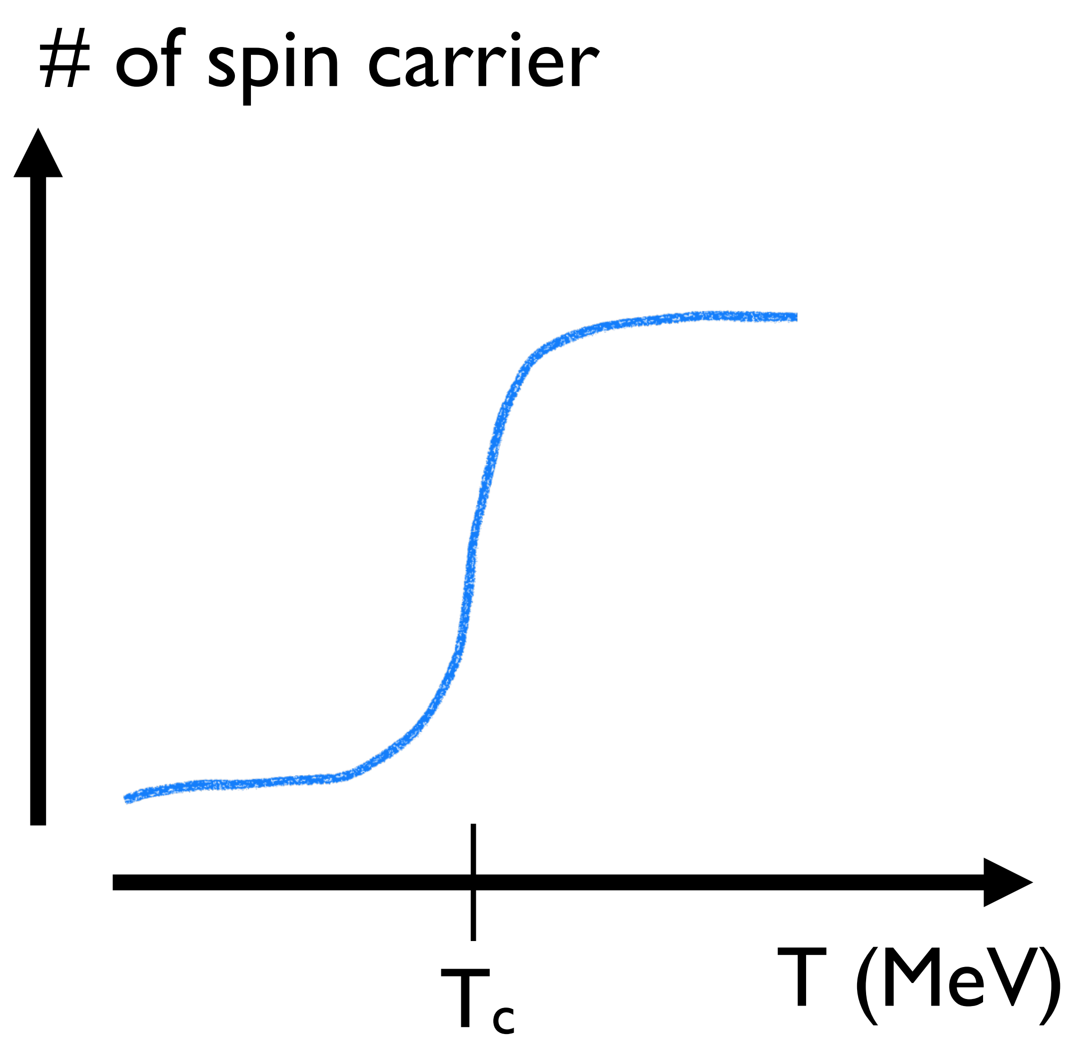

*Baochi Fu, postdoc@Peking U.*

# Background

- Polarization/spin alignment measurement in HIC: new observable degrees of freedom to study QCD matter.

*Review: Becattini, Lisa, 2020*

- probing spin and phase structure of QCD matter.



