Chiral anomaly and hydrodynamics

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Part I Motivation

QFT is to a great extent concentrated on anomalies (which distinguish QFT symmetries from classical FT symmetries)

In particular, in case of axial vector current J_5^α

$$\partial_\alpha J_5^\alpha = C_5 \vec{E} \cdot \vec{B} \qquad (\textit{gauge anomaly})$$

$$\nabla_\alpha \mathcal{J}_5^\alpha = C_{gr} \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} R^\alpha{}_{\beta\mu\nu} \tilde{R}^\beta{}_{\alpha\rho\sigma} \qquad (\textit{gravitational anomaly})$$

where $\vec{E}, \vec{B}$ electromagnetic fields, $R^\alpha{}_{\beta\gamma\delta}$ is Riemann tensor, $C_5 = 1/(4\pi^2)$, $C_{gr} = -1/(768\pi^2)$ for a Weyl spinor

Quantum hydrodynamics

Anomaly in terms of macroscopic quantities
(temperature T, chemical potential μ, 4-velocity of fluid u_α)
Son& Surowka (2009) Ideal fluid, absence of ext. fields:

$$J_5^\alpha = n_5 u^\alpha + C_\omega \omega^\alpha \qquad (\omega^\alpha = 1/2 \epsilon^{\alpha\beta\gamma\delta} u_\beta \partial_\gamma u_\delta)$$

$$C_\omega = \mu^2 C_5 + T^2 C_T \qquad (C_T = 1/12)$$

where C_5 is the same as in front of the anomaly

The C_ω term is called chiral vortical effect

Charge density J_5^0 is a mixture
of microscopic and macroscopic helical motions,
suggesting the possibility of transitions between them.

Encouragement from Phenomenology

–The ratio η/s is smallest for the QGP (close to ideal fluid)

"Global Λ hyperon polarization in nuclear collisions"
STAR Collaboration, Nature 548, 62 (2017)

Quark-Gluon Plasma formed in nuclear collisions as a relativistic fluid at local thermodynamic equilibrium with acceleration and vorticity

Acceleration can be replaced by gravitational field
– QGP as a window to grav. interactions.

First attempted in "Thermal Hadronization and Hawking-Unruh Radiation in QCD",
P. Castorina, D. Kharzeev, H. Satz, Eur.Phys.J.C 52 (2007)

Physics of equilibrium vs physics of gravity

Probably, two most famous examples of similarity are:

$*$ Transport induced by gradient of temperature $\vec{\nabla} T$ identical to that induced by acceleration $\vec{a}_{gr}$

$$\frac{\vec{\nabla} T}{T} \rightarrow -\vec{a}_{gr}$$

as a reflection of universality of the both (Luttinger (1964))

$**$ hypothesis: gravity is not fundamental and could be replaced by macroscpic entropic force (E. Verlinde (2011)) :

$$\vec{F}(X_0)_{entropic} = T\vec{\nabla}_X S(X)|_{X_0}$$

(S is entropy, X is a characteristic of macrostate)

Outline of the talk

- Introduction
- linear in acceleration terms
- higher-order in acceleration terms
- Posiible instabilty at Unruh temperature

References to original papers

“Axial current in rotating and accelerating medium” , G. Prokhorov , O. Teryaev , V. I. Zakharov, 1805.12029;

“Thermodynamics of accelerated fermion gases and their instability at the Unruh temperature”, G. Y. Prokhorov, O. V. Teryaev , V.I. Zakharov 1906.03529 [hep-th]

“Acceleration and rotation in quantum statistical theory” V.I. Zakharov , G.Y. Prokhorov , O.V. Teryaev, Phys.Scripta 95 (2020) 8, 084001

Also e-prints 2009.11402 [hep-th], 2109.06048 [hep-th]

“On chiral vortical effect in accelerated matter”, P.G. Mitkin , V.I. Zakharov 2103.01211 [hep-th]

