NuclearScience Computing CenteratCCNU

Conserved charge fluctuations in strong magnetic fields from lattice QCD

Heng-Tong Ding
Central China Normal University

HTD, S.-T. Li, Q. Shi, X.-D. Wang, arXiv: 2104.06843, Eur.Phys.J.A 57 (2021) 6
HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 126 (2021) 082001

The 6th international conference on Chirality, Vorticity and Magnetic field in Heavy Ion collisions

1-5 Nov, 2021

Strong magnetic fields

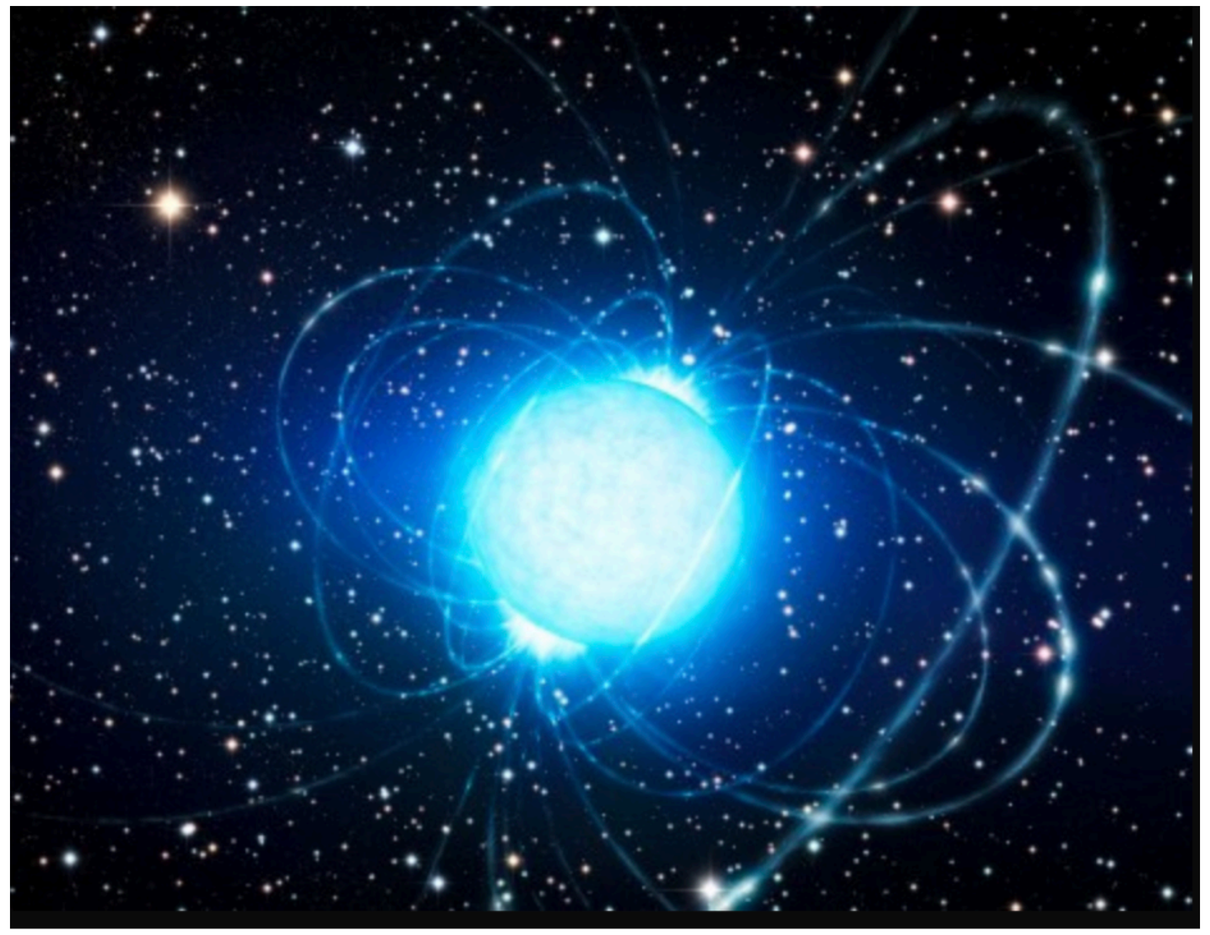
Earth

0.6 Gauss

A common, hand-held magnet

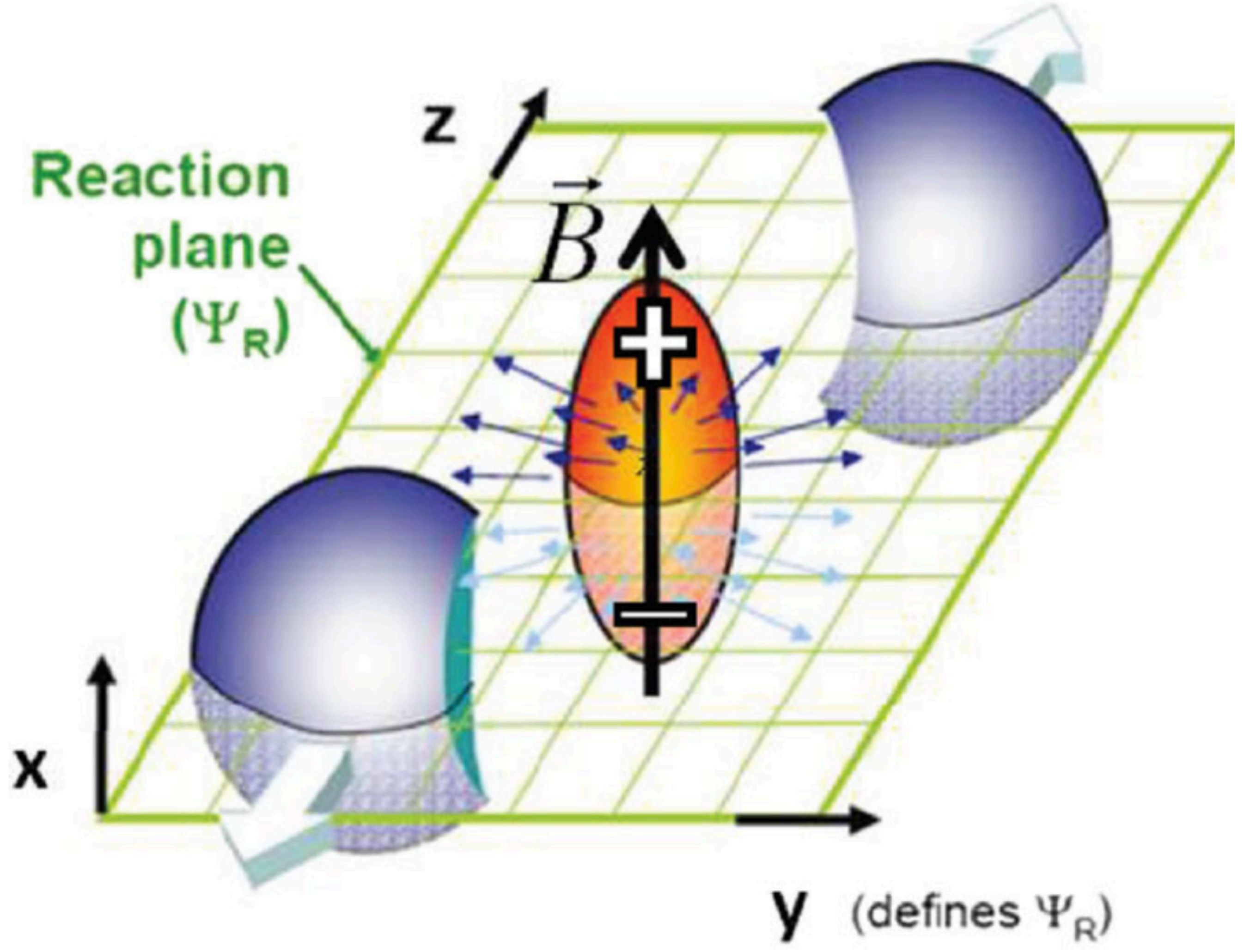
100 Gauss

Magnetar



10^{15} Gauss

Heavy-Ion collision



$10^{17\text{-}18}$ Gauss

$$\Lambda_{QCD}^2 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss}$$

$$1 \text{ Gauss} = 1.95 \times 10^{-14} \text{ MeV}^2$$

Lattice QCD in strong magnetic fields

No sign problem

- B pointing to the z direction & Gauge link multiplied by a U(1) factor

$$u_x(n_x,n_y,n_z,n_\tau) = \begin{cases} \exp[-iqa^2BN_xn_y] & (n_x = N_x - 1) \\ 1 & (\text{otherwise}) \end{cases}$$

$$u_y(n_x,n_y,n_z,n_\tau) = \exp[iqa^2Bn_x],$$

$$u_z(n_x,n_y,n_z,n_\tau) = u_t(n_x,n_y,n_z,n_\tau) = 1.$$

M. D'Elia, S. Mukherjee, F. Sanfilippo, Phys.Rev.D 82 (2010) 051501,
G. Bali et al., JHEP02(2012)044,
…

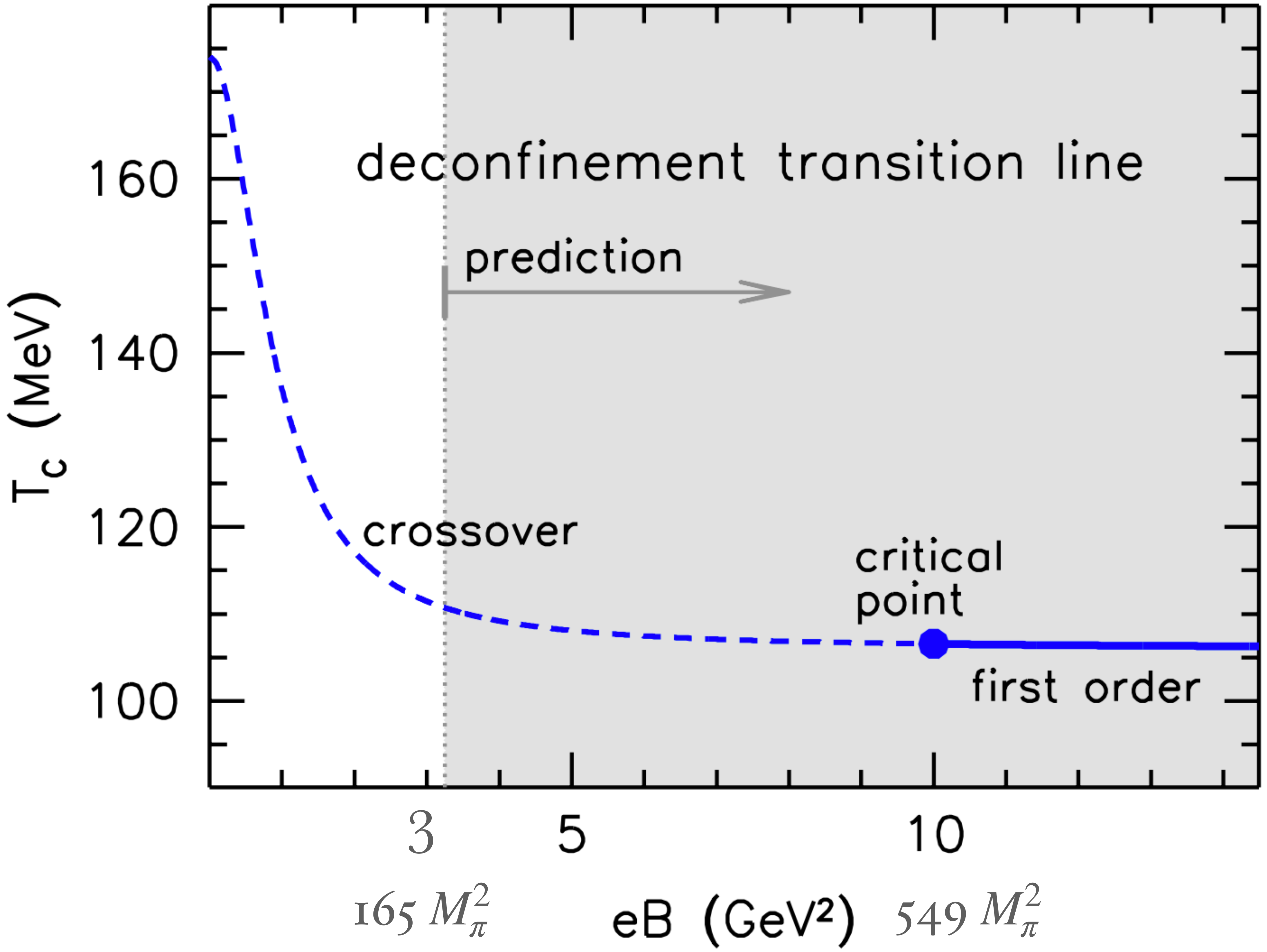
- Quantization of the magnetic field

$$qB = \frac{2\pi N_b}{N_xN_y}a^{-2} \xrightarrow{q_u=2/3e,\ q_d=-1/3e,\ q_s=-1/3e} eB = \frac{6\pi N_b}{N_xN_y}a^{-2}$$

a^{-1}: inverse lattice spacing
N_b: magnetic flux
N_x, N_y, N_z: number of lattice points in x,y,z directions

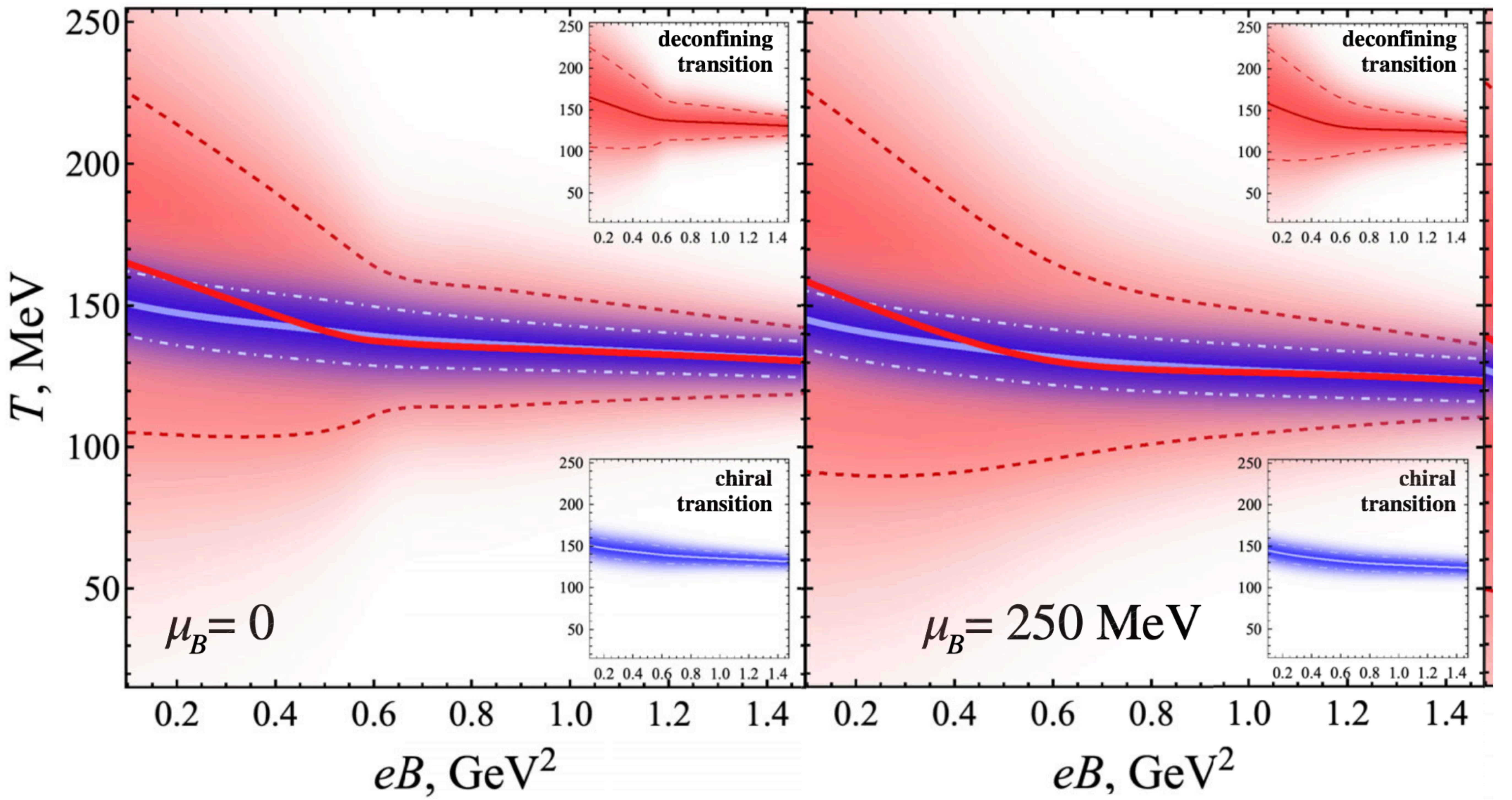
eB induced effects:
Chiral magnetic effects, QCD critical end point…

T-*eB* plane



G. Endrodi, JHEP 1507(2015) 173

Transition at nonzero *eB* and μ_B



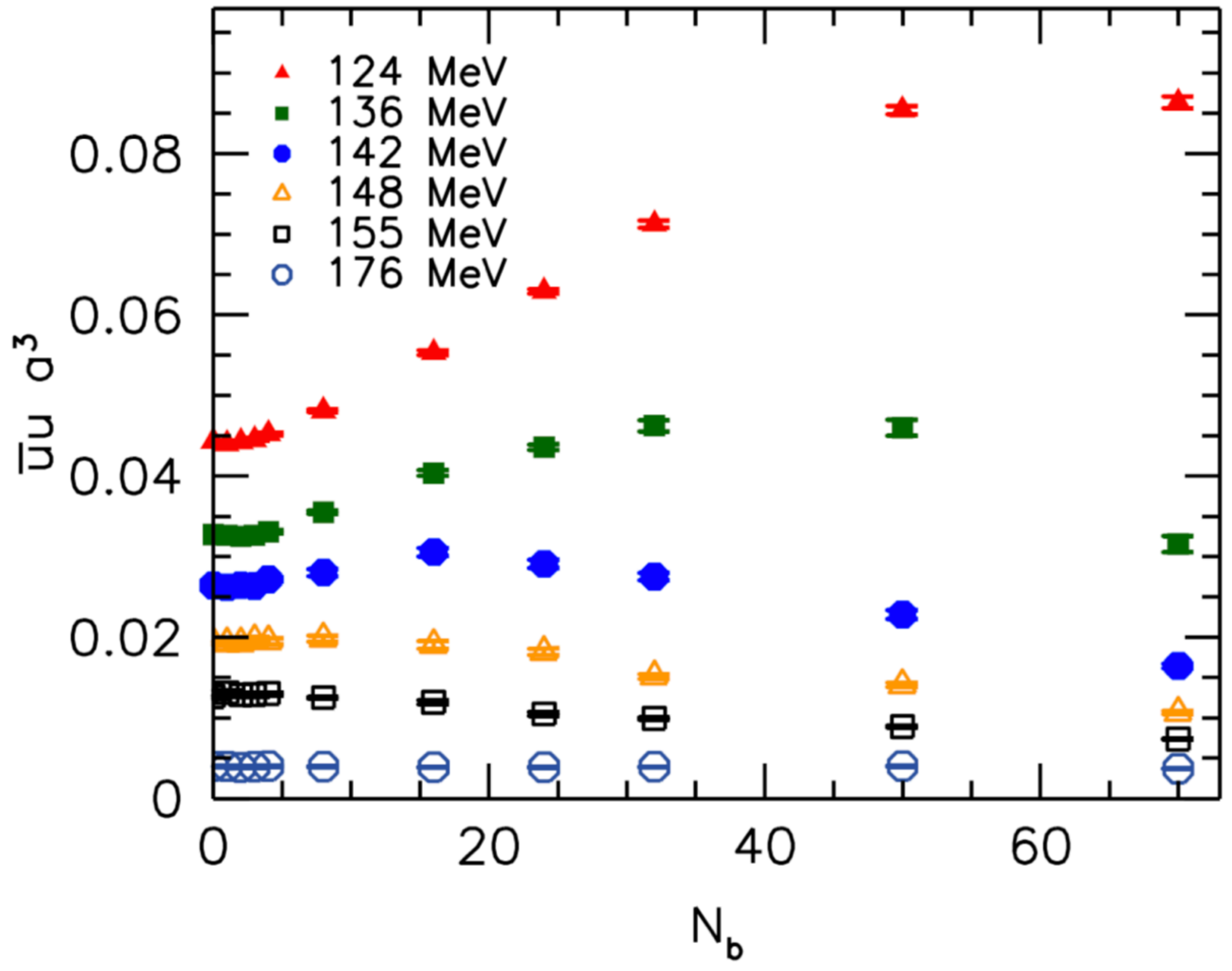
V. Braguta et al., Phys.Rev.D 100 (2019) 114503