# Spin polarization generation

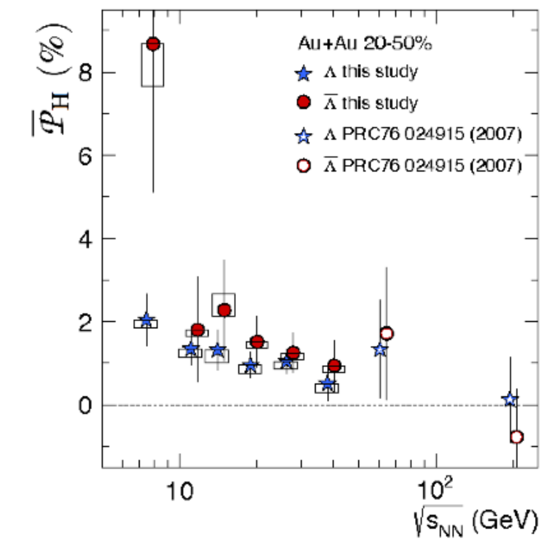
- Rotation (independent of the direction of $\vec{p}$ ):

$$\Delta\epsilon = -\vec{s}\cdot\vec{\Omega} \to \vec{s} \parallel \vec{\Omega}$$

*Landau-Lifshitz volume 5*

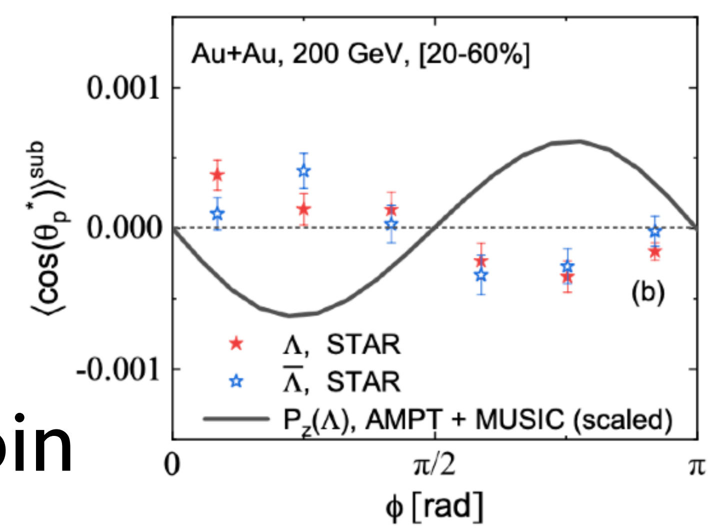
  - At macroscopic scale: vorticity-induced polarization.

- Vorticity effects in heavy-ion collisions:




  - describe the trends of global (phase-space averaged) Λ polarization.

*Xin-Nian Wang, Zuo-Tang Liang, PRL 05'; Becattini et al, Annals Phys 13'*




  - predict qualitatively different behavior in differential measurements. (sign "puzzle")。

But rotation is just one mechanism for spin polarization generation.

*Baochi Fu et. al, PRC21'*

# Spin Hall effect

$$\vec{s} \propto \vec{v} \times \vec{E}$$

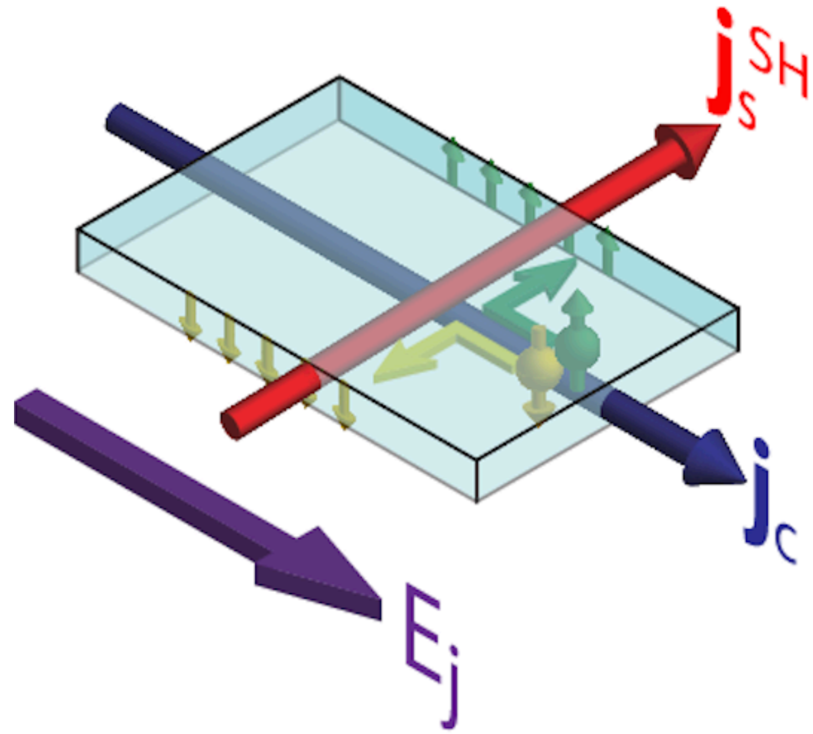



*Fig. from Meyer et al, Nature material 17'*

- Ubiquitous phenomenon: spin-momentum correlation. (c.f. TMD in nucleon structure.)

