Spin and chirality in hydrodynamics

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CRC-TR 211 Strong-interaction matter under extreme conditions

DFG

From quantum field theory to hydrodynamics

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- Want: Polarization dynamics in heavy-ion collisions (see yesterday's talks) from microscopic theory
  $\Longrightarrow$ Dissipative spin hydrodynamics

- Spin is quantum property
  $\Longrightarrow$ starting point: quantum field theory

- Derived kinetic theory for massive spin-1/2 particles from quantum field theory using Wigner-function formalism
  NW, X.-L. Sheng, E. Speranza, Q. Wang, and D. H. Rischke, PRD100, 056018 (2019)
  J.-H. Gao and Z.-T. Liang, PRD100, 056021(2019)
  K. Hattori, Y. Hidaka, and D.-L. Yang, PRD100, 096011 (2019)
  Y.-C. Liu, K. Mameda, and X.-G. Huang, Chin.Phys.C 44 (2020) 9, 094101
  NW, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke, PRL127 (2021) 5, 052301; PRD104 (2021) 1, 016022

- Now: Derive hydrodynamic equations of motion from kinetic theory,
  use method of moments
  G.S. Denicol, H. Niemi, E. Molnar, D.H. Rischke, PRD 85 (2012) 114047

NW, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke, PRL127 (2021) 5, 052301; PRD104 (2021) 1, 016022

- Phase-space distribution function $f(x, p, \mathfrak{s})$, depends on spin variable $\mathfrak{s}^\mu$, exactly related to Wigner function

- **Boltzmann equation** (derived from quantum field theory)

$$p \cdot \partial f(x, p, \mathfrak{s}) = \mathfrak{C}[f]$$

- Nonlocal collision term

$$\mathfrak{C}[f] = \int d\Gamma_1 d\Gamma_2 d\Gamma' \, \mathcal{W} \, [f(x + \Delta_1, p_1, \mathfrak{s}_1) \times f(x + \Delta_2, p_2, \mathfrak{s}_2) - f(x + \Delta, p, \mathfrak{s}) f(x + \Delta', p', \mathfrak{s}')]$$

Particle positions displaced by Δ^μ

- Nonlocal collision term: conversion of orbital angular momentum into spin $\Longrightarrow$ spin alignment with vorticity

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- **Spin hydrodynamics is based on conserved quantities:**

  W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, PRC 97, no. 4, 041901 (2018)
  W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. Speranza, PRD 97, no. 11, 116017 (2018)
  W. Florkowski, F. Becattini, and E. Speranza, APB 49, 1409 (2018)

charge current

$$\partial \cdot N = 0$$

energy-momentum tensor

$$\partial_\mu T^{\mu\nu} = 0$$

+ **total** angular-momentum tensor

$$J^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} + \hbar S^{\lambda,\mu\nu}$$

orbital part   spin tensor

$$\partial_\lambda J^{\lambda,\mu\nu} = 0 \quad \Longrightarrow \quad \hbar \partial_\lambda S^{\lambda,\mu\nu} = T^{[\nu\mu]}$$

$$a^{[\mu} b^{\nu]} \equiv a^\mu b^\nu - a^\nu b^\mu$$

NW, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke, PRL127 (2021) 5, 052301
E. Speranza, NW, EPJA57 (2021) 5, 155

- Definition of energy-momentum and spin tensor depends on choice of pseudo-gauge
  F.W. Hehl, Rept. Math. Phys. 9, 55 (1976)

- Does pseudo-gauge choice affect dynamics? Yes ...
  F. Becattini, W. Florkowski, E. Speranza, PLB789 (2019) 419-425
  K. Fukushima, S. Pu, PLB817 (2021) 136346
  A. Das, W. Florkowski, R. Ryblewski, R. Singh, PRD103 (2021) 9, L091502
  M. Buzzegoli, 2109.12084

- Canonical spin tensor: not conserved even for free fields or in global equilibrium
  $\Longrightarrow$ not consistent with physical picture of spin density

- Hilgevoord-Wouthuyson (HW) currents:
  J. Hilgevoord, S.A. Wouthuysen, Nuclear Physics 40, 1 (1963)

$$
\begin{aligned}
T^{\mu\nu}_{HW} &= \int d\Gamma\, p^\mu p^\nu f(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)\ , \\
S^{\lambda,\mu\nu}_{HW} &= \int d\Gamma\, p^\lambda \left(\frac{1}{2}\Sigma^{\mu\nu}_{\mathfrak{s}} - \frac{\hbar}{4m^2} p^{[\mu}\partial^{\nu]} \right) f(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)
\end{aligned}
$$

