

Updates on the QCD Phase Diagram in a Magnetic Field

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6th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

Stony Brook, November 1-5, 2021

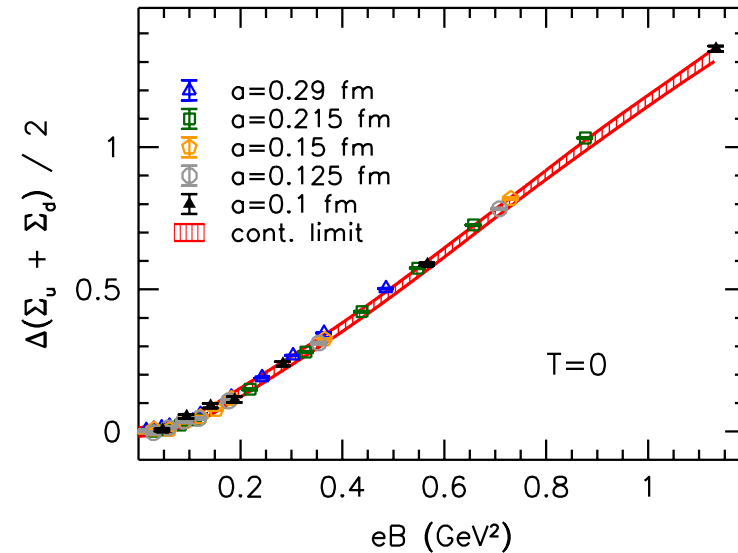
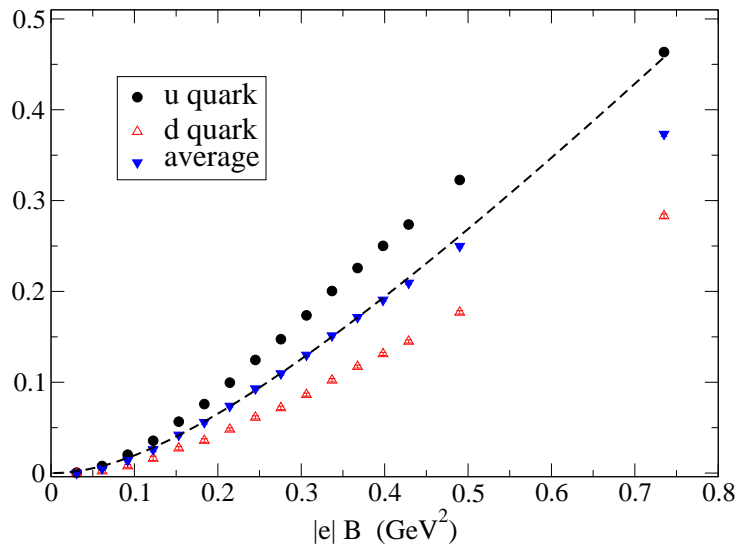
Mostly based on work done in collaboration with Lorenzo Maio, Francesco Sanfilippo and Alfredo Stanzione

OUTLINE

- **Known facts about the QCD in magnetic background: chiral and confining properties, phase diagram**
- **Updates on the properties at $T = 0$ and in the large B limit**
- **New results at $T \neq 0$ in the large B limit: updated phase diagram**

Known facts at $T = 0$ from lattice QCD simulations

Magnetic catalysis has been checked up to moderate values of eB



LEFT: increase in the light quark condensates, $N_f = 2$ QCD, $m_\pi \simeq 200$ MeV, unimproved staggered fermions (M. D'E and Francesco Negro, arXiv:1103.2080)

RIGHT: $N_f = 2+1$ QCD, improved staggered fermions, physical quark masses (G. Bali et al., arXiv:1206.4205)

early studies in pure gauge theories (Buividovich, Chernodub, Luschevskaya and Polikarpov, Phys. Lett. B 682, 484 (2010) and arXiv:1011.3795)

recently confirmed up to $eB \sim 3$ GeV² with HISQ staggered fermions

H. T. Ding, S. T. Li, A. Tomiya, X. D. Wang and Y. Zhang, arXiv:2008.00493

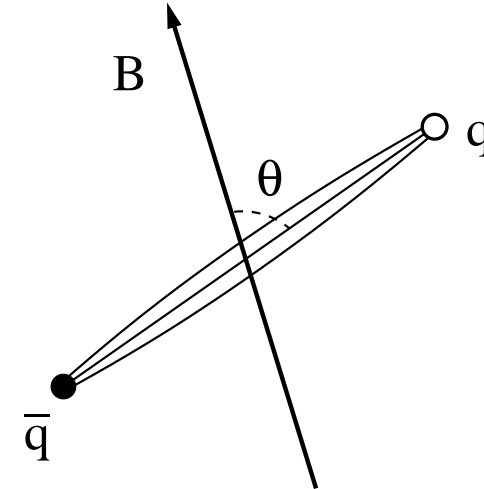
The magnetic field has a significant effect also on purely gluonic quantities

A brief review about the effect of B on confinement

The effects of a magnetic background on the static quark-antiquark potential have been studied in a couple of recent lattice studies

