Lattice results on the QCD phase diagram at finite vorticity

V.V. Braguta

JINR

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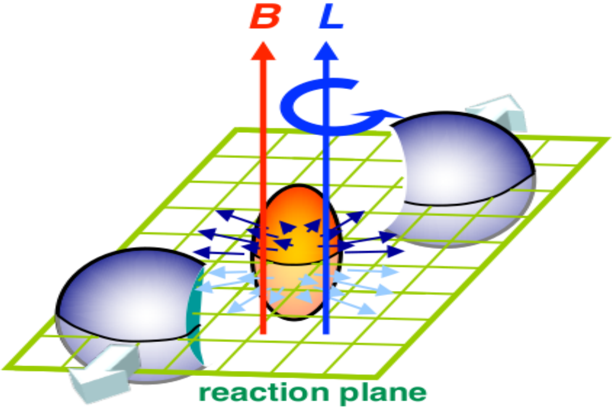
In collaboration with

- A.Yu. Kotov
- D.D. Kuznedelev
- A.A. Roenko
- D.A. Sychev

V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. 112, 9–16 (2020),

Phys.Rev.D 103 (2021) 9, 094515, e-Print: 2110.12302

Rotation of QGP in heavy ion collisions



- QGP is created with non-zero angular momentum in non-central collisions

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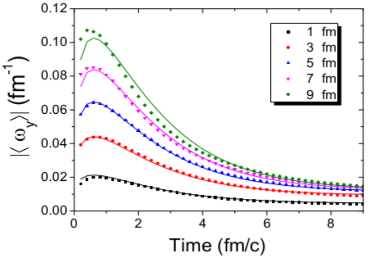
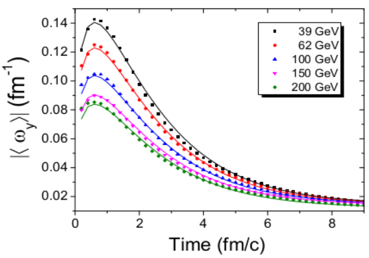
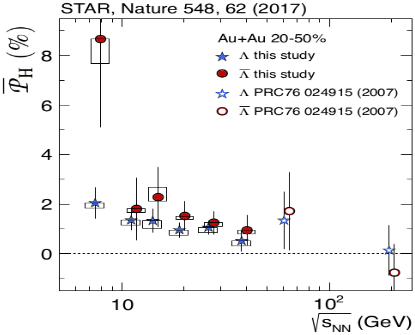




Hydrodynamic simulations (Phys.Rev.C 94, 044910 (2016))

- Au-Au: *left* $\sqrt{s} = 200$ GeV, *right* $b = 7$ fm,
- $\Omega \sim (4 - 28)$ MeV ($\Omega \sim 20$ MeV $\Rightarrow v \sim c$ at distances 7 fm)
- Relativistic rotation of QGP

Rotation of QGP in heavy ion collisions



Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar{\Lambda}}) \frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim 10$ MeV
- Relativistic rotation of QGP

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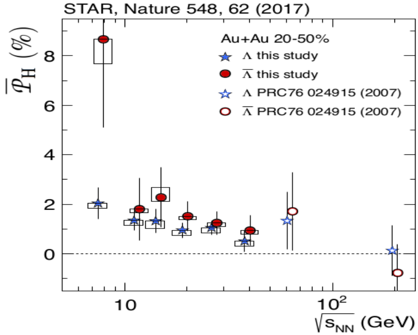

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- $\Omega \sim 10$ MeV
- Relativistic rotation of QGP

How relativistic rotation influences QCD?

Recent theoretical works

- A. Yamamoto, Y. Hirono, Phys.Rev.Lett. 111 (2013) 081601
- A. Yamamoto, Eur.Phys.J.A 57 (2021) 6, 211
- S. Ebihara, K. Fukushima, K. Mameda, Phys. Lett. B 764 (2017) 94–99
- M.N. Chernodub, S. Gongyo, Phys.Rev.D 95 (2017) 9, 096006
- M.N. Chernodub, S. Gongyo, JHEP 01 (2017) 136
- H. Zhang, D. Hou, J. Liao, e-Print: 1812.11787 [hep-ph]
- Y. Jiang, J. Liao, Phys.Rev.Lett. 117 (2016) 19, 192302
- Xun Chen, Lin Zhang, Danning Li, Defu Hou, Mei Huang, e-Print: 2010.14478
- Y. Fujimoto, K. Fukushima, Y. Hidaka, Phys.Lett.B 816 (2021) 136184
- M.N. Chernodub, Phys.Rev.D 103 (2021) 5, 054027

...