Dipole-moment tensor $\Sigma^{\mu\nu}_{\mathfrak{s}} \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha \mathfrak{s}_\beta$

NW, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke, PRL127 (2021) 5, 052301
E. Speranza, NW, EPJA57 (2021) 5, 155

- Boltzmann equation $\Longrightarrow$ Equations of motion

$$\partial_\mu T^{\mu\nu}_{HW} = \int d\Gamma \, p^\nu \, \mathfrak{C}[f] = 0 \,,$$

$$\hbar \, \partial_\lambda S^{\lambda,\mu\nu}_{HW} = \int d\Gamma \, \frac{\hbar}{2} \Sigma^{\mu\nu}_{\mathfrak{s}} \, \mathfrak{C}[f] = T^{[\nu\mu]}_{HW} \,.$$

- HW spin tensor conserved for free fields or in global equilibrium: $\mathfrak{C} = 0$

- Nonzero nonlocal collision term $\Longleftrightarrow T^{[\nu\mu]}_{HW} \neq 0 \Longleftrightarrow$ Spin not conserved

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- Decompose currents with respect to fluid velocity u^μ

$$
\begin{aligned}
N^\mu &= n u^\mu + n^\mu\,, \\
T^{\mu\nu}_{\text{sym}} &= \epsilon u^\mu u^\nu - \Delta^{\mu\nu}(P_0+\Pi) + u^{(\mu} h^{\nu)} + \pi^{\mu\nu}\,, \\
S^{\lambda,\mu\nu} &= u^\lambda \tilde{\mathfrak{N}}^{\mu\nu} + \Delta^\lambda_\alpha \tilde{\mathfrak{P}}^{\alpha\mu\nu} + 2u_{(\alpha}\tilde{\mathfrak{H}}^{\lambda)\mu\nu\alpha} + \tilde{\mathfrak{Q}}^{\lambda\mu\nu} \\
&\quad + \frac{\hbar}{2m}\partial^{[\nu}\left[\epsilon_0 u^{\mu]} u^\lambda - \Delta^{\mu]\lambda}(P_0+\Pi) + \pi^{\mu]\lambda}\right]
\end{aligned}
$$

$$
\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu,\ \tilde{A}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} A_{\alpha\beta},\ a^{(\mu} b^{\nu)} \equiv a^\mu b^\nu + a^\nu b^\mu
$$

- 4+10+24 degrees of freedom $\leftrightarrow$ 1+4+6 equations of motion
  $\Longrightarrow$ need additional equations of motion to close system of equations
- All components can be related to moments of distribution function, e.g., spin-energy tensor

$$
\tilde{\mathfrak{N}}^{\mu\nu} \equiv -\frac{1}{2m}\epsilon^{\mu\nu\alpha\beta} u_\alpha \langle E_p^2\, \mathfrak{s}_\beta \rangle
$$

$\Longrightarrow$ derive additional equations of motion from kinetic theory

use method of moments
G.S. Denicol, H. Niemi, E. Molnar, D.H. Rischke, PRD 85 (2012) 114047

Equilibrium and scale ordering

- Nonlocal collision term vanishes only in global equilibrium
  $\Longrightarrow$ How to define "local" equilibrium as starting point of hydrodynamic expansion?
- Issue about local equilibrium:
  Nonlocality scale of collision term smaller than scale on which dissipation happens
- Treat vorticity on different scale than other gradients
  S. Li, M. Stephanov, H.-U. Yee, PRL127 (2021) 8, 082302
- Impose ordering of scales

$$\Delta < l_{\text{mfp}} \ll l_{\text{hydro}}\ , \qquad \Delta \ll l_v < l_{\text{hydro}}$$

needed for molecular chaos — allows for definition of local equilibrium

with

$$\textit{expansion in spin degrees of freedom}\quad \frac{\Delta}{l_v} \sim \frac{l_{\text{mfp}}}{l_{\text{hydro}}} \quad \textit{hydrodynamic expansion}$$

Δ: nonlocality of microscopic collision, l_{mfp}: mean free path
l_{hydro}: scale of inverse dissipative gradients, l_v: scale of inverse vorticity

- Local equilibrium:
  neglect $\mathcal{O}(\Delta/l_{\text{hydro}})$ $\Longrightarrow$ Nonlocal collision term vanishes if spin potential equal to thermal vorticity