*In global polarization, such a correlation has been largely washed out after momentum average.*

*Differential measurement probes the spin-momentum correlation (NB: differential spin polarization $\neq$ local vorticity).*

# Spin Hall effect and Berry curvature

*Murakami, Nagaosa, Shou-Cheng Zhang, Science 2003'*

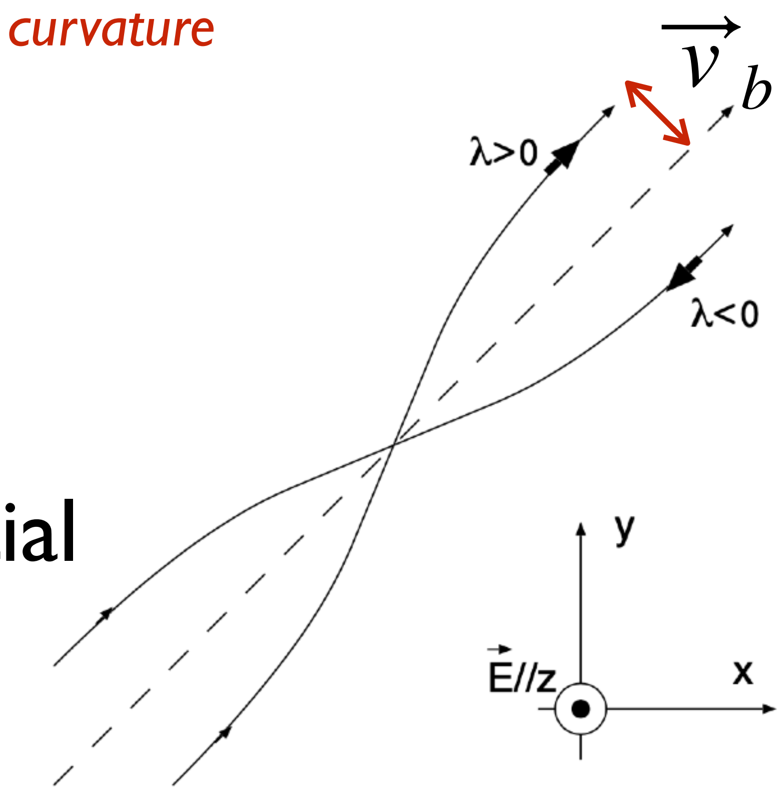
- Berry curvature modifies effective velocity ⇒ SHE

$$\dot{\vec{x}} = \vec{v} + \lambda \vec{F} \times \vec{b}_k(\hat{p}) \qquad \vec{b} = \frac{\hat{p}}{2p^2}$$

helicity

Berry curvature

Shifted velocity $\vec{v}_b$




- Effective force $\vec{F}$ can be:
  - electric field $\vec{E}$ and chemical potential gradient $T\nabla(\mu/T)$(spin Hall effect).
  - T-gradient (spin Nernst effect);
  - and more ….
- A class of spin phenomena generated by Berry curvature effects.

*Macroscopically, the gradient of hydro. field (e.g. flow and energy/charge density) leads to spin-momentum correlation.*

# Theory

- Chiral kinetic theory and its generalization, e.g., axial kinetic theory incorporate Berry curvature effects and can describe spin-momentum correlation.

*see Stephanov's talk and refs therein.*

- Another systematic approach is response theory:
  - expansion in gradient.
  - relating expansion coefficients to correlators $\langle O(x) T^{\mu\nu}(x') \rangle$.
- Consider axial Wigner function:

$$\mathscr{A}^{\mu}(t, \overrightarrow{x}, \overrightarrow{p}) = \int d^3\overrightarrow{y}\, e^{-i\overrightarrow{y}\cdot\overrightarrow{p}} \langle \bar{\psi}(t, \overrightarrow{x} - \frac{1}{2}\overrightarrow{y})\, \gamma^{\mu}\gamma^{5}\psi(t, \overrightarrow{x} + \frac{1}{2}\overrightarrow{y}) \rangle$$

# The derivative expansion

Shuai Liu, YY, JHEP 21

- The most general expression for axial Wigner function $\mathscr{A}^\mu$ consistent with symmetries (for a neutral fluid):

$$u \cdot \mathscr{A} = \tilde{c}_\omega p \cdot \omega \,,$$

$$\mathscr{A}^\mu_\perp = c_\omega \omega^\mu - c_T \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \partial_\lambda \log T + g_\sigma \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho{}_\lambda + g_\omega Q^{\mu\nu} \omega_\nu$$

vorticity effects — spin Nernst effect — shear-induced polarization (SIP)

$$\vec{s} \propto \hat{p} \times \nabla \log T$$

Spin-momentum correlation

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$$

vorticity

$$\sigma_{\mu\nu} = \frac{1}{2} \left( \partial_\mu u_\nu + \partial_\mu u_\nu - \text{trace} \right)$$

shear

$$Q^{\mu\nu} = -\frac{p_\perp^\mu p_\perp^\nu}{p_\perp^2} - \frac{1}{3} \Delta^{\mu\nu}, \ldots$$

quadrupole

- Flow gradient and momentum quadrupole coupling, have long been overlooked.