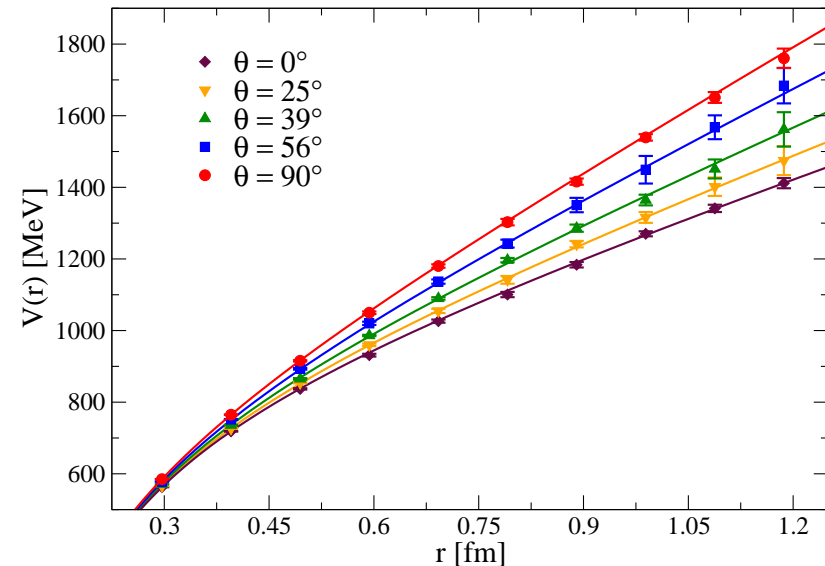
C. Bonati, MD, M. Mariti, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo, PRD 94, no.9, 094007 (2016), arXiv:1607.08160

C. Bonati, MD, M. Mariti, M. Mesiti, F. Negro and F. Sanfilippo, PRD 89, no.11, 114502 (2014), arXiv:1403.6094



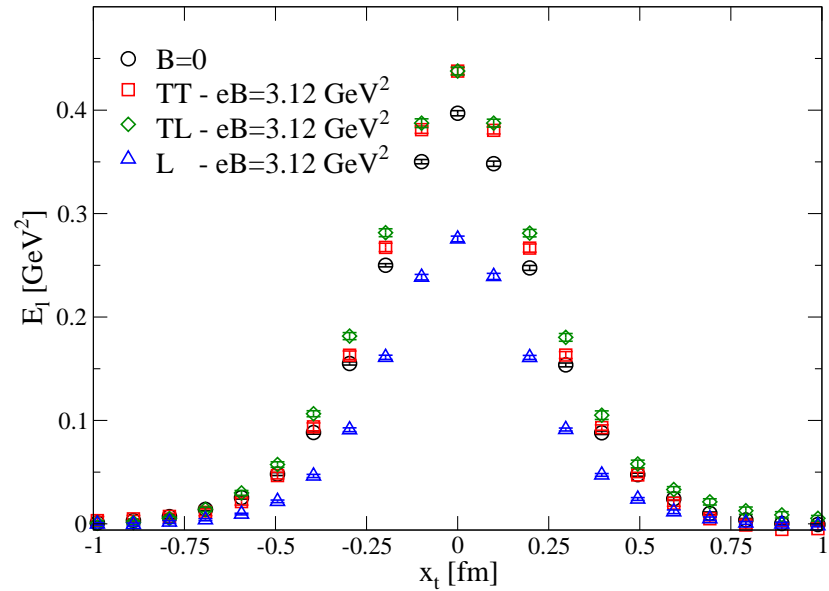
The potential becomes anisotropic, with a reduction of the string tension in the direction parallel to B , and an increase in the direction orthogonal to it

$$eB \simeq 1 \text{ GeV}^2 \implies$$



The modifications are clearly visible also at the level of the flux tube profile

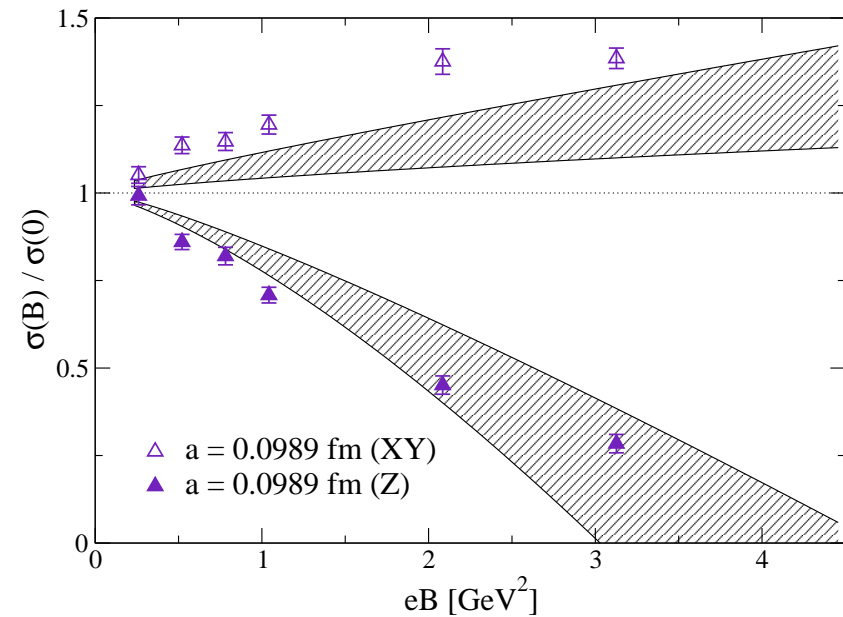
C. Bonati, S. Calì, M. D'Elia, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo, PRD 98, no.5, 054501 (2018), arXiv:1807.01673



The continuum extrapolated results for σ predicted a vanishing longitudinal string tension for $eB \sim 4 \text{ GeV}^2$