- Expand distribution function up to first order in $\hbar$ and gradients

$$f = f_{\text{eq}} + \delta f = f_{0p}\left[1 + \frac{\hbar}{4}\Omega_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu} + \phi_p + \mathfrak{s}\cdot\zeta_p\right],$$

equilibrium, deviations from equilibrium

Zeroth-order distribution function

$$f_{0p} \equiv \frac{1}{(2\pi\hbar)^3} e^{-\beta_0 u\cdot p + \alpha_0}$$

inverse temperature β_0, chemical potential α_0,
spin potential: thermal vorticity + dissipative corrections

$$\Omega_{\mu\nu} = \varpi_{\mu\nu} + \mathcal{O}\left(\frac{1}{l_{\text{hydro}}}\right)$$

- Define dissipative irreducible moments

$$\rho_n^{\mu_1\cdots\mu_l} \equiv \langle E_p^n\, p^{\langle\mu_1}\ldots p^{\mu_l\rangle}\rangle_\delta$$

$$\tau_n^{\mu,\mu_1\cdots\mu_l} \equiv \langle E_p^n\, \mathfrak{s}^\mu p^{\langle\mu_1}\ldots p^{\mu_l\rangle}\rangle_\delta$$

$A^{\langle\mu_1\cdots\mu_n\rangle}$: traceless, symmetric projection orthogonal to u^μ

- δf can be expressed in basis of dissipative moments

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- Matching conditions (Landau frame)

$$u_\mu N^\mu = u_\mu N^\mu_{eq} \, , \qquad u_\mu T^{\mu\nu}_{\mathsf{sym}} = u_\mu T^{\mu\nu}_{\mathsf{sym},eq}$$

- Additional matching condition in presence of spin

$$u_\lambda J^{\lambda,\mu\nu} = u_\lambda J^{\lambda,\mu\nu}_{eq}$$

$\Longrightarrow$ Defines spin potential near local equilibrium

- Conservation laws for N^μ, $T^{\mu\nu}$, and $J^{\lambda,\mu\nu}$

$\Longrightarrow$ equations of motion for thermodynamic potentials α_0, β_0, u^μ, $\Omega^{\mu\nu}$

- Boltzmann equation $\Longrightarrow$ exact equations of motion for spin moments $\tau_n^{\mu,\mu_1\cdots\mu_l}$, e.g.,

$$
\begin{aligned}
&\dot{\tau}_r^{\langle\mu\rangle} - \mathfrak{C}_{r-1}^{\langle\mu\rangle} = \\
&\left[\xi_r^{(0)}\theta + \frac{G_{2(r+1)}}{D_{20}}\Pi\theta - \frac{G_{2(r+1)}}{D_{20}}\pi^{\lambda\nu}\sigma_{\lambda\nu} - \frac{G_{3r}}{D_{20}}\partial\cdot n\right]\omega_0^\mu - \frac{\hbar}{2m}I_{(r+1)1}\Delta_\lambda^\mu\nabla_\nu\tilde{\Omega}^{\lambda\nu} \\
&-\frac{\hbar}{2m}\tilde{\Omega}^{\langle\mu\rangle\nu}\left[I_{(r+1)1}I_\nu - I_{(r+2)1}\frac{\beta_0}{\epsilon_0+P_0}\left(-\Pi\dot{u}_\nu + \nabla_\nu\Pi - \Delta_{\nu\lambda}\partial_\rho\pi^{\lambda\rho}\right)\right] \\
&+r\,\dot{u}_\nu\tau_{r-1}^{\langle\mu\rangle,\nu} + (r-1)\sigma_{\alpha\beta}\tau_{r-2}^{\langle\mu\rangle,\alpha\beta} - \Delta_\lambda^\mu\nabla_\nu\tau_{r-1}^{\lambda,\nu} - \frac{1}{3}\left[(r+2)\tau_r^{\langle\mu\rangle} - (r-1)m^2\tau_{r-2}^{\langle\mu\rangle}\right]\theta \\
&-\frac{\hbar}{2m}I_{(r+1)0}\epsilon^{\mu\nu\alpha\beta}u_\nu\dot{\Omega}_{\alpha\beta}
\end{aligned}
$$

$\nabla^\mu \equiv \Delta^\mu_\nu\partial^\nu$, $\theta\equiv\nabla\cdot u$, $\sigma^{\mu\nu}\equiv\nabla^{\langle\mu}u^{\nu\rangle}$, $\omega_0^\mu\equiv-(1/2)\epsilon^{\mu\nu\alpha\beta}u_\nu\Omega_{\alpha\beta}$, $I^\mu\equiv\nabla^\mu\alpha_0$