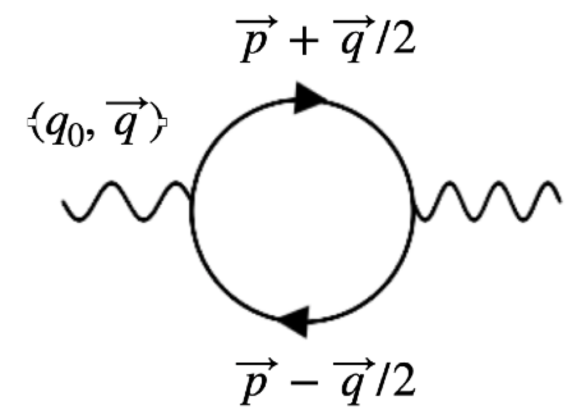
- All those effects are non-dissipative (based on T-parity).

# One-loop

Shuai Liu, YY, 2103.09200, JHEP 21

- The expansion coefficients can be computed systematically from microscopic theories.

$$\int_{\vec{y}} e^{i\vec{y}\cdot\vec{p}} \langle \bar{\psi}(t, \vec{x} - \frac{1}{2}\vec{y})\, \gamma^\mu \gamma^5 \psi(t, \vec{x} + \frac{1}{2}\vec{y})\, T^{\alpha\beta}(0,0) \rangle$$




- For general fermion mass at one-loop:

$$\mathscr{A}^\mu_\perp = (-n'_{FD}) \left[ \omega^\mu + \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \partial_\lambda \log \beta + \frac{-p_\perp^2}{(p\cdot u)} \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho{}_\lambda \right] + \# \times Q^{\mu\nu} \omega_\nu$$

vorticity effects    spin Nernst effect    SIP    $\propto (g\text{-}2)$

- Chiral kinetic theory analysis is consistent with one loop calculations.

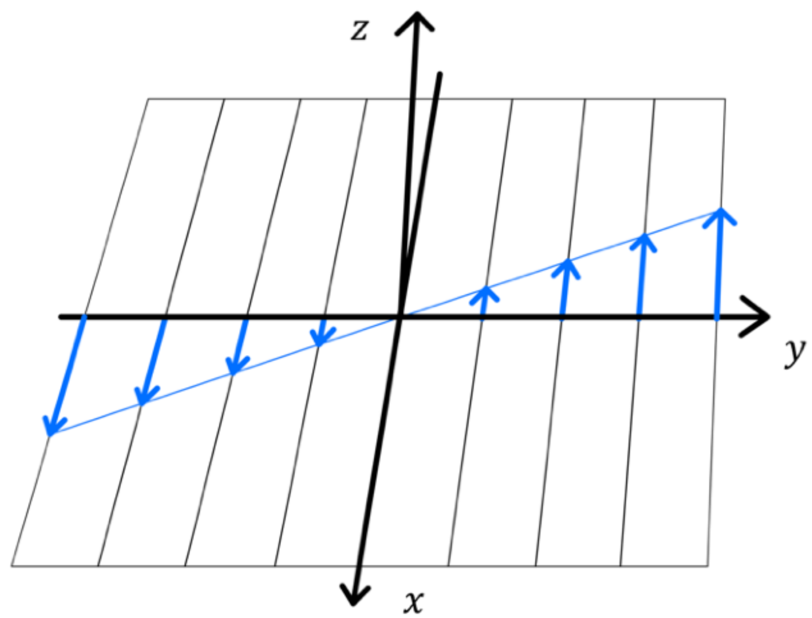
*SIP has been derived almost simultaneously by Becattini et al, 2103.10917, PLB 21 via a different method*

