Chiral Magnetic Effect in Holography

6th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

Stony Brook University

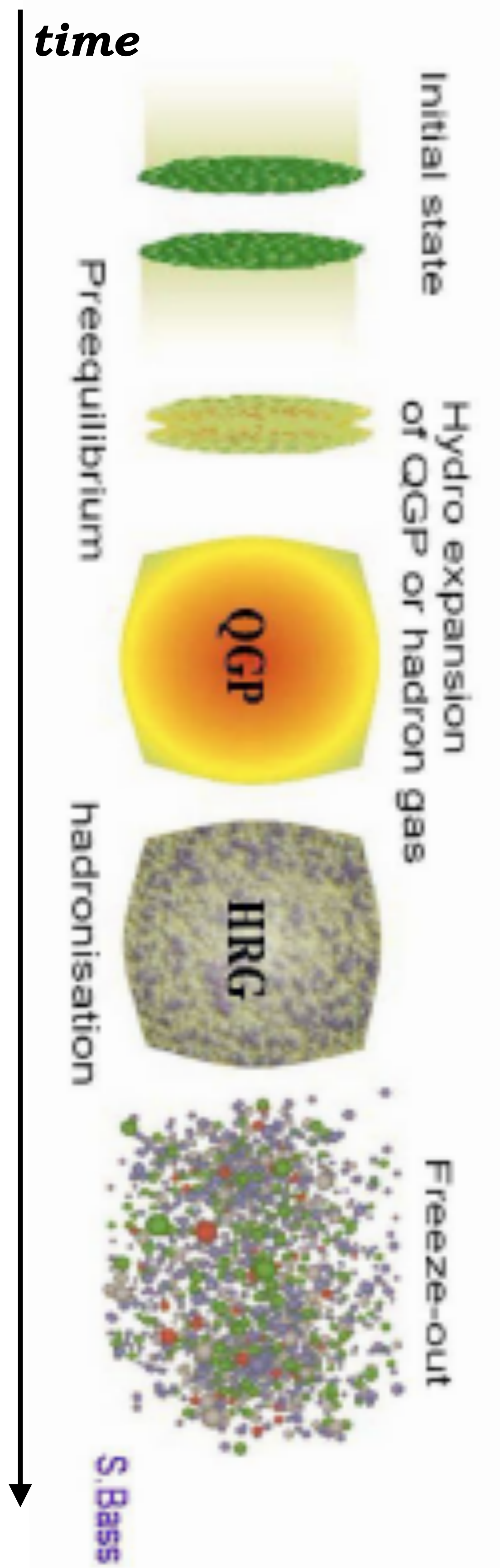
November 4th, 2021

Matthias Kaminski

University of Alabama

Outline

Disclaimer: references incomplete



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HYDRODYNAMICS

Chiral Magnetic Effect (CME) from chiral anomaly

[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

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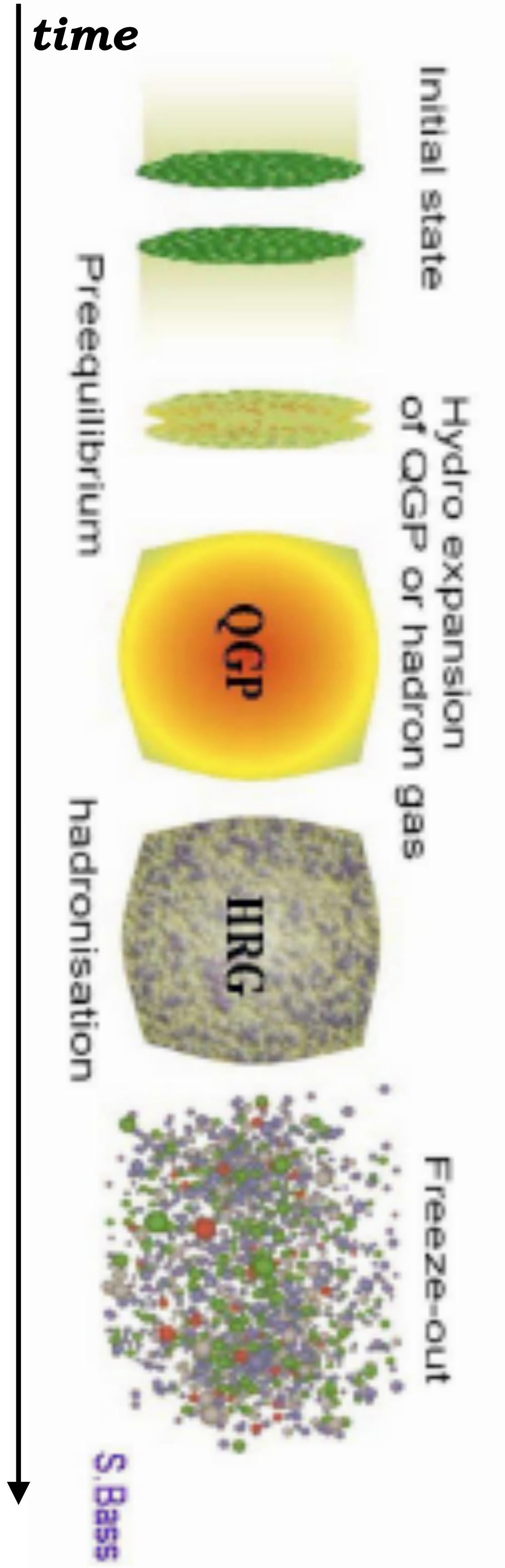
$$\xi_B = C\,\mu \qquad \nabla_\mu J_A^\mu = C\, E\cdot B$$



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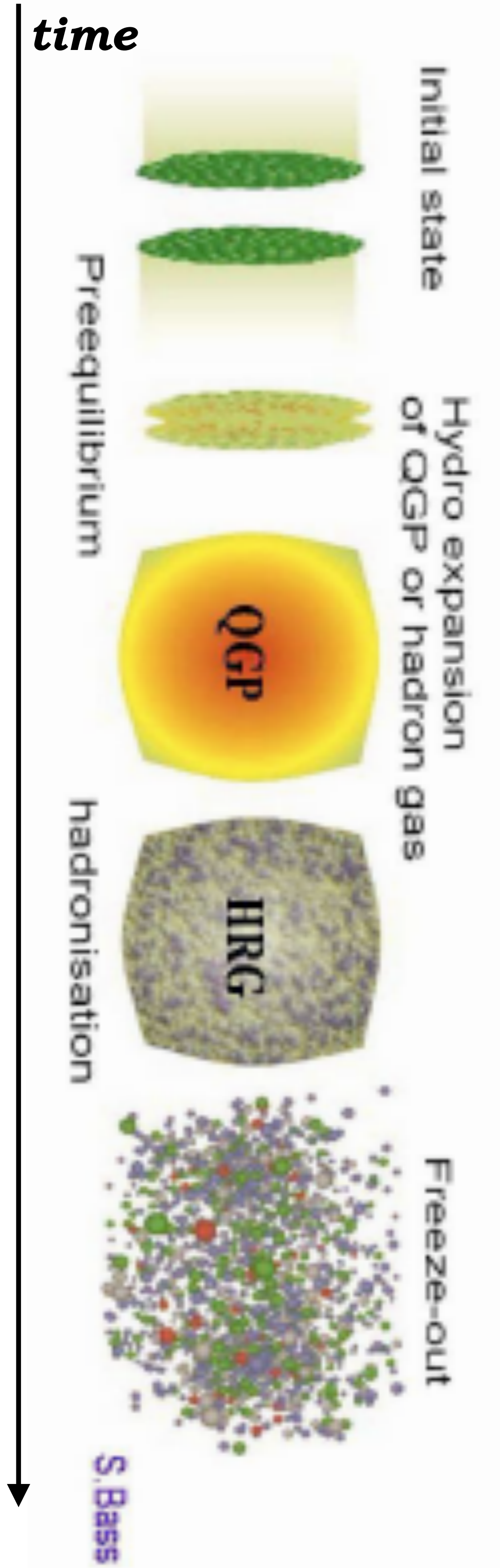

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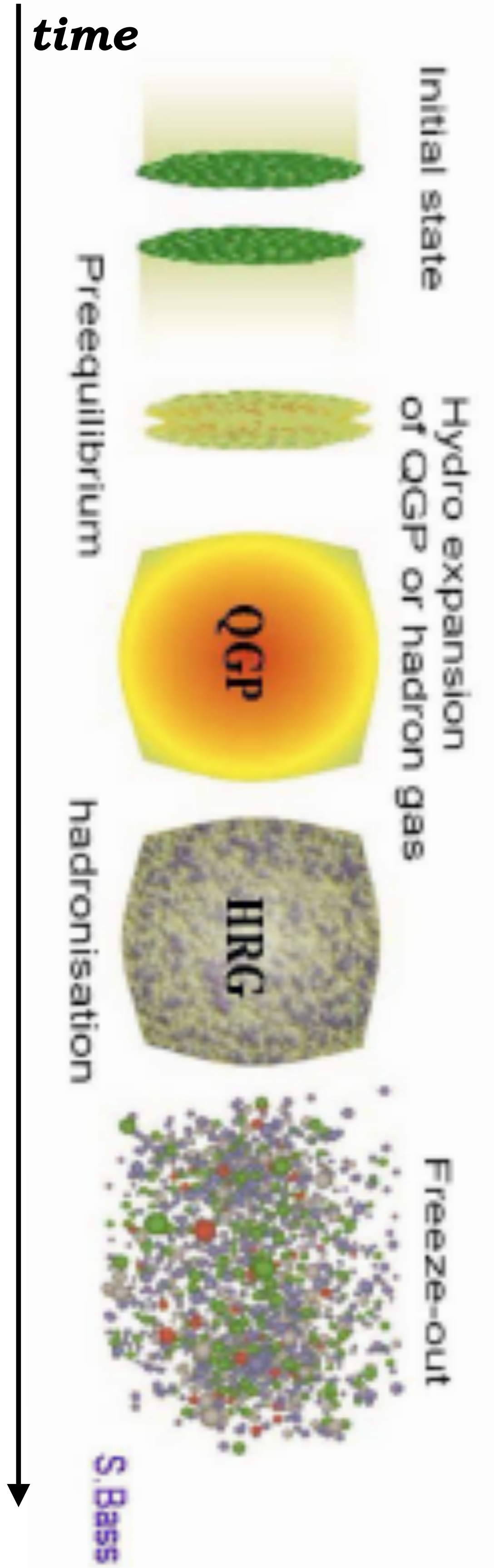

HOLOGRAPHY

CME far from equilibrium, strong B

- **non-expanding plasma**
  [Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]
- **expanding plasma**
  [Cartwright,Kaminski,Schenke; to appear (2021)]

Frequency dependence of CME

[Amado,Landsteiner,Pena_Benitez; JHEP (2011)]
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CME near equilibrium

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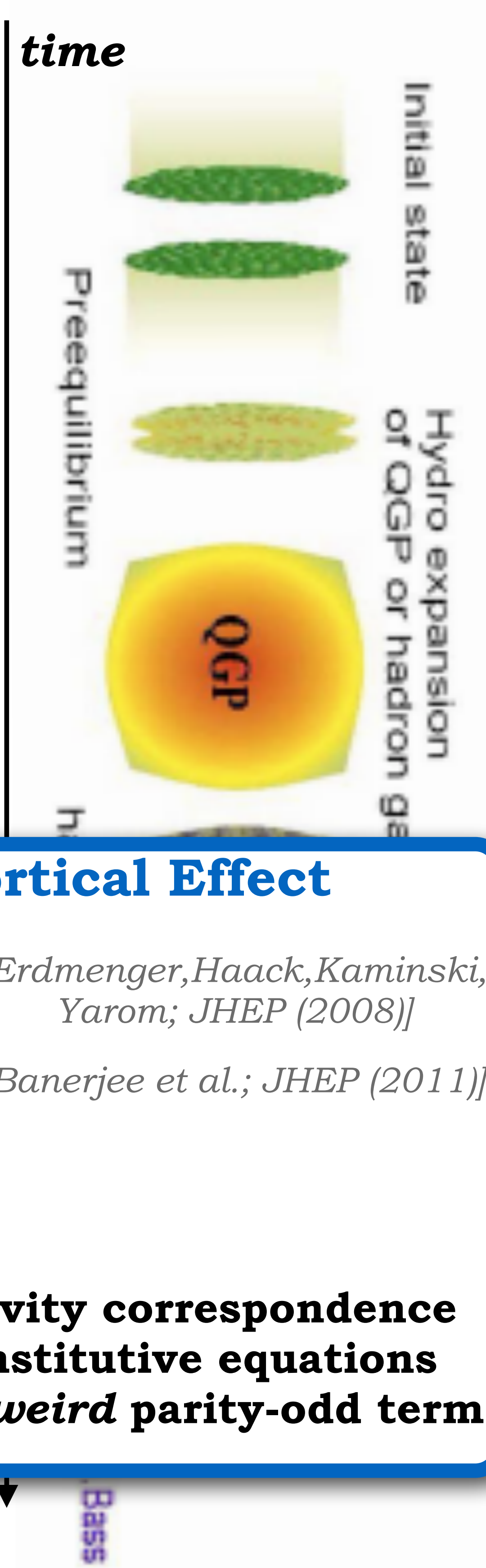
time



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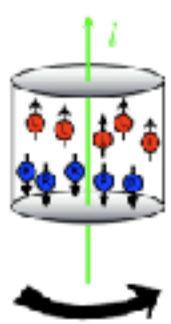
hydro and holo in parallel

Chiral Vortical Effect



[Erdmenger,Haack,Kaminski, Yarom; JHEP (2008)]

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- **fluid/gravity correspondence**
- **gives constitutive equations**
- **contain *weird* parity-odd term**