- Collision term

$$
\mathfrak{C}_{r-1}^{\mu,\langle\mu_1\cdots\mu_n\rangle} \equiv \int d\Gamma\, E_p^{r-1}\mathfrak{s}^\mu p^{\langle\mu_1}\cdots p^{\mu_n\rangle}\mathfrak{C}[f]
$$

Truncation procedure and dynamical moments

- Infinite number of coupled equations $\Longrightarrow$ need truncation procedure
- **Idea** (Israel-Stewart): Approximate moments which do not appear in conservation laws by those which do appear
- **Conventional hydrodynamics:**

$n^\mu \equiv \rho_0^\mu,$	$h^\mu \equiv \rho_1^\mu,$	$\Pi \equiv -\frac{m^2}{3}\rho_0,$	$\pi^{\mu\nu} \equiv \rho_0^{\mu\nu}$
particle diffusion	heat flux	bulk viscous pressure	shear stress

$\Longrightarrow$ 14-moment approximation

- **Spin hydrodynamics:** additional dynamical moments from spin tensor

$\mathfrak{n}^\nu \equiv \tau_2^\nu,$	$\mathfrak{p}^\mu \equiv \tau_0^\mu,$	$\mathfrak{z}^{\lambda\mu} \equiv \tau_1^{(\langle\mu\rangle,\lambda)},$	$\mathfrak{q}^{\lambda\mu\nu} \equiv \tau_0^{\nu,\mu\lambda}$
spin energy	spin pressure	spin diffusion	spin stress

$\Longrightarrow$ 14+24-moment approximation

- Dynamical spin moments determined by form of spin tensor
  $\Longrightarrow$ Pseudo-gauge dependence enters!

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- General form of collision terms

$$\mathfrak{C}_{r-1}^{\mu,\langle\mu_1\cdots\mu_l\rangle} = \sum_{n=0}^{N_l} B_{rn}^{(l)} \tau_n^{\mu,\langle\mu_1\cdots\nu_l\rangle} + \int [d\Gamma]\, \mathcal{W}\, E_p^{r-1} p^{\langle\mu_1} \cdots p^{\mu_l\rangle} \mathfrak{s}^\mu f_{0p} f_{0p'}$$
$$\times \left[-\frac{\hbar}{4}(\Omega_{\alpha\beta} - \varpi_{\alpha\beta})\Sigma_{\mathfrak{s}}^{\alpha\beta} + \frac{1}{2}\partial_{(\beta}\beta_{\alpha)}\Delta^\beta p^\alpha \right]$$

dissipative spin moments
difference between spin potential and thermal vorticity
from nonlocal collision term, thermal shear,

$$\Delta^\mu \equiv -\frac{\hbar}{2m(p\cdot\hat{t}+m)}\epsilon^{\mu\nu\alpha\beta} p_\nu \hat{t}_\alpha \mathfrak{s}_\beta$$