Part II Linear in acceleration effects

Interesting effect claimed first by

"Chiral and Gravitational Anomalies on Fermi Surfaces"
G. Basar, D. E. Kharzeev , I. Zahed, 1307.2234 [hep-th]

Consider motion of levels of the Fermi sphere at finite μ caused by external grav. field, a la Nielsen&Ninomiya

$$\partial_\alpha J_5^\alpha = \frac{\mu^2}{2\pi^2}(\vec{a}_{gr} \cdot \vec{\Omega})$$

where $\vec{a}_{gr}$ is the grav. acceleration, $\vec{\Omega}$ is the angular velocity

Upon substitution $\vec{a_{gr}} \to -\vec{\nabla} T/T$ looks as a novel chiral thermal effect

Interpretation as a puzzle

Gravimagnetic fields, (analogy between magnetic field and field of rotation),

$$\vec{B}_{gr} = 2\epsilon\vec{\Omega}, \quad \vec{E}_{gr} = -\epsilon\vec{\nabla}\phi_{gr}$$

where ϵ is energy of test particle

Analog (with all the coefficients) of the gauge anomaly
But: there is no place for such an anomaly in gravitational case since it is not "gauge invariant"

Need another explanation.
Guiding principle: linear terms fixed by flat-space physics

Effective theory

To describe equilibrium, effective interaction is introduced

$$\hat{H}_{eff} \; = \; \mu\hat{Q} \;\; or, \; in \; hydro \;\; \mathcal{L}_{eff} \; = \; \mu u_\alpha J^\alpha$$

where u_α is 4-velocity of element of a fluid, $\hat{Q}$ is conserved charge and μ is the associated chemical potential

Compare L_{eff} with corresponding gauge-potential interaction $\mathcal{L}_{fund} = eA_\alpha J^\alpha$

Clearly, effect of the thermodynamic interaction is calculable from known anomaly through the substitution:

$$eA_\alpha \;\to\; \mu u_\alpha$$

In this way we rederive directly the chiral vortical effect:
$(J_5^\alpha)_{vortical} \; = \; C_\omega \mu^2 \omega^\alpha$

Extra conservation law

Thermodynamic, effective interaction applies in infrared and is absent in the ultraviolet. Anomalous current is infrared sensitive while its divergence is ultraviolet sensitive and cannot be changed by thermodynamics

The way out: impose conservation of the vortical current in the media

$$\partial_\alpha(\mu^2\omega^\alpha) \ = \ 0$$

Conservation law of fluid helicity, non-Noether in nature known, since long, to hold in case of ideal fluid
(The helical motion is manifest in expression for $\mu^2\omega^0$)

Cnt'd

In presence of gravity the conservation law takes on form

$$\nabla_\alpha(\mu^2\omega^\alpha) \sim \partial_\alpha(\sqrt{-g}\mu^2\omega^\alpha) = 0$$

where ∇_α is the covariant derivative.

This equation unifies ordinary derivative $\partial_\alpha J^\alpha$ and gravimagnetic "anomaly". The physical meaning is that equations of motion in accelerated frame in presence of gravity are the same as in rest frame without gravity.

Flat-space phenomenology

However, phenomenology is made in flat-space terms, and in flat-space interpretation the current is not conserved (see above)

Thus equivalence principle imitates non-conservation of the current (through transformation law of element of volume) .

Unification of anomalies?

What we are getting (P.G. Mitkin+VZ 2103.01211)

$$\partial_\alpha J_5^\alpha = C_5(\vec{\Omega} \cdot \vec{a_{gr}}) + C_{gr} R\tilde{R}$$

A kind of unification of anomalies since C_5 and C_{gr} enter same equation. Can rewrite

$$(\partial_\alpha - a_\alpha) J_5^\alpha = C_{gr} R\tilde{R} \qquad \text{where} \quad a_\alpha \equiv u^\beta \partial_\beta u_\alpha$$

Strange idea: $R\tilde{R}$ is to be constructed on the same a_α

The idea seems to be true at least in one particular case of p-wave superfluidity (G. Volovik, 2104.01020)