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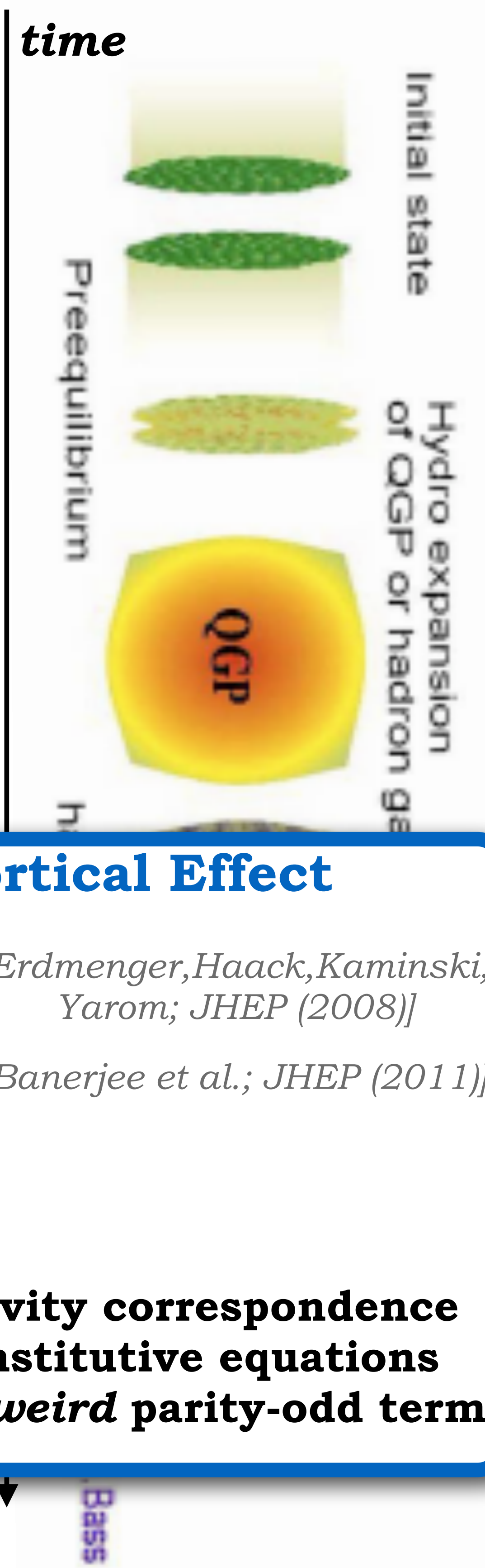

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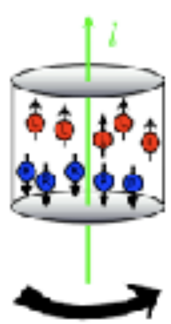

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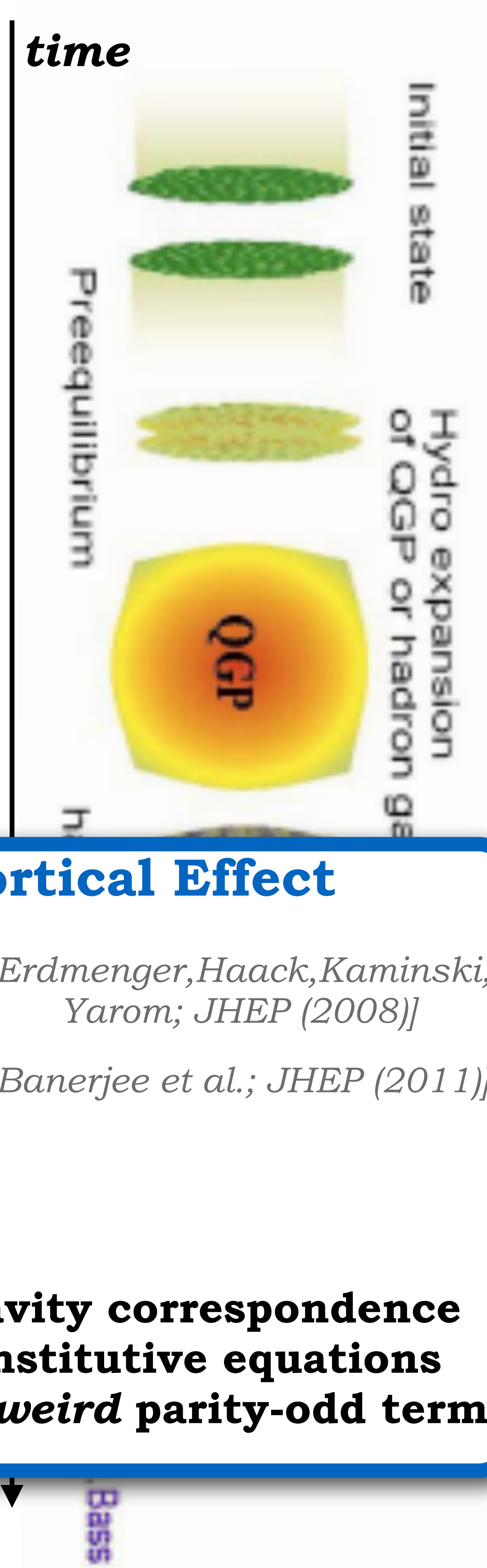

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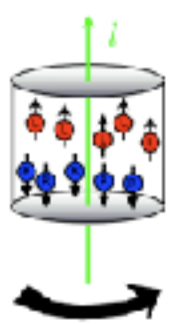

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- **fluid/gravity correspondence**
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Near equilibrium - weak $B \propto \mathcal{O}(\partial)$

Method

HYDRODYNAMICS

1. **Construct hydrodynamic constitutive relations:** e.g. $\langle J^\mu \rangle = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \frac{\mu}{T} \right]$

   charge current

2. **Calculate poles of hydrodynamic correlation functions:** e.g. $\langle J^x J^x \rangle = \frac{i\omega^2 \sigma}{\omega + iDk^2} + \ldots$

   charge diffusion

   yields hydro dispersion relations, e.g. $\omega_{\text{hydro}}(k) = -\, iDk^2 + \ldots$

   charge diffusion dispersion

HOLOGRAPHY

3. **Calculate same poles in holographic model:**

   [Starinets; PRD (2002)]
   [Kovtun,Starinets; PRD (2005)]

   those are quasinormal mode (QNM) frequencies $\omega_{\text{QNM}}(k)$

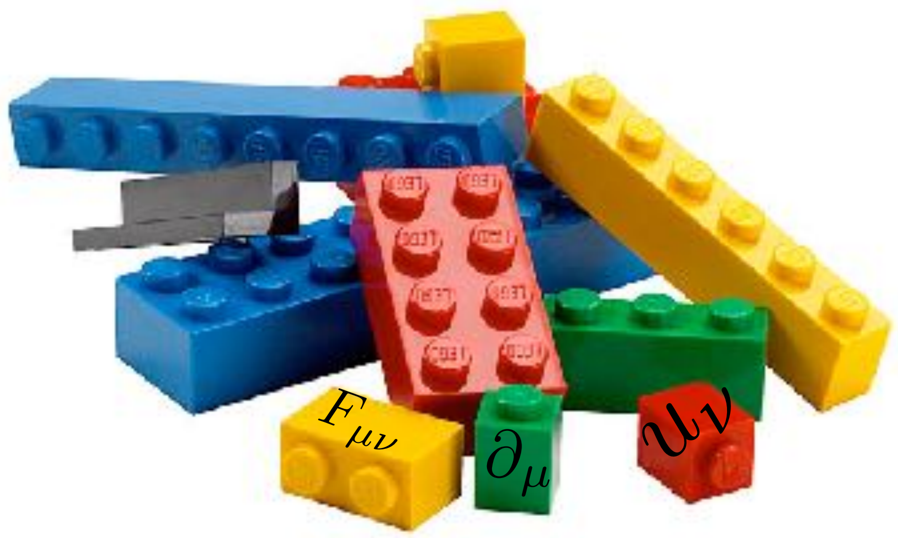
4. **Check holo result against hydro prediction (small *k*):** $\omega_{\text{QNM}}(k) \approx \omega_{\text{hydro}}(k)$

   ➡ **agreement**

Hydrodynamics - construct constitutive eq's

see Xu-Guang Huang's talk

1. Constitutive equations: all (pseudo)vectors and (pseudo)tensors under Lorentz group



Examples: $\nabla_\nu u^\nu$

$$\underset{\textit{vorticity}}{\omega^\mu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \nabla_\lambda u_\rho$$

2. Restricted by conservation equations

Example: $\nabla_\mu j^\mu_{(0)} = \nabla_\mu (n u^\mu) = 0$

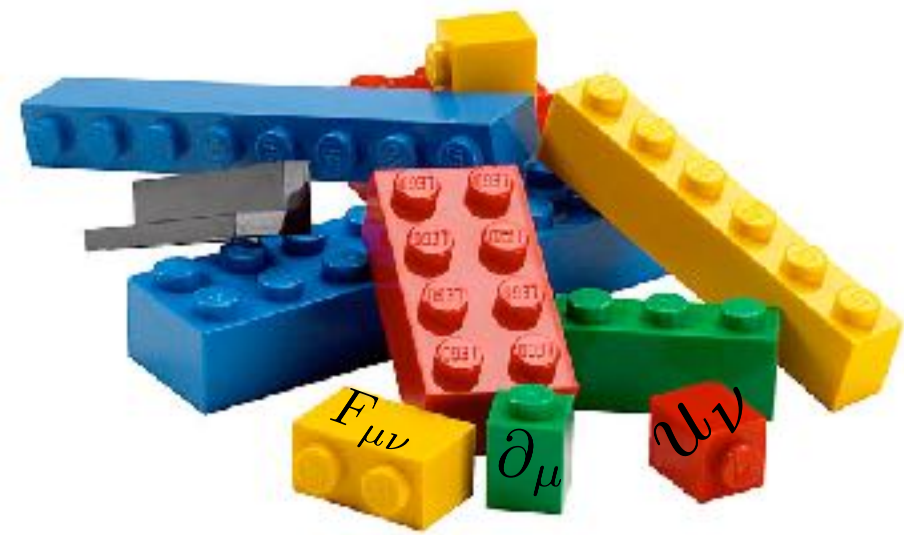
3. Further restricted by positivity of local entropy production: $\nabla_\mu J^\mu_s \geq 0$ *[Landau, Lifshitz]*

➡ **hydrodynamic 2-point functions (+ their poles)**

[Ammon, Kaminski et al.; JHEP (2017)]

Note: for generating functional approach equivalent to entropy restrictions see
[Jensen, Kaminski, Kovtun,Meyer, et al.; PRL (2012)]
[JHEP (2011)]
[Banerjee et al. JHEP (2012)]

Hydrodynamics - construct constitutive eq's



Side note
The big advantage of this systematic construction is that it yields all possible terms in the constitutive relations. For example:

$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$	*Thermal vorticity* Adimensional in natural units
$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$	*Thermal shear* Adimensional in natural units

taken from Francesco Becattini's talk

See also the construction of hydrodynamics with spin as a slow mode which contains both these contributions:

[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2021)]

see Xu-Guang Huang's talk

Dispersion relations: weak *B* hydrodynamics

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\,T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\,J^{\alpha}\rangle$, $\langle J^{\mu}\,T^{\alpha\beta}\rangle$, $\langle J^{\mu}\,J^{\alpha}\rangle$:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 1 modes under SO(2) rotations around *B*

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0 / n_0$$
$$\tilde{c}_P = T_0 (\partial \mathfrak{s} / \partial T)_P$$

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i \Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i \Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

➡ **a chiral magnetic wave**

[Kharzeev,Yee; PRD (2011)]

$$v_0 = \frac{2 B T_0}{\tilde{c}_P n_0} \left(\tilde{C} - 3 C \mathfrak{s}_0^2 \right)$$