- Invert matrix $B_{rn}^{(l)}$ to obtain final form of equations of motion

Obtain equations of motion of all dynamical spin moments, e.g.,

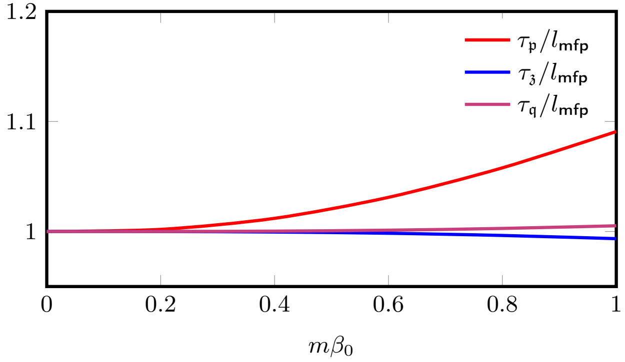
$$\tau_{\mathfrak{q}} \Delta^{\mu}_{\rho} \Delta^{\nu\lambda}_{\alpha\beta} \frac{d}{d\tau} \mathfrak{q}^{\langle\rho\rangle\alpha\beta} + \mathfrak{q}^{\langle\mu\rangle\nu\lambda}$$
$$= -\,\mathfrak{d}^{(2)} \beta_0 \sigma_{\rho}{}^{\langle\nu} \epsilon^{\lambda\rangle\mu\alpha\rho} u_{\alpha} + \mathfrak{K}^{(2)}_{\Omega I} \tilde{\Omega}^{\langle\mu\rangle\langle\nu} I^{\lambda\rangle} + \mathfrak{K}^{(2)}_{\nabla\Omega} \Delta^{\nu\lambda}_{\alpha\beta} \nabla^{\alpha} \tilde{\Omega}^{\mu\beta} - \mathfrak{K}^{(2)}_{\omega\sigma} \sigma^{\nu\lambda} \omega^{\mu}_{0}$$
$$- \mathfrak{K}^{(2)}_{\Omega\Pi} \tilde{\Omega}^{\langle\mu\rangle\langle\nu} \left(-\Pi \dot{u}^{\lambda\rangle} + \nabla^{\lambda\rangle} \Pi - \Delta^{\lambda\rangle}_{\alpha} \partial_{\beta} \pi^{\alpha\beta} \right) + \mathfrak{g}^{(2)}_{1} \mathfrak{z}^{\mu\langle\nu} F^{\lambda\rangle} + \mathfrak{g}^{(2)}_{2} \mathfrak{z}^{\mu\langle\nu} I^{\lambda\rangle}$$
$$+ \mathfrak{g}^{(2)}_{3} \Delta^{\mu}_{\rho} \Delta^{\nu\lambda}_{\alpha\beta} \nabla^{\beta} \mathfrak{z}^{\rho\alpha} + \mathfrak{g}^{(2)}_{4} \mathfrak{q}^{\langle\mu\rangle\nu\lambda} \theta + \mathfrak{g}^{(2)}_{5} \mathfrak{q}^{\langle\mu\rangle\rho\langle\nu} \sigma^{\lambda\rangle}_{\rho} + 2\tau_{\mathfrak{q}} \mathfrak{q}^{\langle\mu\rangle\rho\langle\nu} \omega^{\lambda\rangle}_{\rho}$$
$$+ \mathfrak{g}^{(2)}_{6} \mathfrak{p}^{\langle\mu\rangle} \sigma^{\nu\lambda} - 6\mathfrak{g}^{(2)}_{7} \mathfrak{q}^{\rho\mu}{}_{\rho} \sigma^{\nu\lambda} + \mathfrak{g}^{(2)}_{8} F^{\mu} \mathfrak{z}^{\langle\nu\lambda\rangle} + \mathfrak{g}^{(2)}_{9} \mathfrak{p}^{\langle\nu} \nabla^{\lambda\rangle} u^{\mu} + \mathfrak{g}^{(2)}_{10} \mathfrak{q}^{\rho\langle\nu}_{\ \ \rho} \nabla^{\lambda\rangle} u^{\mu}$$

relaxation time
transport coefficients

and similar equations for $\mathfrak{p}^{\langle\mu\rangle}$ and $\mathfrak{z}^{\lambda\mu}$

$\Longrightarrow$ closed set of relaxation equations

- Calculate relaxation times for spin moments in dependence of $m\beta_0$
- Relaxation times of same order as those for spin-independent dissipative moments (Π, n^μ, $\pi^{\mu\nu}$)



Calculations on spin relaxation times in different context can be found in

J. I. Kapusta, E. Rrapaj, S. Rudaz, PRC 101, 024907, 031901, 102 6, 064911 (2020)
A. Ayala, D. de la Cruz, L. Hernández, J. Salinas, PLB 801, 135169, PRD 102 5, 056019 (2020)

- Observable in heavy-ion collisions: Pauli-Lubanski vector

  F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, AP338, 32 (2013)
  F. Becattini, arXiv:2004.04050
  E. Speranza, NW, EPJA 57 (2021) 5, 155
  L. Tinti, W. Florkowski, arXiv:2007.04029

$$\Pi^\mu(p) = \frac{1}{2\mathcal{N}} \int d\Sigma_\lambda p^\lambda dS\, \mathfrak{s}^\mu f(x,p,\mathfrak{s})$$

- Equilibrium:

$$\Pi^\mu_{\textsf{eq}}(p) = -\frac{\hbar}{4m\mathcal{N}} \int d\Sigma_\lambda p^\lambda \tilde{\Omega}^{\mu\nu}(E_p u_\nu + p_{\langle\nu\rangle})$$