$F\tilde{F} \sim R\tilde{R}$ with "potential" A_α equal to a spin connection

Conclusions to part II

- Two-step anomaly: at first step conserved, flat-space Noether current becomes conserved non-Noether current (extra conservation law)
- Once external grav. field is switched on, ordinary derivative becomes covariant derivative
- In flat-space terms, at second step the full current is not conserved while a Noether current is still conserved, with inclusion of gravity

III Higher-order terms

$*$ Back to equilibrium of accelerated and rotated medium.
Statistically, effective, or macroscopic interaction

$$\hat{H}_{eff} = \vec{\Omega} \cdot \hat{\vec{M}} + \vec{a} \cdot \hat{\vec{K}}$$

where $\vec{M}$ is angular momentum and $\vec{K}$ is the boost
$**$On other hand, in FT

$$\hat{H}_{fund} = \frac{1}{2} \hat{\Theta}^{\alpha\beta} h_{\alpha\beta}$$

where $\Theta^{\alpha\beta}$ is the energy momentum tensor, $h_{\alpha\beta}$ is the grav. potentials accommodating the same $\vec{\Omega}$, $\vec{a}$
$***$ Evaluate "external probes", $< \Theta^{\alpha\beta} >, < J_5^{\alpha} >$ for quantum particles. Expect results to be the same (duality)

More on statistical approach

- The scheme known to work in case of pure rotation. Inclusion of acceleration is recent, see "Thermodynamic equilibrium with acceleration and the Unruh effect" F. Becattini 1712.08031 [gr-qc]
- Statistical averaging involves density operator $\hat{\rho}$ where $\hat{\rho} = \frac{1}{Z} \exp\Big(- b_\alpha \hat{P}^\alpha + \bar{\omega}_{\alpha\beta} \hat{J}^{\alpha\beta} \Big)$ where $\hat{J}^{\alpha\beta}$ are generators of the Lorentz transformations $\bar{\omega}_{\alpha\beta} = \partial_\alpha (u_\beta / T) - \partial_\beta (u_\alpha / T)$,
- The boost operators $\hat{K}^\alpha$ are conserved but do not commute with $\hat{H}$. A novel feature!

Statistics-gravity duality at work

Evaluate energy density Θ_{00} of quantum massless spinors as function of independent a, T exploiting ‘novel’ density operator (G. Prokhorov, O. Teryaev, VZ+references)

$$\rho_{vac} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2}$$

First ever evaluation of vac. energy without subtractions.

$$get \qquad \rho_{vac}(T_{Unruh}) = 0$$

as is expected from general covariance

One-loop exact evaluation of the Unruh temperature

On the other side of duality

Energy density of same quantum particles in geometrical terms (metrics determined by external gravitational field)

metric is Euclidean Rindler space with boundary and conical singularity on the boundary

$$\epsilon_{vac} = \Big(\frac{7\pi^2 T^4}{60} + \frac{T^2}{24r^2} - \frac{17}{960\pi^2 r^4}\Big)$$

where r is the distance along the cone, related to acceleration (the result known since long)

Statistical calculation in flat space fits exactly field theory on a manifold with a boundary

Further examples of duality

The ϵ_{vac} is special since there is no free parameters at all
Typically, the picture is more complicated:
Infrared side is fixed by effective imteraction
Ultraiolet side is fixed by fundamental interaction
Upon matching the asymptotes predictive power is still left.

Overall impression that duality works

Turn to the case of gravitational anomaly

Axial current on gravitational background

Slowly rotating, accelerated gas of massless fermions, $\vec{a} \,||\, \vec{\Omega}$

Axial current $\vec{J}_5$, by dimension and polynomiality

$$\vec{J}_5 \;=\, c_T T^2 \vec{\Omega} \;+\; c_a a^2 \vec{\Omega}$$

c_T term is thermal contribution, calculable in terms of Fermi distribution

c_a term is vacuum contribution, exists in absence of medium

Statistical approach misses the anomaly,

or correct vacuum component

Forced to switch to a less ambitious form of duality.