Inverse magnetic catalyses and Tpc

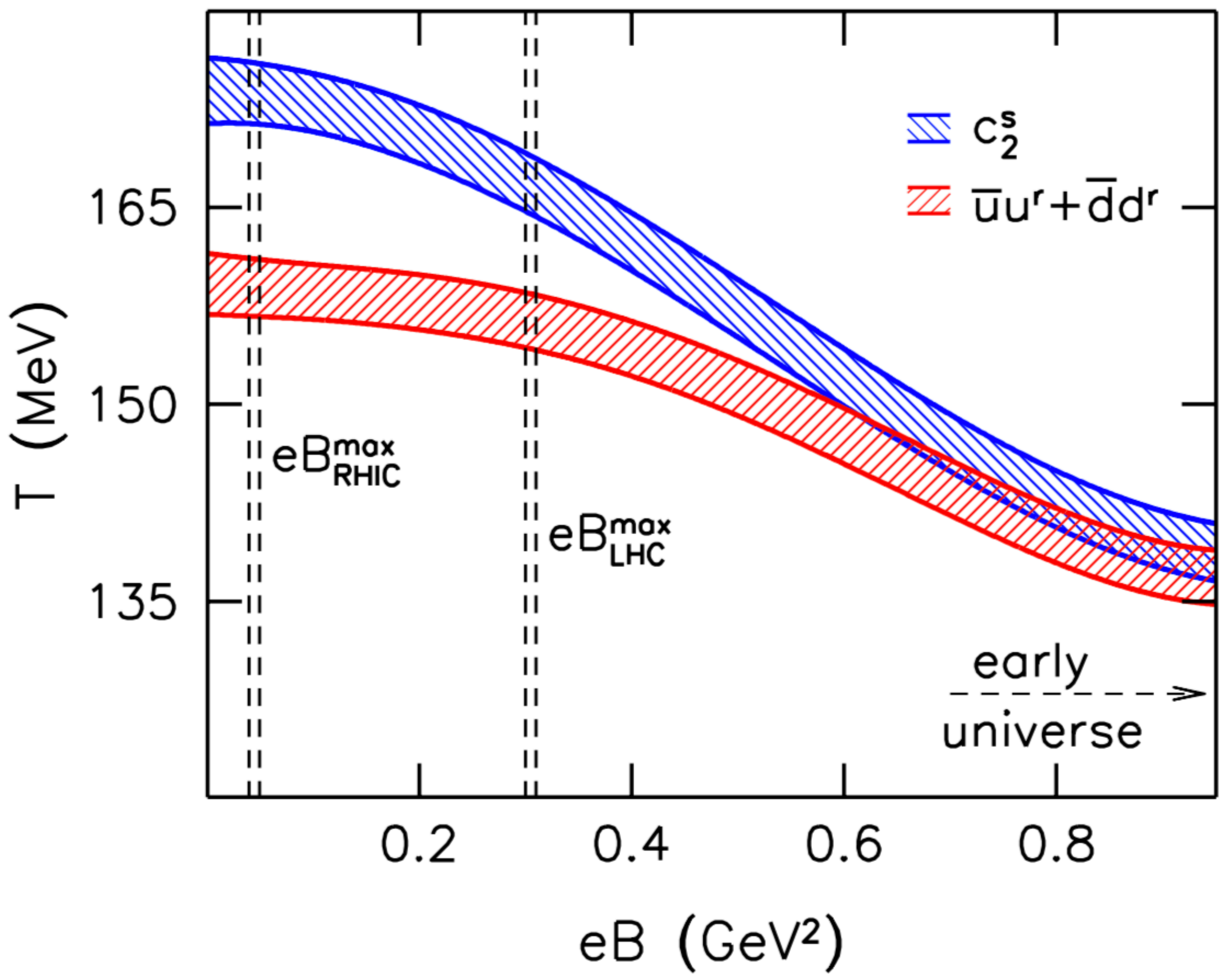
Continuum extrapolated lattice QCD results with physical pion mass

Bali et al., JHEP02(2012)044

Inverse magnetic catalysis (IMC)



$eB\uparrow\ T_{pc}\downarrow$



See recent reviews e.g.
Gaoqing Cao,
arXiv:2103.00456
Andersen et al., Rev. Mod. Phys. 88(2016)02001

Reduction of T_{pc} always associated with IMC? Not necessarily! See more in Massimo's talk

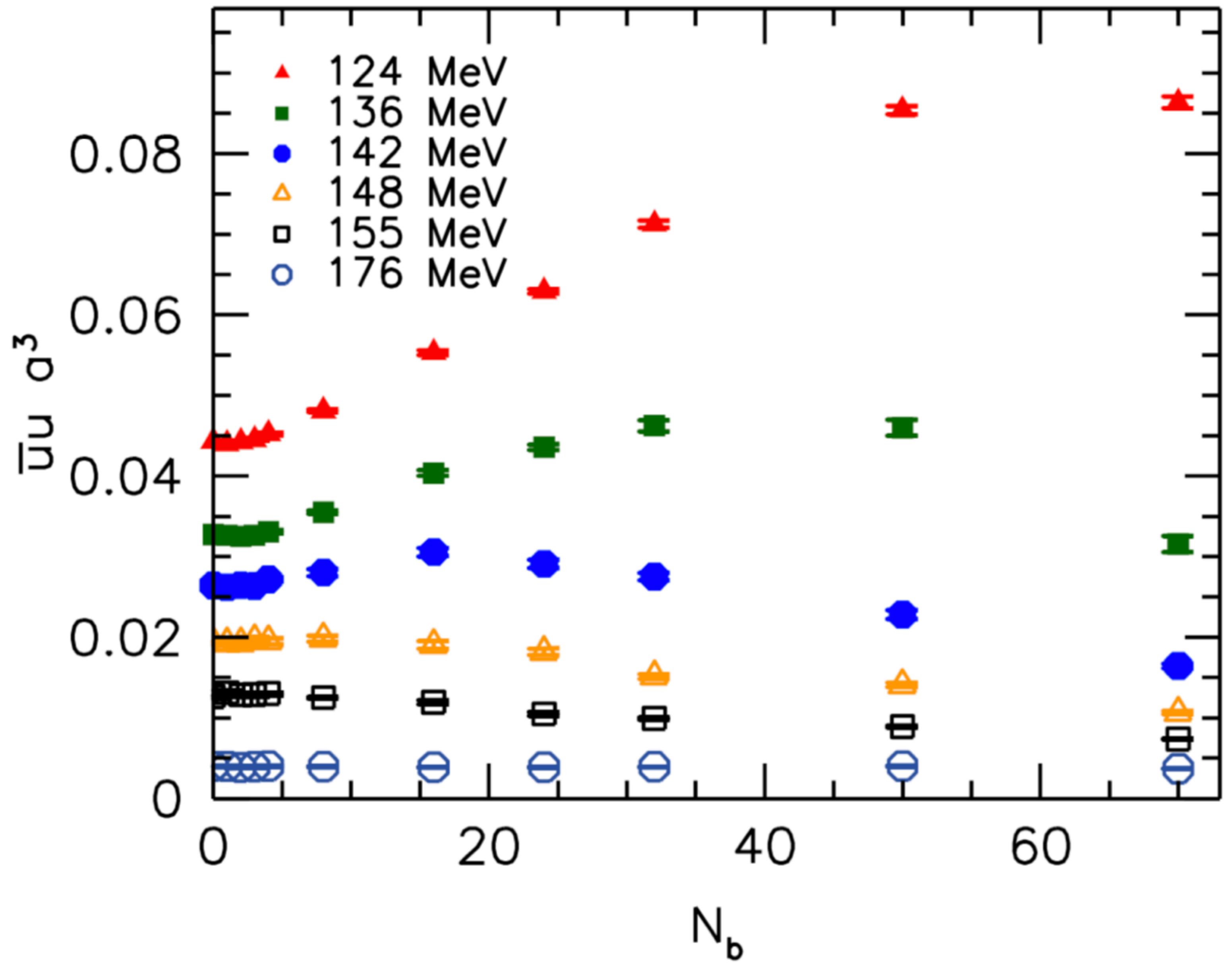
Role of hadrons?

Inverse magnetic catalyses and Tpc

Continuum extrapolated lattice QCD results with physical pion mass

Bali et al., JHEP02(2012)044

Inverse magnetic catalysis (IMC)



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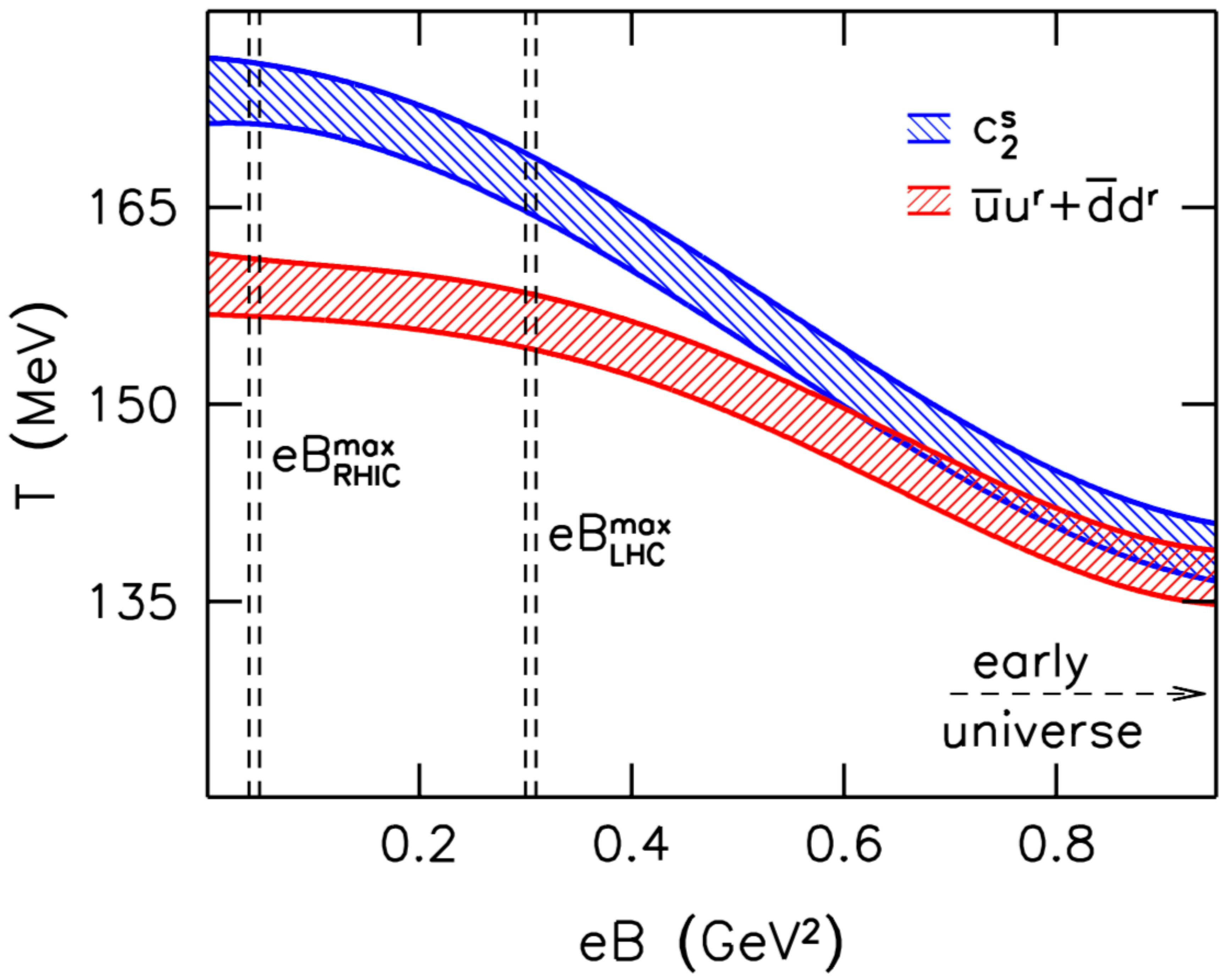
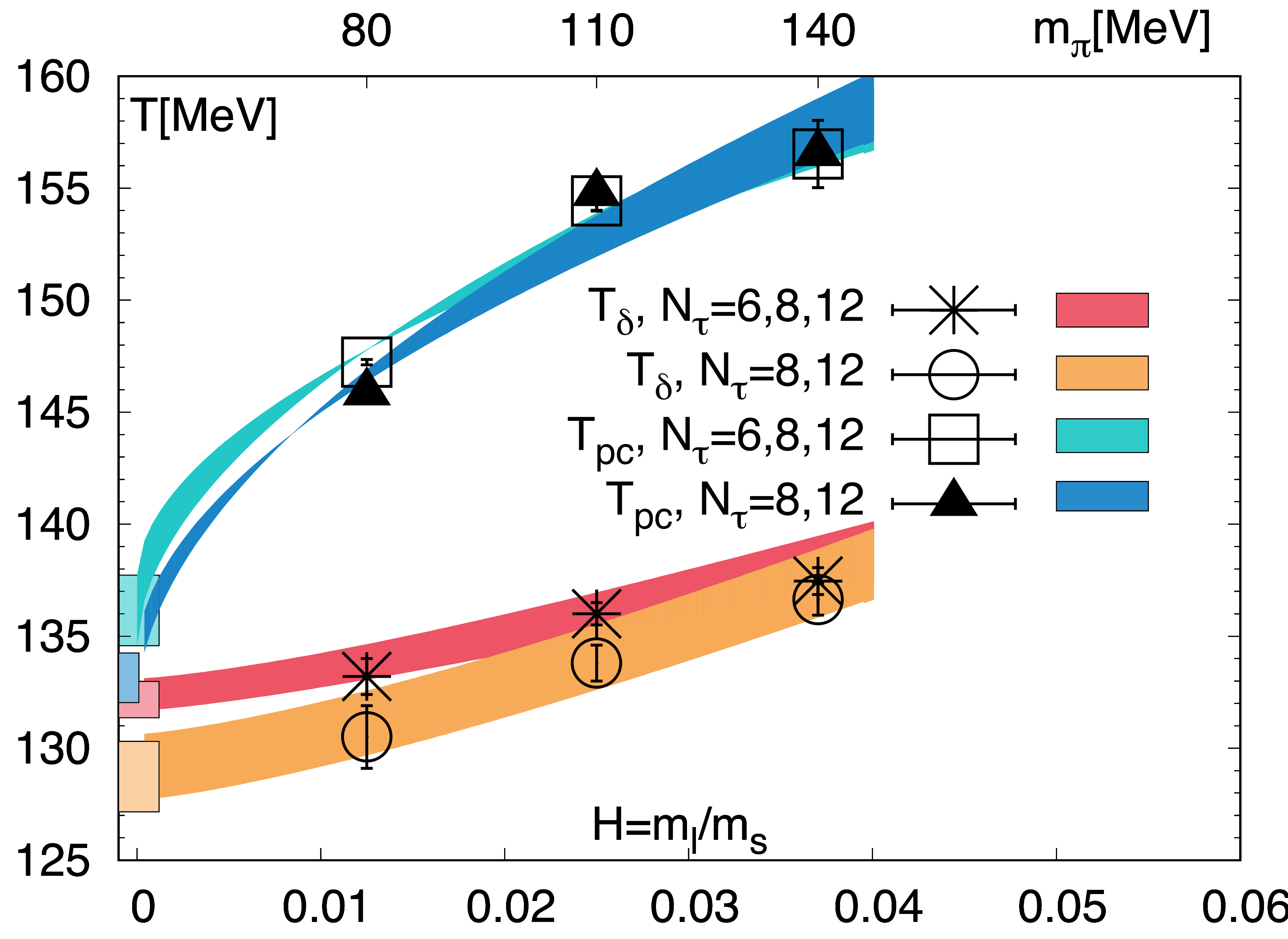


eB=0, N_f=2+1 QCD



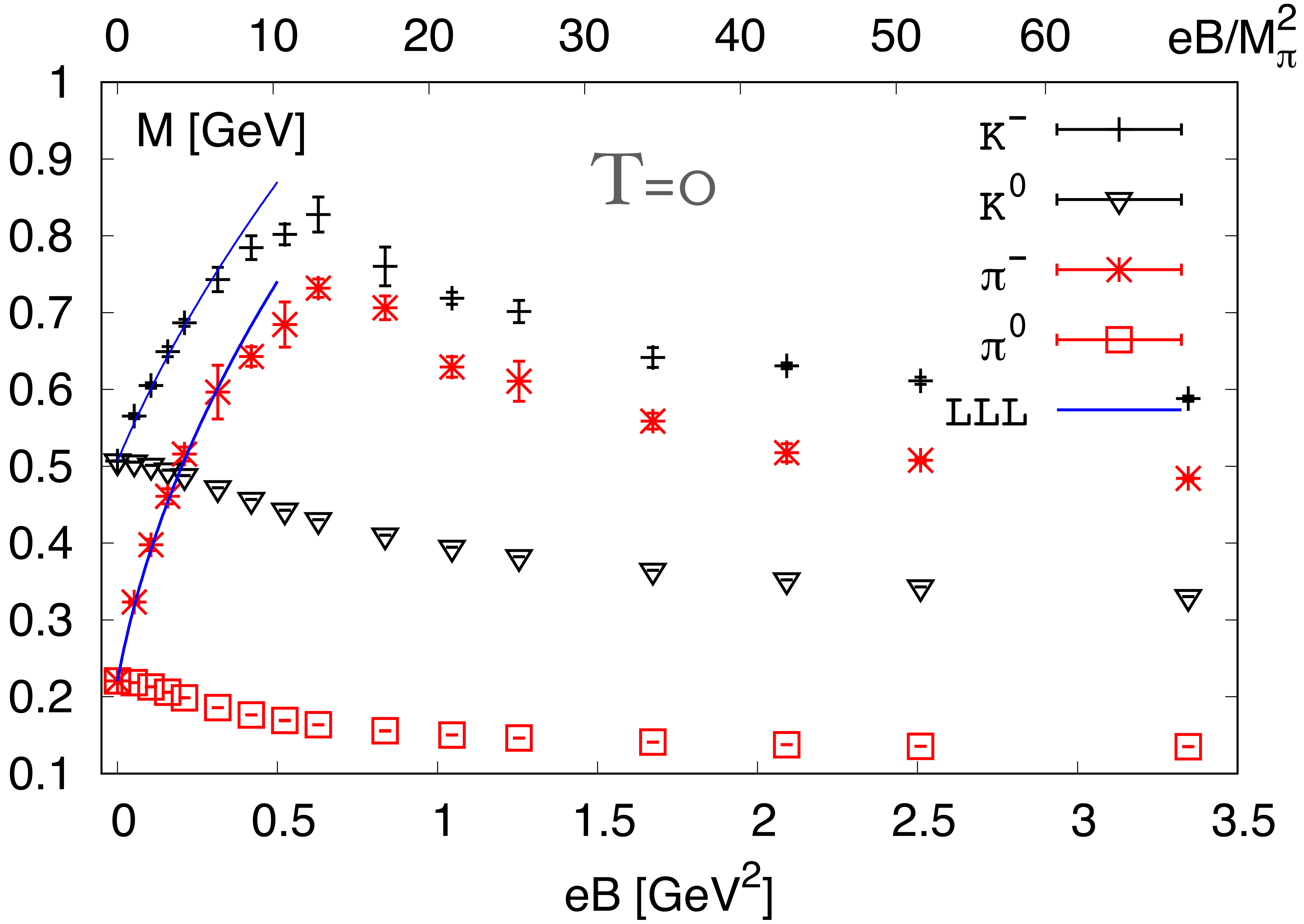
HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys. Rev. Lett. 123 (2019) 062002
HTD, arXiv:2002.11957

Reduction of T_{pc} always associated with IMC? Not necessarily! See more in Massimo's talk

Role of hadrons?