- Express dissipative corrections through dynamical spin moments:

$$\delta\Pi^\mu(p) = \frac{1}{2\mathcal{N}}\left(g^\mu_\nu - \frac{p^\mu p_\nu}{p^2}\right)\int d\Sigma_\lambda p^\lambda \Bigg\{ \chi_\mathfrak{p}\mathfrak{p}^{\langle\nu\rangle} - 6\chi_\mathfrak{n}\mathfrak{q}^{\rho\nu}{}_\rho + \mathfrak{x}_\mathfrak{p} u^\nu \mathfrak{z}^\lambda{}_\lambda$$
$$+ \left[\chi_\mathfrak{z}\mathfrak{z}^{\nu\alpha} + \left(\mathfrak{x}_\mathfrak{q}\mathfrak{q}^{\lambda\alpha}{}_\lambda + \mathfrak{x}_\mathfrak{p}\mathfrak{p}^{\langle\alpha\rangle}\right)u^\nu\right] p_{\langle\alpha\rangle} + \left(\chi_\mathfrak{q}\mathfrak{q}^{\langle\nu\rangle\alpha\beta} + \mathfrak{x}_\mathfrak{z} u^\nu \mathfrak{z}^{\langle\alpha\beta\rangle}\right) p_{\langle\alpha}p_{\beta\rangle} \Bigg\}$$

Navier-Stokes limit

- So far: kept terms up to second order in equations of motion
  $\Longrightarrow$ transient hydrodynamics
- Now: keep only first-order terms $\Longrightarrow$ Navier-Stokes limit
- No additional dynamical quantities,
  everything can be expressed in terms of α_0, β_0, u^μ, $\Omega^{\mu\nu}$ and their derivatives
- Full expression of Pauli-Lubanski vector lengthy
- From nonlocal collision term: contributions independent of spin potential

$$\delta\Pi^\mu(p) \simeq \frac{1}{2\mathcal{N}}\left(g^\mu_\nu - \frac{p^\mu p_\nu}{p^2}\right)\int d\Sigma_\lambda p^\lambda \chi_\sigma \sigma_\rho{}^{\langle\alpha}\epsilon^{\beta\rangle\nu\tau\rho} u_\tau p_{\langle\alpha} p_{\beta\rangle} + \ldots$$

$\Longrightarrow$ contribution from shear to local polarization, vanishes after momentum integration
Effects of shear important for description of local Λ polarization

B. Fu, S. Y.F. Liu, L. Pang, H- Song, Y. Yin, PRL127 (2021) 14, 142301
F. Becattini, M. Buzzegoli, A. Palermo, arXiv:2103.10917
F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, I. Karpenko, arXiv:2103.14621

What comes next?

- Numerical implementation of second-order equations of motion?
  $\Longrightarrow$ Causality and stability?

- So far: No work in that direction done in dissipative spin hydrodynamics
  But: **Causality and stability for chiral hydrodynamics**
  E. Speranza, F. Bemfica, M. Disconzi, J. Noronha, 2104.02110

- So far: spin-1/2 fermions
  $\Longrightarrow$ What happens for higher spins? $\rightarrow$ ϕ-polarization in heavy-ion collisions

- First step:
  Derive kinetic theory for massive spin-1 particles from Wigner-function formalism
  D. Wagner, NW, E. Speranza, D. H. Rischke, in preparation

Ideal chiral hydrodynamics from kinetic theory

E. Speranza, F. Bemfica, M. Disconzi, J. Noronha, 2104.02110

- Constitutive equations in ideal chiral hydrodynamics
  e.g., Chen, Son, Stephanov, PRL 115, no.2, 021601 (2015); Yang, PRD 98, no.7, 076019 (2018)

$$
\begin{aligned}
T^{\mu\nu} &= \varepsilon u^\mu u^\nu + P\Delta^{\mu\nu} + \xi_T(\omega^\mu u^\nu + \omega^\nu u^\mu) \\
J_V^\mu &= n_V u^\mu + \xi_V\, \omega^\mu \\
J_A^\mu &= n_A u^\mu + \xi_A\, \omega^\mu
\end{aligned}
$$

J_V^μ, J_A^μ - Vector- and axial-vector current, ω^μ vorticity vector

- Equations of motion

$$
\partial_\mu T^{\mu\nu} = 0\,, \qquad \partial \cdot J_V = \partial \cdot J_A = 0
$$

- One can prove:
  solution of system of equations either does not exist or is not unique
  $\Longrightarrow$ cannot be numerically implemented