$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

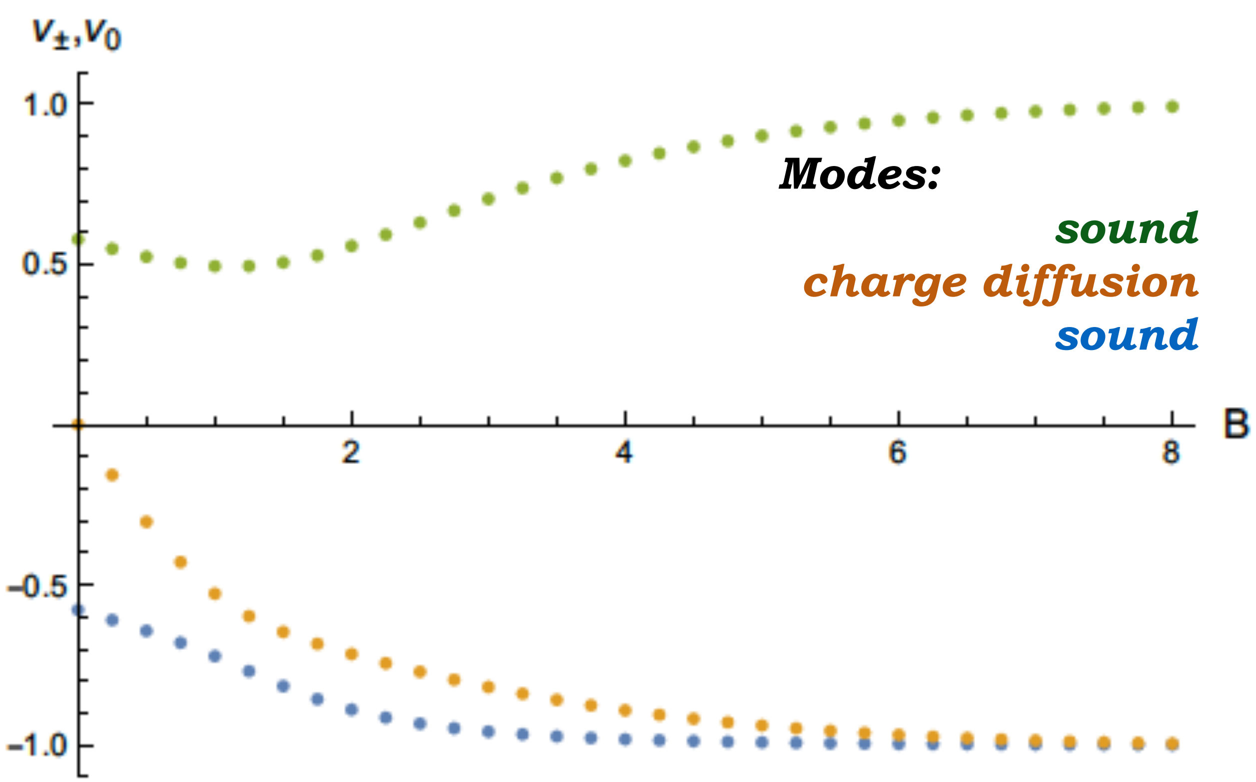
Holographic result: hydrodynamic poles

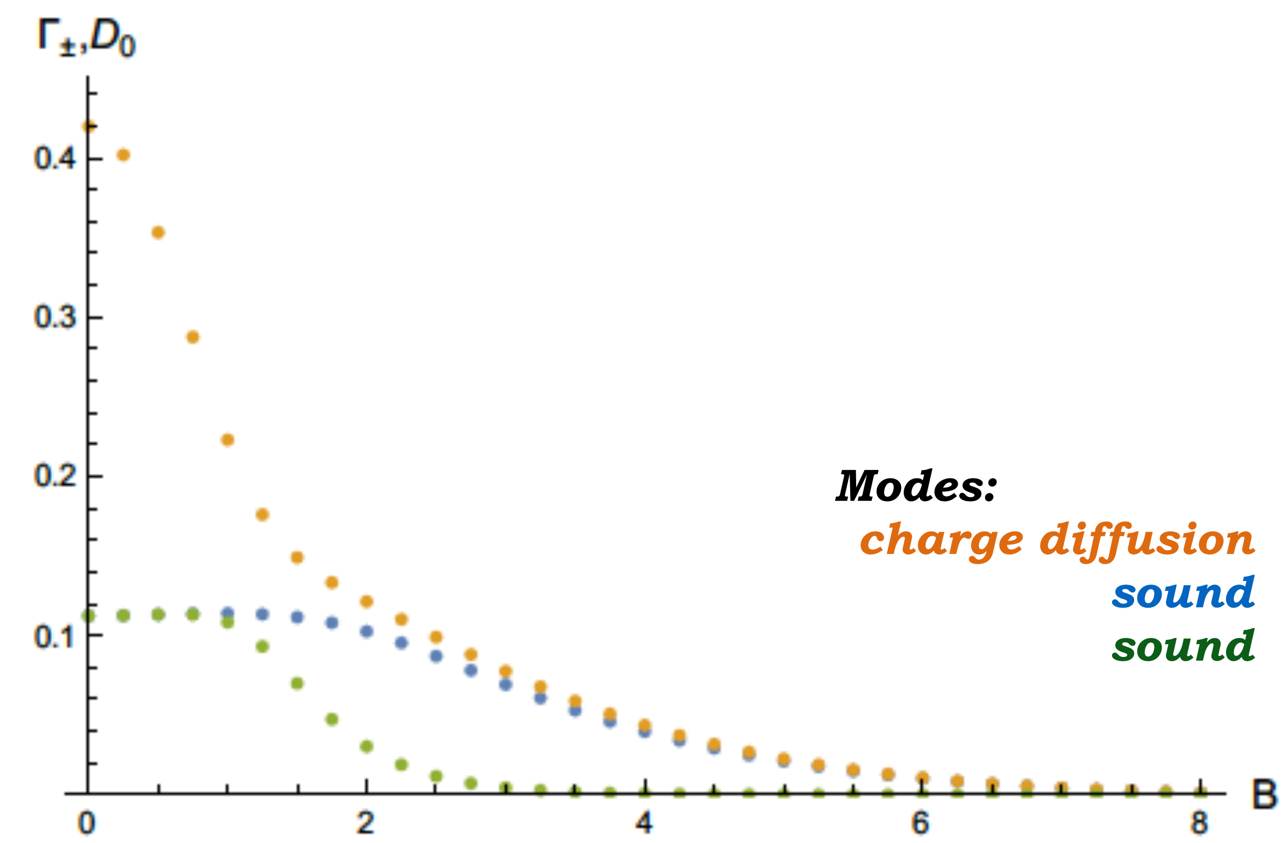
[Ammon, Kaminski et al.; JHEP (2017)]

Fluctuations around charged magnetic black branes (QNMs)

- Weak *B*: **holographic results are in “agreement” with hydrodynamics.**
- Strong *B*: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate:**

(i) at the speed of light **and (ii) without attenuation**





confirming conjectures and results in probe brane approach *[Kharzeev,Yee; PRD (2011)]*

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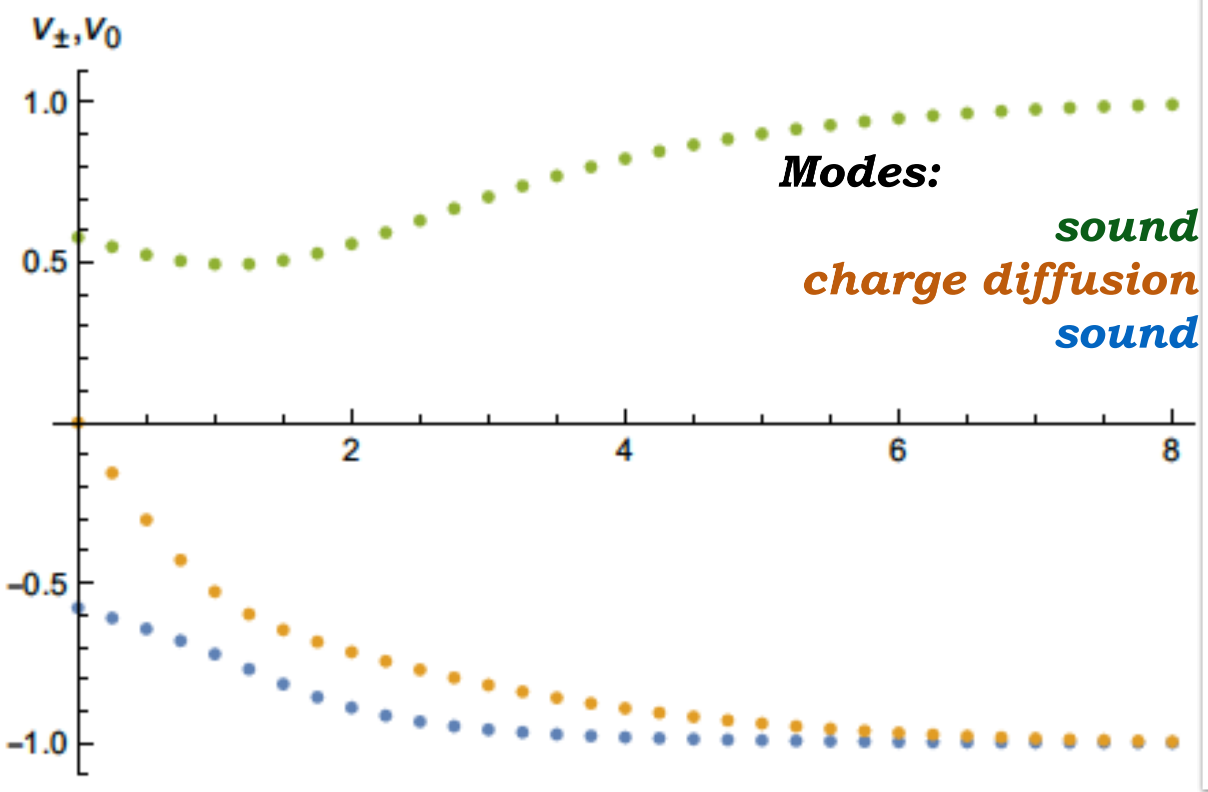

RECALL: weak B hydrodynamic pole

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

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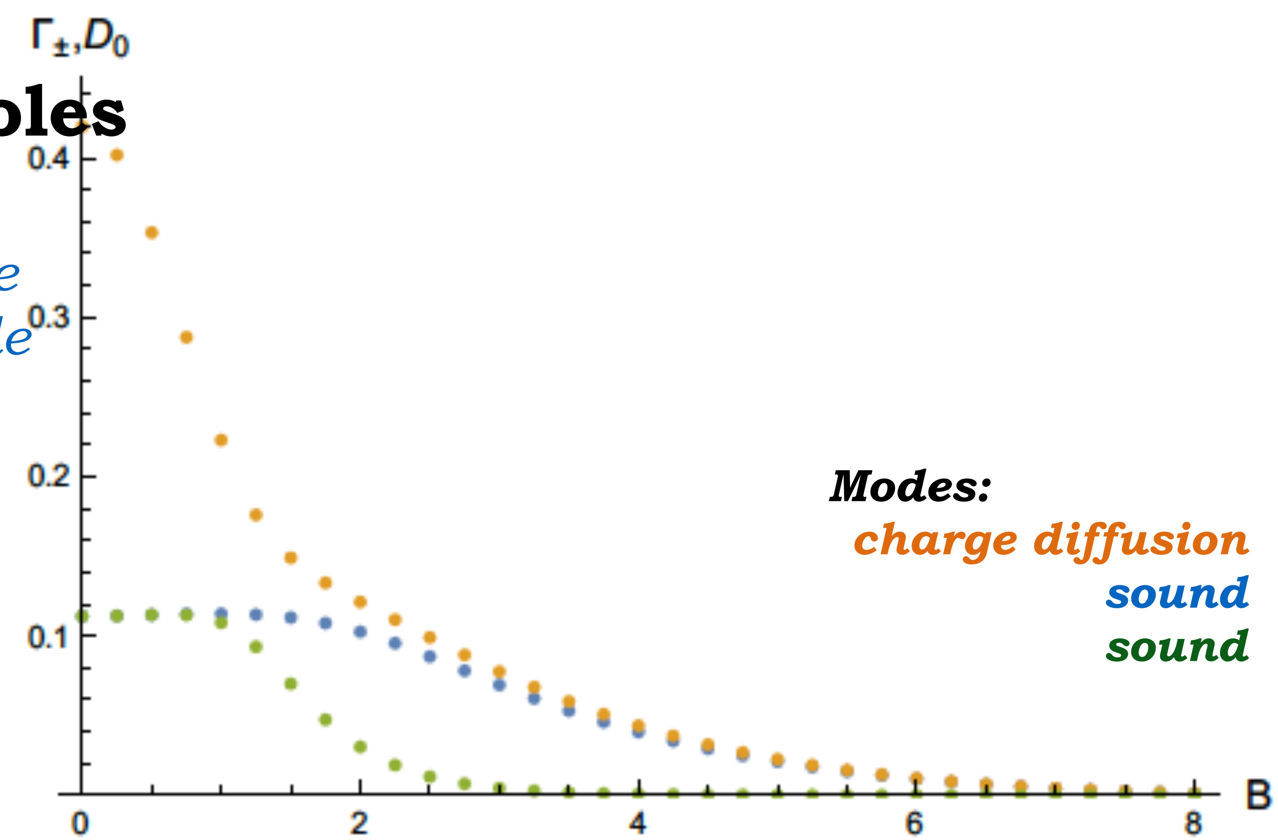

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Details: weak B hydro/holo comparison

Spin-1 modes

No knowledge of anisotropic (B-dependent) transport coefficients — take B=0 values of this model instead

except zero charge: [Finazzo, Critelli, Rougemont, Noronha; PRD (2016)]

weak B hydro prediction:

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

calculate from holography

We find agreement between hydrodynamic prediction and holographic model for small values of B, increasing deviations for larger B.

Real part of spin-1 modes matches exactly even at large B!

Near equilibrium - strong $B \propto \mathcal{O}(1)$

Applicability range *hydrodynamic limit, small gradients*

$$\frac{\omega}{T} \ll 1, \qquad \frac{|\vec{k}|}{T} \ll 1$$

temperature is largest scale

$$B \sim \mathcal{O}(1) \qquad B \ll T^2$$

example: RHIC

$T \approx 300\,\text{MeV}$

$B \approx (140\,\text{MeV})^2$

$B \approx 10^4\,\text{MeV}^2 \ll T^2 \approx 10^5\,\text{MeV}^2$

Method

HYDRODYNAMICS

1. **Construct hydrodynamic constitutive relations**
2. **Calculate hydrodynamic correlation functions and derive Kubo formulae**

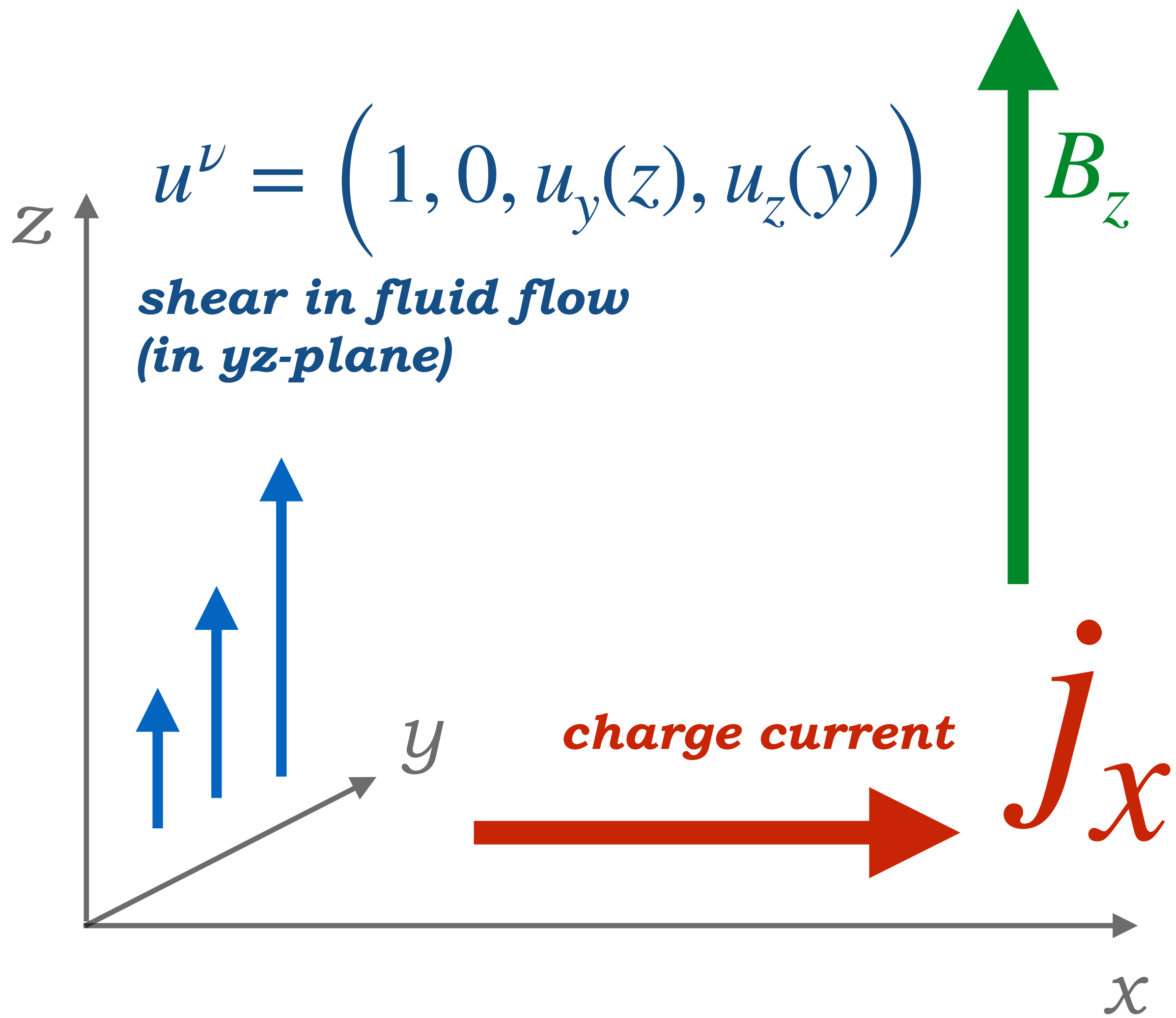
Reminder: shear viscosity $\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, dx\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$

HOLO

3. **Calculate same correlation functions in holographic model**
4. **Plug holo correlators into Kubo formulae to extract transport coefficients**

Hydrodynamics - novel transport effect

Shear-induced Hall conductivity c_{10}



In constitutive relation:

$$j_x \sim c_{10}(\partial_y u_z + \partial_z u_y)$$

Kubo formula:

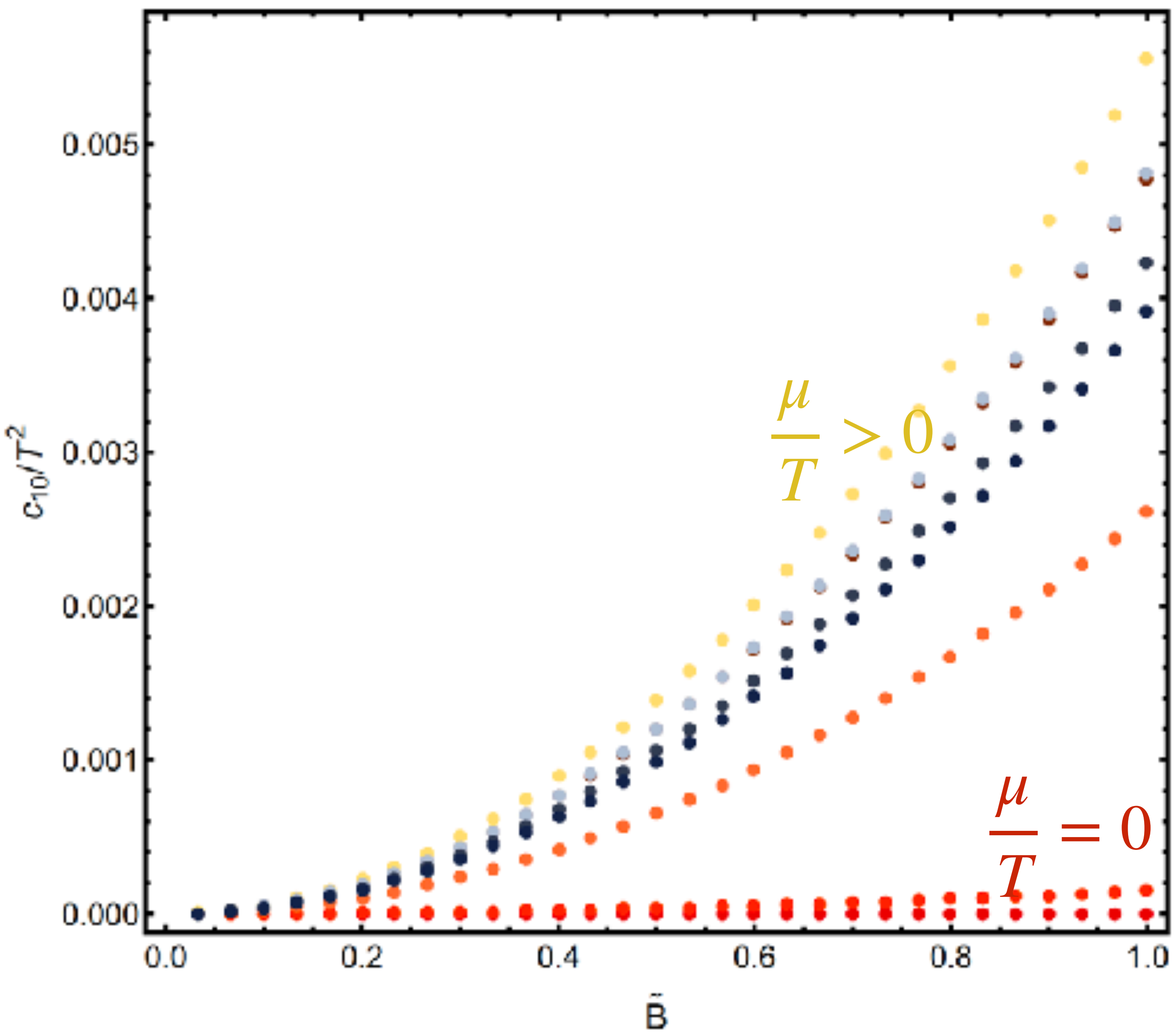
$$c_{10} \sim \frac{1}{\omega} \mathrm{Im}\, G_{T^{tx}\, T^{yz}}$$

- ➡ novel Hall response
- ➡ non-dissipative
- ➡ interplay: shear-charge

Holographic model - novel transport effect

Shear-induced Hall conductivity c_{10}

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]



- ➡ **not zero, finite**
- ➡ **Kubo formulae reasonable**
- ➡ **This is not the only coefficient …**

All coefficients in strong B hydrodynamics

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Thermodynamic $\left(\lim\limits_{\mathbf{k}\to 0}\lim\limits_{\omega\to 0}\right)$, non-dissipative					
helicity 1					
M_2	perp. magnetic vorticity susceptibility	$T^{zz}T^{yz}$ (2.30)	+	-	+
M_5	magneto-vortical susceptibility	$T^{tx}T^{yz}$ (2.30,2.31)	+	-	+
ξ	chiral vortical conductivity	$J_x T_{ty}$ (2.38,2.39)	+	+	+
ξ_B	chiral magnetic conductivity	$J^x J^y$ (2.38,2.39)			
ξ_T	chiral vortical heat conductivity	$T^{tx}T^{ty}$ (2.38,2.39)	+	-	+
helicity 0					
M_1	magneto-thermal susceptibility	$J^t T^{xx}$ (2.32)	+	+	-
M_3	magneto-acceleration susceptibility	$J^t T^{tt}$ (2.32)	+	+	-
M_4	magneto-electric susceptibility	$J^t J^t$ (2.32)	+	-	-

CME

dissipative, hydrodynamic $\left(\lim\limits_{\omega\to 0}\lim\limits_{\mathbf{k}\to 0}\right)$					
coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
helicity 2					
$\eta_\perp$	perp. shear viscosity	$T_{xy}T_{xy}$ (2.55)	+	+	-
helicity 1					
$\eta_\parallel$	parallel shear viscosity	$T^{xz}T^{xz}$ (2.59a)	+	+	-
$\tilde{\eta}_\parallel$	parallel Hall viscosity	$T_{yz}T_{xz}$ (2.59b)	+	-	+
$c_8 \propto c_{15}$	shear-induced conductivity	$T_{tx}T_{xz}$, $T_{tx}T_{yz}$ (2.57)	+	+	+
$\rho_\perp$	perp. resistivity	$J^x J^x$ (2.54)	+	+	-
$\tilde{\rho}_\perp$	Hall resistivity	$J^x J^y$ (2.55e)	+	+	-
$\sigma_\parallel$	long. conductivity	$J^z J^z$ (2.53a)	+	+	-
$\sigma_\perp$	perp. conductivity	$\rho_{ab} \equiv (\sigma^{-1})_{ab} = \rho_\perp \delta_{ab} + \tilde{\rho}_\perp \epsilon_{ab}$	+	+	-
helicity 0					
η_1	bulk viscosity	$\mathcal{O}_1\mathcal{O}_1$ (2.55c)	+	+	-
η_2	bulk viscosity	$\mathcal{O}_2\mathcal{O}_2$ (2.55d)	+	+	-
ζ_1	bulk viscosity	$T^{ij}(T^{xx}+T^{yy})$(2.55a)	+	+	-
ζ_2	bulk viscosity	$3\zeta_2 - 6\eta_1 = 2\eta_2$	+	+	-
c_4	expan.-induced long. cond.	$J_x T_{xx}$ (2.57)	+	-	-
c_5	expan.-induced long. cond.	$J_z T_{zz}$ (2.57)	+	-	-
c_3		$c_5 = -3(c_3 + c_4)$	+	-	-

Non-dissipative Hydrodynamic $\left(\lim\limits_{\omega\to 0}\lim\limits_{\mathbf{k}\to 0}\right)$					
coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
helicity 2					
$\tilde{\eta}_\perp$	transverse Hall viscosity	$T_{xy}(T_{xx} - T_{yy})$(2.55f)	+	-	+
helicity 1					
$c_{10} \propto c_{17}$	shear-induced Hall cond.	$T^{tx}T^{xz}$, $T^{tx}T^{yz}$ (2.60,2.62a,2.62b)	+	+	+
$\tilde{\sigma}_\perp$	Hall conductivity	$J^x J^x$, $J^x J^y$ (2.54,2.53b,2.53c)	+	-	+

➡**interesting**

➡**nightmare**

➡**more background noise for CME?**

Good news: CME prevails!

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

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ξ	chiral vortical conductivity	$J_x T_{ty}$ (2.38,2.39)	+	+	+
ξ_B	chiral magnetic conductivity	$J^x J^y$ (2.38,2.39)	**CME**		
ξ_T	chiral vortical heat conductivity	$T^{tx}T^{ty}$ (2.38,2.39)	+	-	+
helicity 0					
M_1	magneto-thermal susceptibility	$J^t T^{xx}$ (2.32)	+	+	-
M_3	magneto-acceleration susceptibility	$J^t T^{tt}$ (2.32)	+	+	-
M_4	magneto-electric susceptibility	$J^t J^t$ (2.32)	+	-	-

➡Kubo formula unchanged (covariant vs consistent)

$$\xi_B = \lim_{k\to 0}\lim_{\omega\to 0}\frac{i}{k}\langle J^x J^y\rangle$$

➡Value unchanged (fixed by anomaly) $\xi_B = C\,\mu$

$$\nabla_\mu J^\mu_A = C\,E\cdot B$$

➡Frequency-dependence unchanged by *B*

[Li,Yee; PRD (2018)]
[Koirala; PhD thesis (2020)]
[Amado,Landsteiner,Pena_Benitez; JHEP (2011)]

Chiral effects in vector and axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD *U(1)*)

$$J_V^\mu = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu$$

chiral magnetic effect

Axial current (e.g. QCD axial *U(1)*)

$$J_A^\mu = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B_A^\mu$$

chiral vortical effect

chiral separation effect

➡**phenomenology needs *both* currents**

Holographic model with axial current only

➡ **use as holographic dual to charged state in strong *B***

➡ **values for transport coefficients in *N*=4 Super-Yang-Mills**

[Ammon, Kaminski et al.; JHEP (2017)]

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; arXiv:2012.09183]

Einstein-Maxwell-Chern-Simons action

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

5-dimensional Einstein-Maxwell action encodes N=4 Super-Yang-Mills theory with axial U(1) gauge symmetry

5-dimensional Chern-Simons term encodes chiral anomaly

Charged magnetic black branes dual to charged thermal state with *B*

[D'Hoker, Kraus; JHEP (2010)]

- charged magnetic analog of Reissner-Nordstrom black brane
- asymptotically AdS_5

Holographic model with two currents

Einstein-Maxwell-Chern-Simons action with two gauge fields A_μ and V_μ

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R - 2\Lambda - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu} - \frac{L^2}{4}F^{(5)}_{\mu\nu}F^{\mu\nu}_{(5)} + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\left(3F_{\nu\rho}F_{\sigma\tau} + F^{(5)}_{\nu\rho}F^{(5)}_{\sigma\tau}\right)\right)$$

gravitational coupling κ

Einstein-Hilbert

Maxwell

"axial Maxwell"

Chern-Simons coupling α

Chern-Simons term encoding chiral anomaly

5D vector gauge field A_μ ⟷ *4D conserved vector current*

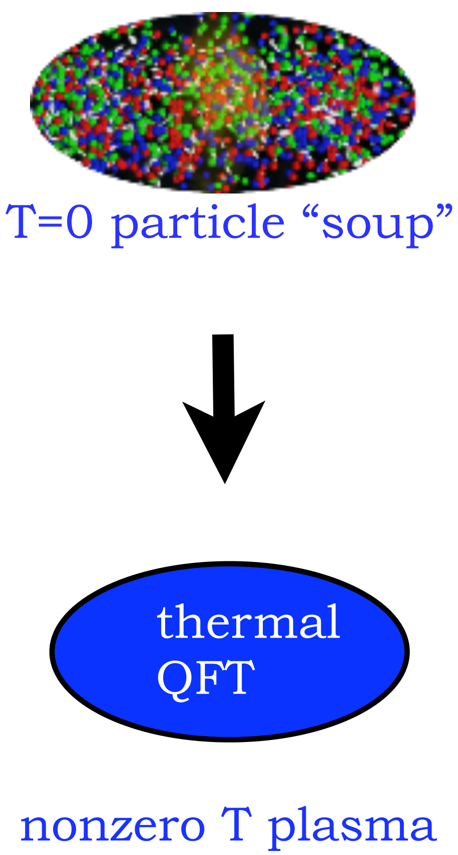
$$J^\mu_V = \cdots + \xi_V\omega^\mu + \xi_{VV}B^\mu + \xi_{VA}B^\mu_A$$

5D axial gauge field V_μ ⟷ *4D anomalous axial current*

$$J^\mu_A = \cdots + \xi\omega^\mu + \xi_B B^\mu + \xi_{AA}B^\mu_A$$

Far from equilibrium

Thermalization
in field theory:



Far from equilibrium

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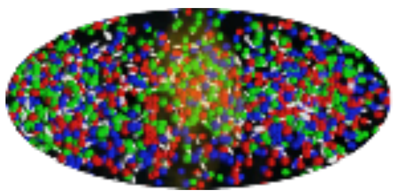
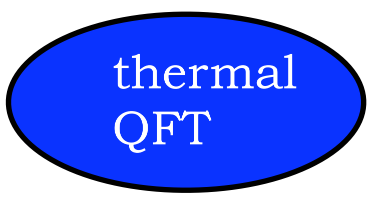
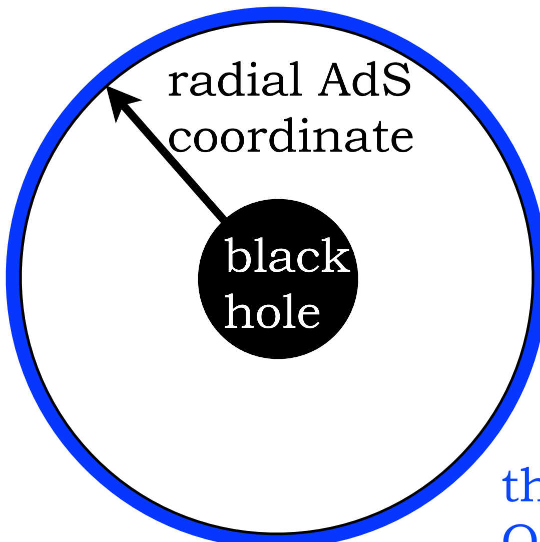