Masses of $\pi^{0,\pm}$ and $K^{0,\pm}$ and energy density

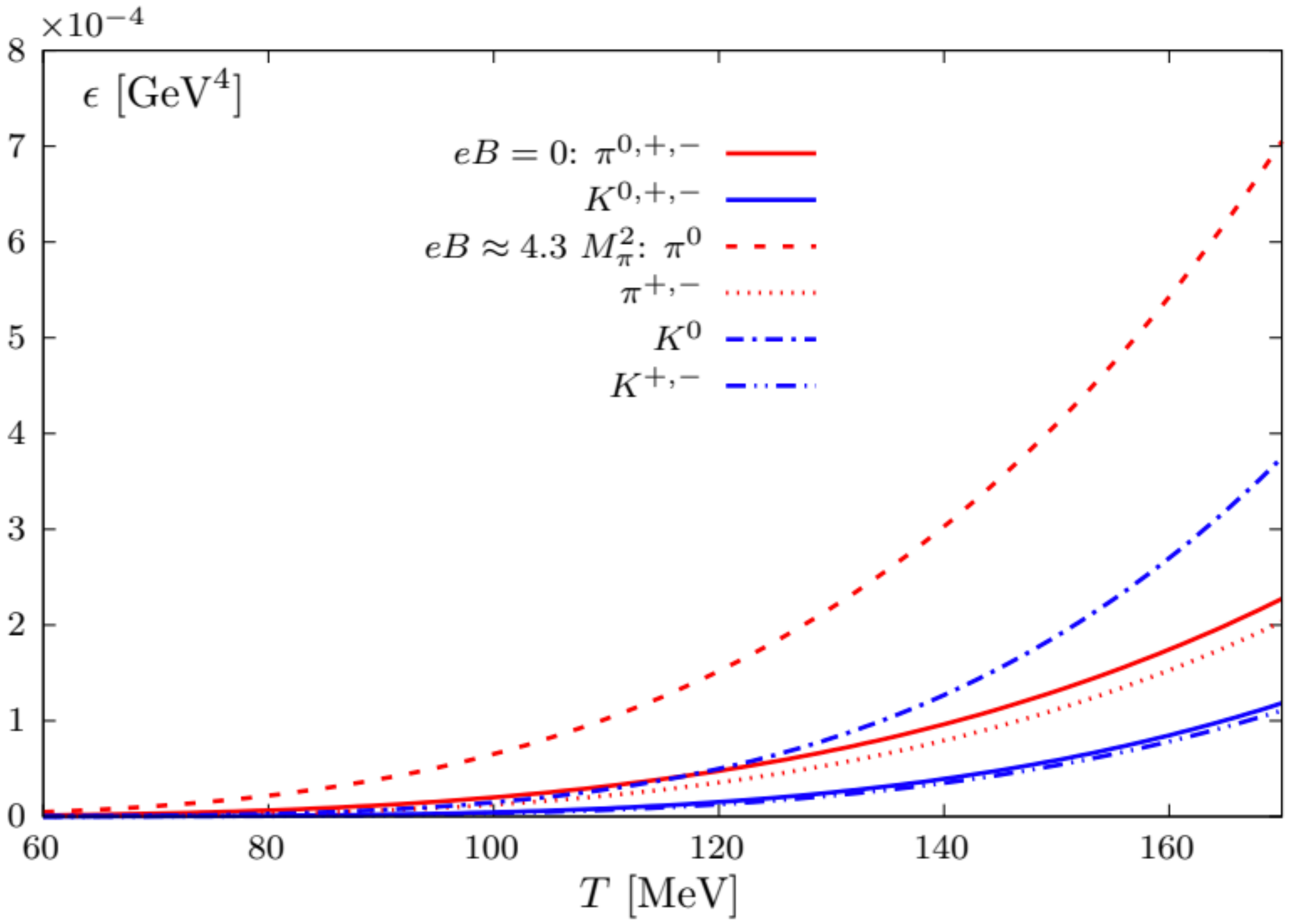
$N_{f=2+1}$ QCD, $M_\pi(eB=0) \approx 220$ MeV,
$32^3 \times 96$ lattices with $a^{-1} \approx 1.7$ GeV and HISQ action



HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001

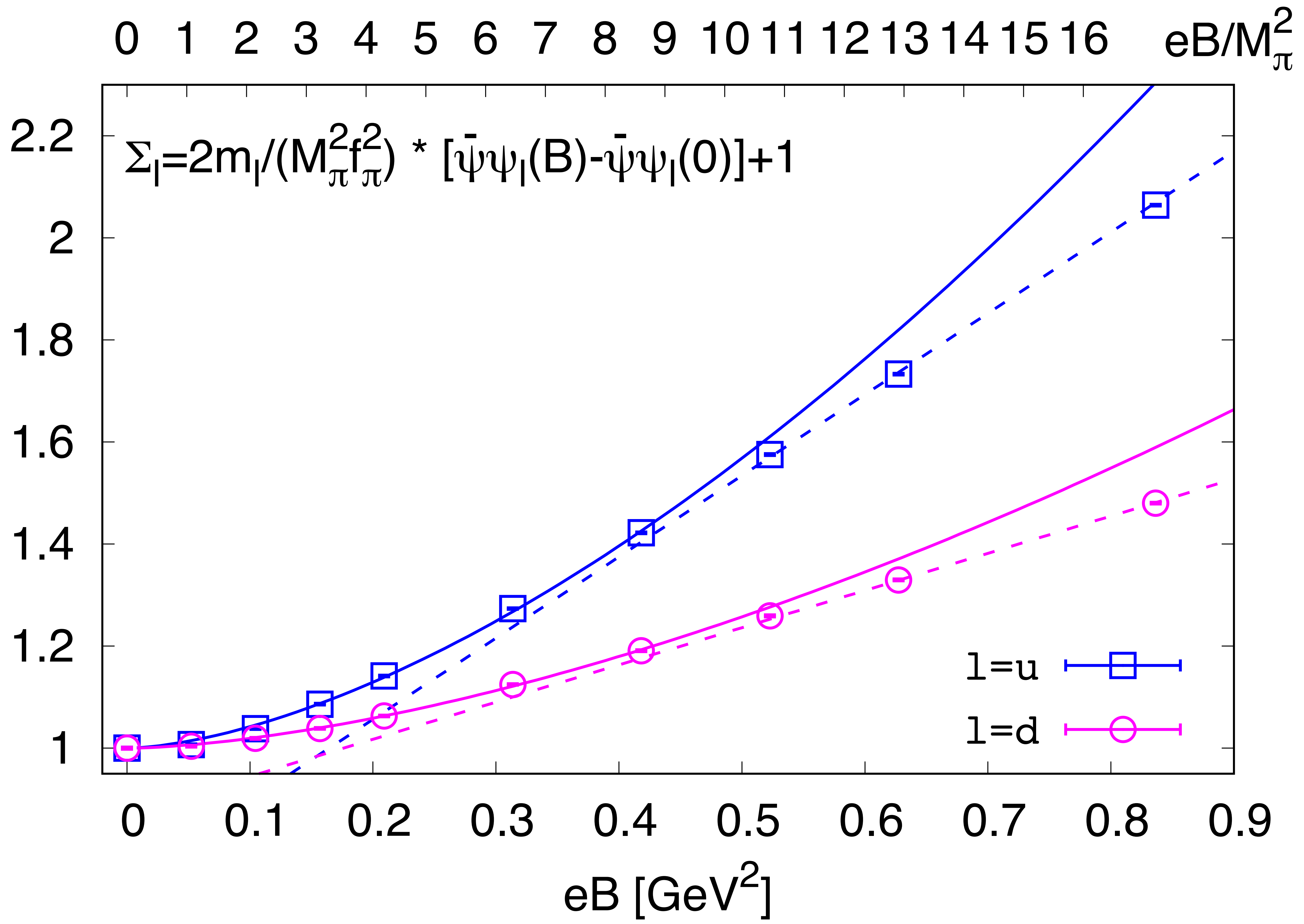
See quenched LQCD results in Bali et al., PRD 97 (2018) 034505,
Luschevskaya et al., NPB 898 (2015) 627

Energy density in Hadron resonance gas model



HTD, S.-T. Li, Q. Shi, A. Tomiya, X.-D. Wang, Y. Zhang, arXiv: 2011.04870

Isospin symmetry breaking at $eB\neq0$ manifested in chiral condensates

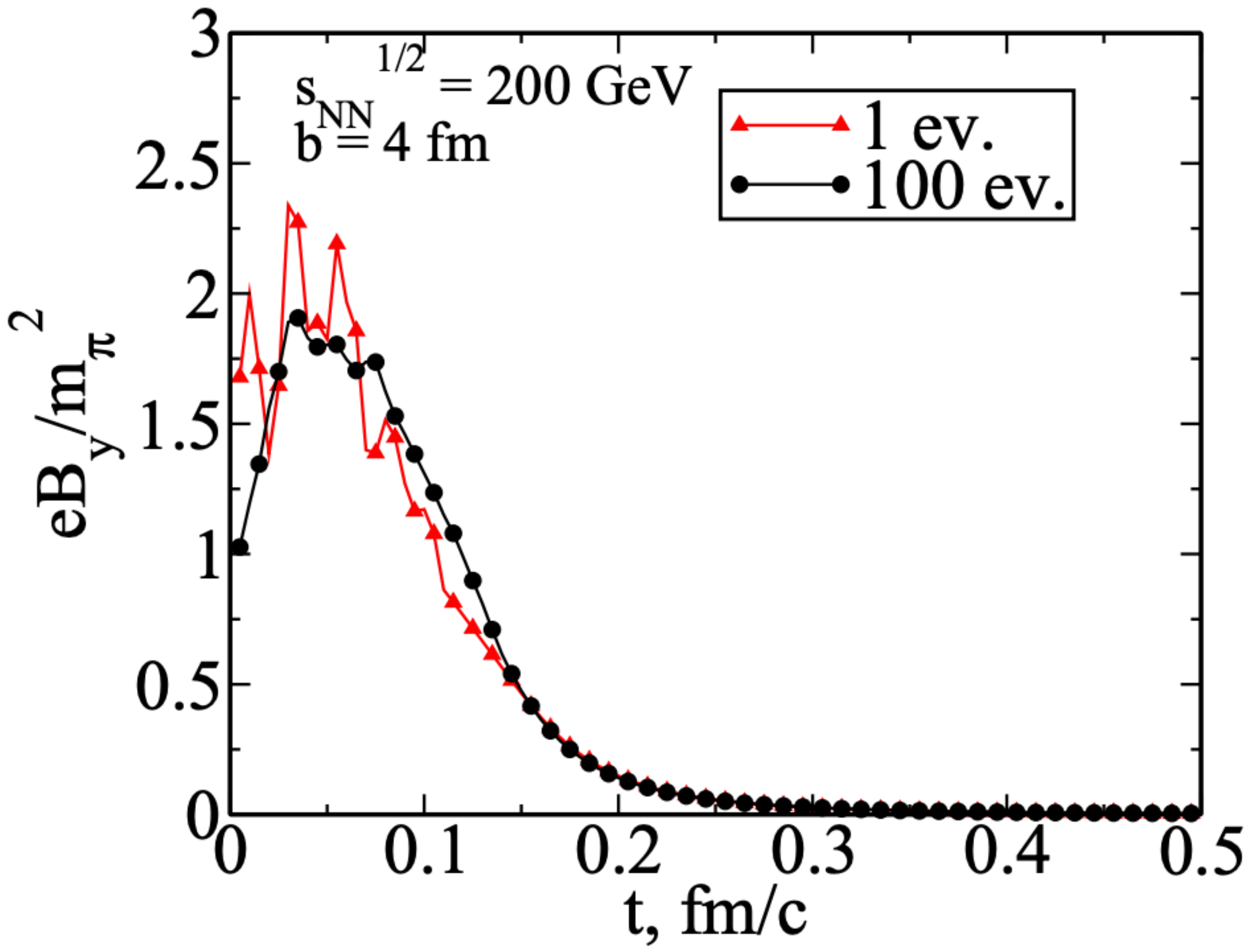


Not accessible in HIC experiments

HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

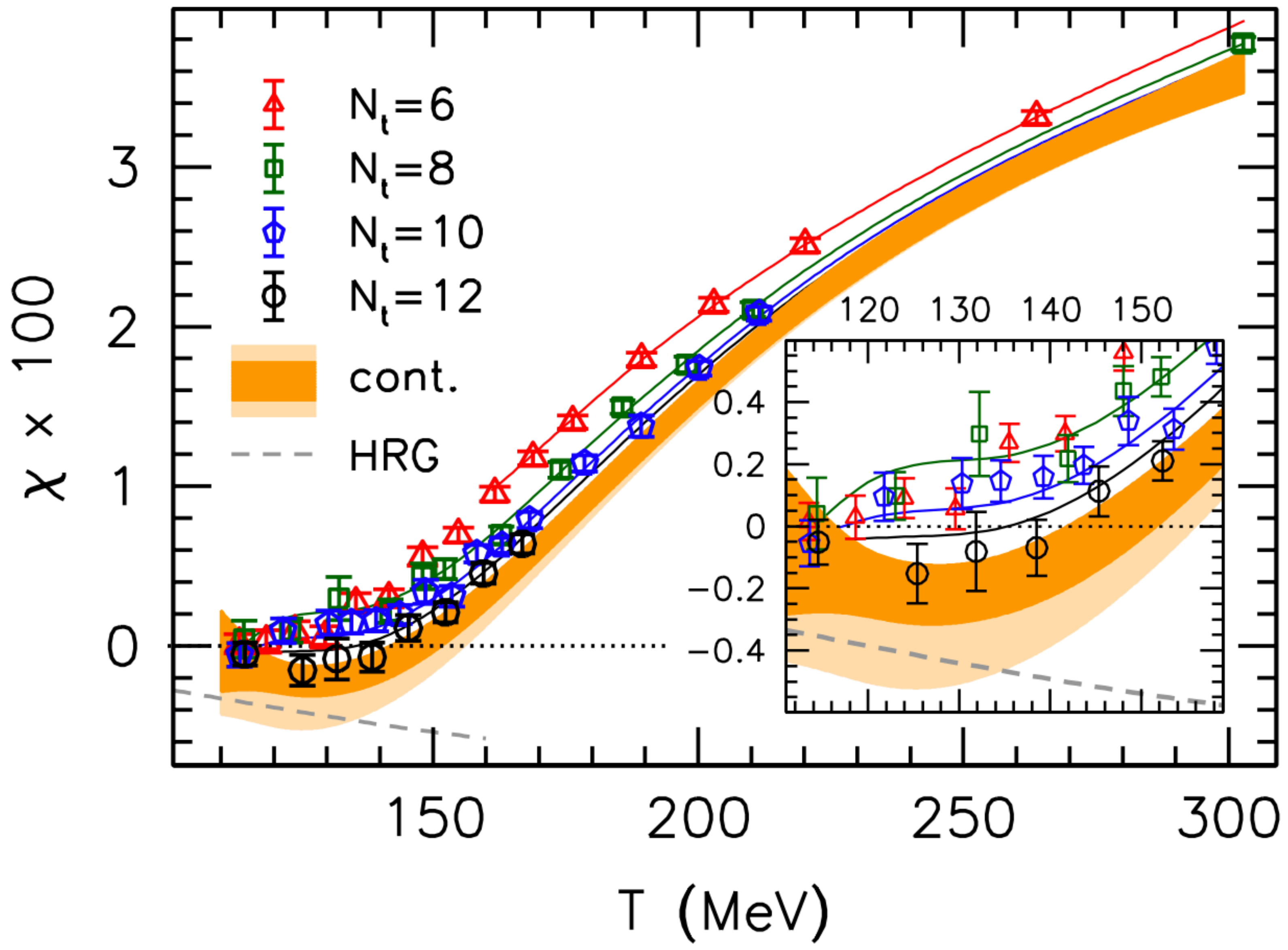
See also in reviews e.g. M. D'Elia, Lect.NotesPhys.871(2013)181