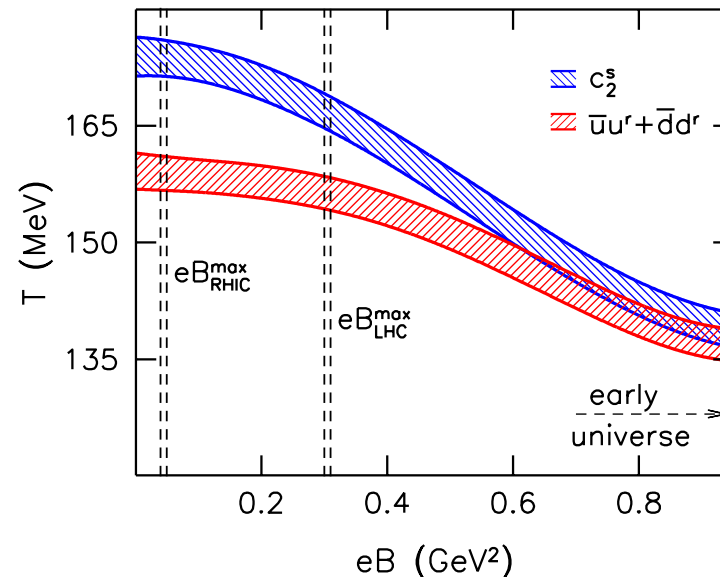
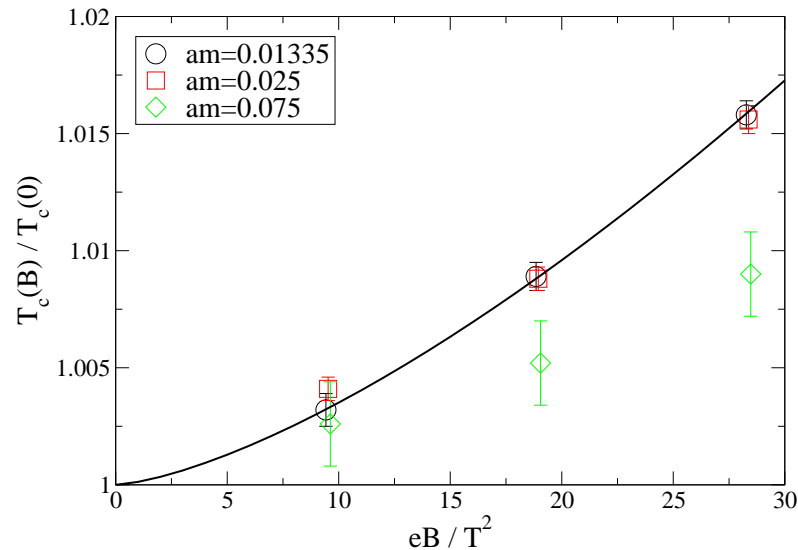
This is however, outside the range explored for the continuum extrapolation, $eB \lesssim 1 \text{ GeV}^2$

Is that really true?



Known facts at $T \neq 0$ from lattice QCD simulations

Early lattice results on the QCD phase diagram in a magnetic background produced contrasting results: $T_c(B)$ increasing vs decreasing



$N_f = 2$ standard staggered fermions, plaquette gauge action, $m_\pi \simeq 200$ MeV, $a \simeq 0.3$ fm
 MD, S. Mukherjee and F. Sanfilippo, PRD 82, 051501 (2010), arXiv:1005.5365

$N_f = 2 + 1$ stout improved staggered fermions, Symanzik improved gauge action, physical quark masses, continuum extrap.

G. S. Bali *et al*, JHEP 02, 044 (2012), arXiv:1111.4956

Decreasing behaviour confirmed by later studies, together with a substantial strengthening of the transition

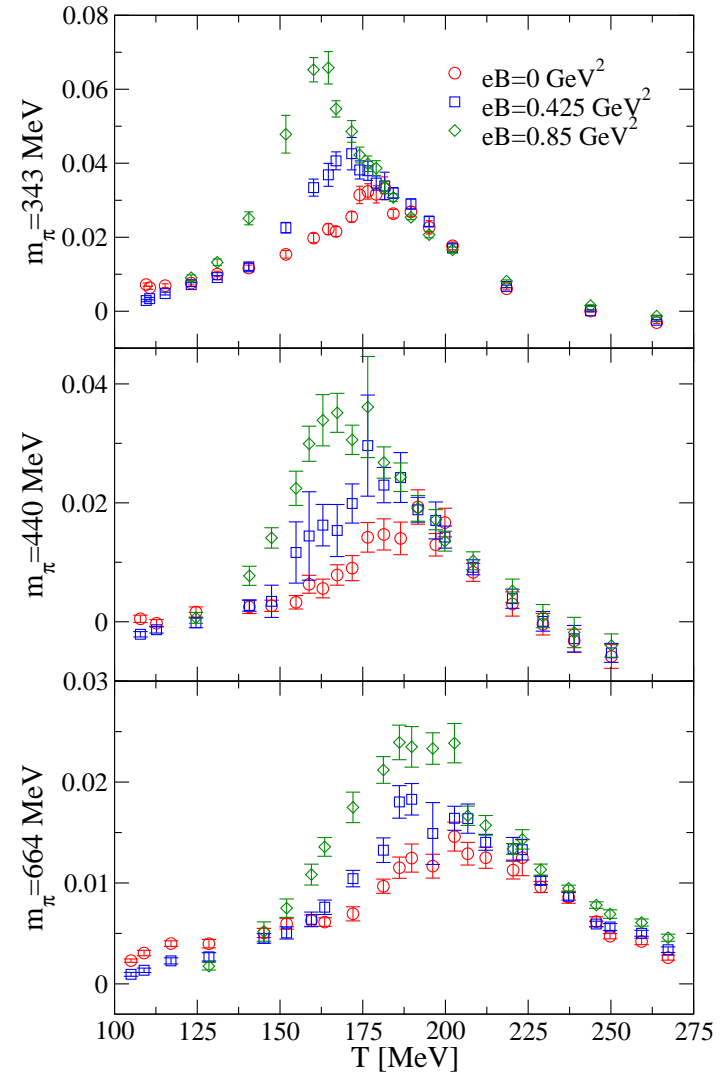
Early results affected by lattice artefacts, the decrease of T_c confirmed also for heavier pion masses

(MD, F. Manigrasso, F. Negro and F. Sanfilippo, PRD 98, no.5, 054509 (2018), arXiv:1808.07008)

(G. Endrodi, M. Giordano, S. D. Katz, T. G. Kovács and F. Pittler, JHEP 07, 007 (2019), arXiv:1904.10296)