T=0 particle "soup"

correspondence



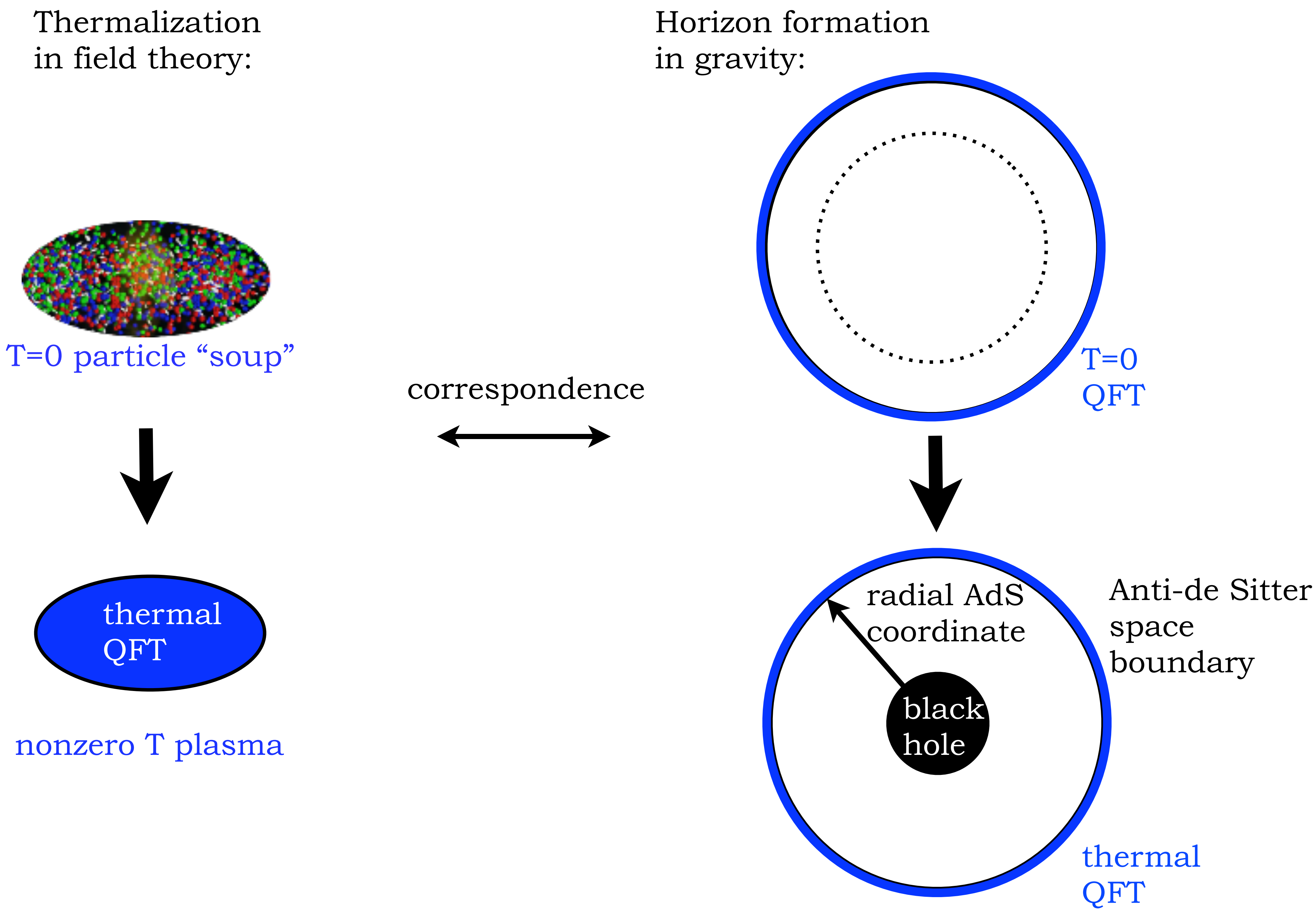
nonzero T plasma

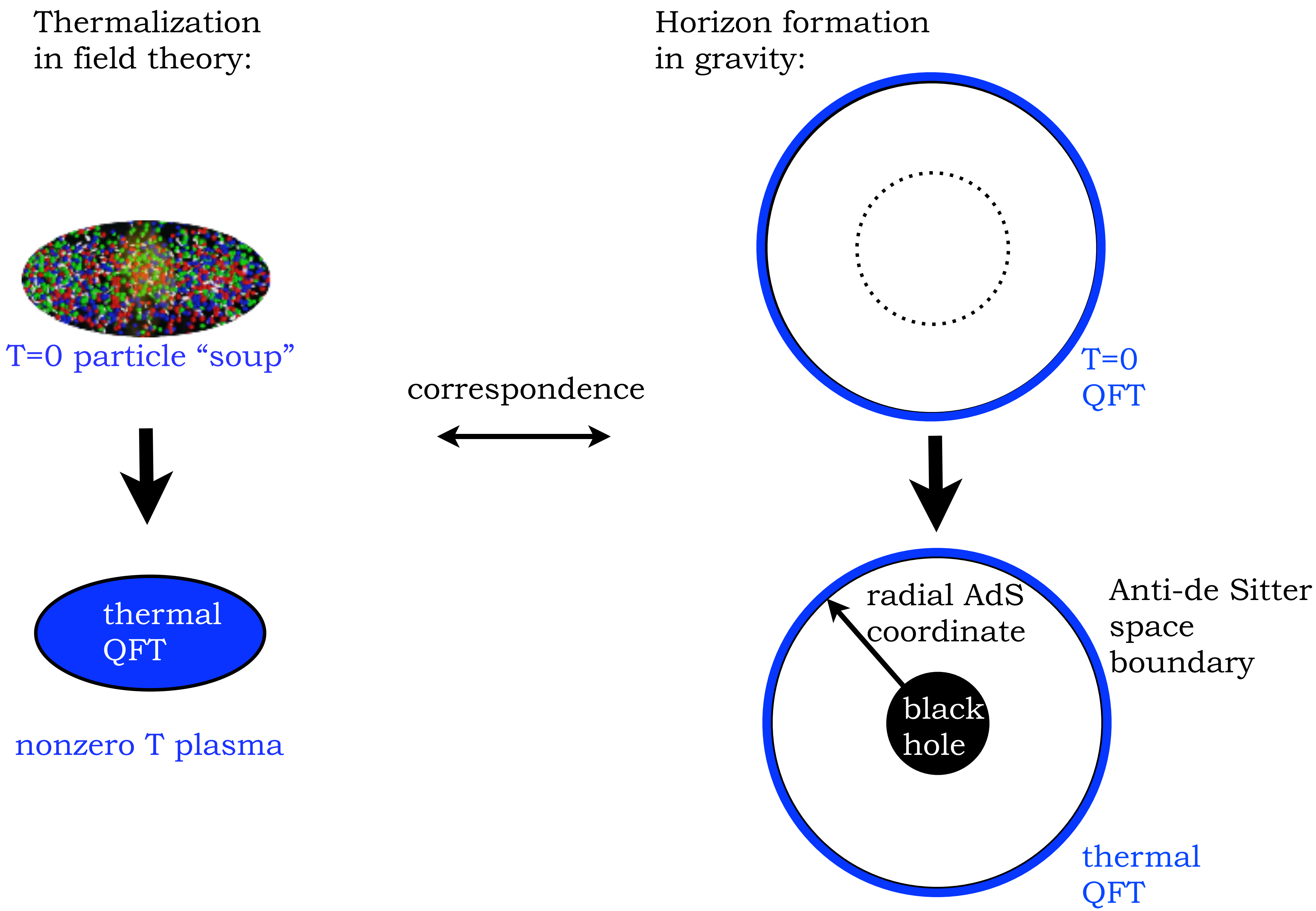


Anti-de Sitter
space
boundary

thermal
QFT

Far from equilibrium

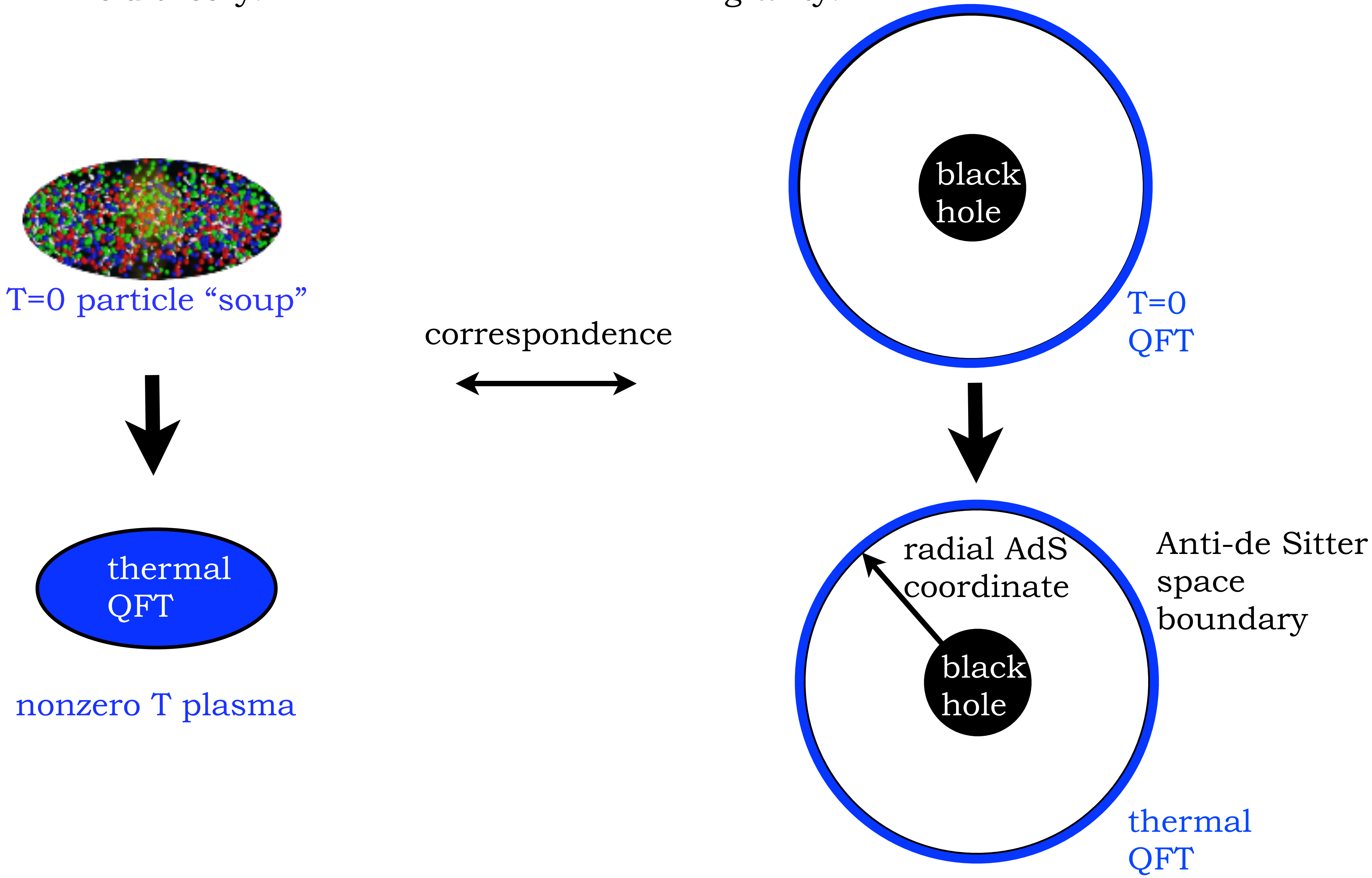



Thermalization
in field theory:
Horizon formation
in gravity:
T=0 particle “soup”
correspondence
T=0
QFT
thermal
QFT
radial AdS
coordinate
Anti-de Sitter
space
boundary
black
hole
nonzero T plasma
thermal
QFT

Far from equilibrium

Thermalization
in field theory:

Horizon formation
in gravity:



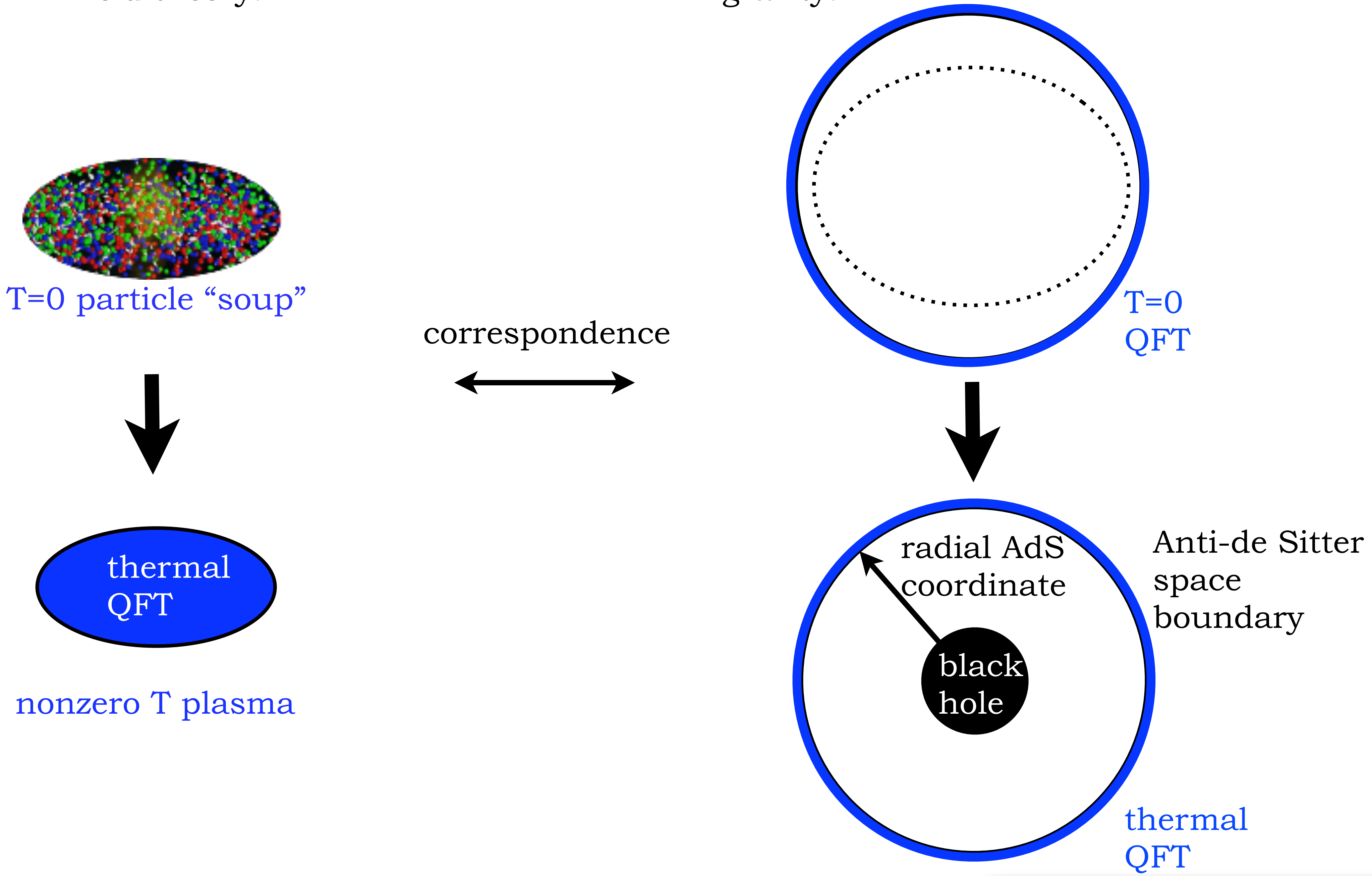
Far from equilibrium

[Janik,Peschanski; (2006)]