Magnetic field created in the early stage of HIC



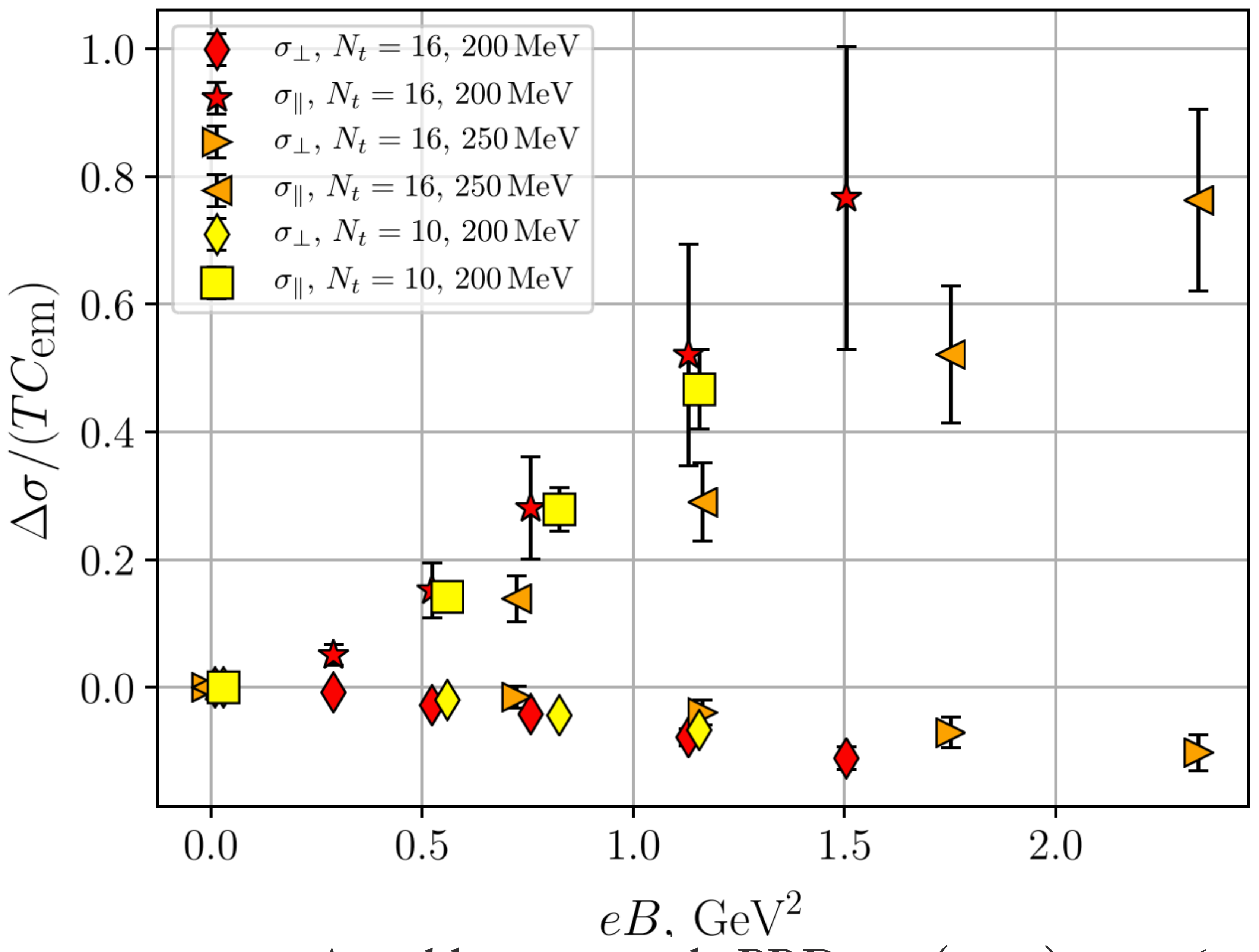
Skokov, Illarionov and Toneev, IJMPA 24 (2009) 5925

Magnetic susceptibility



Bali, Endrodi, Piemonte, JHEP 07 (2020) 183

Difference to electromagnetic conductivity at eB=0



Astrakhantsev et al., PRD 102 (2020) 054516

t=0: RHIC: $eB \sim m_\pi^2$
LHC: $eB \sim 15 m_\pi^2$

T> 155 MeV: Paramagnetic
T<155 MeV: Diamagnetic

See more in Massimo D'Elia's talk

Parallel: ↑

Transverse: ↓

Fluctuations of net baryon number, electric charge and strangeness

- Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in LQCD

See recent reviews:
LQCD: HTD, F. Karsch, S. Mukherjee, Int. J. Mod. Phys. E 24 (2015) no.10, 1530007
Exp.: X.-F. Luo & N. Xu, Nucl. Sci. Tech. 28 (2017) 112

$$\hat{\chi}_{ijk}^{uds} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k} \right|_{\mu_{u,d,s}=0}$$

$$\hat{\chi}_{ijk}^{\mathrm{BQS}} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_\mathrm{B}/T\right)^i \partial\left(\mu_\mathrm{Q}/T\right)^j \partial\left(\mu_\mathrm{S}/T\right)^k} \right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_\mathrm{B} + \frac{2}{3}\mu_\mathrm{Q}\,, \quad \mu_d = \frac{1}{3}\mu_\mathrm{B} - \frac{1}{3}\mu_\mathrm{Q}\,, \quad \mu_s = \frac{1}{3}\mu_\mathrm{B} - \frac{1}{3}\mu_\mathrm{Q} - \mu_\mathrm{S}\,.$$

- At eB=/=0 a lot more need to be explored

HRG: G. Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301
Bhattacharyya et al., EPL115(2016)62003
PNJL: W.-J. Fu, Phys. Rev. D 88 (2013) 014009

High T: Ideal gas limit

- **At eB=0:** $\varepsilon^2 = m^2 + |\vec{p}|^2$ Kapusta & Gale, Finite-temperature field theory: Principles and applications

$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{20} + \sum_{f=u,d,s} \left[\frac{1}{2}\hat{\mu}_f^2 + \frac{1}{4\pi^2}\hat{\mu}_f^4\right]$$

- **At eB=/=0:** $\varepsilon_l^2 = p_z^2 + m^2 + 2qB(l + 1/2 - s_z)$ HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \sum_{f=u,d,s} \frac{3|q_f|B}{\pi^2 T^2}\left[\frac{\pi^2}{12} + \frac{\hat{\mu}_f^2}{4} + 2\frac{\sqrt{2|q_f|B}}{T}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\cosh\left(k\hat{\mu}_f\right) \times \mathrm{K}_1\left(\frac{k\sqrt{2|q_f|Bl}}{T}\right)\right]$$

K_1: first-order modified Bessel function of the second kind

High T: Ideal gas limit

$$\frac{\chi_2^{\mathrm{B}}}{eB} = \frac{4}{9\pi^2}\left(\frac{1}{2} + \hat{b}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}(-1)^{k+1}k\times\left[\sqrt{2}\,\mathrm{K}_1\left(k\,\hat{b}\sqrt{2l}\right) + \mathrm{K}_1\left(k\,\hat{b}\sqrt{l}\right)\right]\right)$$

$$\frac{\chi_{11}^{\mathrm{BQ}}}{eB} = \frac{4}{9\pi^2}\left(\frac{1}{4} + \hat{b}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}(-1)^{k+1}k\times\left[2\sqrt{2}\,\mathrm{K}_1\left(k\,\hat{b}\sqrt{2l}\right) - \mathrm{K}_1\left(k\,\hat{b}\sqrt{l}\right)\right]\right)$$

$$\hat{b} = \sqrt{2eB/3}/T$$

$$\sqrt{eB}/T \to \infty$$

Quantity	Value
χ_2^u/eB	$1/\pi^2$
$\chi_2^{d/s/S}/eB$	$1/(2\pi^2)$
$\chi_{11}^{ud}/eB = \chi_{11}^{us}/eB = \chi_{11}^{ds}/eB{=}0$	0
χ_2^{B}/eB	$2/(9\pi^2)$
χ_2^{Q}/eB	$5/(9\pi^2)$
$\chi_{11}^{\mathrm{BQ}}/eB$	$1/(9\pi^2)$
$\chi_{11}^{\mathrm{QS}}/eB = -\chi_{11}^{\mathrm{BS}}/eB = \chi_2^{\mathrm{S}}/3eB$	$1/(6\pi^2)$

$$eB = 0$$

$$\chi_2^{\mathrm{B}} = \chi_{11}^{\mathrm{QS}} = -\chi_{11}^{\mathrm{BS}} = \chi_2^{\mathrm{Q}}/2 = \chi_2^{\mathrm{S}}/3 = 1/3$$

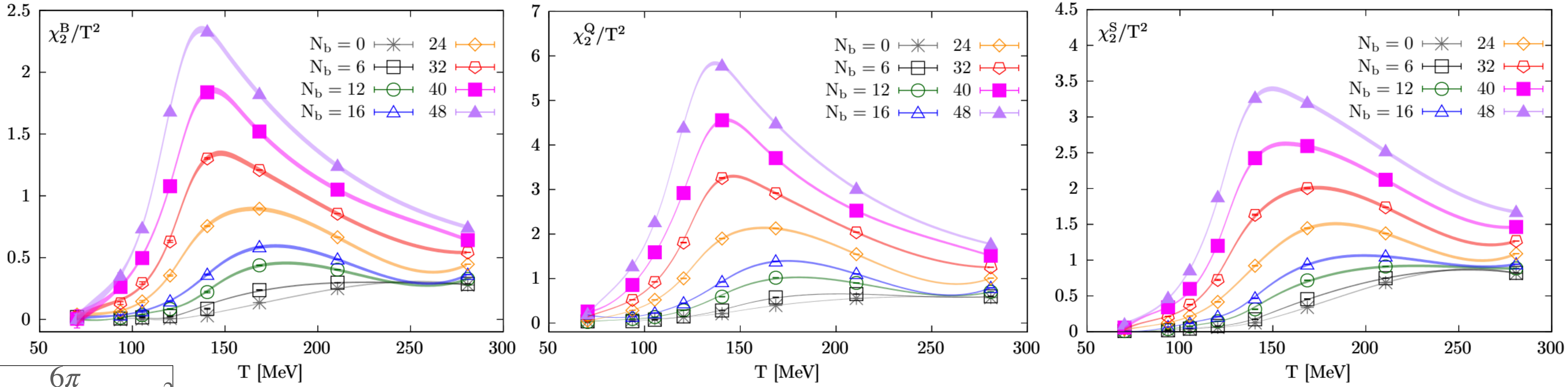
$$\chi_{11}^{\mathrm{BQ}} = 0\,.$$

Holds at both eB=0 and eB=/=0 with T→∞

$$\chi_{11}^{\mathrm{BS}}/\chi_2^{\mathrm{S}} = -\chi_{11}^{\mathrm{QS}}/\chi_2^{\mathrm{S}} = -\frac{1}{3}$$

2nd order fluctuations of net baryon number, electric charge and strangeness

N_f=2+1 QCD, $M_\pi(eB=0) \approx 220$ MeV, with $a^{-1} \approx 1.7$ GeV and HISQ action, fixed a approach ($T = a^{-1}/N_\tau$)



$$eB = \frac{6\pi}{N_x N_y} N_b a^{-2}$$

HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

Peak locations shift to lower T in stronger *eB*

Consistent with the reduction of T_{pc} in a stronger magnetic field

2nd order correlations of net baryon number, electric charge and strangeness

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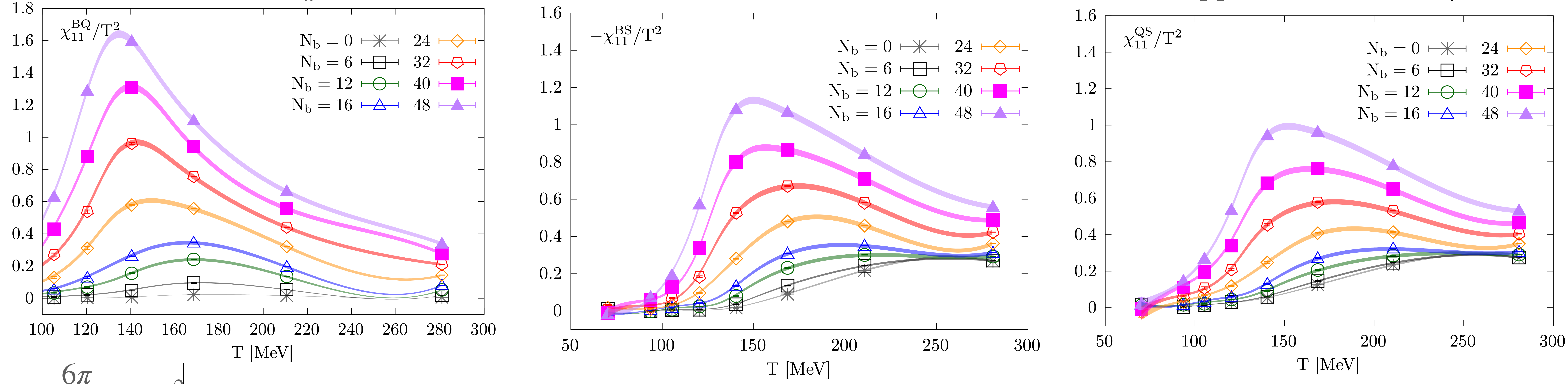


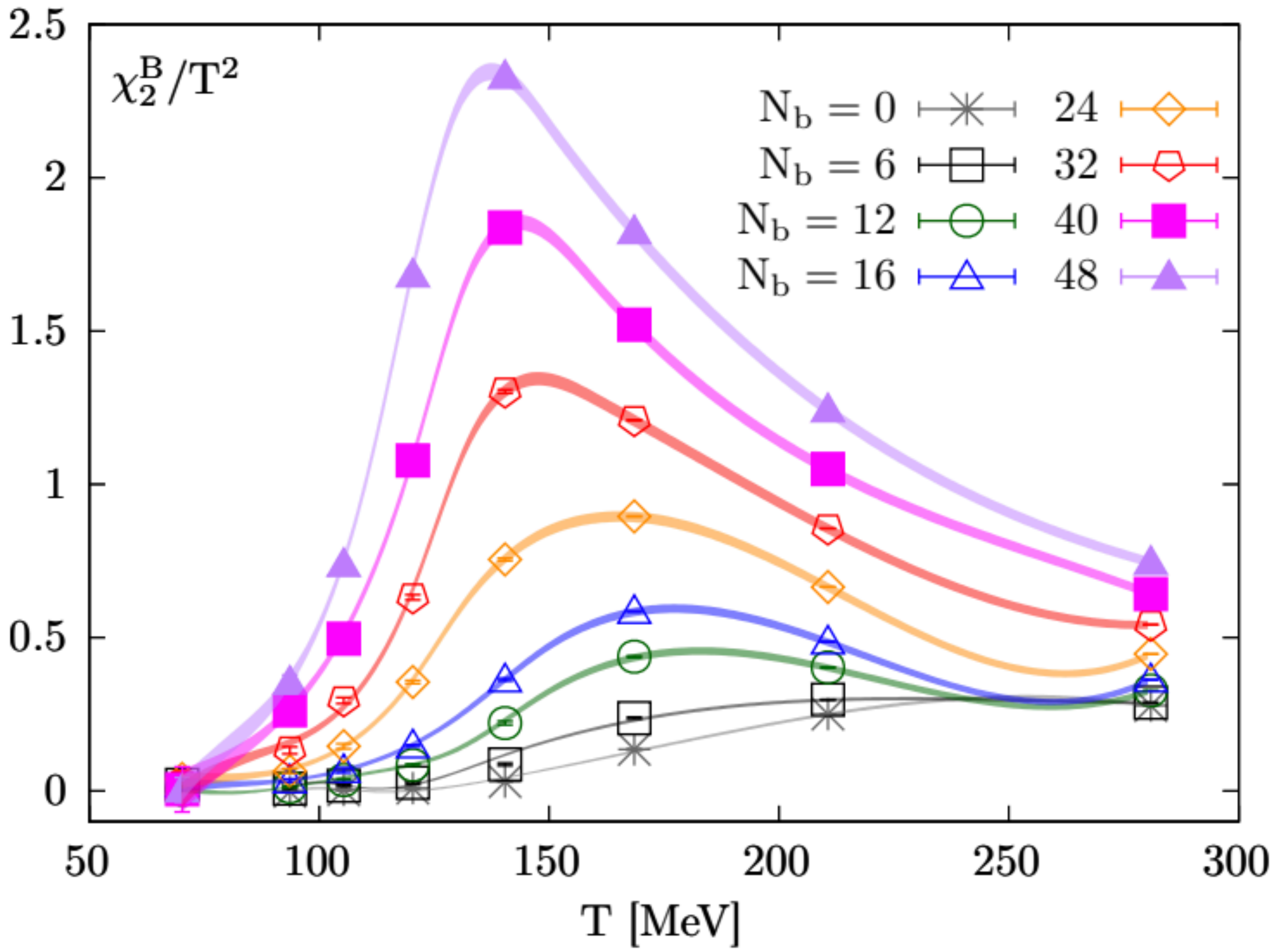
$$eB = \frac{6\pi}{N_x N_y} N_b a^{-2}$$

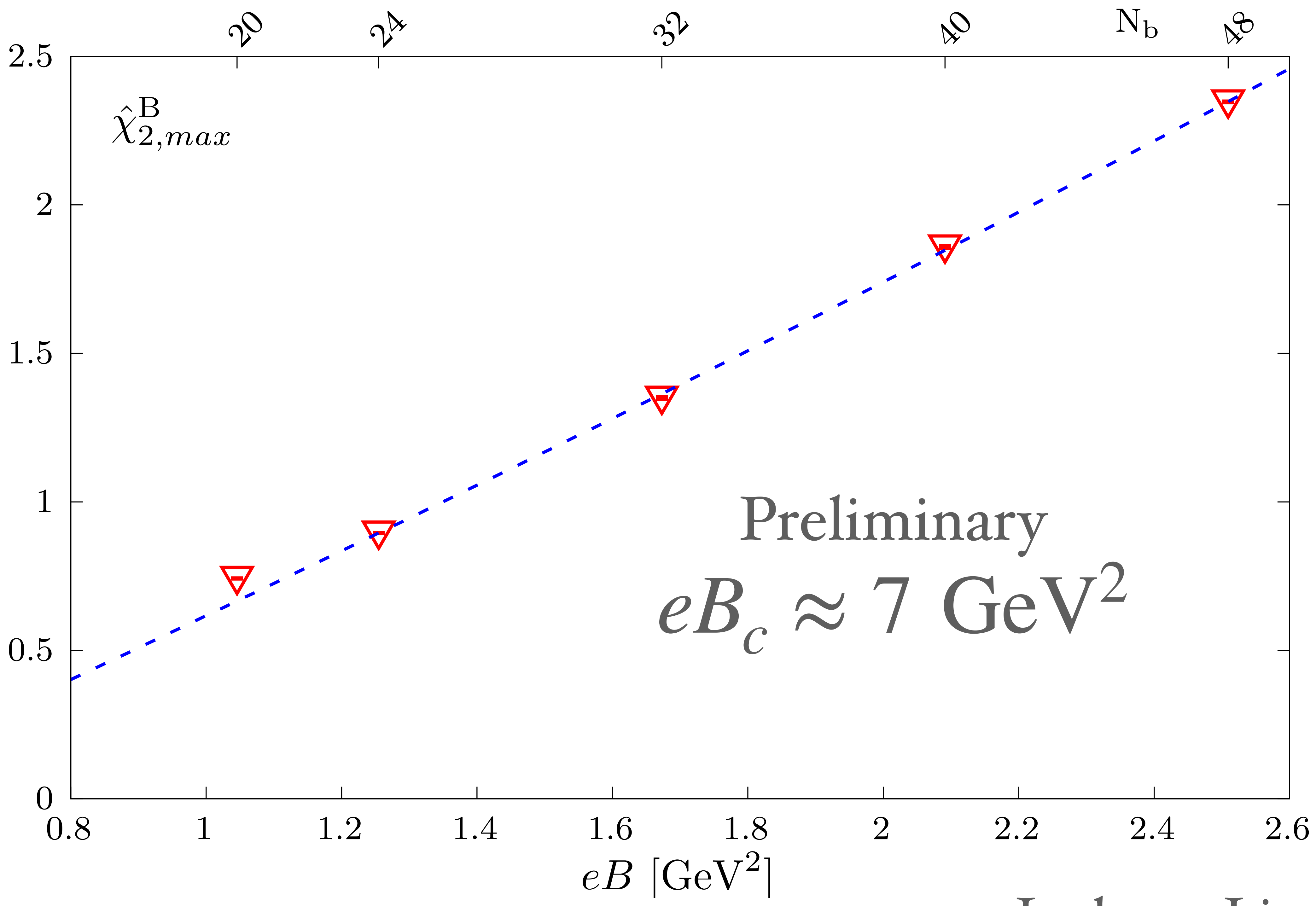
HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

Similar to the 2nd order diagonal fluctuations

Signal for a critical end point in the T-eB plane of QCD phase diagram?