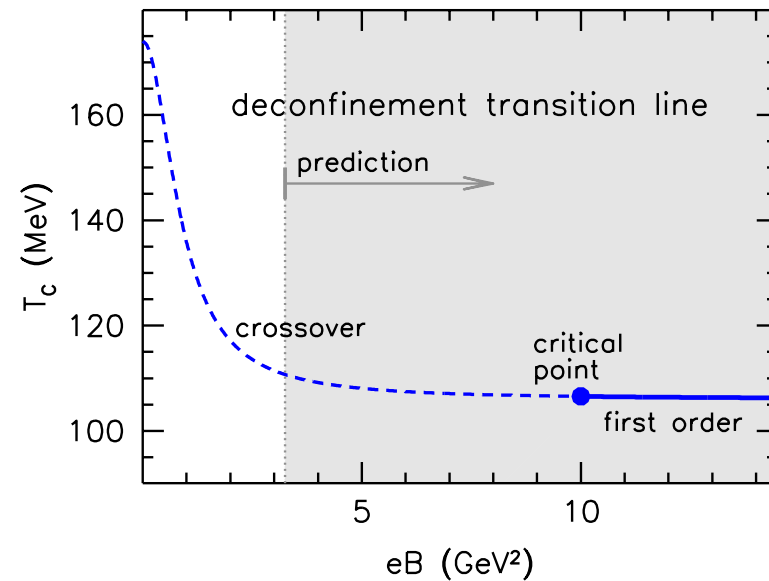
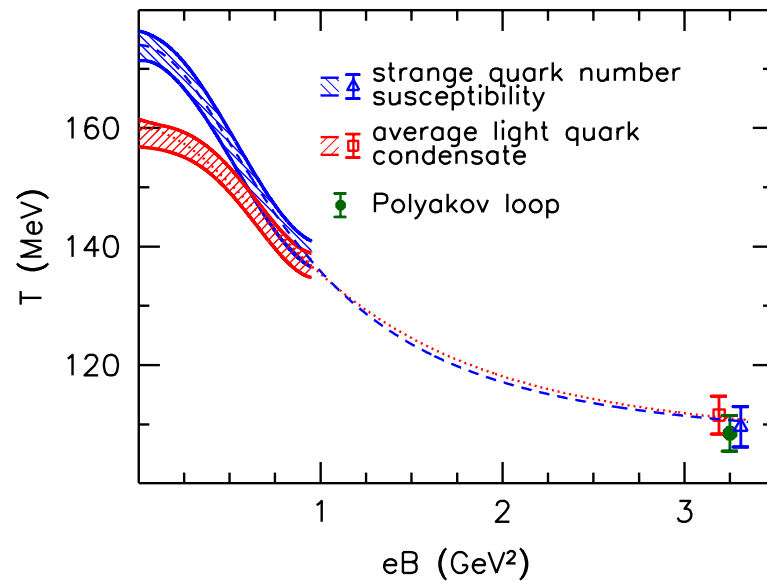
Renormalized chiral susceptibility for different temperatures, magnetic fields and pion masses

T_c decreases with B for all pion masses, likely up to the quenched limit, and the transition strengthens



Later studies have extended lattice simulations of the phase diagram up to $eB \simeq 3 \text{ GeV}^2$ and speculated about the possible presence of a critical endpoint for $eB \simeq 10 \text{ GeV}^2$, where the transition would turn into first order

likely relevant for the Early Universe



from G. Endrodi, JHEP 07, 173 (2015) [arXiv:1504.08280 [hep-lat]]

Direct confirmations of first order only obtained with unimproved staggered fermions

H. T. Ding, C. Schmidt, A. Tomiya and X. D. Wang, PRD 102, 054505 (2020) [arXiv:2006.13422 [hep-lat]].

Looking for updates

- is there a critical magnetic field B_c , at $T = 0$, where confining properties of QCD get disrupted? (anisotropic deconfinement?)
- What is fate of $T_c(B)$ for large magnetic fields? And what the fate of the order of the phase transition?

Recently, we started some efforts in this direction:

MD, L. Maio, F. Sanfilippo, A. Stanzione, arXiv:2109.07456 and work in progress

- $N_f = 2 + 1$ QCD with physical quark masses and two large magnetic fields, $eB = 4$ and 9 GeV^2
- 2-level stout improved stag. fermions, Symanzik tree level improved gauge action
- three different lattice spacings, $a = 0.057, 0.086, 0.114 \text{ fm}$, spatial size mostly kept fixed around 3 fm
- additional UV effects expected from large B : maximum flux across a plaquette sets $eB \lesssim 2\pi/a^2 \sim 18 \text{ GeV}^2$ for $a = 0.114 \text{ fm}$, so 9 GeV^2 is borderline ...

Updates on $T = 0$ results

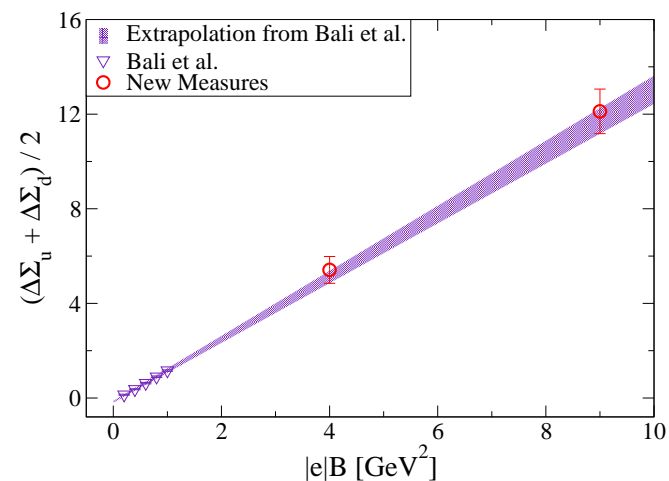
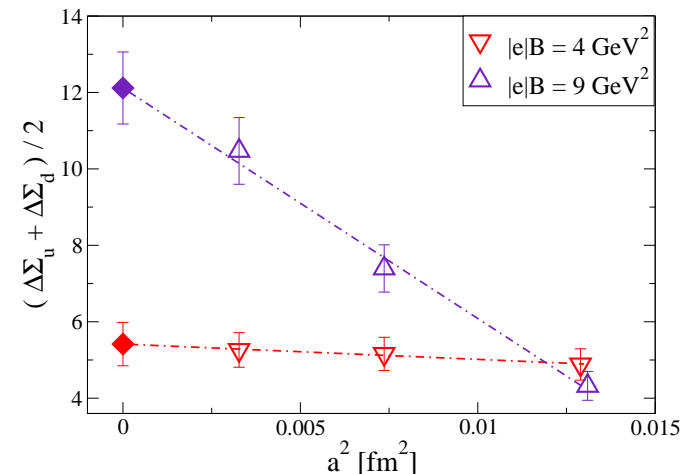
MD, L. Maio, F. Sanfilippo, A. Stanzione, arXiv:2109.07456