[Chesler, Yaffe; PRL (2009)]

Thermalization in field theory:

Horizon formation in gravity:



➡**solve time-dependent Einstein equations**

Far from equilibrium - non-expanding plasma

Initial state:

- Energy and axial charge corresponding to (T,μ_5) in final state
- Magnetic field is uniform and constant in time
- Dynamical pressure anisotropy vanishes
- CME current is absent

	"RHIC"
T	300MeV
μ_5	10 (100) MeV
B	1 (0.1) m_π^2

Matching couplings to QCD:

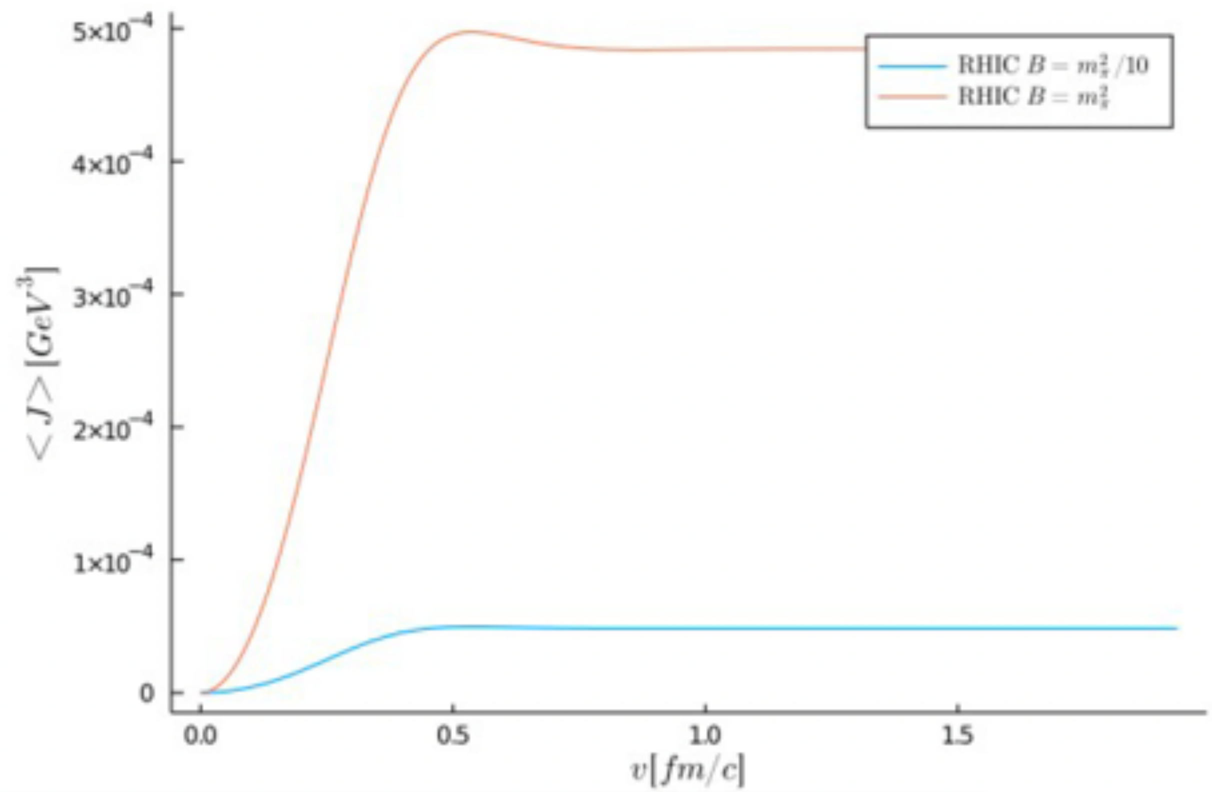
➔ Gravitational coupling: match to entropy

$$s_{BH} = \frac{4\pi^2 T^3}{2\kappa^2} \qquad s_{SB} = 4\left(\nu_b + \frac{7}{4}\nu_f\right)\frac{\pi^2 T^3}{90}$$

$$s_{BH} = \frac{3}{4} s_{SB} \qquad \Longrightarrow \kappa^2 \approx 12.5$$

➔ Chern Simons coupling: match to anomaly

$$\frac{\alpha}{2\kappa^2} = A_{QCD} = \frac{1}{8\pi^2} \qquad \Longrightarrow \alpha \approx 0.316$$



➡ **CME more likely to be seen at RHIC than at LHC**

➡ **lifetime of *B* crucial**

Far from equilibrium - non-expanding plasma

[Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

taken from Karl Landsteiner's talk

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- Magnetic field is uniform and constant in time
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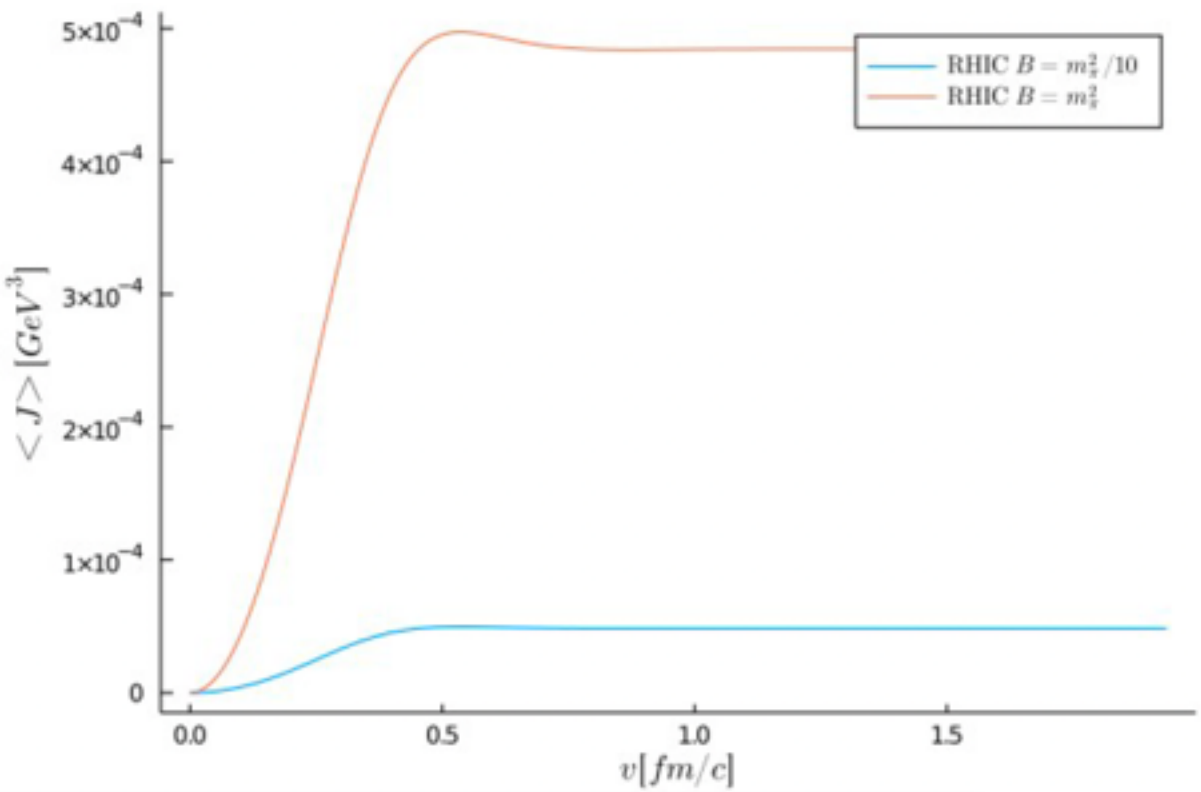


➡CME more likely to be seen at RHIC than at LHC

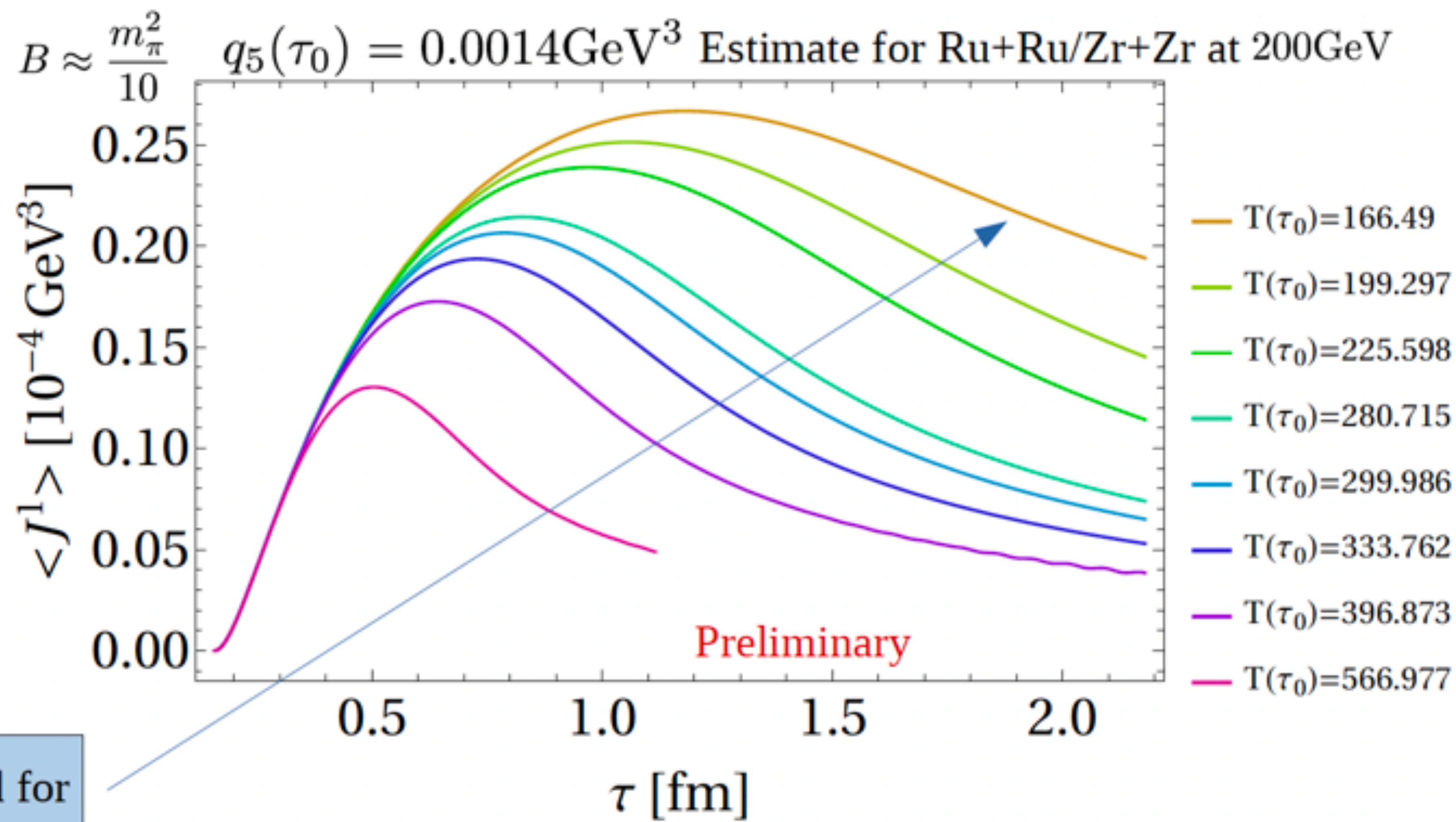
➡lifetime of *B* crucial

Far from equilibrium - expanding plasma

Initial state:
constant *B*,
pressure anisotropy

time-dependent μ_5,
plasma expanding along beam line

Matching to QCD:
SUSY value for α
L=1fm fixes κ



CME signal is enhanced for decreasing initial energy

➡CME more likely to be seen at low energies

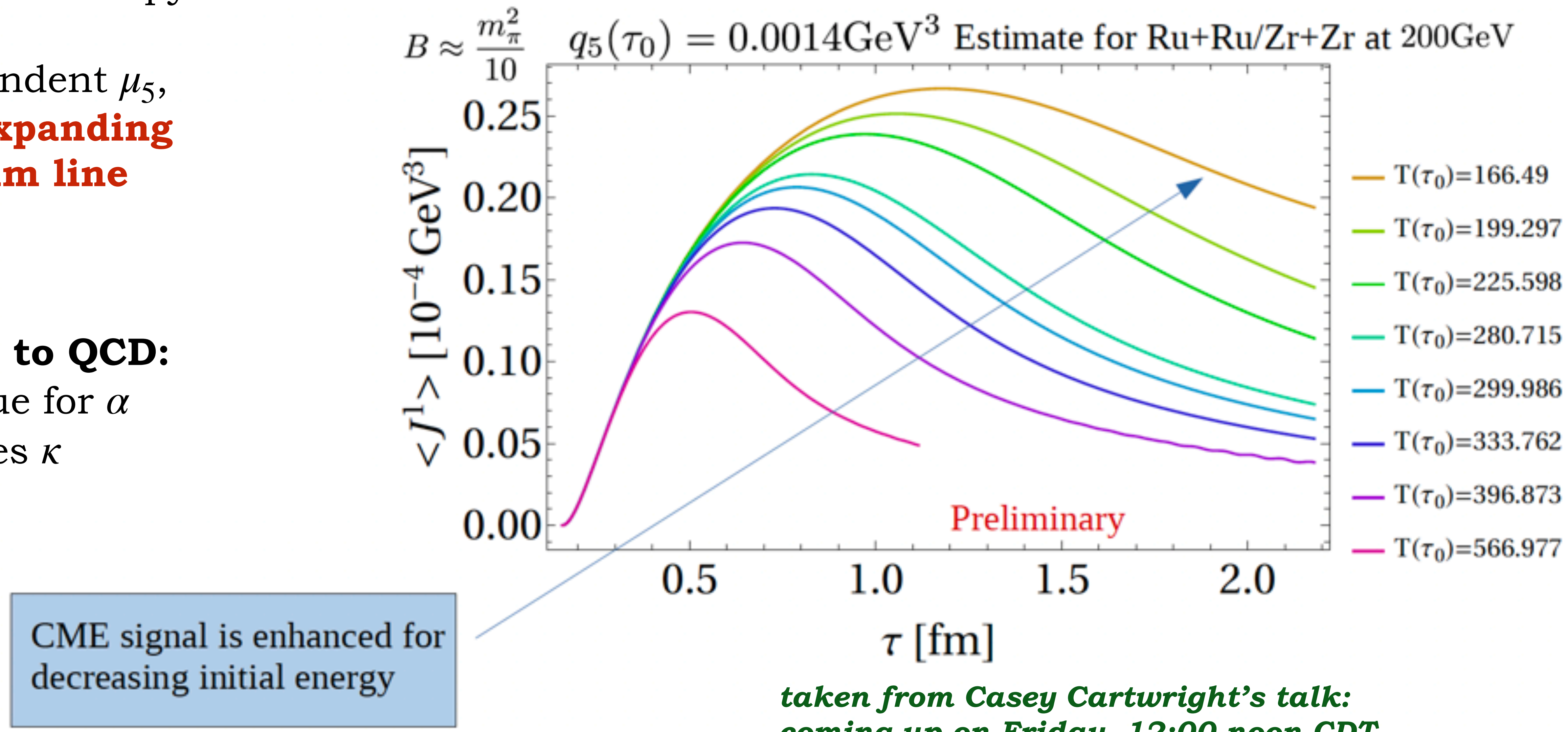
Far from equilibrium - expanding plasma

[Cartwright,Kaminski,Schenke; to appear (2021)]