A rough estimate of a CEP in T-eB plane





Junhong Liu, work in progress

At eB=0:

$$\chi_n^B \propto (-2\kappa_q)^{n/2} h^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(z)$$

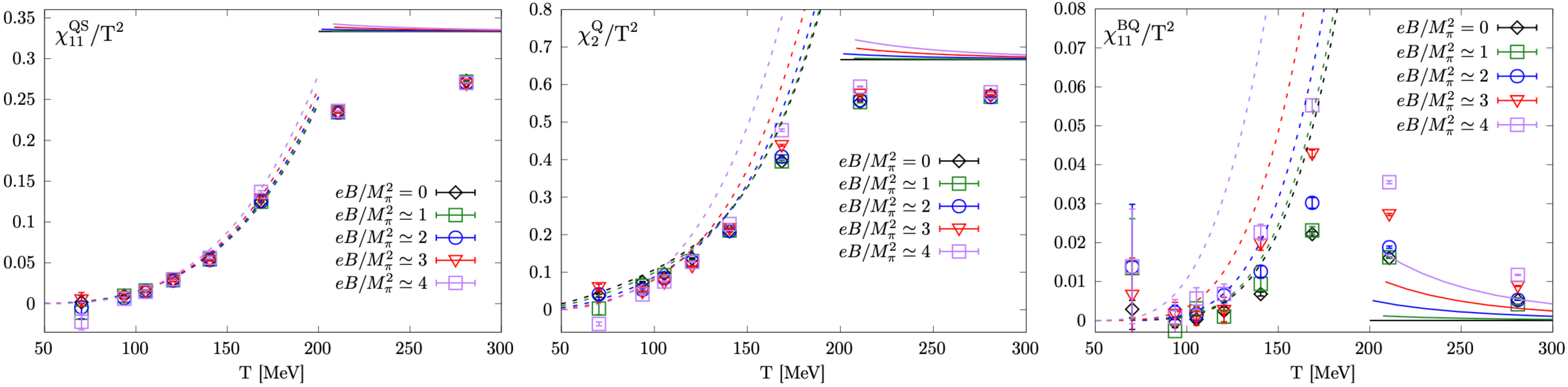
Friman et al., Eur. Phys. J. C 71(2011)1694

$$\chi^B_{2,max} = b\,(eB_c - eB)^{(1-\alpha)/\beta\delta} + d$$

	$\beta\delta$	α	$(1-\alpha)/\beta\delta$
Z(2)	1.5654	0.1088	0.5693
O(4)	1.8468	-0.2268	0.6643

Comparison to HRG and idea gas limit

N_f=2+1 QCD, $M_\pi(eB=0) \approx 220$ MeV, with $a^{-1} \approx 1.7$ GeV and HISQ action, fixed a approach ($T = a^{-1}/N_\tau$)



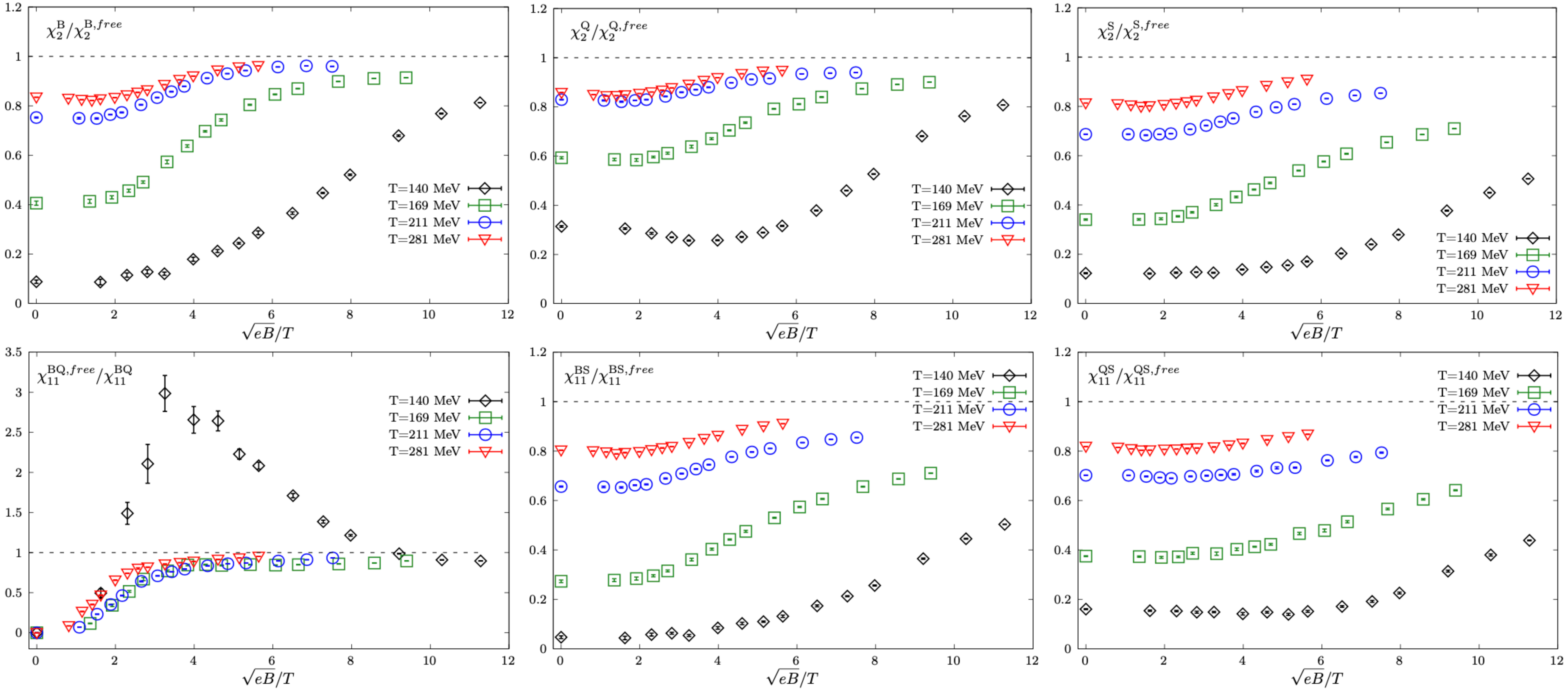
Low T: Kaons dominated Pions dominated Baryons dominated

HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

Comparison to the ideal gas limit

N

Nf=2+1 QCD, $M_\pi(eB=0) \approx 220$ MeV, with $a^{-1} \approx 1.7$ GeV and HISQ action, fixed a approach ($T = a^{-1}/N_\tau$)



HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

Isospin symmetry breaking at nonzero eB with physical pion mass

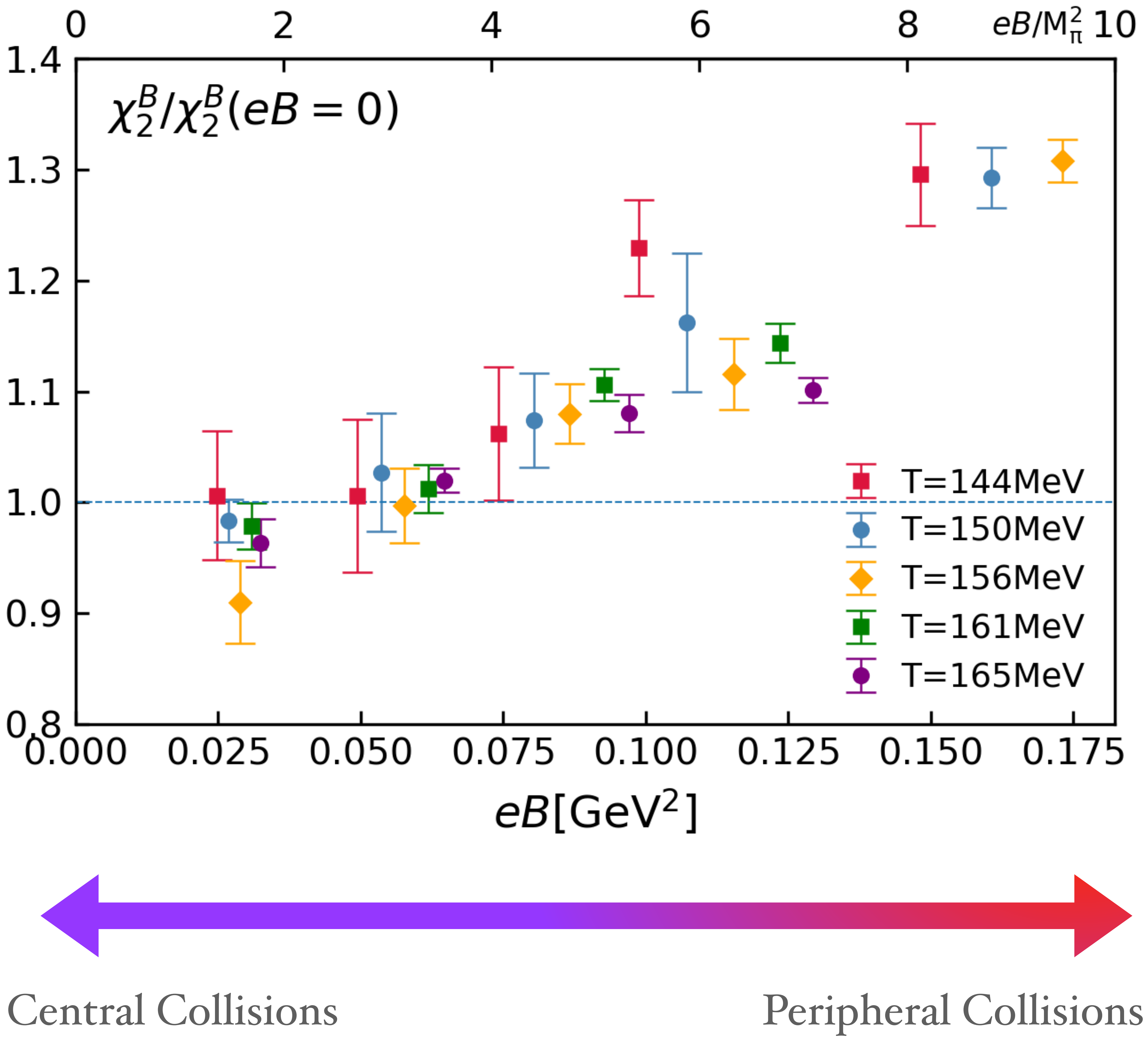
N_f=2+1 QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 157$ MeV, $32^3 \times 8$ lattices with HISQ action

HTD, Sheng-Tai Li, Jun-Hong Liu, Xiao-Dan Wang, work in progress

At eB=0: $\chi_2^u/\chi_2^d = 1, \quad 2\chi_{11}^{BQ} - \chi_{11}^{BS} = \chi_2^B, \quad 2\chi_{11}^{QS} - \chi_{11}^{BS} = \chi_2^S$

Ratio X(*eB*)/X(*eB*=0) for 2nd order diagonal fluctuations

$N_{f=2+1}$ QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 157$ MeV, $32^3 \times 8$ lattices with HISQ action



X(eB)/X(eB=0) : Rcp like observable

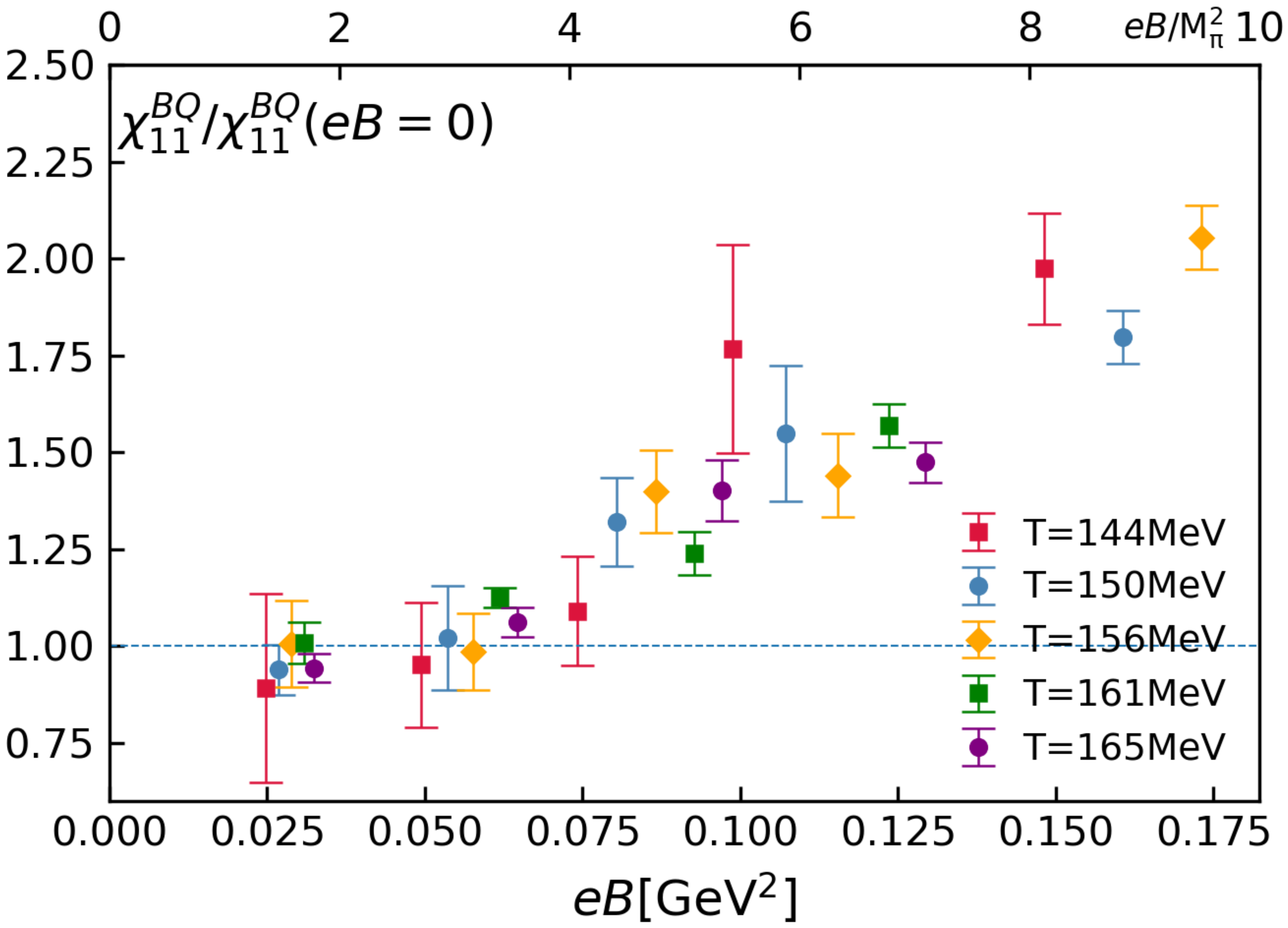
At $eB \simeq M_\pi^2$: deviation from unity is mild

At $eB \simeq 10M_\pi^2$: ~1.3

Note: $T_{pc}(eB \simeq 10M_\pi^2)/T_{pc}(eB=0) \sim 99\,\%$

Ratio X(*eB*)/X(*eB*=0) for 2nd order off-diagonal fluctuations

N_f=2+1 QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 157$ MeV, $32^3 \times 8$ lattices with HISQ action