Common features

- Mostly the studies are carried out in NJL (chiral transition)
- **Critical temperature of the chiral phase transition drops with angular velocity**
- Explanation: polarization of the chiral condensate (Phys.Rev.Lett. 117 (2016) 19, 192302)
- **Critical temperature of the confinement transition drops with angular velocity**

Study of rotating QGP

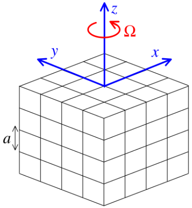
- Our aim: study rotating QCD within lattice simulations
- Rotating QCD at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \text{Tr} \exp\left[-\beta \hat{H}\right]$$

- Euclidean action

$$S_G = -\frac{1}{2g_{YM}^2} \int d^4x \sqrt{g_E}\, g_E^{\mu\nu} g_E^{\alpha\beta} F_{\mu\alpha}^{(a)} F_{\nu\beta(a)}$$

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \text{Tr}\big[(1 - r^2\Omega^2) F_{xy}^a F_{xy}^a + (1 - y^2\Omega^2) F_{xz}^a F_{xz}^a +$$

$$+(1 - x^2\Omega^2) F_{yz}^a F_{yz}^a + +F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a -$$

$$-2iy\Omega(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{xz} F_{zy}\big]$$

Details of the simulations

- *Ehrenfest–Tolman effect:* **In gravitational field the temperature is not constant in space at thermal equilibrium**

$$T(r)\sqrt{g_{00}} = const = 1/\beta$$

$$T(r)\sqrt{1 - r^2\Omega^2} = 1/\beta$$

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- We use the designation $T = T(r = 0) = 1/\beta$

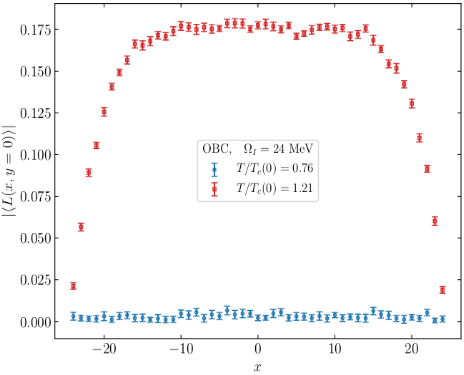
Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
- **Neumann b.c.:**
  - Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$

- *The dependence on boundary conditions is the property of all approaches*
- *One can expect that boundary conditions influence our results considerably, but their influence is restricted due to the screening*

Screening of boundary conditions



Details of the simulations

Sign problem

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \mathrm{Tr}\big[(1 - r^2\Omega^2) F^a_{xy} F^a_{xy} + (1 - y^2\Omega^2) F^a_{xz} F^a_{xz} +$$

$$+(1 - x^2\Omega^2) F^a_{yz} F^a_{yz} + +F^a_{x\tau} F^a_{x\tau} + F^a_{y\tau} F^a_{y\tau} + F^a_{z\tau} F^a_{z\tau} -$$

$$-2iy\Omega(F^a_{xy} F^a_{y\tau} + F^a_{xz} F^a_{z\tau}) + 2ix\Omega(F^a_{yx} F^a_{x\tau} + F^a_{yz} F^a_{z\tau}) - 2xy\Omega^2 F_{xz} F_{zy}\big]$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω ($\Omega < 50$ MeV)

Details of the simulations

The critical temperature

- Polyakov line

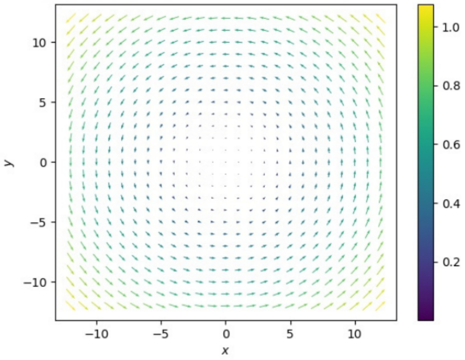
$$L = \left\langle \mathrm{Tr}\mathcal{T} \exp\left[ig \int_{[0,\beta]} A_4 \, dx^4 \right] \right\rangle$$