# Shear-induced polarization (SIP)

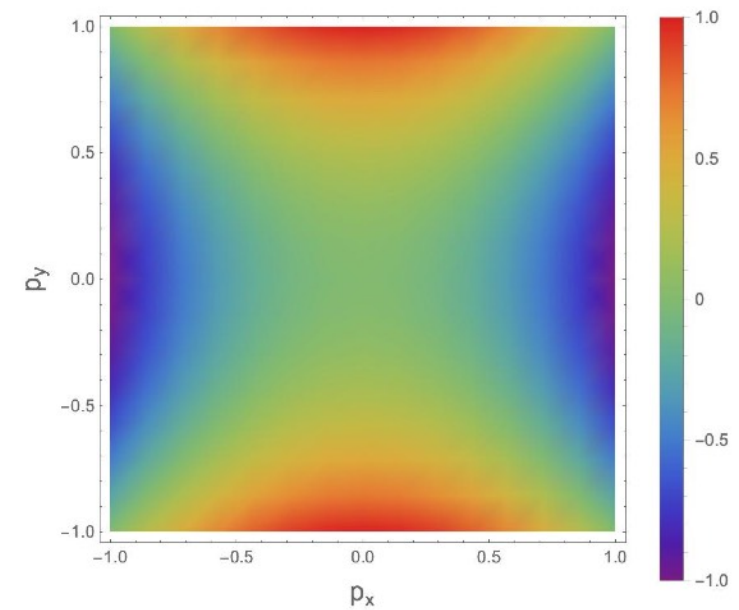
Spin polarization (of a neutral fluid)

=[Vorticity]+[T-gradient]+[Shear]

# Illustration




A standard shear flow profile:
$\omega^z \neq 0\,, \sigma^{xy} \neq 0$

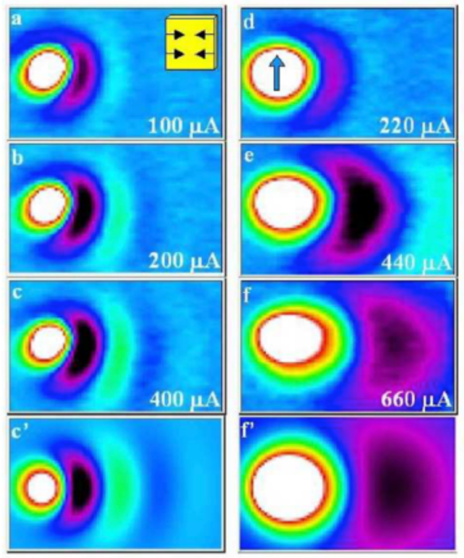



Spin polarization along z-direction in phase space from SIP.

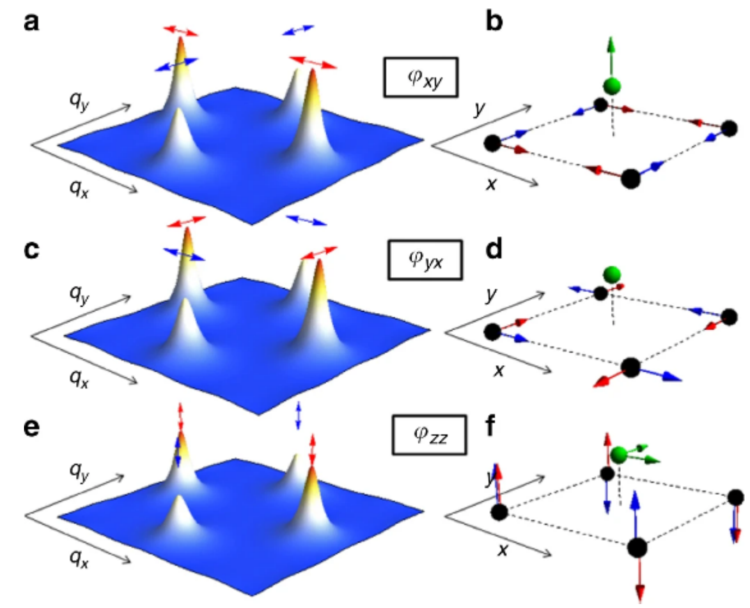
$$\mathscr{A}^i_{SIP} \propto \epsilon^{ikj} Q_{jl} \sigma^l{}_k\,, \qquad Q_{ij} = \hat{p}_i \hat{p}_j - \frac{1}{3}\delta_{ij}$$

Shear-induced polarization (SIP): imaging anisotropy in a fluid into anisotropy in spin space.