Ideal chiral hydrodynamics in Landau frame

E. Speranza, F. Bemfica, M. Disconzi, J. Noronha, 2104.02110

- Shift fluid velocity such that it becomes eigenvector of $T^{\mu\nu}$
- Drop terms of order $\mathcal{O}(\partial^2)$
- Resulting constitutive equations:

$$
\begin{aligned}
T^{\mu\nu} &= \varepsilon u^\mu u^\nu + P\Delta^{\mu\nu} \\
J_V^\mu &= n_V u^\mu + \xi_V\, \omega^\mu \\
J_A^\mu &= n_A u^\mu + \xi_A\, \omega^\mu
\end{aligned}
$$

- Impose nontrivial conditions on ξ_V, ξ_A
  $\Longrightarrow$ equations of motion have unique solution and are causal and stable
  $\Longrightarrow$ numerical implementation possible

- Proca fields V^μ interacting with electromagnetic fields A^μ

- Starting point: Maxwell-Proca Lagrangian
  H. C. Corben. J. Schwinger, PR58, 953 (1940)

$$\mathcal{L}_P = \hbar\left(-\frac{1}{2}V^{*\mu\nu}V_{\mu\nu} + \frac{m^2}{\hbar^2}V^{*\mu}V_\mu\right) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - iq\kappa F_{\mu\nu}V^\mu V^{*\nu}$$

  Field strength tensors $V^{\mu\nu} \equiv D^\mu V^\nu - D^\nu V^\mu\ , \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$,
  covariant derivative $D^\mu \equiv \partial^\mu + (iq/\hbar)A^\mu$

- Magnetic moment $\mu \equiv (1+\kappa)q\hbar/2m$
  $\Longrightarrow$ Choosing $\kappa = 1$ sets gyromagnetic ratio $g = 2$
  S. Ferrara, M. Porrati, V. L. Telegdi, PRD46, 3529 (1992)

D. Wagner, NW, E. Speranza, D. H. Rischke, in preparation

- **Idea:** Calculate Wigner function and its equations of motion in presence of electromagnetic fields by $\hbar$-expansion in collisionless regime
  NW, X.-L. Sheng, E. Speranza, Q. Wang, and D. H. Rischke, PRD100, 056018 (2019)

  $\Longrightarrow$ extend formalism used for Dirac fields to Proca fields

- Spin-one Wigner function:

$$W^{\mu\nu}(x,k) \equiv \frac{1}{(2\pi\hbar)^4} \int d^4v \, e^{-\frac{i}{\hbar}k^\alpha v_\alpha} \left\langle : V^{*\mu}\left(x+\frac{v}{2}\right) U_{+-} V^{\nu}\left(x-\frac{v}{2}\right) : \right\rangle$$

gauge link

- Proca equation
  $\Longrightarrow$ mass-shell relations, constraints and kinetic equations for Wigner function

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D. Wagner, NW, E. Speranza, D. H. Rischke, in preparation

- Decompose Wigner function into symmetric and antisymmetric part
  Huang, Mitkin, Sadofyev, Speranza, JHEP 10 (2020) 117

$$W^{\mu\nu} = W_S^{\mu\nu} + W_A^{\mu\nu}$$

- Decompose both parts with respect to k^μ

$$\begin{aligned} W_S^{\mu\nu} &= E^{\mu\nu} f_E + K^{\mu\nu} f_K + \frac{k^{(\mu}}{2\sqrt{k^2}} F_S^{\nu)} + F_K^{\mu\nu} \\ W_A^{\mu\nu} &= \frac{k^{[\mu}}{2\sqrt{k^2}} F_A^{\nu]} + \epsilon^{\mu\nu\alpha\beta} \frac{k_\alpha}{m} G_\beta \end{aligned}$$

$E^{\mu\nu} \equiv k^\mu k^\nu / \sqrt{k^2}$, $K^{\mu\nu} \equiv g^{\mu\nu} - E^{\mu\nu}$
$F_S^\mu k_\mu = F_A^\mu k_\mu = G^\mu k_\mu = F^\mu_{K,\mu} = 0\,, \quad k_\mu F_K^{\mu\nu} = 0\,, \quad F_K^{\mu\nu} = F_K^{\nu\mu}$

independent components, determined by mass-shell conditions and kinetic equations
dependent components, fixed by constraint equations