- Susceptibility of the Polyakov line

$$\chi = N_s^2 N_z \left(\langle |L|^2 \rangle - \langle |L| \rangle^2 \right)$$

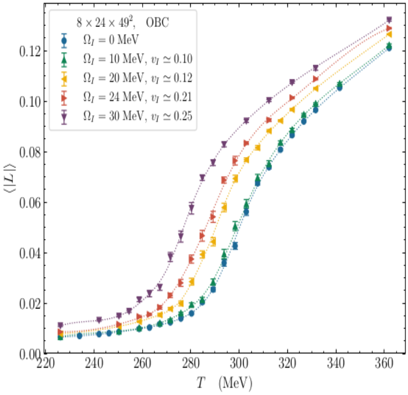
- T_c is determined from Gaussian fit of the $\chi(T)$

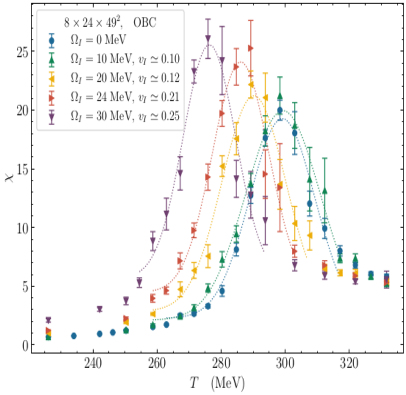
Rotation at zero temperature



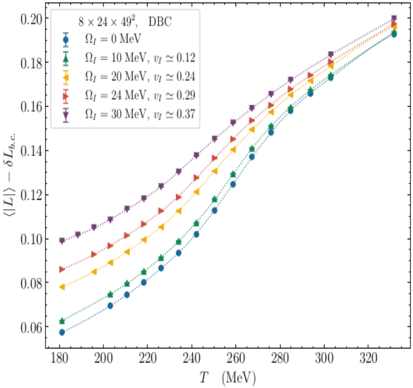
- $\langle trF_{\mu\nu}^2 \rangle \neq 0, \quad \langle T_{\mu\nu} \rangle = \epsilon g_{\mu\nu}, \quad \epsilon \sim \langle trF_{\mu\nu}^2 \rangle$
- In rotating frame $\langle T_{0i} \rangle \neq 0$
- The ground state of our system is "rotating vacuum"

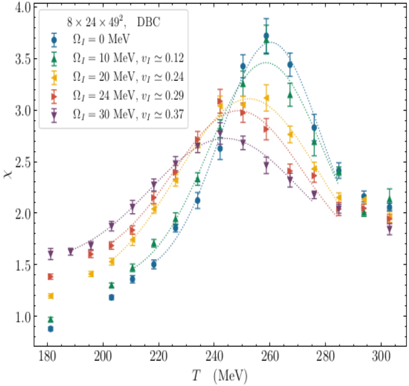
Results of the calculation (Neumann b.c.)



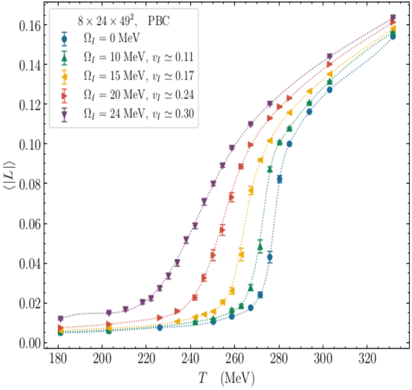


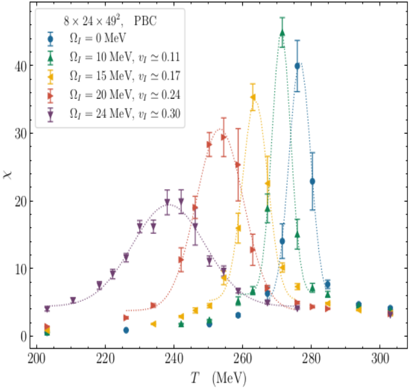
Results of the calculation (Dirichlet b.c.)



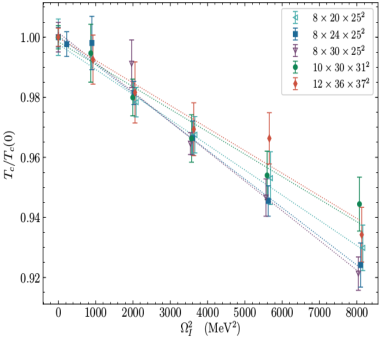


Results of the calculation (Periodic b.c.)