Central Collisions
Smaller eB

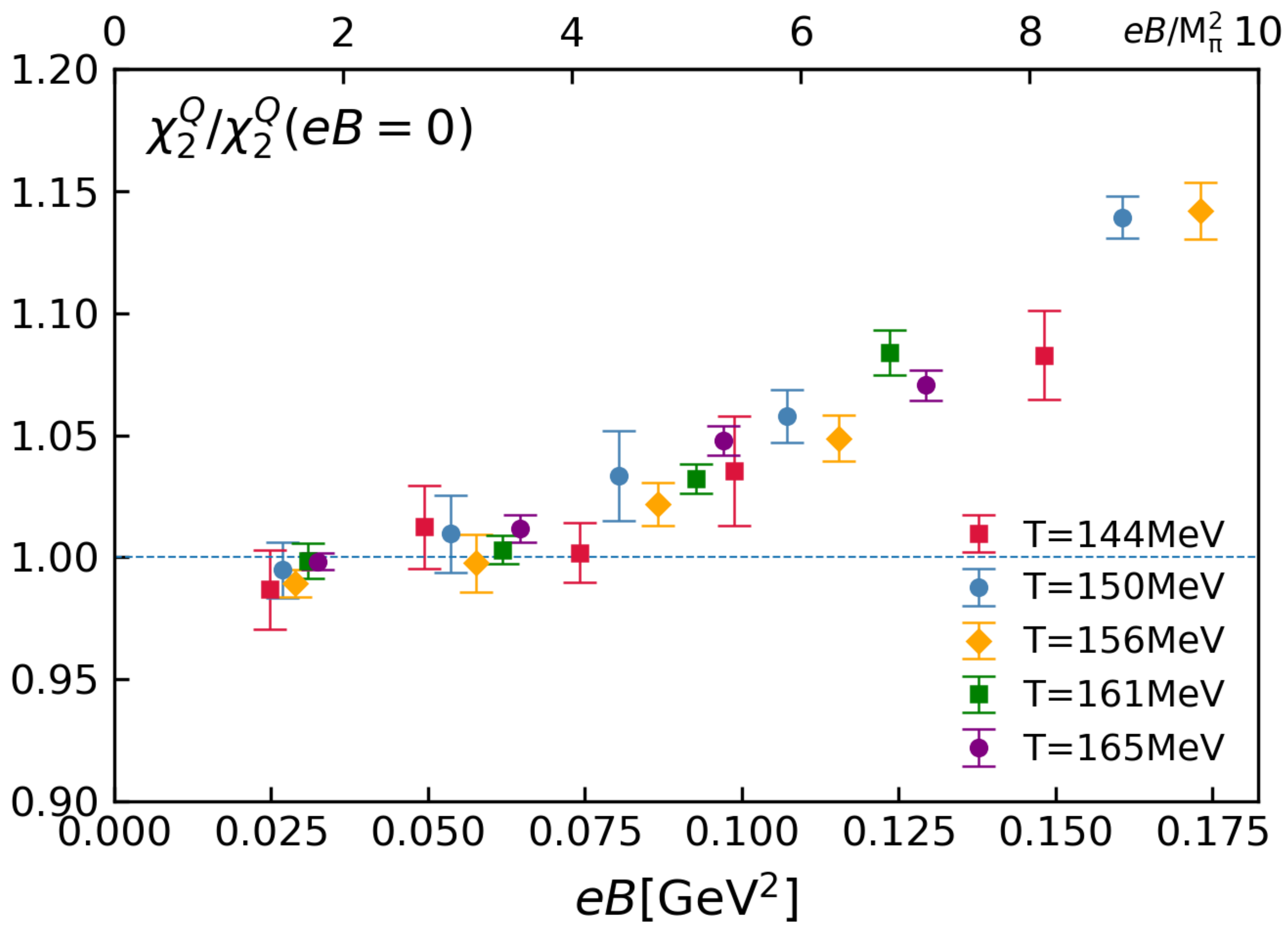
Peripheral Collisions
Larger eB

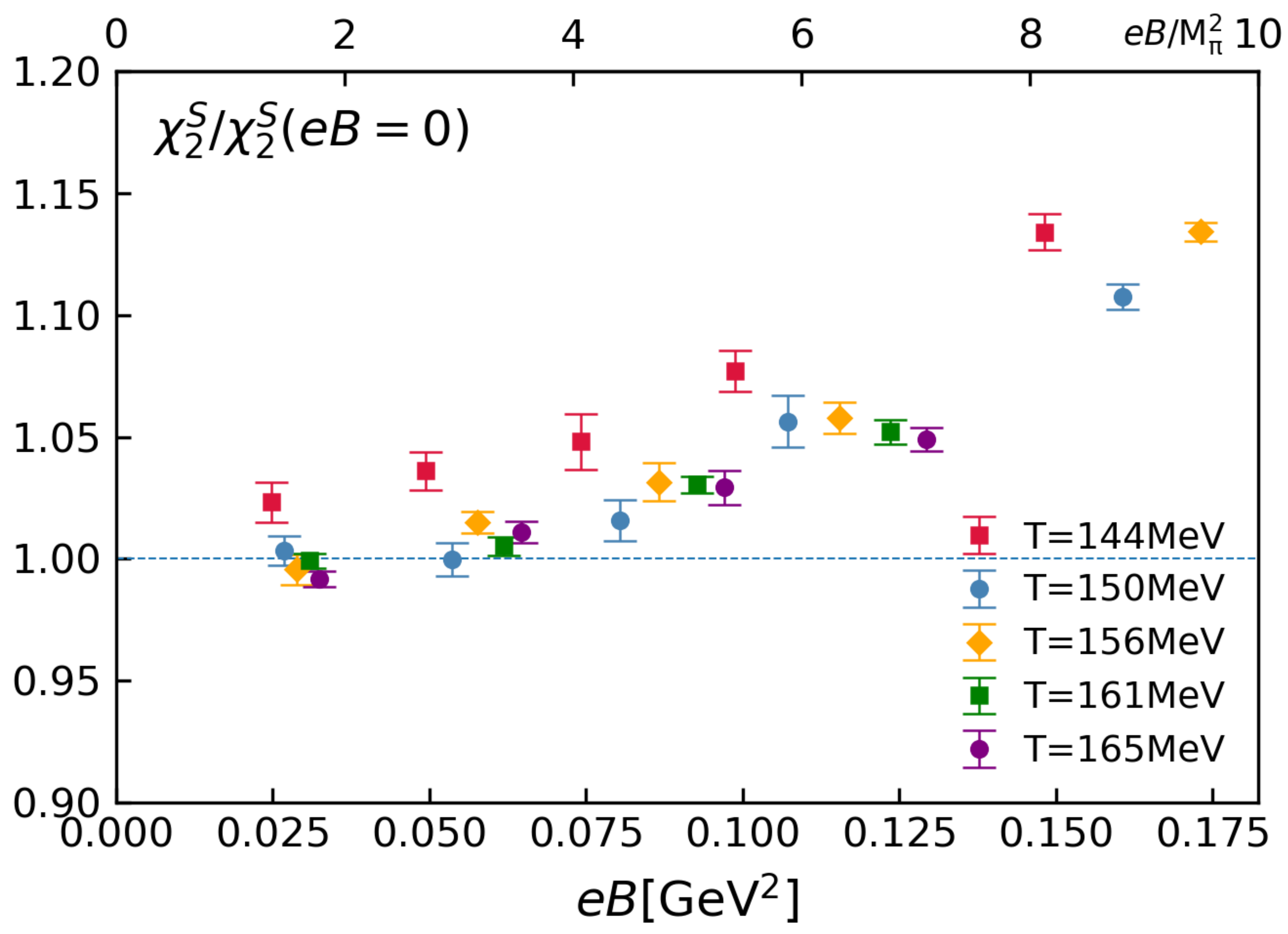
X(eB)/X(eB=0) : Rcp like observable

At $eB \simeq M_\pi^2$: deviation from unity is mild

At $eB \simeq 10M_\pi^2$: ~2 !

Note: $T_{pc}(eB \simeq 10M_\pi^2)/T_{pc}(eB=0) \sim 99\,\%$

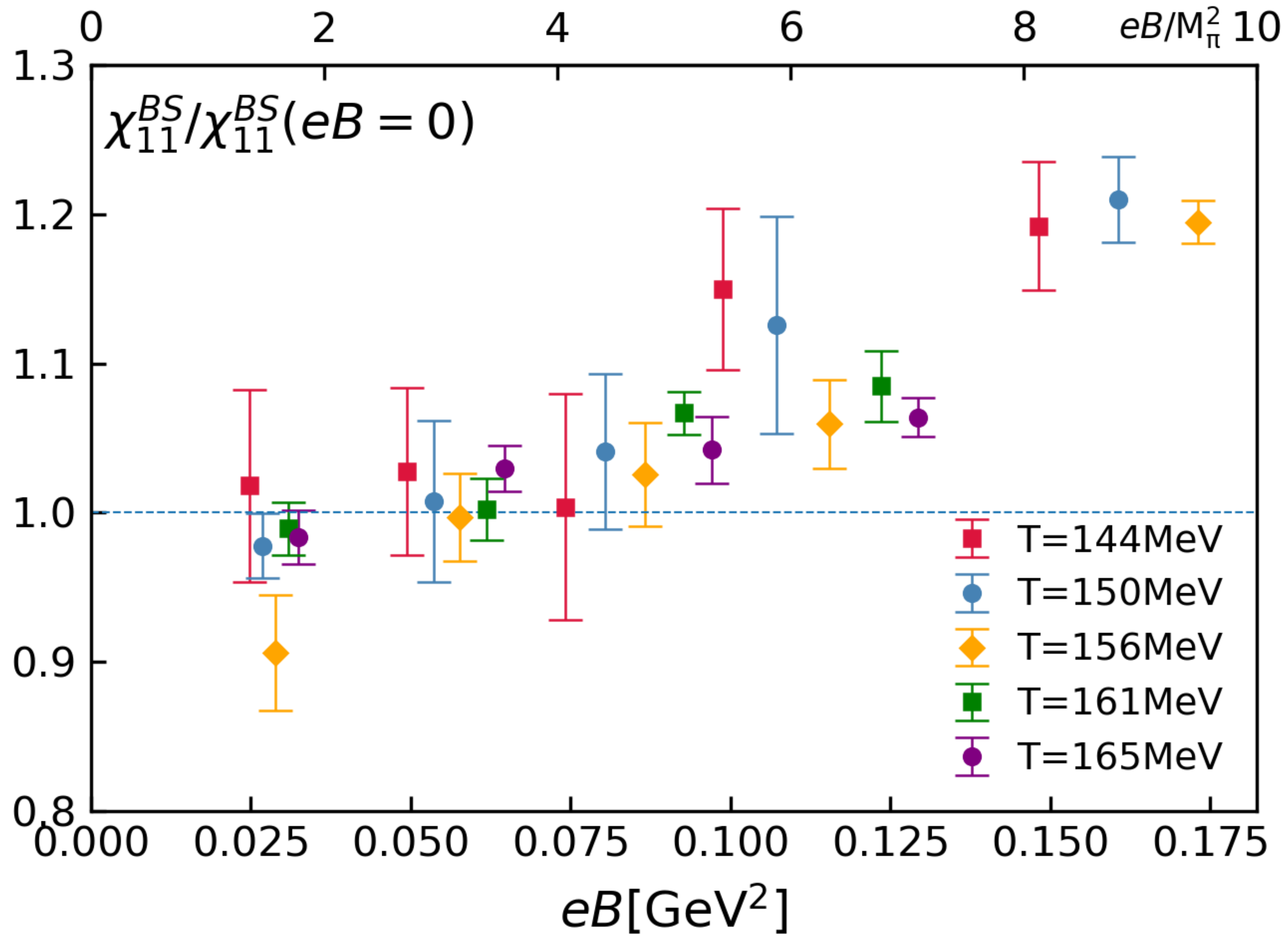


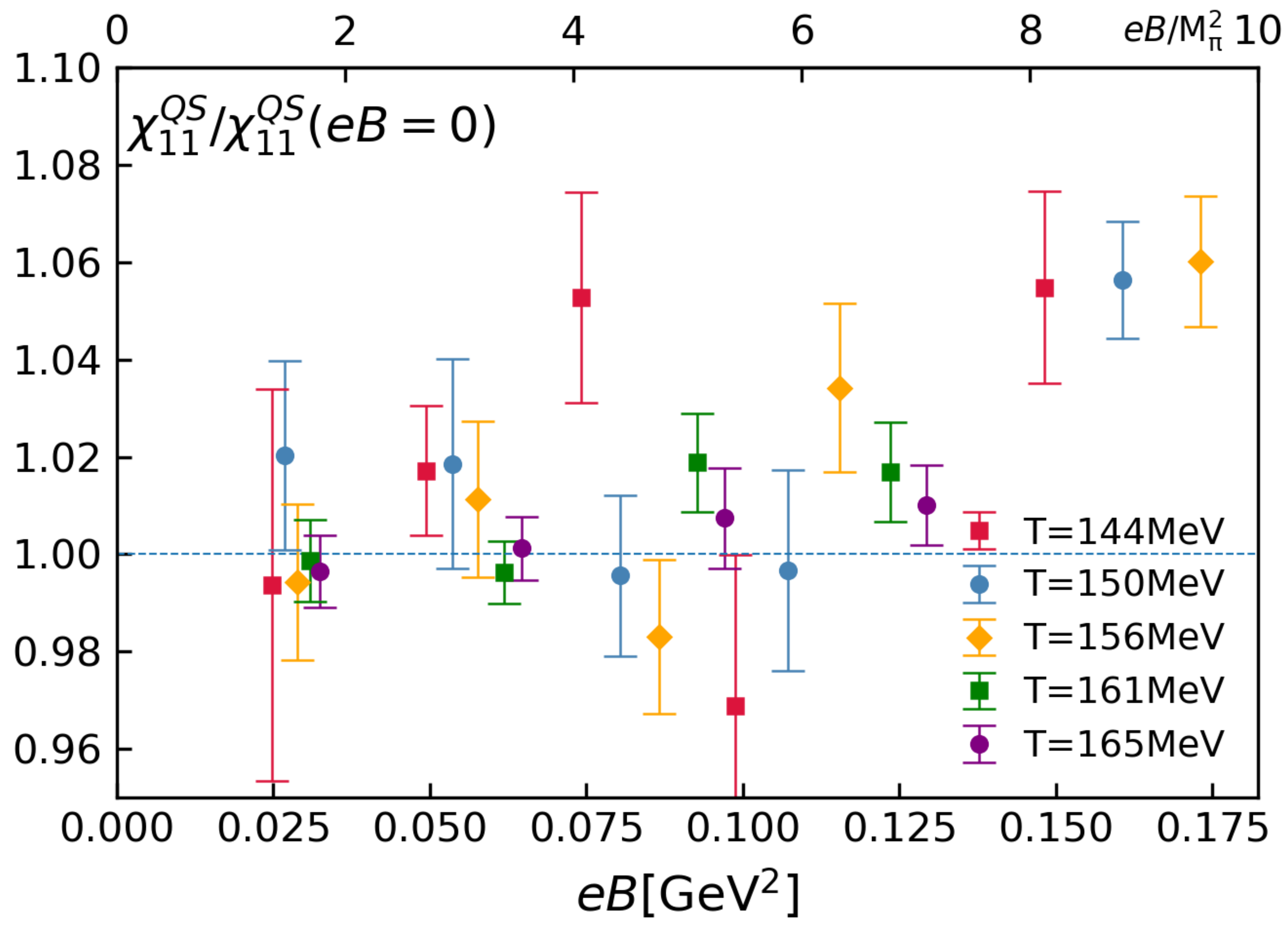


At $eB \simeq 10M_\pi^2$:

$\chi_2^{\mathrm{Q}}/\chi_2^{\mathrm{Q}}(eB=0) \sim 1.15$

$\chi_2^{\mathrm{S}}/\chi_2^{\mathrm{S}}(eB=0) \sim 1.15$



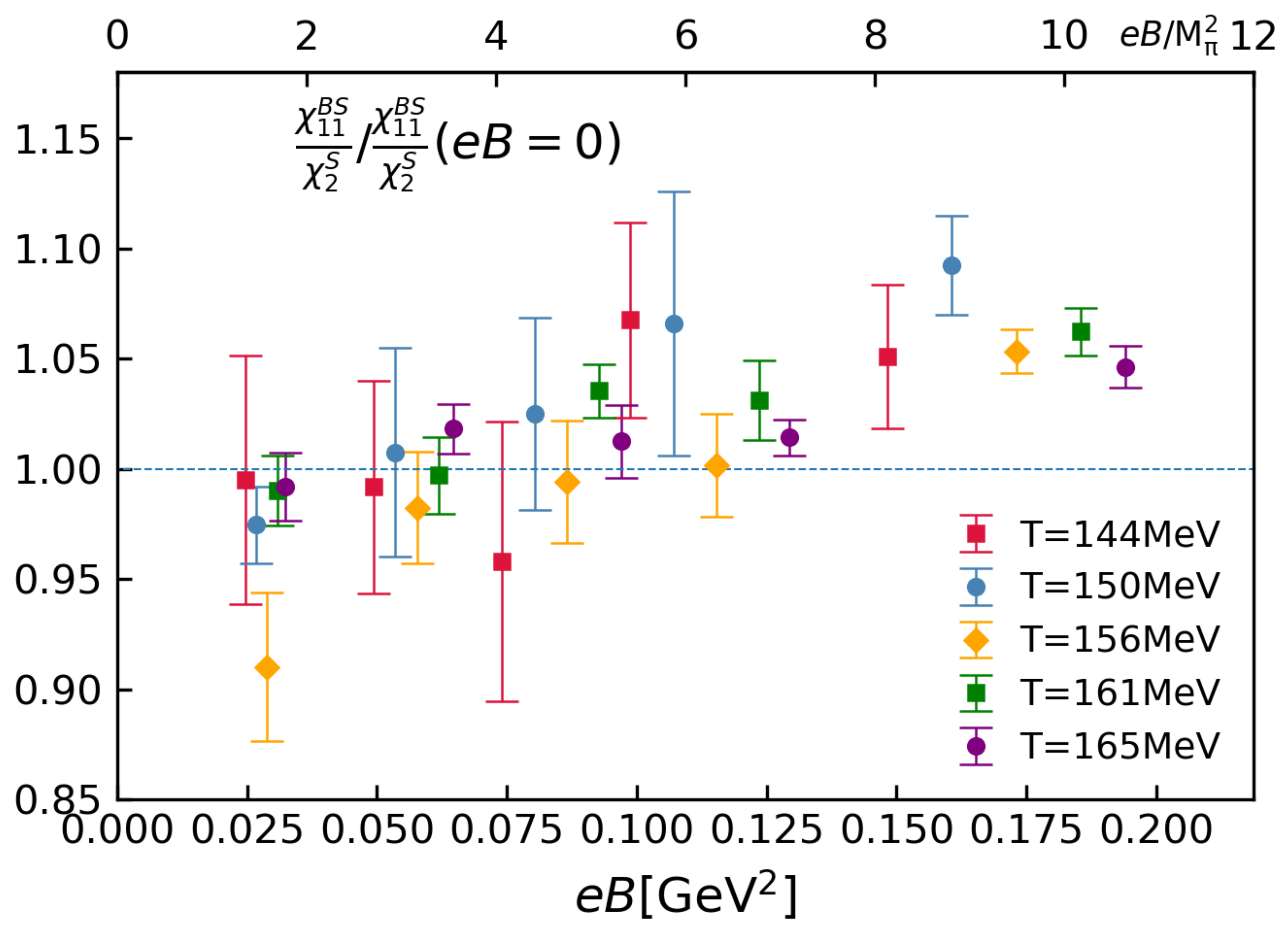


$\chi_{11}^{\mathrm{BS}}/\chi_{11}^{\mathrm{BS}}(eB=0) \sim 1.2$

$\chi_2^{\mathrm{QS}}/\chi_2^{\mathrm{QS}}(eB=0) \sim 1.06$

Lattice QCD meets experiment

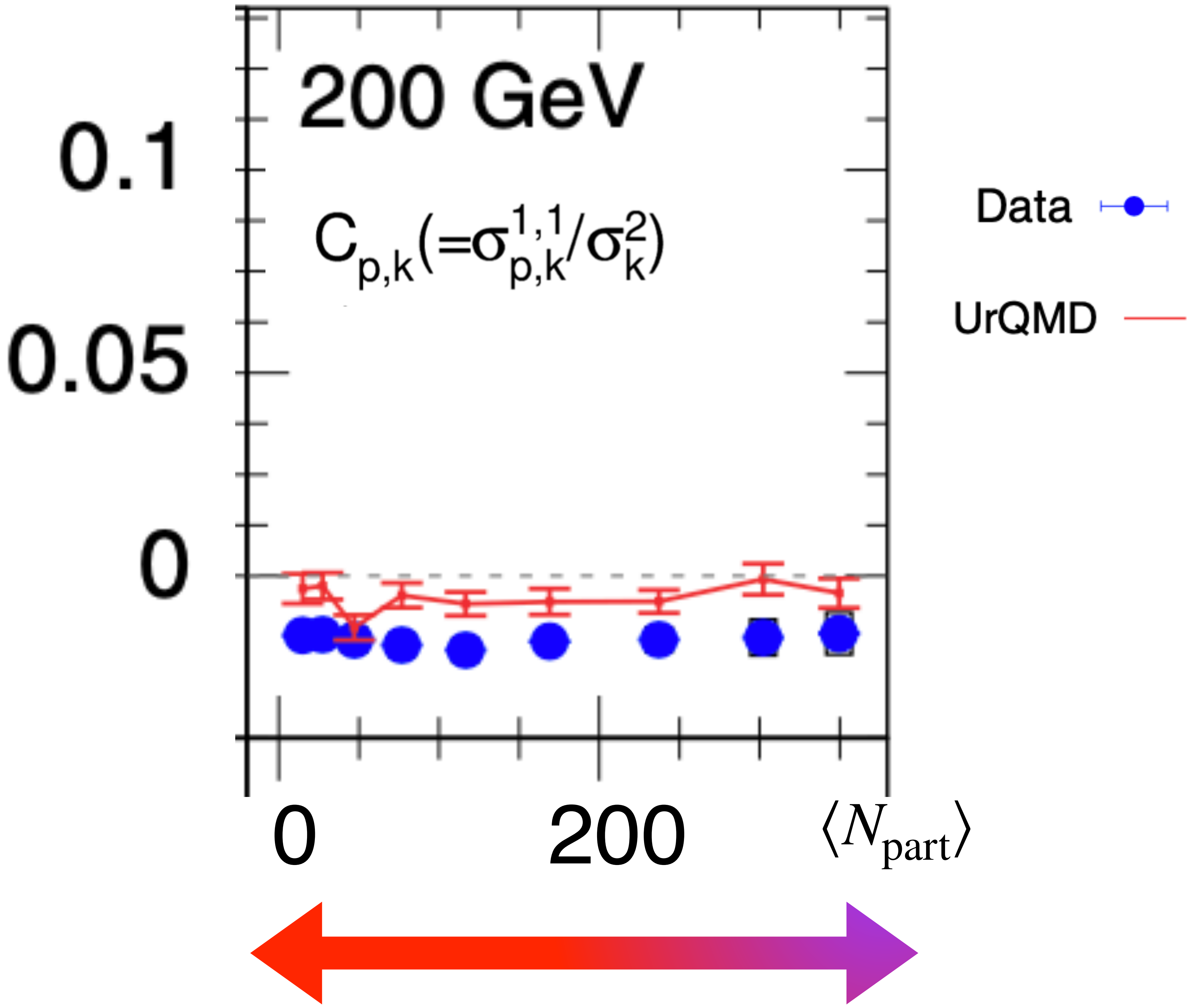
Lattice QCD



$M_\pi(eB=0) \approx 135$ MeV

$T_{pc}(eB=0) \approx 157$ MeV on Nt=8 lattices

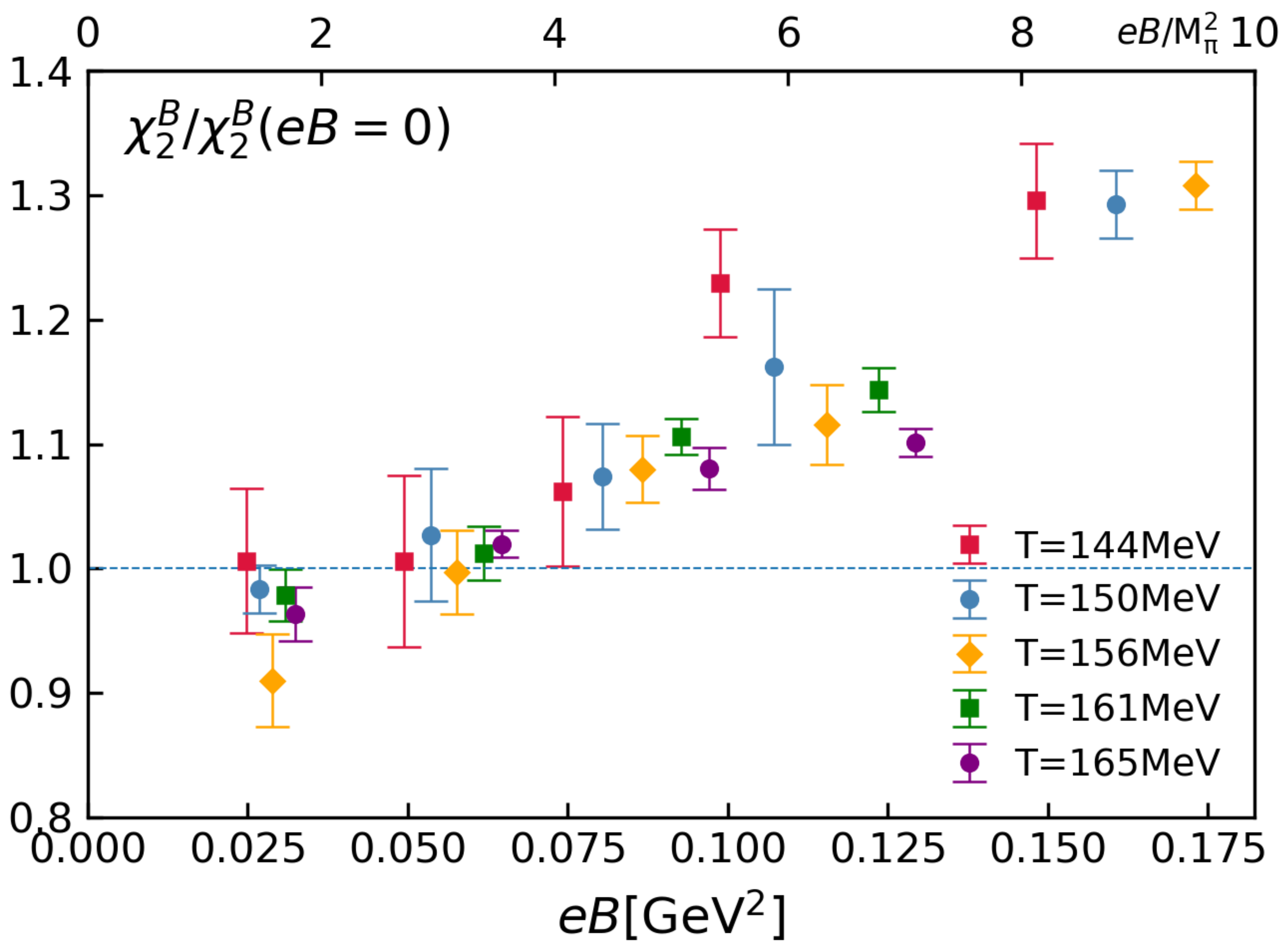
Proxy of χ_{11}^{BS}/χ_2^S



STAR, *Phys.Rev.C* 100 (2019) 1, 014902

Lattice QCD meets experiment

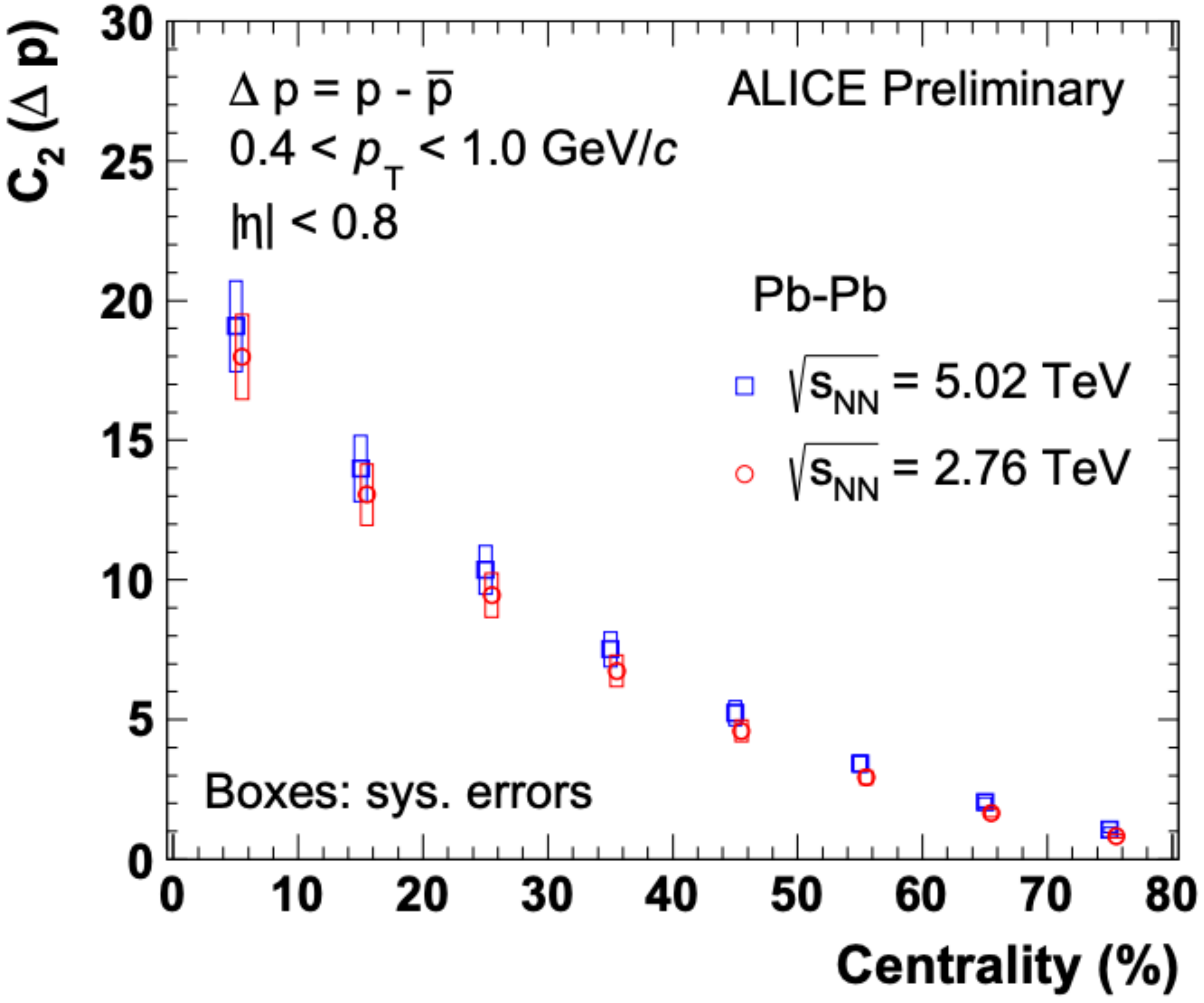
Lattice QCD



$M_\pi(eB=0) \approx 135$ MeV

$T_{pc}(eB=0) \approx 157$ MeV on Nt=8 lattices

Proxy of χ_2^B



$C_2 = VT^3\chi_2^B$

Volume!