Initial state:
constant B,
pressure anisotropy

time-dependent μ_5,
plasma expanding along beam line

Matching to QCD:
SUSY value for α
L=1fm fixes κ



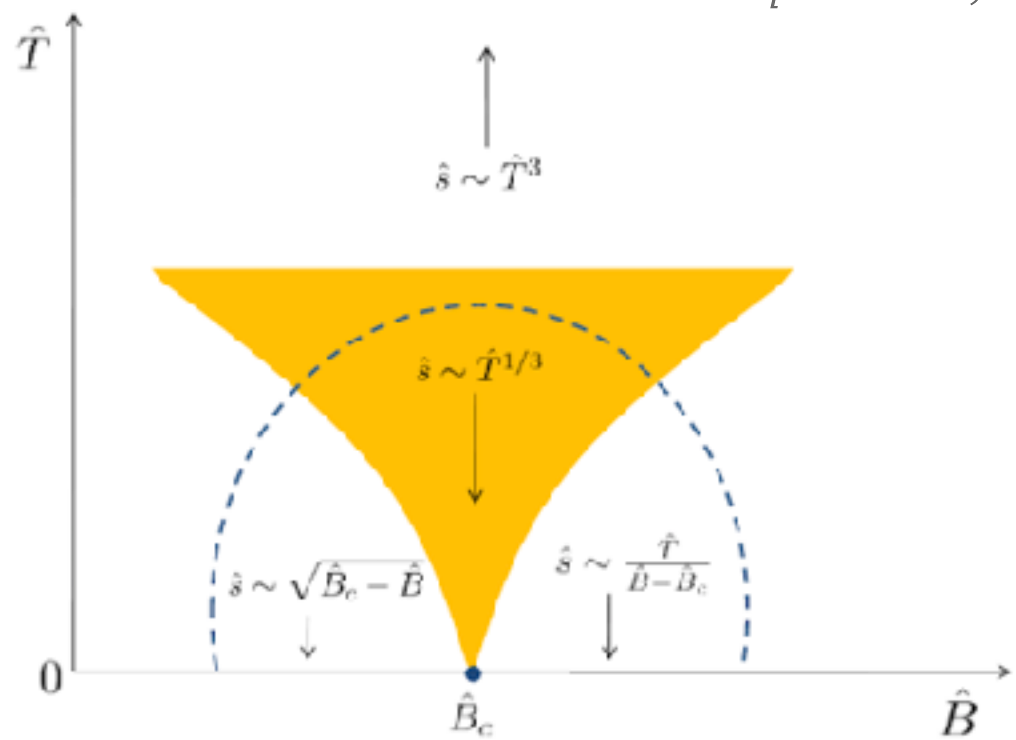
taken from Casey Cartwright's talk: coming up on Friday, 12:00 noon CDT

➡CME more likely to be seen at low energies

Need to be careful with parameter choice

➡ **this holographic model has a quantum critical point**

[D'Hoker, Kraus; JHEP (2010)]



➡ **in addition: instability exists for range of Chern-Simons coupling, leading to phase transition with helical magnetic field configuration**

Need to be careful with parameter choice

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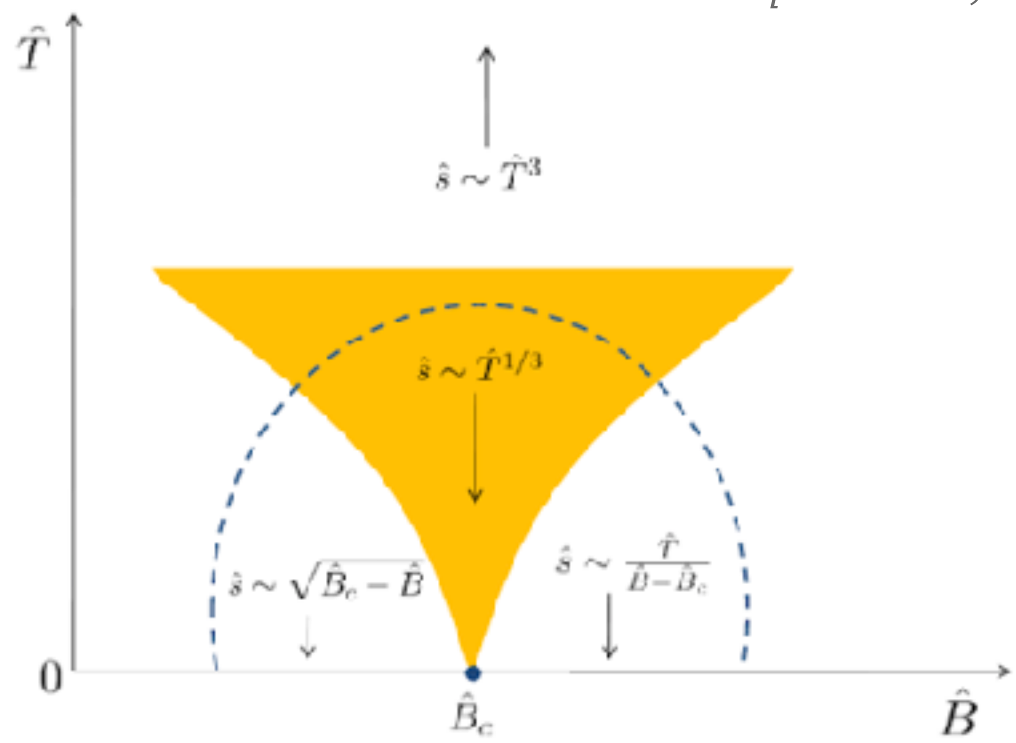


➡ **in addition: instability exists for range of Chern-Simons coupling, leading to phase transition with helical magnetic field configuration**

[Ammon,Leiber,Macedo; JHEP (2016)]

Summary

Hydrodynamics confirmed by holographic model

- constructed (3+1)D hydrodynamics: charged chiral fluids in strong and weak *B*
- strong *B:*
  - ✦ 5 novel hydro transport coefficients (+3 thermo) at leading and sub-leading order in the hydrodynamic expansion
  - ✦ Kubo formulae for 25 transport coefficients
- weak *B*: heavily modified dispersion relations
- **CME survives unaltered by all this!**

Far from equilibrium holography

- CME more likely to be seen at low energies
- lifetime and time-evolution of *B* crucial

Outlook

- Please discuss with me!

Collaborators on these projects

Perimeter, Canada
Juan Hernandez

Thank you for listening!

Friedrich-Schiller University of Jena, Germany

Prof. Dr. Martin Ammon

Dr. Julian Leiber

Dr. Sebastian Grieninger (now IFT Madrid)

BNL
Dr. Bjoern Schenke

University of Alabama, Tuscaloosa, USA

Dr. Roshan Koirala

Markus Garbiso

Dr. Jackson Wu

Jana Ingram

Casey Cartwright

APPENDIX

CPT symmetries

quantity	C	P	T
t	+	+	-
x^i	+	-	+
r	+	+	+
T, h_{tt}, T^{tt}	+	+	+
μ_A, A_t, J^t	+	-	+
μ_V, V_t, J_V^t	-	+	+
A_i, J^i	+	+	-
V_i, J_V^i	-	-	-
A_r	+	-	-
V_r	-	+	-
u^i, h_{ti}, T^{ti}	+	-	-
h_{ij}, T^{ij}	+	+	+
B^i	+	-	-
B_V^i	-	+	-
E^i	+	+	+
E_V^i	-	-	+
$dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \wedge dx^\kappa$	+	-	-
$\int_i^f A \wedge F \wedge F$	+	+	+
$\int_i^f V \wedge F_V \wedge F_V$	-	-	+
u^t	+	+	+
generating functional W (axial $U(1)_A$)	+	+	+

EFT calculation: chiral hydrodynamics with magnetic field

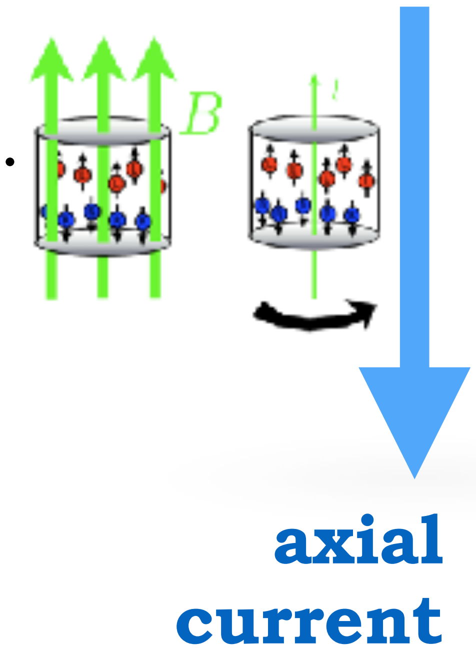
For any theory with chiral anomaly

$$\partial_\mu {J_A}^\mu = C\,\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

[Son,Surowka; PRL (2009)]
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]

Axial current with weak external B field: $\boxed{B \sim \mathcal{O}(\partial)}$

$$\langle {J_A}^\mu \rangle = n u^\mu + \sigma E^\mu - \sigma T \Delta^{\mu\nu}\nabla_\nu \left(\frac{\mu}{T}\right) + \underline{\xi_B B^\mu} + \xi_V \Omega^\mu + \dots$$



Energy momentum tensor with weak external B field:

$$\langle T^{\mu\nu} \rangle = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu} + u^\mu q^\nu + u^\nu q^\mu + \tau^{\mu\nu}$$

Definitions and properties:

$$\tau^{\mu\nu} = -\eta \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3}\nabla_\lambda u^\lambda g_{\alpha\beta}\right) - \zeta \Delta^{\mu\nu}\nabla_\lambda u^\lambda$$

$$E^\mu = F^{\mu\nu} u_\nu$$

$$B^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda} u_\nu F_{\rho\lambda}$$

$q^\mu = \xi_V B^\mu + \xi_3 \omega^\mu$ $\quad \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ $\quad u^\mu u_\mu = -1$ $\quad u_\mu \tau^{\mu\nu} = 0,\ u_\mu \nu^\mu = 0$, and $u_\mu q^\mu = 0$.

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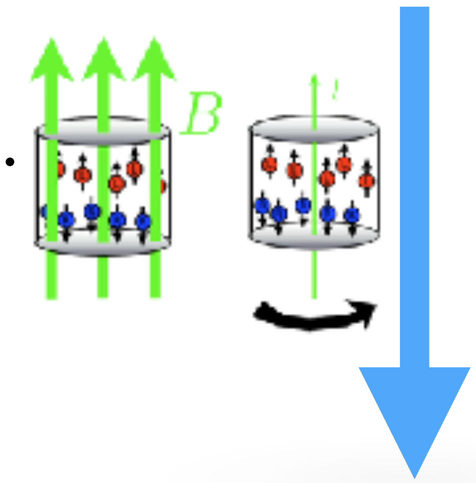
(ideal) charge flow — *conductivity term* — *charge diffusion* — *chiral magnetic conductivity term* — *chiral vortical conductivity term*



axial current

Energy momentum tensor with weak external B field:

$$\langle T^{\mu\nu} \rangle = \underbrace{\epsilon u^\mu u^\nu + P \Delta^{\mu\nu}}_{\text{ideal fluid}} + \underline{u^\mu q^\nu + u^\nu q^\mu}_{\text{heat current}} + \tau^{\mu\nu}$$

ideal fluid — *heat current*

Definitions and properties:

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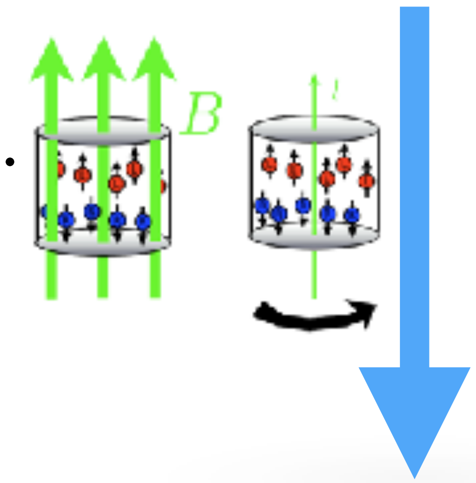

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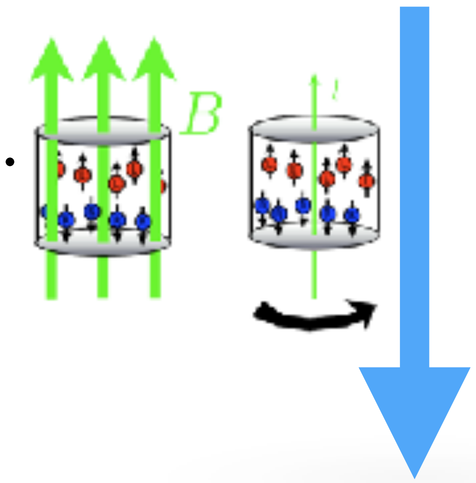