$\Longrightarrow$ Solve these equations order by order in $\hbar$

Results: equations of motion I

D. Wagner, NW, E. Speranza, D. H. Rischke, in preparation

- Scalar distribution function $V \equiv$ on-shell part of f_K

$$0 = \delta(k^2 - m^2c^2)\left[k \cdot \hat{\nabla}^{(0)} 3 \left(V + \frac{1}{3}\frac{q\hbar}{4k^2} F^{\mu\nu}\bar{\Sigma}_{\mu\nu}\right) + \frac{q\hbar}{2}(\partial^\gamma F^{\alpha\beta})\partial_{k,\gamma}\bar{\Sigma}_{\alpha\beta}\right]$$
$$-\delta'(k^2 - m^2c^2) q\hbar F^{\alpha\beta} k \cdot \hat{\nabla}^{(0)}\bar{\Sigma}_{\alpha\beta}$$

$$\hat{\nabla}^{(0)\mu} \equiv \partial_x^\mu - qF^{\mu\nu}\partial_{k\nu}$$

- Dipole-moment tensor

$$\bar{\Sigma}^{\mu\nu} \equiv -iW^{\mu\nu}_{A,\text{on-shell}} - 2\hbar\frac{q}{k^2}\frac{k_\rho k^{[\mu}}{k^2}F^{\nu]\rho}V^{(0)} + \frac{\hbar q}{k^2}F^{\mu\nu}V^{(0)}$$

$$0 = \delta(k^2 - m^2c^2)\Big[k \cdot \hat{\nabla}^{(0)}\bar{\Sigma}^{\mu\nu} - qF_\rho{}^{[\mu}\bar{\Sigma}^{\nu]\rho}$$
$$-\frac{q\hbar}{2}(\partial^\gamma F_\rho{}^{[\mu})\partial_{k,\gamma}\left(\mathcal{F}_K^{\nu]}{}_\rho + g^{\nu]\rho}V\right) - \frac{q\mu_0}{k^2}N_\alpha \mathcal{F}_K^{\alpha[\mu}k^{\nu]}\Big]$$
$$+2\delta'(k^2 - m^2c^2)q\hbar F_\rho{}^{[\mu}\left[k \cdot \hat{\nabla}^{(0)}\mathcal{F}_K^{\nu]\rho} + qF_\sigma{}^{(\nu]}\mathcal{F}_K^{\rho)\sigma} + K^{\nu]\rho}k \cdot \hat{\nabla}^{(0)}V\right]$$

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- Tensor polarization $\mathcal{F}_K^{\mu\nu} \equiv$ on-shell part of $F_K^{\mu\nu}$

$$
\begin{aligned}
0 \;=\; & \delta(k^2 - m^2c^2)\left[k \cdot \hat{\nabla}^{(0)} \mathcal{F}_K^{\rho\sigma} + qF_\alpha{}^{(\rho} \mathcal{F}_K^{\sigma)\alpha} + \frac{q\hbar}{2} K_{\mu\nu}^{\rho\sigma} (\partial^\gamma F^\mu{}_\alpha) \partial_{k,\gamma} \bar{\Sigma}^{\nu\alpha}\right] \\
& + \delta'(k^2 - m^2c^2) q\hbar K_{\mu\nu}^{\rho\sigma} F_\beta{}^\mu \left(k \cdot \hat{\nabla}^{(0)} \bar{\Sigma}^{\nu\beta} - qF_\lambda{}^{[\nu} \bar{\Sigma}^{\beta]\lambda}\right)
\end{aligned}
$$

- Scalar distribution and dipole moment: analogous to spin-1/2

  Tensor polarization: new degrees of freedom for spin 1
  $\Longrightarrow$ Effects in heavy-ion collisions?

- Kinetic theory is starting point to derive dissipative hydrodynamics for spin-1 particles

Conclusions and outlook

- Derived second-order spin hydrodynamics from kinetic theory using method of moments
- Dissipative corrections to Pauli-Lubanski vector
  $\Longrightarrow$ depend on all dynamical spin moments
  $\Longrightarrow$ Navier-Stokes limit: contributions from spin potential and shear
  $\Longrightarrow$ Need numerical implementation to compare to experiment
- Presence of vorticity in spin/ chiral hydrodynamics leads to causality issues even in ideal case
  E. Speranza, F. Bemfica, M. Disconzi, J. Noronha, 2104.02110
  $\Longrightarrow$ Causality and stability analysis of second-order equations of motion needed
- Derived kinetic theory for spin-1 particles
  D. Wagner, NW, E. Speranza, D. H. Rischke, in preparation
  $\Longrightarrow$ Derive spin-1 kinetic theory with particle collisions and spin-1 hydrodynamics
- Include electromagnetic fields
  $\Longrightarrow$ dissipative spin magnetohydrodynamics