# Observation?




*strain-induced polarization in n-type GaAs, Crooker and Smith, PRL, 04'*

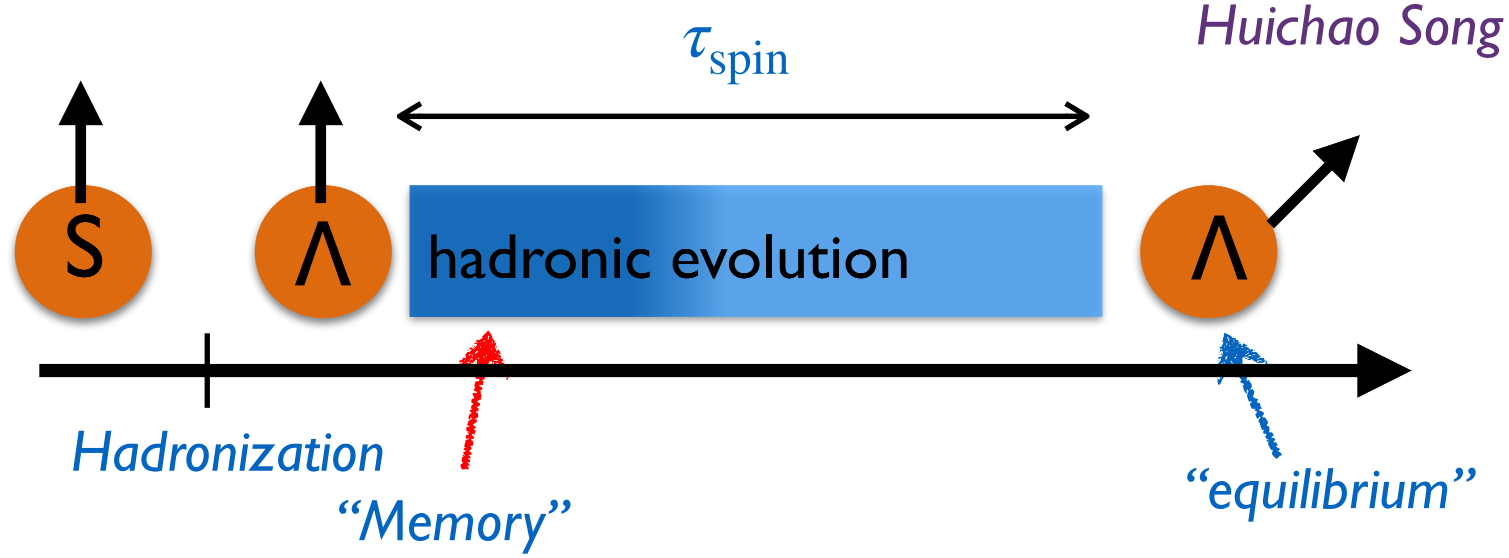



*strain-induced polarization in BaFe2As2, Kissikov et al, Nature communication, 18'*

- The cousin effect, strain-induced polarization has been observed in crystals and liquid crystals.
- Shear-induced polarization (SIP): generic in fluids.
  - Can we/did we see SIP in heavy-ion collisions?
  - What can we learn ?