Matching anomaly and Hawking radiation

Constant c_a is determined by field theory at $T = 0$. Namely, in a simple enough geometry with intrinsic rotation Ω and acceleration a, chiral gravitational anomaly:

$$\partial_\alpha J_5^\alpha = c_a \partial_\alpha \left(a^2 \Omega^\alpha \right)$$

At spatial infinity $a_\infty = 0$, The difference between the currents at infinity and at finite a is uniquely fixed by the anomaly. Thus, c_a is related to C_{gr}
c_T term is like subtraction constant,
to be yet determined at this point.

Axial current, cnt'd

Coefficient c_T determined by pure thermal field theory (a=0). The result of a standard calculation $\vec{J}_5 = \vec{\Omega}(T^2/6)$

Combining the two terms (M. Stone (2018), rephrased)

$$\vec{J}_5 \;=\; \vec{\Omega}\Big(\frac{T^2}{6} - \frac{a^2}{6(2\pi)^2}\Big)$$

The current is vanishing at $T \;=\; T_{Unruh}$ as it should vanish provided that Minkowski vacuum is stable under rotation

Conclusions to part III

Understanding of higher-order (in acceleration) terms is much more limited

- A few examples of similarity, or duality between statistical calculations in flat space and gravitational calculations, including on space with boundary
- The validity of this approach is, however, limited. Need to match full-scale BH physics

IV Instabilities

It is argued in (G. Prokhorov et al. (2019)) that below T_U expression for ρ_{vac} given above is no longer valid. Instead:

$$\rho_{vac}(T < T_U) = \frac{127\pi^2 T^4}{60} - \frac{11 T^2 a^2}{24} \tag{1}$$

$$-\frac{17 a^4}{960\pi^3} - \pi T^3 a + \frac{T a^3}{4\pi} . \tag{2}$$

Note that

$$\rho_{vac}(T \to T_U) = \rho_{vac}(T_U \leftarrow T) \tag{3}$$

$$\frac{\partial}{\partial T}\rho_{vac}(T \to T_U) = \frac{\partial}{\partial T}\rho_{vac}(T_U \leftarrow T), \tag{4}$$

while the second derivative from ρ_{vac} with respect to temperature is not continuous at $T = T_U$.

Comparison to field theory

At black-hole horizon $T = T_{Unruh}$
Going to $T < T_{Unruh}$ corresponds crossing into the BH
The results above fit nicely field theory
"Vacua on the Brink of Decay" G. L. Pimentel, A. M. Polyakov, G. M. Tarnopolsky 1803.09168
where one considers manifold instability due to negative energy levels of quantum particles

Higher spins

We considered axial current fir spin-1/2 particles and found out that the current vanishes at $T = T_U$ due to the balance between the thermal and vacuum components, $J^5 \sim \Omega(T^2 - a^2/(4\pi^2))$

For higher spins the balance seems difficult:

$$c_a \sim (2S^3 - S), \qquad c_T \sim S ,$$

in apparent contradiction with the univesality of T_U

Higher spins, cnt'd

It is not necessarily a problem, because the effective interaction $\delta H \sim \vec{\Omega} \cdot \vec{S}$ induces negative modes, or instability of Minkowski vacuum beginning with $S \geq 3/2$. The crucial case is $S = 1$ (photon) with modes $\epsilon \geq 0$ Recently it was shown that regularization with infinitesimal photon mass results in $J_5(T = T_U) = 0$.

Conclusions

We had four parts:

- “Introduction”–hopes that gravitational effects might be relevant to phenomenology
- “Linear in acceleration terms”-governed by equivalence principle, extra conservation law of fluid helicity; a kind of two-step anomaly
- “Higher orders in acceleration”–studied mostly for non-interacting gas (not ideal fluid). Possible extension of the equivalence principle. Merge wth standard BH physics
- Possible instability at $T = T_U$, Higher spins, as example of work in progress