increase of the renormalized light chiral condensate simulations on $24^3 \times 48$, $32^3 \times 64$, $48^3 \times 96$ lattices

lattice artefacts significant for $eB = 9 \text{ GeV}^2$, minimum phase around a plaquette is $\simeq 2\pi/3$ for the up quark on the coarsest lattice

Nevertheless, after continuum extrapolation magnetic catalysis \sim linear with eB confirmed up to $eB \sim 9 \text{ GeV}^2$

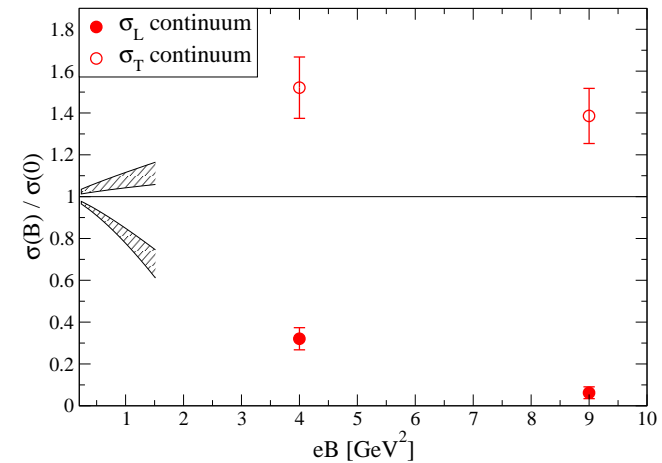
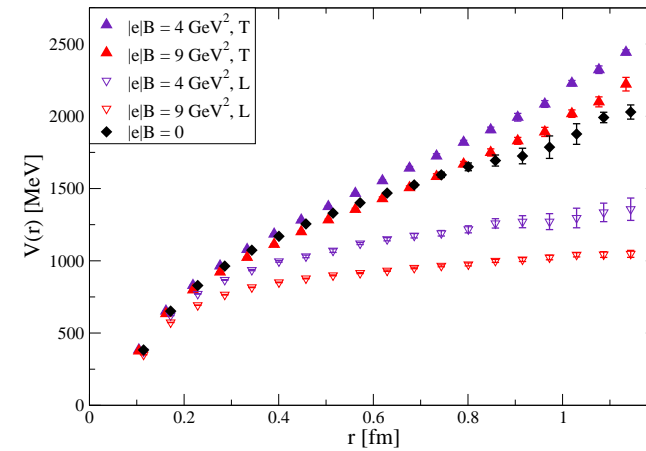
similar results up to $eB \sim 3 \text{ GeV}^2$ with HISQ staggered fermions H. T. Ding, S. T. Li, A. Tomiya, X. D. Wang and Y. Zhang, arXiv:2008.00493



Results for the static potential (longitudinal vs transverse) on the finest lattice

Contrary to previous extrapolations, the longitudinal string tension does not seem to vanish, neither at 4 GeV^2 , nor at 9 GeV^2

After proper continuum extrapolation, the transverse string tension seems to saturate, the longitudinal string tension is strongly suppressed, could vanish at some larger magnetic field



Updates on the finite B - finite T Phase Diagram

MD, L. Maio, F. Sanfilippo, A. Stanzione, in progress

Finite T simulations performed at fixed cut-off

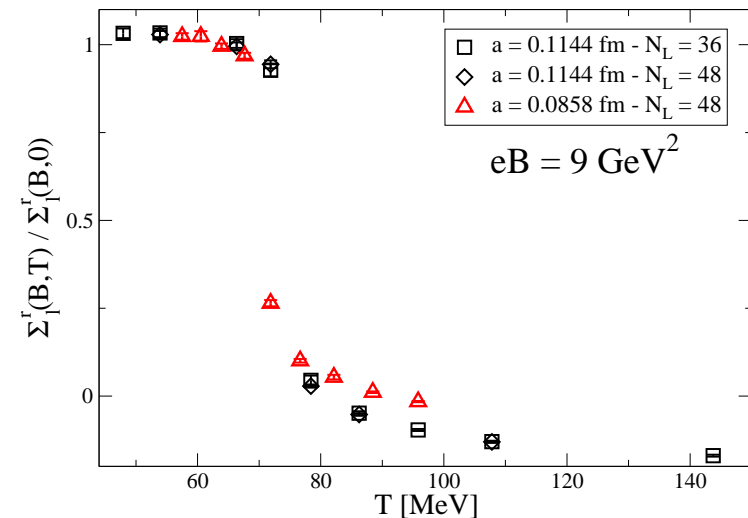
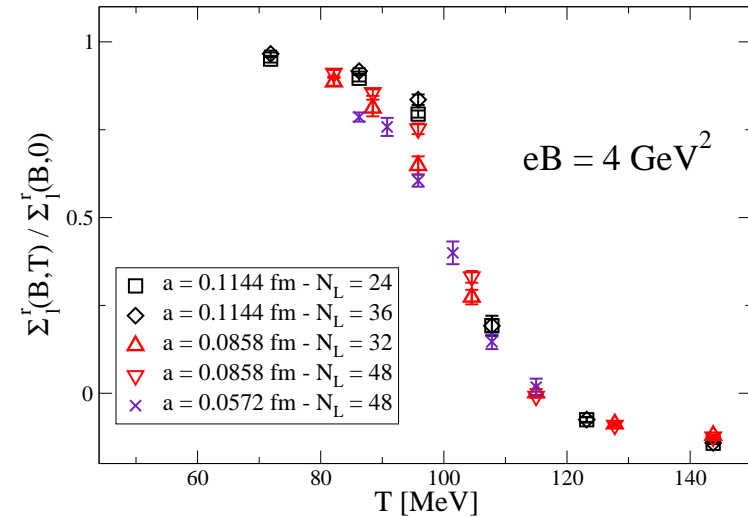
$$T = 1/(N_t a)$$