## Phenomenology

*Baochi Fu, Shuai Liu, Longgang Pang, Huichao Song and YY, PRL 21*

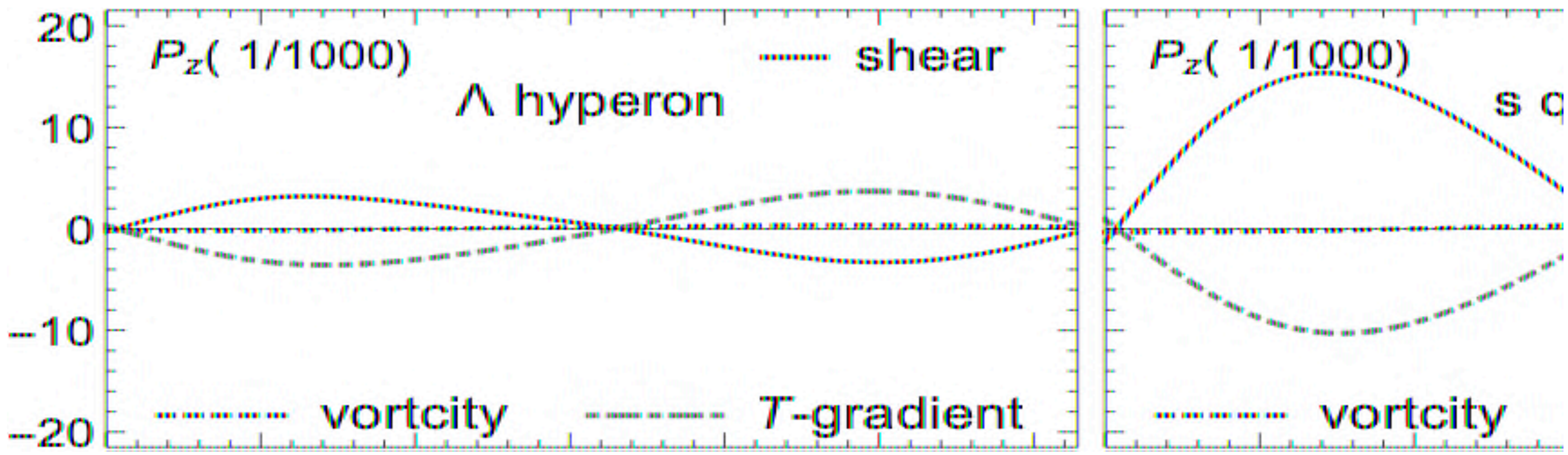


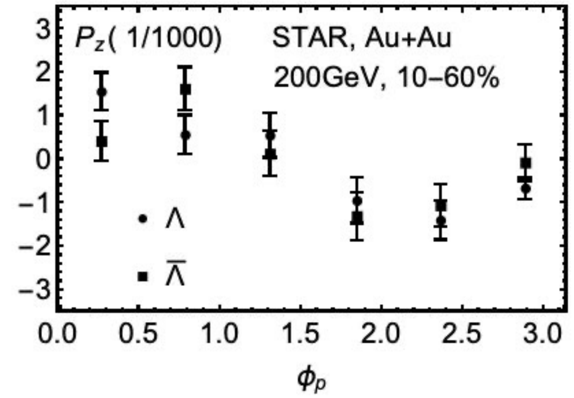

- Hydro. profile from AMPT (initial condition)+MUSIC.
- Two benchmark scenarios:
  - "Lambda equilibrium": Λ is born in equilibrium.
  - "Strange memory": Λ inherits the polarization of s-quarks

NB: spin-dependent d.o.f. might not be in equilibrium even if spin-independent ones are.

# Differential longitudinal polarization (similar story for $P_y$)

*Baochi Fu, Shuai Liu, Longgang Pang, Huichao Song and YY, PRL 21*




vs transverse azimuthal angle $\phi_p$

Spin polarization=[Vorticity]+[T-gradient]+[Shear]

- SIP gives a "right sign" while T-gradient leads to a "wrong sign".

*see also Becattini et al, 2103.14621; Cong Yi et al, 2106.00283.*

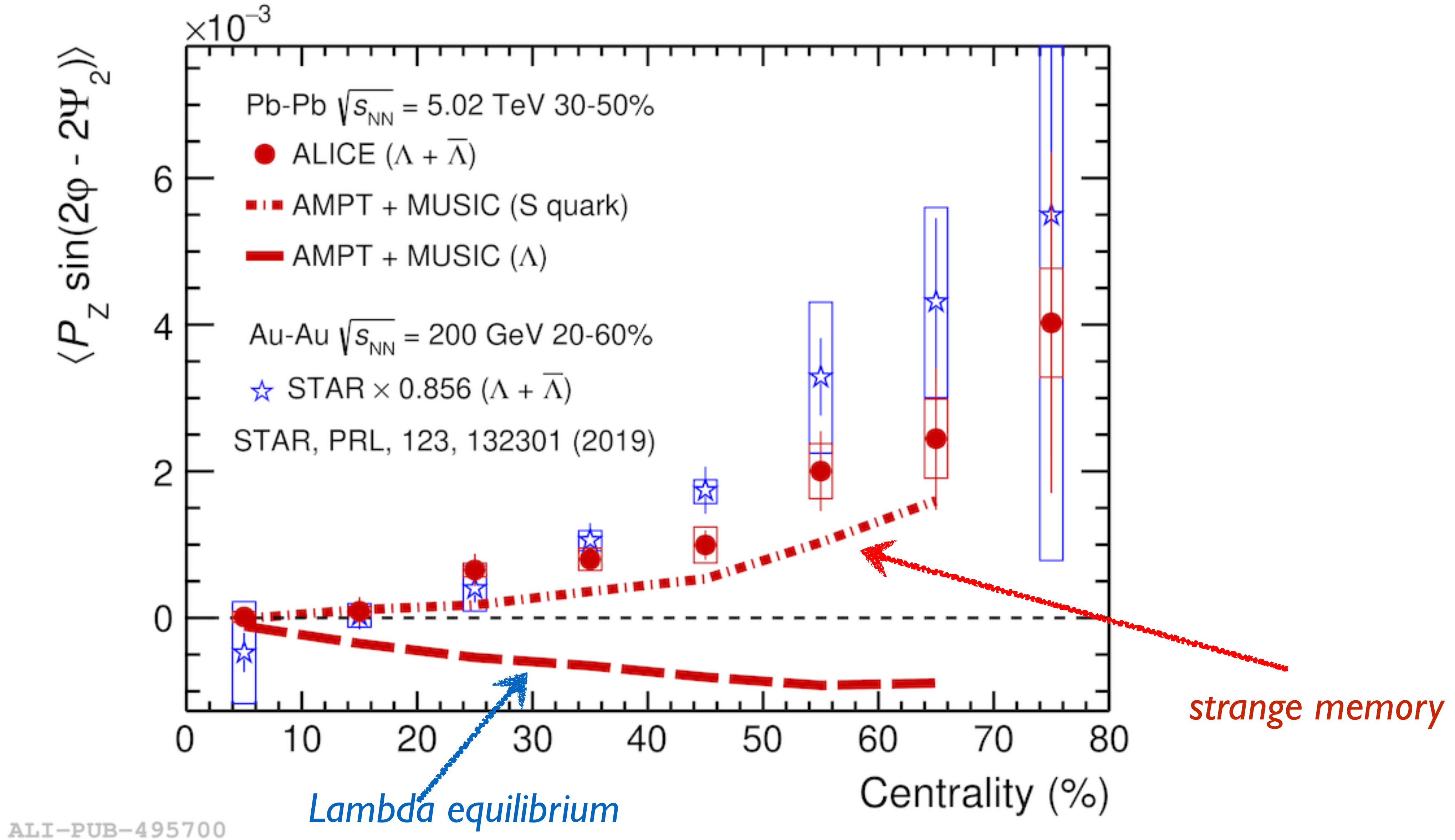
- "Strange memory" scenario: SIP determines the qualitative feature of differential polarization in