Results of the calculation

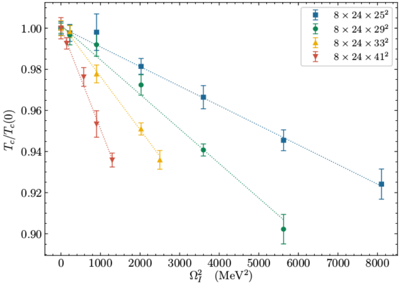


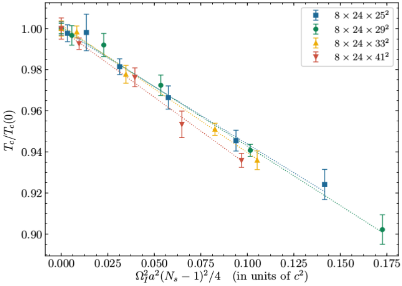
- The results can be well described by the formula ($C_2 > 0$)

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2 \Rightarrow \frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

- **The critical temperature rises with angular velocity**
- The results weakly depend on lattice spacing and the volume in z-direction

Dependence on the transverse size

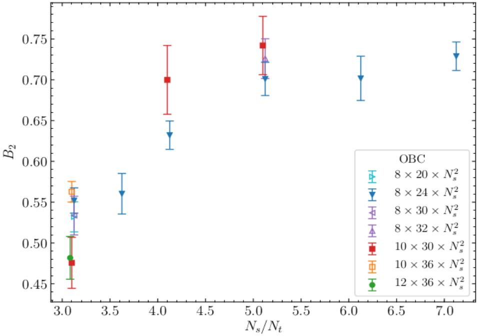




- The results can be well described by the formula

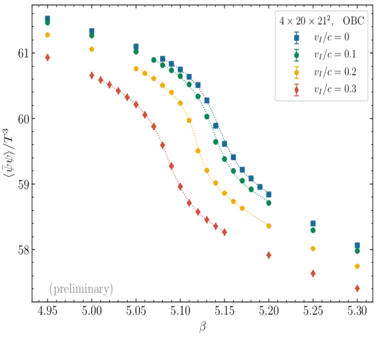
$$\frac{T_c(\Omega)}{T_c(0)} = 1 - B_2 v_I^2, \quad v_I = \Omega_I(N_s-1)a/2, \quad C_2 = B_2(N_s-1)^2 a^2/4$$

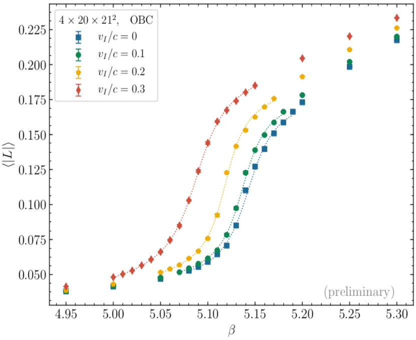
Dependence on the transverse size



- The value of B_2 depends on b.c.
  - **Periodic b.c.:** $B_2 \sim 1.3$
  - **Dirichlet b.c.:** $B_2 \sim 0.5$
  - **Neumann b.c.:** $B_2 \sim 0.7$

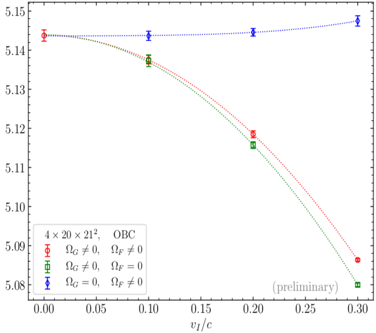
Simulation with fermions (preliminary)





- Lattice simulation with Wilson fermions: $m_\pi \simeq 700\,\text{MeV}$
- Critical couplings of both transitions coincide
- Critical temperatures are increased (at least for $m_\pi \simeq 700\,\text{MeV}$)

Simulation with fermions (preliminary)



- One can introduce angular velocities for gluons Ω_G and fermions Ω_F
- $\Omega_F \neq 0, \Omega_G = 0$ decreases critical temperatures (**in agrement with NJL**)
- $\Omega_F = 0, \Omega_G \neq 0$ increases critical temperatures

- $\Omega_G = \Omega_F \neq 0$ gluons win (for heavy pion mass)

Conclusion

- Lattice study of rotating gluodynamics has been carried out
- Critical temperature of the confinement/deconfinement transition rises with rotation
- First results on rotating QCD
- QCD with heavy pions: competition between fermions and gluons ($m_\pi \sim 700$ MeV gluons win): Critical temperatures rise

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THANK YOU!