keeping a fixed and changing N_t , three sets of lattice spacings

Renormalized light chiral condensate, normalized by $T = 0$ values

the inflection point moves to lower and lower temperatures as eB increases

at $eB = 9 \text{ GeV}^2$ a jump seems to appear (first order transition?)



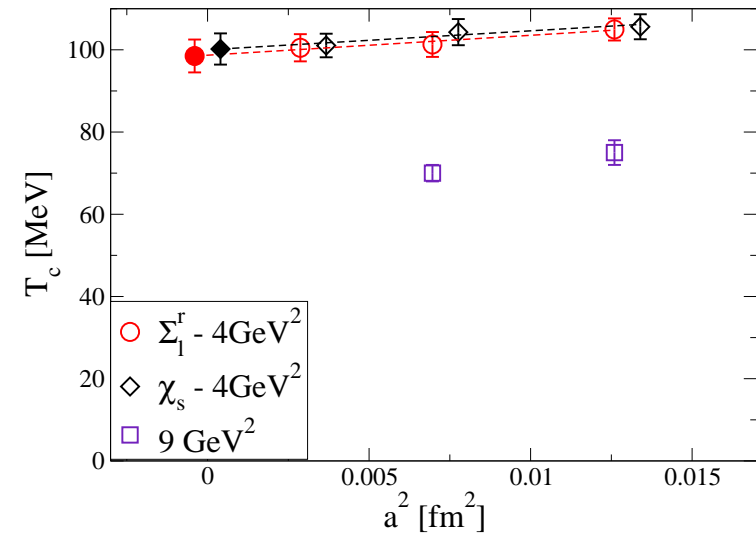
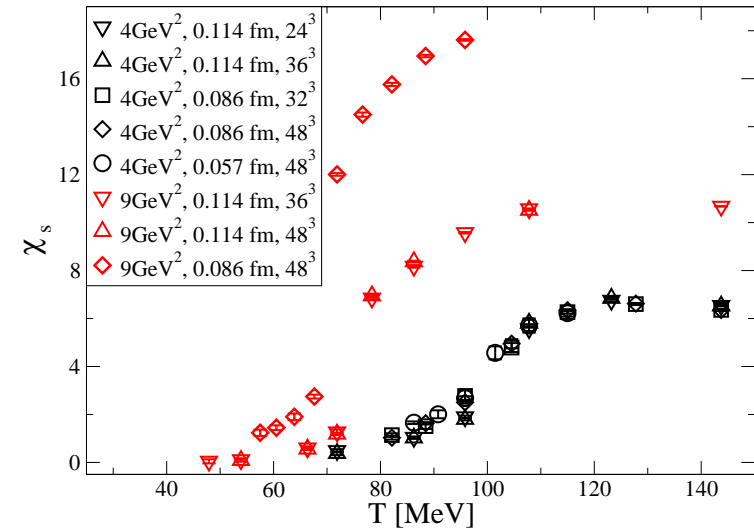
Similar results from the strange quark number susceptibility

large cutoff effects at 9 GeV^2 , but the jump is stable or even stronger on the finer lattice

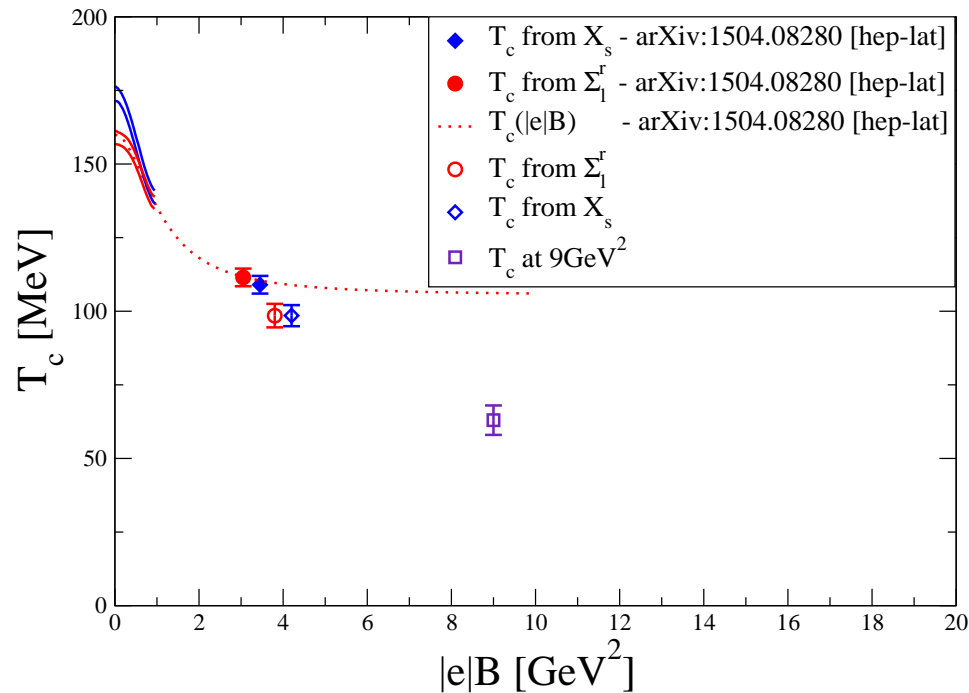
We also observe a significant increase of quark number fluctuations with eB , confirming results reported in