  - Macroscopic manifestation of Berry phase effects?

LHC

*ALICE collaboration: 2107.11183;*

*benchmark calculations by Baochi Fu&Huicaho Song*




- Tantalizing yet inconclusive evidence for SIP.
- The observation of SIP in QGP might be its first detection among all kinds of fluid!

# Spin Hall effect induced by $\mu_B$ gradient

- SHE (induced by baryon density gradient) separates Lambda and anti-Lambda.

$$\overrightarrow{P}_{\pm} \propto \pm \hat{p} \times \nabla \mu_B$$

*Theory: Son, Yamamoto, PRD 12; Di-Lung Yang, Hattori, Yoshimasa, PRD 19' … ; First phenomenological study: Liu-YY, 2006.12421, PRD21.*

*Shen et al, 2106.08125 on its effect on global spin polarization (splitting).*

- Since its first detection in 2004, all known SHE materials (semi-conductors, metals, insulators) are not exceeding room temperature and are microscopically described by QED.

- Observation of SHE in HIC where QCD matter at trillion degrees is created with spin carriers interacting through strong interaction?

  - *Looking for SHE at difference in $\Lambda, \overline{\Lambda}$ differential polarization at lower beam energy (e.g. BESII) and forward rapidity; prediction is coming!*

*Baochi Fu, Longgang Pang, Huichao Song, YY, to appear.*

# Summary and outlook

## Summary

- Differential spin polarization: probes the spin-momentum correlation of QCD matter and related to Berry curvature of spin carriers .
  - Response theory analyses the effects of hydro. gradient on spin polarization systematically.
- Shear-induced polarization (SIP): important@RHIC and LHC.
- Spin Hall effect via gradient : new probe of baryon-rich QCD matter.
- Not covered: Berry curvature and topology in color superconducting phase. *Noriyuki Sogabe, YY, to appear.*

Ultimate goal: spin and phase structure of QCD matter.

# Back-up

How can a kinetic equation account for anomalous particle creation?

Also, how can classical equation account for quantum anomaly?

Two words: **Berry monopole**

*From Stephanov's slides on chiral kinetic theory at the RBRC Workshop on P- and CP-Odd Effects in Hot and Dense Matter (2012), the precedent of the chirality workshop series.*

Although I will cover different topics from quantum anomaly, i.e. spin-momentum correlation in QCD matter and color super-conductor, the key word in my talk today is still Berry phase.

Many ideas are inspired by stimulation environment in this workshop series in past years.

## *Can Λ spin flipping rate be small?*

Quark model+vector meson dominance➡ nucleon (N)-hyperon interaction is mediated by ω meson which only couples with constituent u and d quark.

*Jennings, PLB1990; Cohen-Weber PRC 1991*

However, spin of Λ is carried by s quark. So

(spin-dependent) N-Λ interaction<<(spin-dependent) N-N interaction.

This picture explains the puzzling experimental results

$$\text{N-}\Lambda \approx \frac{1}{40}\,\text{N-N}$$

*S. Ajimura et al. PRL 2001*

Under this picture, Λ spin flip rate could be (much) smaller than its equilibration rate => worthy checking in future.