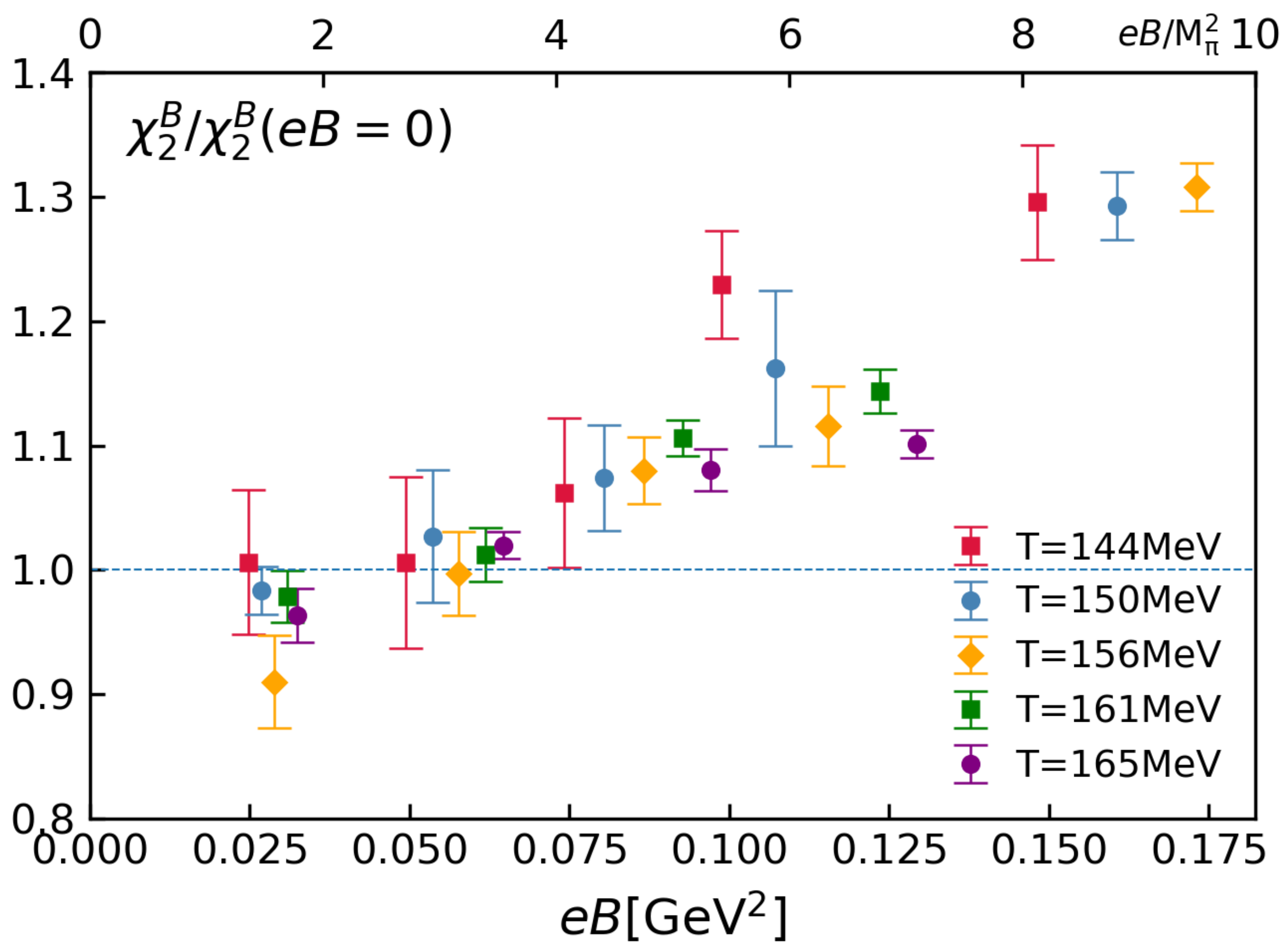
Smaller eB

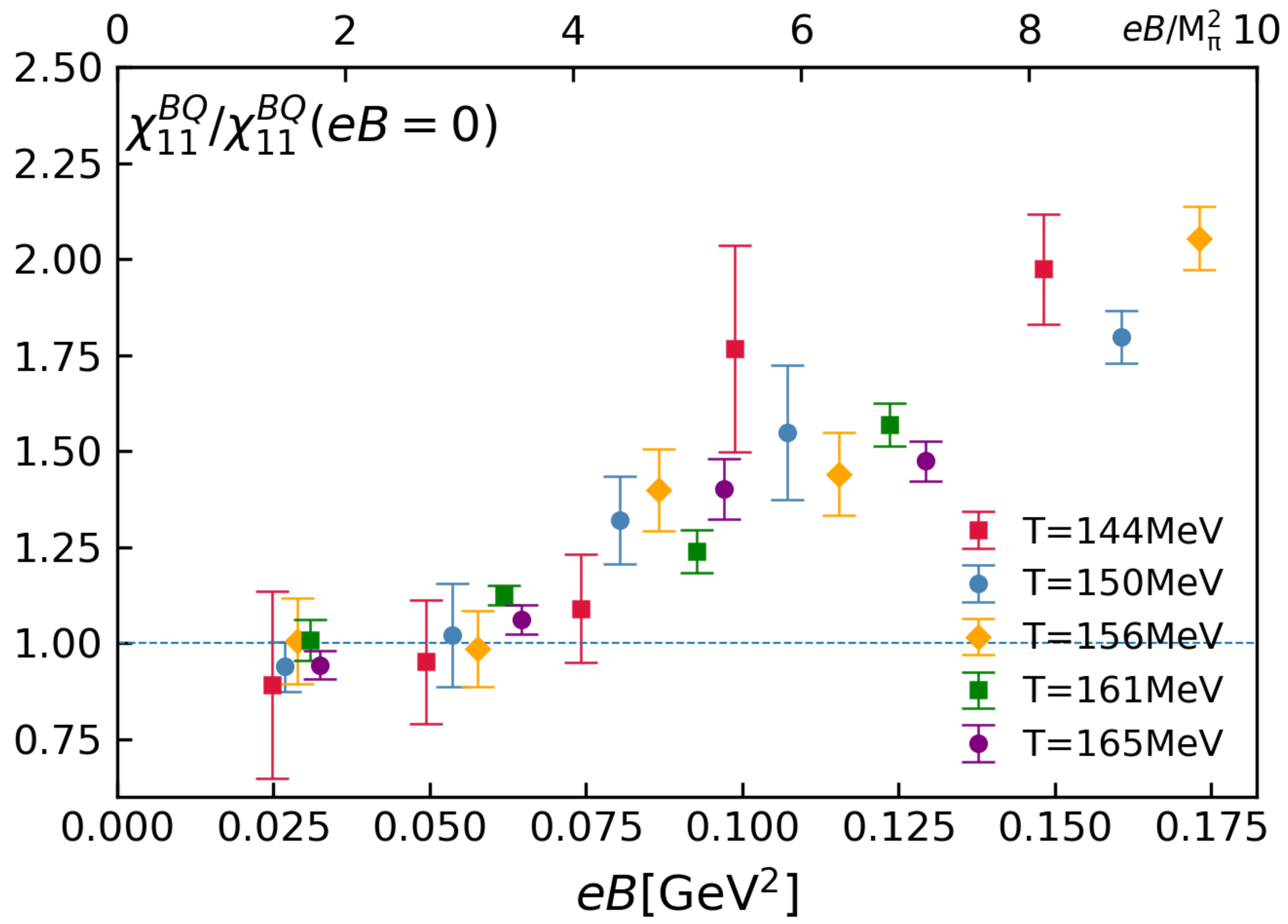
Larger eB

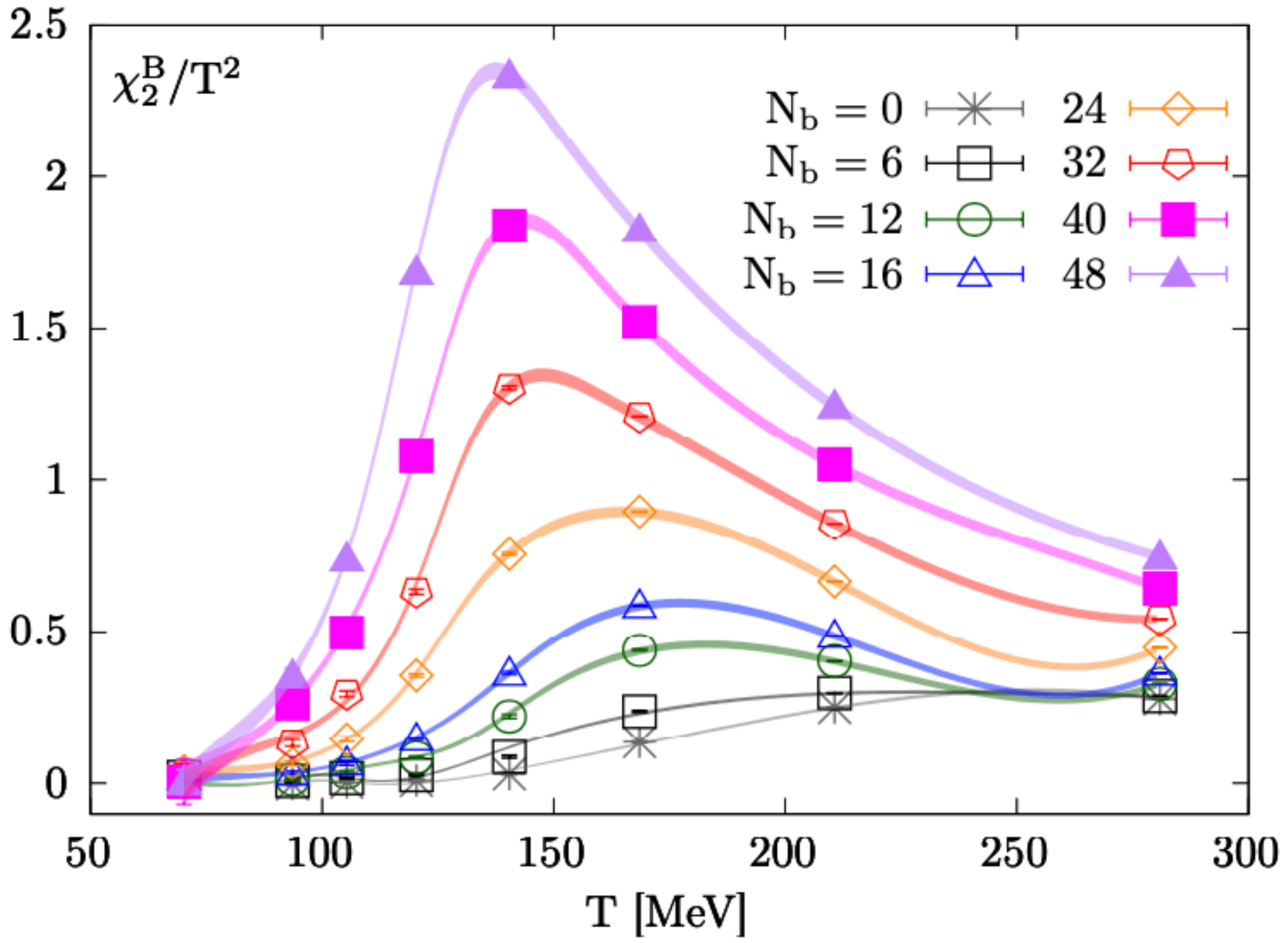
ALICE: *Nucl.Phys.A* 982 (2019) 851

Summary & Outlook

- The 2nd order fluctuations and correlations of B,Q & S are strongly affected by *eB*
- Could be useful to 1) detect the existence of a magnetic field in HIC; 2) analogy to study the QCD critical end point in the $T - \mu_B$ plane







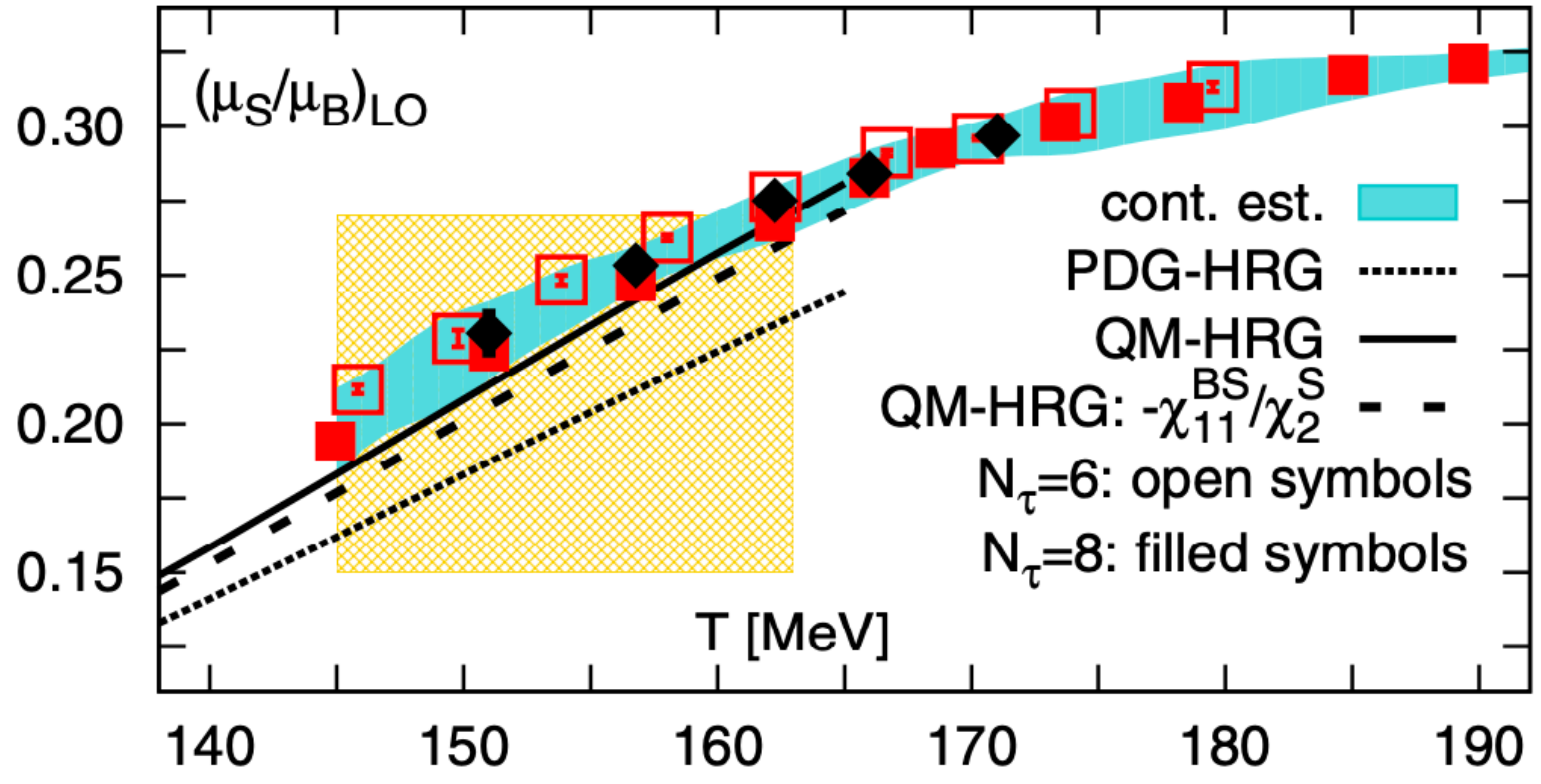
- A continuum estimate of 2nd order fluctuations with $N_\tau = 12$ lattices is ongoing

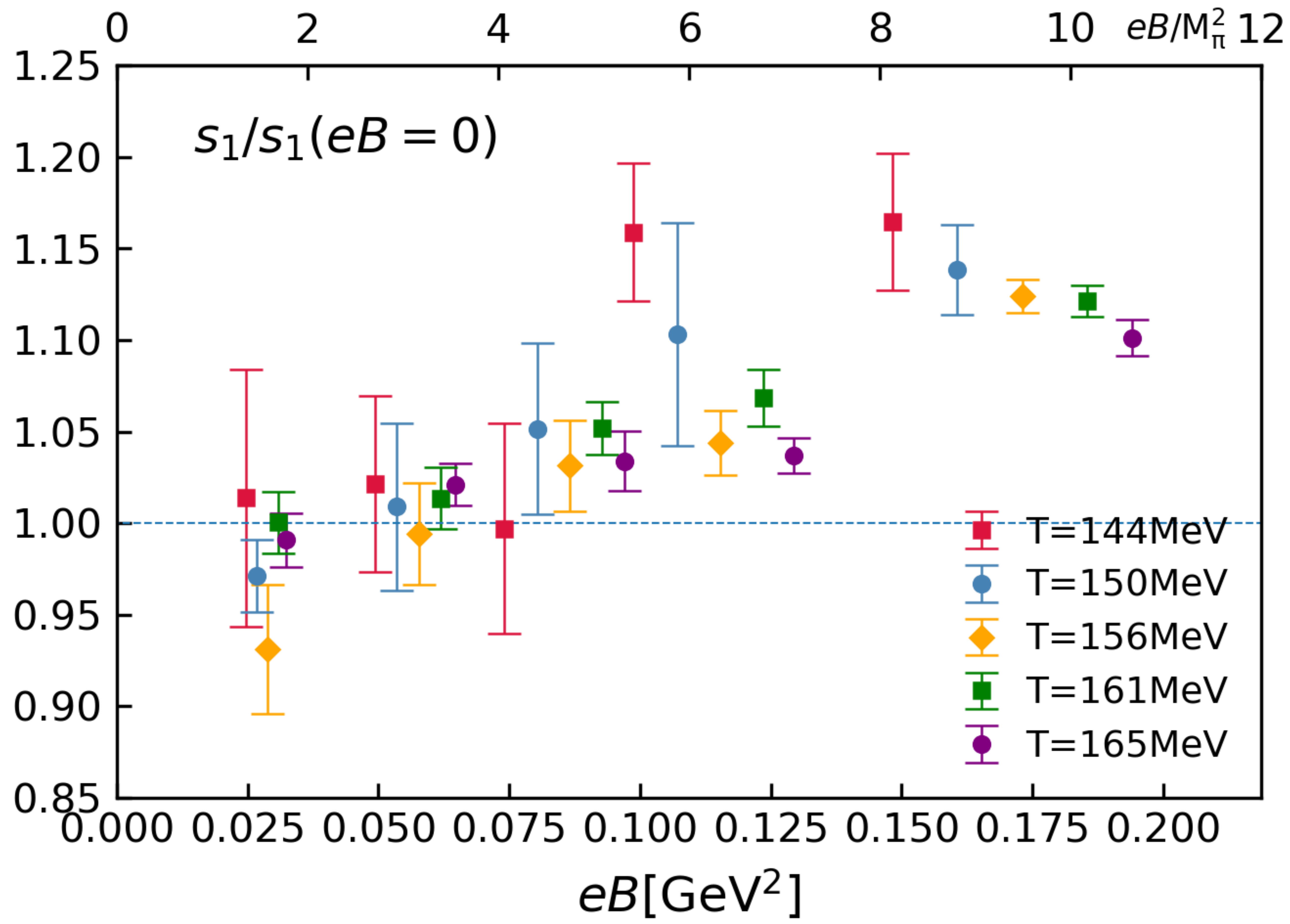
HTD, S.-T. Li, J.-H. Liu, X.-D. Wang in preparation

Backup

$$\left(\frac{\mu_S}{\mu_B}\right)_{\rm LO} \equiv s_1(T) = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S}\frac{\mu_Q}{\mu_B}.$$

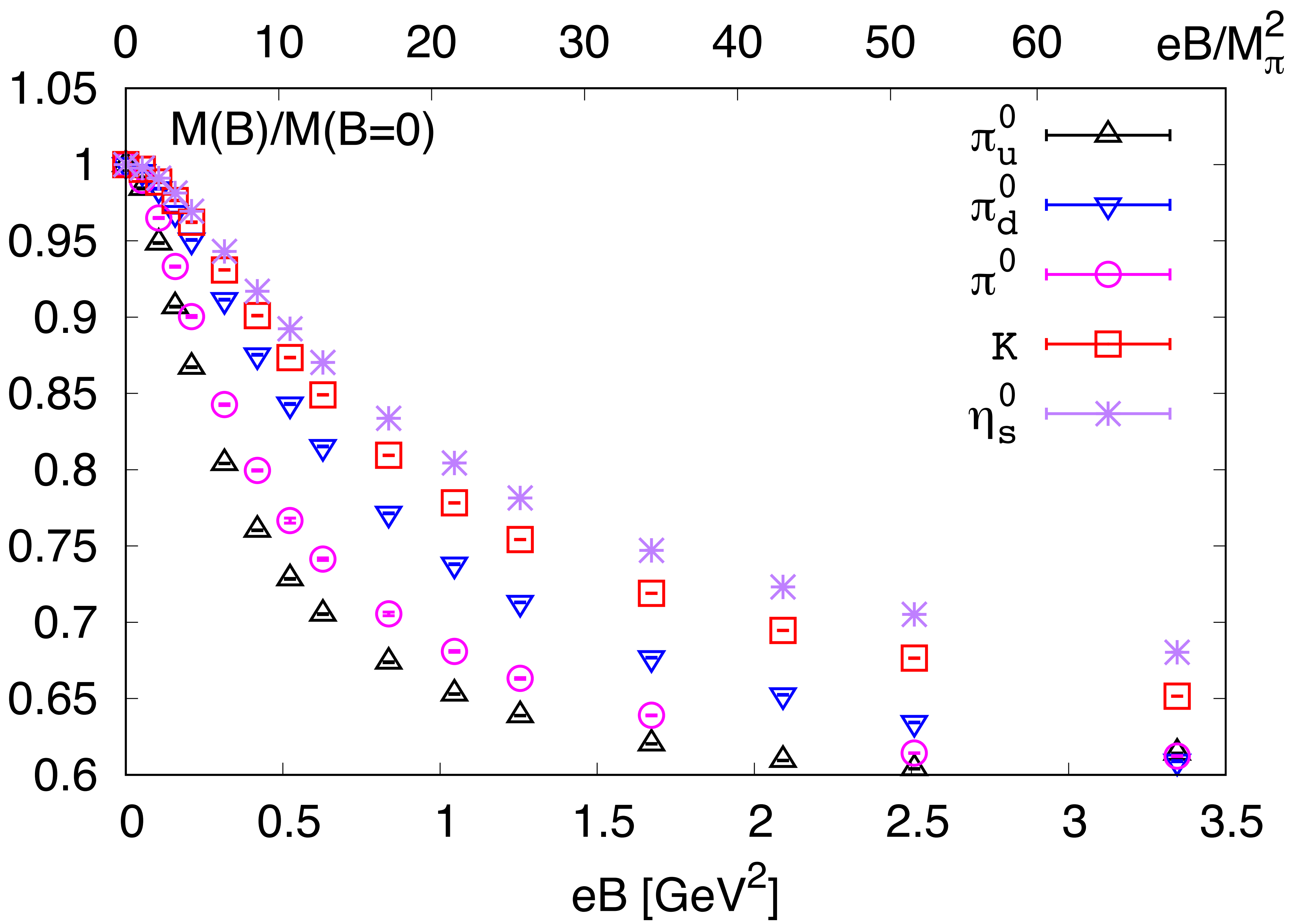
eB=0

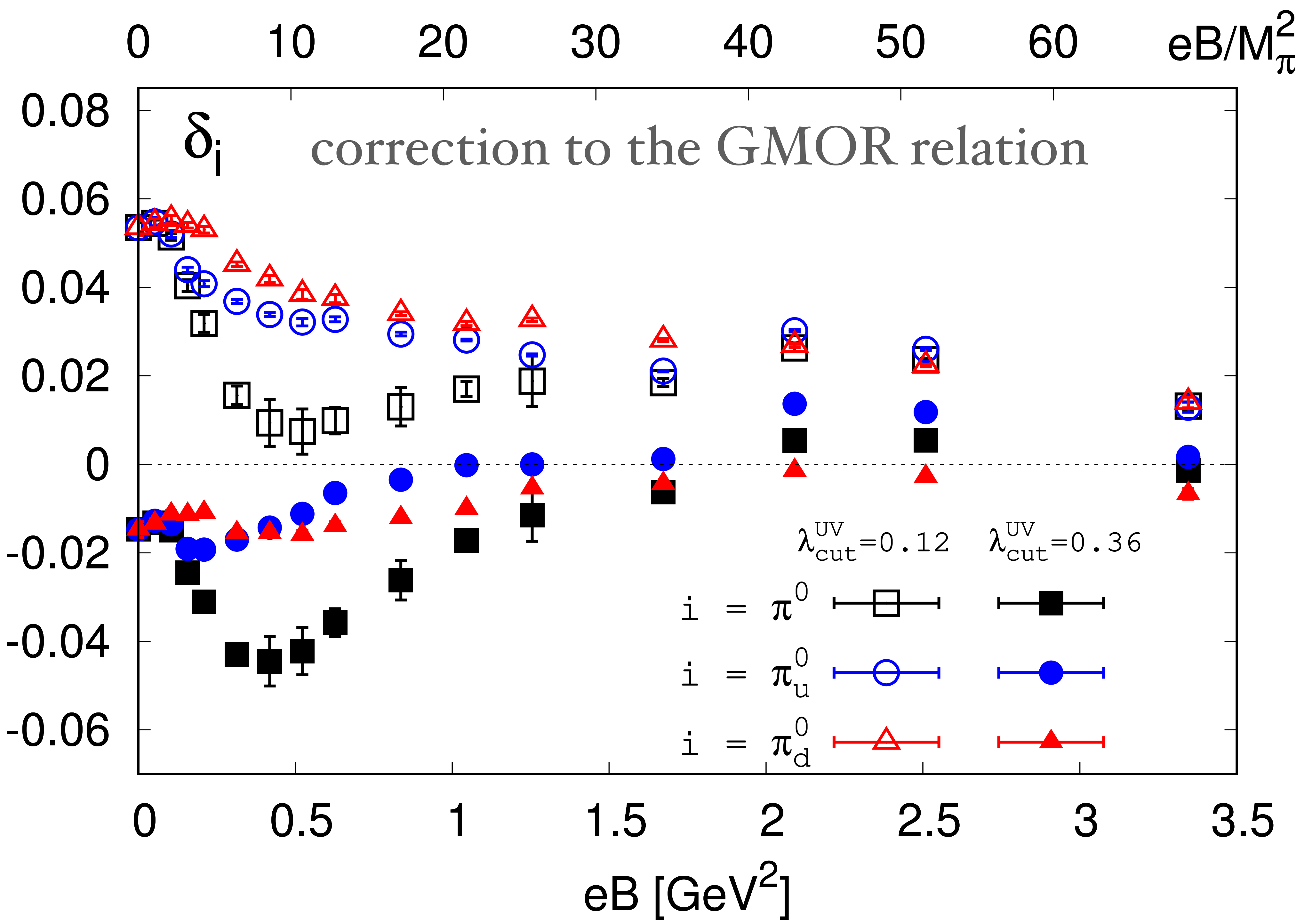




Neutral pion mass & Gell-Mann-Oakes-Renner relation with $eB \neq 0$

N_f=2+1 QCD, $M_\pi(eB=0) \approx 220$ MeV, on $32^3 \times 96$ lattices with $a^{-1} \approx 1.7$ GeV^{-1} and HISQ action at T=0





HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001

neutral pion remains as a Goldstone boson with eB up to ~3.5 GeV^2

T_{pc} decreases with eB regardless of (inverse) magnetic catalysis