H. T. Ding, S. T. Li, Q. Shi, A. Tomiya, X. D. Wang and Y. Zhang, arXiv:2011.04870

The critical temperature extrapolates smoothly to the continuum limit and is observable-independent



Updated phase diagram - a first sketch



main observations:

- the steady decrease of T_c continues ... hitting the ground somewhere?
- moreover, it seems that at 9 GeV^2 the transition is first order, but we just see a large jump on discrete temperature mesh. We need to check more carefully ...

Finite size scaling analysis around 9 GeV²

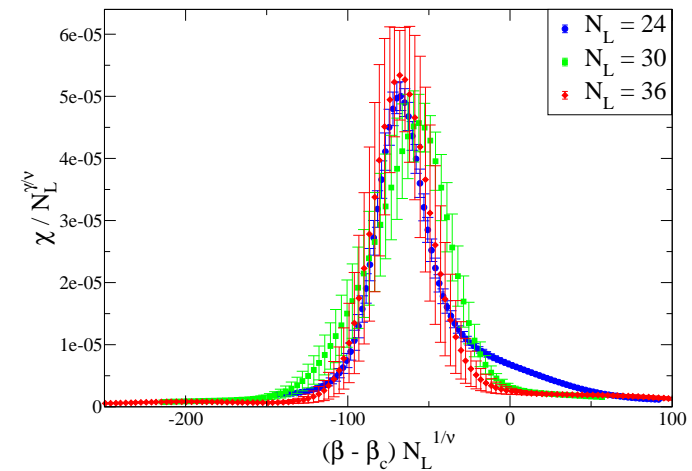
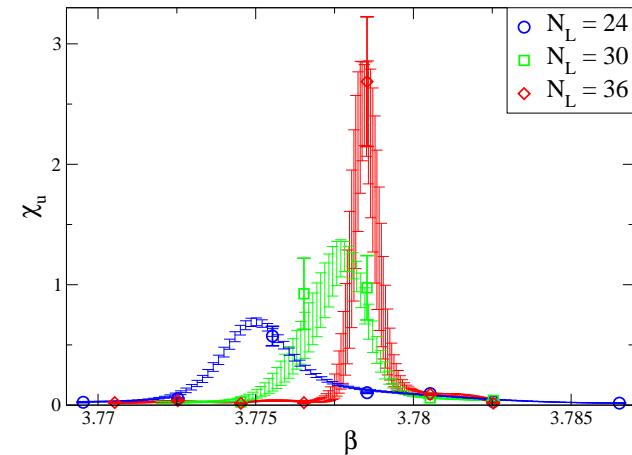
In order to fine tune T and perform a multi-histogram analysis, we give up the fixed cut-off and the line-of-constant-physics setup, changing just the inverse gauge coupling β around there

This is of course irrelevant in order to assess if there is a first order transition around there

FSS analysis for the unrenormalized, disconnected up-quark chiral susceptibility

$$\chi_u / N_L^{\gamma/\nu} = \phi \left((\beta - \beta_c) N_L^{1/\nu} \right)$$

correct scaling observed with first order indexes, $\nu = 1/3$ and $\gamma = 1$



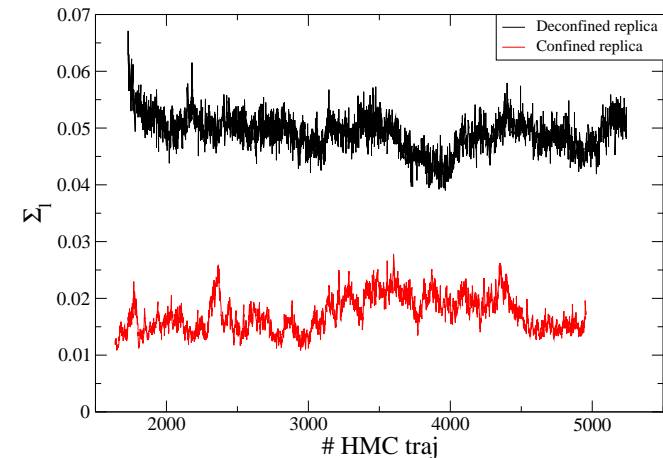
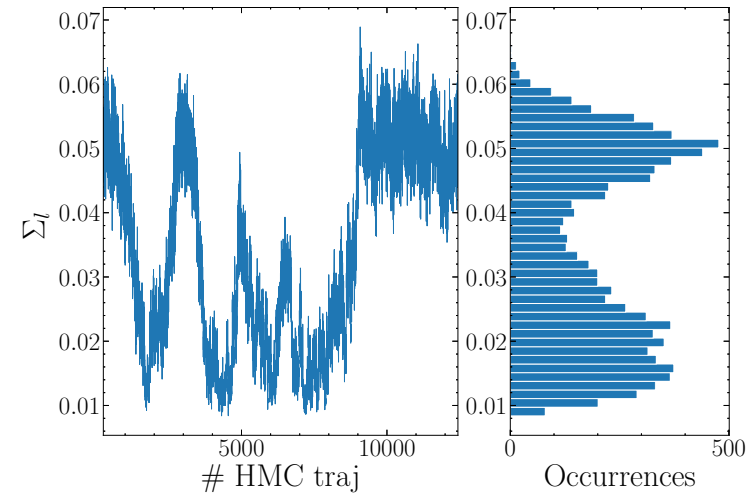
Other smoking guns for a first order transition at 9 GeV²

Double peak in the distribution of the light chiral condensate observed at the transition point on a $24^3 \times 20$ lattice

Similar bistability observed in other observables, including the pure gauge action

On a larger lattice, $36^3 \times 20$, twin pair of runs starting from different sides of the transition with identical parameters stay separated

⇒ **strong metastability!**



What about the confining properties on the two sides of the transition?

Static potential at $T = 86$ MeV and $eB = 4$ and 9 GeV^2 , i.e. on the two sides of the transition

The confined phase is still strongly anisotropic, $\sqrt{\sigma_T} = 475(15)$ MeV, $\sqrt{\sigma_L} = 215(15)$ MeV

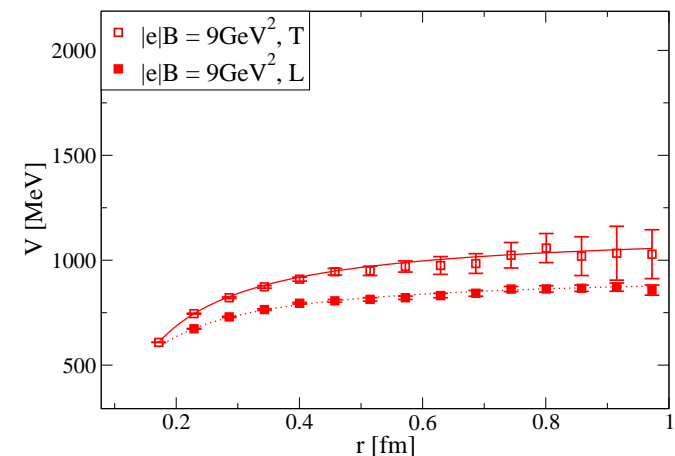
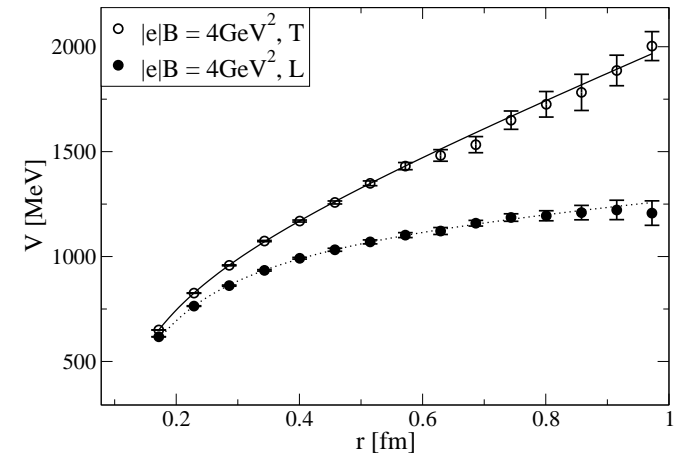
In the deconfined phase, the potential is well fitted by a purely Coulombic term.

Confinement before the transition is strongly anisotropic, but deconfinement is isotropic.

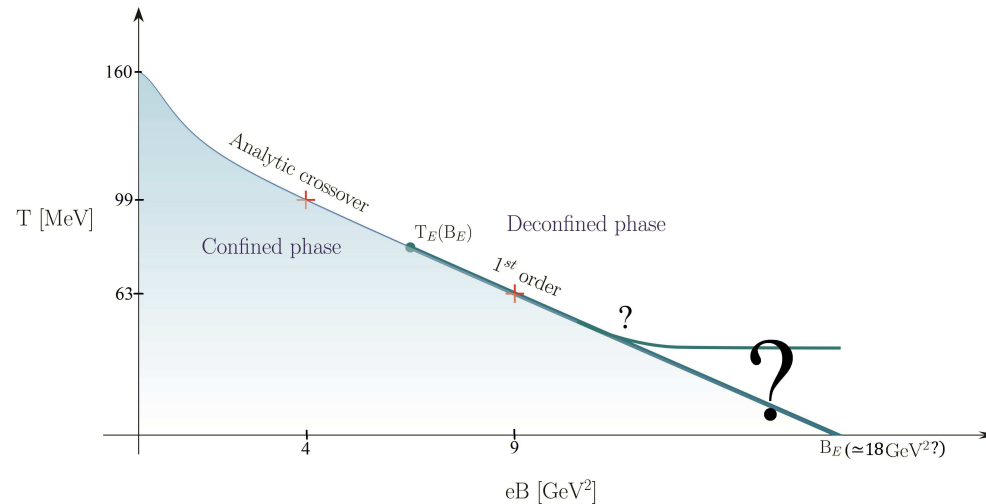
Anyway strong anisotropies likely affect other observables in the deconfined phase, e.g.:

electric conductivity

N. Astrakhantsev et al., arXiv:1910.08516



Updated phase diagram: new facts and new speculations



- T_c decreases at least down to 60 MeV
- The transition becomes first with a critical endpoint $65 \text{ MeV} < T_E < 95 \text{ MeV}$, $4 \text{ GeV}^2 < eB_E < 9 \text{ GeV}^2$
- The transition at large eB is deconfining, with the string tension staying anisotropic in the confined phase and vanishing isotropically in the deconfined phase
- Does $T_c(B)$ hit the ground at some large $eB_c \sim 20 \text{ GeV}^2$, or not? Would that be a natural scale for $N_f = 2 + 1$ QCD ?

Perspective

- The critical endpoint in the $B - T$ plane is likely not interesting for heavy ion collisions (eB_E too large), but could be extremely interesting for the physics of the Early Universe
- Future simulations could locate the critical endpoint more precisely
- Properties of the two metastable phases should be better clarified by measuring other interesting observables
- Studies at $T = 0$ and larger magnetic field could help clarifying if a finite B_c exists along that axis, and if $T_c(B)$ indeed hits the ground at B_c