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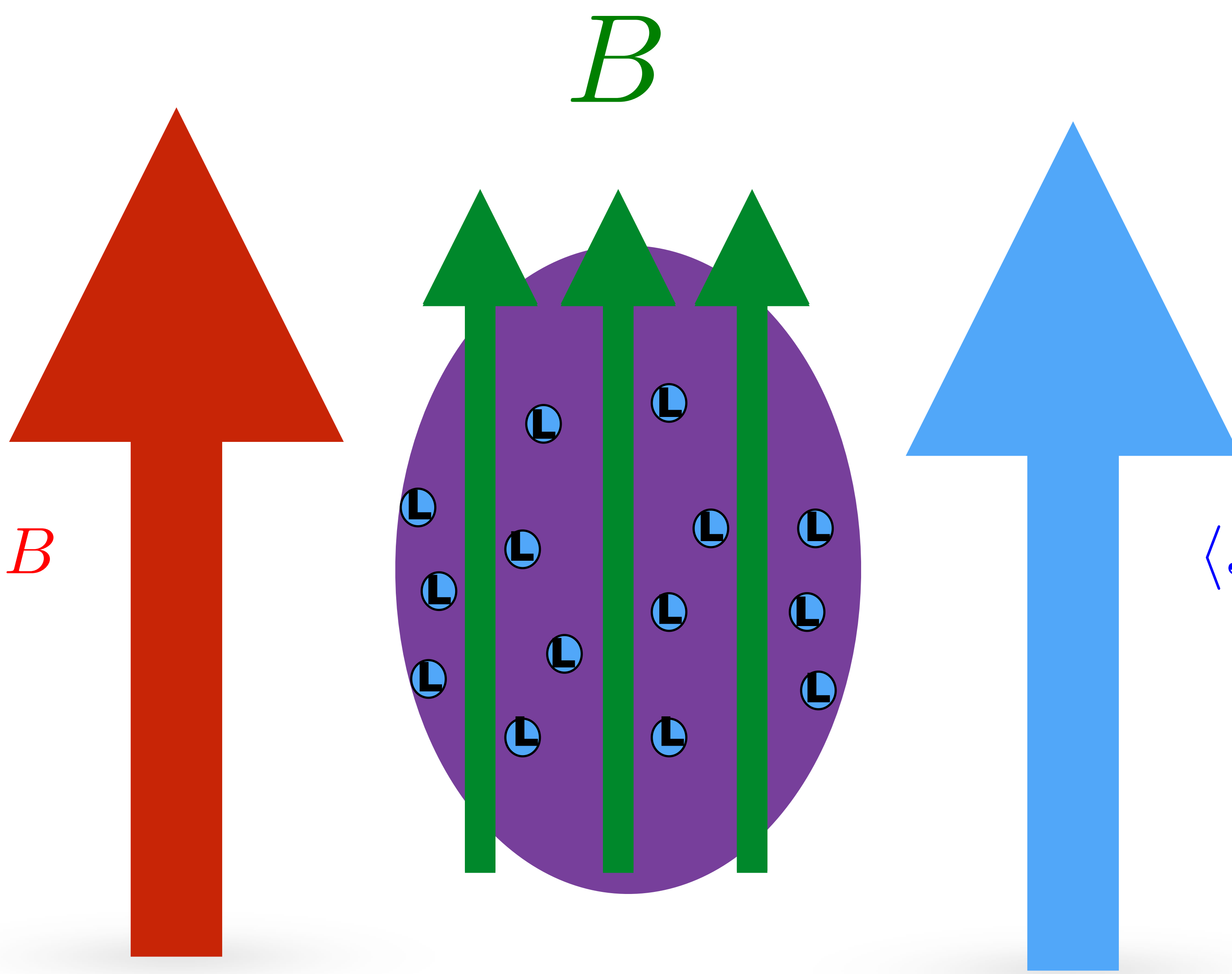
ideal fluid — *heat current*

chiral effects measured in Weyl semi metals
e.g. [Huang et al; PRX (2015)]

neutron stars?
[Kaminski et al.; PLB (2014)]

Now calculate hydrodynamic 1- and 2-point functions and determine their poles!
[Landau, Lifshitz]
[Kadanoff; Martin]

Currents in equilibrium



$$\langle T^{0z} \rangle = \xi_V^{(0)} B$$

heat current

$$\langle J^z \rangle = \xi_B^{(0)} B$$

axial current

EFT result III: weak B details

Weak B hydrodynamics - poles of 2-point functions:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$ *former charge diffusion mode*

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$
$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$
former sound modes

$$w_0 = \epsilon_0 + P_0$$
$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$
$$c_s^2 = (\partial P/\partial\epsilon)_{\mathfrak{s}}$$

damping coefficients:

$$\Gamma_\pm = \frac{3\zeta + 4\eta}{6w_0} + c_s^2\,\frac{w_0\,\sigma}{2n_0^2}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)^2 \qquad D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0}$$

velocities:

$$v_\pm = \pm c_s - B\,\frac{c_s^2}{n_0}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)\left[3C T_0 \mathfrak{s}_0 + \frac{\alpha_P T_0^2}{\tilde{c}_P}(\tilde{C} - 3C\mathfrak{s}_0^2) + \frac{1}{2}\xi_B^{(0)} - \frac{n_0}{w_0}\xi_V^{(0)}\right] + B\,\frac{1}{w_0}\,c_s^2\,\xi_V^{(0)}, \qquad v_0 = \frac{2BT_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

chiral conductivities:

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2$$

known from entropy current argument
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

Hydrodynamics - Kubo formulae

Two types :

1.) Thermodynamic transport coefficients

$$\frac{1}{k_z}\mathrm{Im}\, G_{T^{xz}T^{yz}}(\omega=0, k_z\hat{\mathbf{z}}) = -2\, B_0^2 \boxed{M_2}$$

novel: perpendicular magnetic vorticity susceptibility

+ ***3 novel thermodynamic transport coefficients*** M_1, M_3, M_4

+ ***3 chiral conductivities (CME / CVE / CTE) in equilibrium,***
$\propto$ ***chiral anomaly,***
measured negative magnetoresistance in 3D Weyl semimetals

+ ... *e.g. [Huang et al; PRX (2015)]*

2.) Hydrodynamic transport coefficients

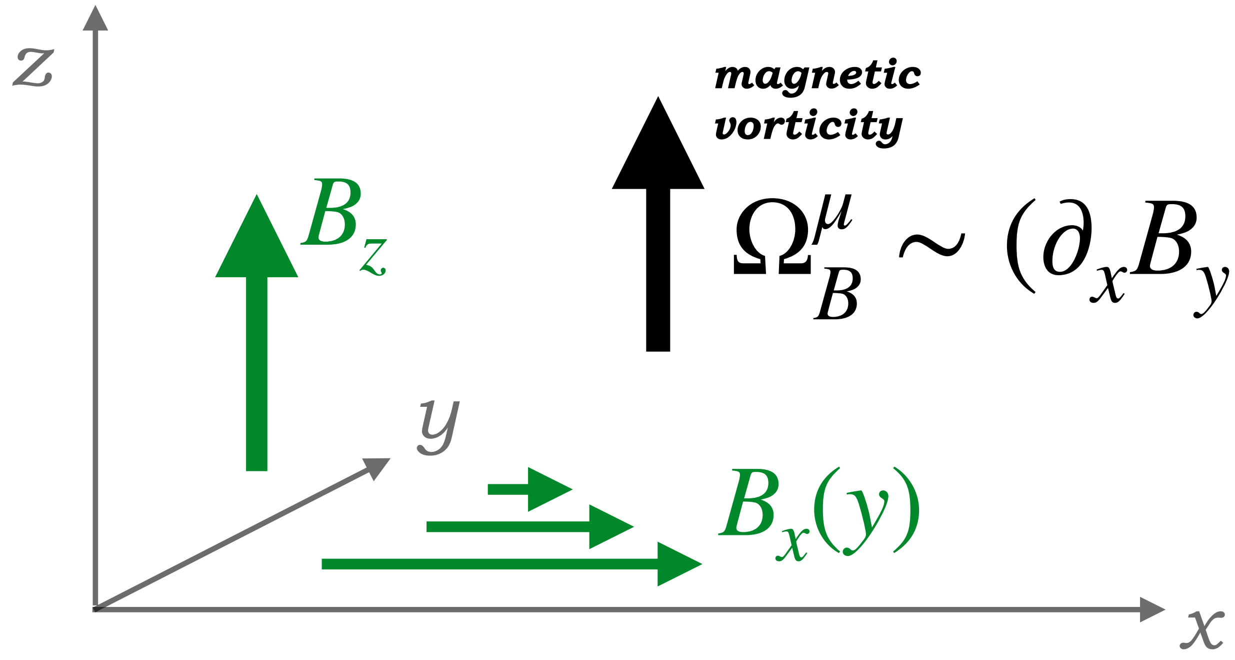
— boring example :

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^z J^z}(\omega, \mathbf{k}{=}0) = \sigma_{||} + \cdots$$

parallel charge conductivity

+ ***3 novel hydrodynamic transport coefficients***

Perpendicular magnetic vorticity susceptibility M_2



$$\Omega^\mu_B \sim (\partial_x B_y - \partial_y B_x)$$

$$\begin{aligned}\mathcal{E}_{\text{eq}} \sim \mathcal{P}_{\text{eq}} &\sim M_2 B\cdot\Omega_B \\ &\sim -M_2 B_z \partial_y B_x(y)\end{aligned}$$

magnetic vorticity :

$$\Omega^\mu_B = \epsilon^{\mu\nu\rho\sigma} u_\nu \nabla_\rho B_\sigma$$

$$u^\nu = (1,0,0,0) + \mathcal{O}(\partial)$$

Recall tensor structure in W_s **:**

$$s_2 = \epsilon^{\mu\nu\rho\sigma} u_\mu B_\nu \nabla_\rho B_\sigma$$

Hydrodynamics - 3 more novel coefficients

Shear-induced conductivity c_8

$$j_x \sim c_8(\partial_x u_z + \partial_z u_x)$$

Expansion-induced conductivities c_4 and c_5

$$j^\mu \sim \hat{b}^\mu(c_4 \nabla \cdot u + c_5 \hat{b}^\alpha \hat{b}^\beta \partial_\alpha u_\beta)$$

➡**Fluid flow gradients create charge currents**

More thermodynamic transport coefficients

Magneto-thermal susceptibility M_1 :

$$\mathcal{E}_{\rm eq} \sim M_1 B^\mu \partial_\mu \left(\frac{B^2}{T^4}\right)$$

Magneto-acceleration susceptibility M_3 :

$$\mathcal{E}_{\rm eq} \sim \mathcal{P}_{\rm eq} \sim M_{3,B^2}\, B\cdot a$$

Magneto-electric susceptibility M_4 :

$$\mathcal{E}_{\rm eq} \sim M_{4,T}\, B\cdot E\,, \qquad \mathcal{P}_{\rm eq} \sim M_{4,B^2}\, B\cdot E$$

Magneto-vortical susceptibility M_5 :

$$\mathcal{E}_{\rm eq} \sim \mathcal{P}_{\rm eq} \sim M_5 B\cdot\Omega$$

Strong B thermodynamics

$$\boxed{B \sim \mathcal{O}(1)}$$

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly : $\langle T^{\alpha\beta}\rangle = \epsilon\, u^\alpha u^\beta + p\Delta^{\alpha\beta} + \tau^{\alpha\beta}$

Energy momentum tensor:

equilibrium heat current

$$\langle T^{\mu\nu} \rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

"magnetic pressure shift"

Axial current:

$$\langle J^\mu \rangle = \left(n_0, 0, 0, \underline{\xi_B^{(0)} B}\right) + \mathcal{O}(\partial)$$

equilibrium charge current

➡ new contributions to thermodynamic equilibrium observables

previous work:

[Kovtun; JHEP (2016)]

[Jensen, Loganayagam, Yarom; JHEP (2014)]

[Israel; Gen.Rel.Grav. (1978)]

Update: strong B hydrodynamics

[Hernandez, Kovtun; JHEP (2017)]

Spin-1 modes

Anisotropic transport coefficients

strong B: $\omega = \pm\frac{B_0 n_0}{w_0} - \frac{iB_0^2}{w_0}(\sigma_\perp \pm i\tilde{\sigma}) - iD_c k^2$

parity-odd

weak B: $\omega = \mp\frac{B n_0}{\epsilon_0 + P_0} - ik^2\frac{\eta}{\epsilon_0 + P_0} + k\frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2\sigma}{\epsilon_0 + P_0}$

Agreement in form

Exact agreement in real part!

Spin-0 modes

strong B: $\omega = \pm k v_s - i\frac{\Gamma_{s,\parallel}}{2}k^2\,,$

$\omega = -iD_\parallel k^2\,,$

Anisotropic transport coefficients

$$D_\parallel = \frac{\sigma_\parallel w_0^2}{n_0^2\chi_{11} + w_0^2\chi_{33} - 2n_0 w_0 \chi_{13}}$$

parity-odd

weak B: $\omega_0 = v_0 k - iD_0 k^2 + \mathcal{O}(\partial^3)$

$\omega_+ = v_+ k - i\Gamma_+ k^2 + \mathcal{O}(\partial^3)$

$\omega_- = v_- k - i\Gamma_- k^2 + \mathcal{O}(\partial^3)$

Agreement in form

$$D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0} \qquad v_0 = \frac{2BT_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$

1. Hydrodynamics - Formalism

Universal **effective field theory (EFT)**

[Baier, Romatschke, Romatschke, Son, Starinets, Stephan; JHEP (2008)]

- expansion in gradients of fields
- systematic construction
- generating functional
  [Jensen, Kaminski, Kovtun, Meyer, et al.; PRL (2012)]
  [JHEP (2011)]
  [Banerjee et al. JHEP (2012)]

[Previously: *[Landau, Lifshitz]* phenomenological]

Hydrodynamic limit $\frac{\omega}{T} \ll 1, \quad \frac{|\vec{k}|}{T} \ll 1$

- fields
- constitutive equations
- conservation equations
- sources
  [Luttinger]

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Hydrodynamic limit $\frac{\omega}{T} \ll 1, \quad \frac{|\vec{k}|}{T} \ll 1$

- fields $T(x), \quad n(x), \quad u^\alpha(x)$
  - $T(x)$: *temperature*
  - $n(x)$: *charge density*
  - $u^\alpha(x)$: *fluid velocity*
- constitutive equations

$$\langle j^\alpha \rangle = n\, u^\alpha + \nu^\alpha$$

ideal hydro ($n\, u^\alpha$), *derivative corrections* (ν^α)

- conservation equations

$$\nabla_\alpha \langle j^\alpha \rangle = 0$$

e.g. continuity: $\partial_t n + \vec{\nabla} \cdot \vec{j} = 0$

- sources *[Luttinger]* $g_{\alpha\beta}(x), \quad A_\alpha(x)$
  - $g_{\alpha\beta}(x)$: *metric*
  - $A_\alpha(x)$: *gauge field*

Hydrodynamics - Correlators

Variation of the ...

... constitutive relations (1-point functions)

$$G^R_{T^{\mu\nu},J^\alpha} = \frac{\delta}{\delta A_\alpha} T^{\mu\nu}_{\text{on-shell}}[A,g]$$

$$G^R_{J^\mu T^{\alpha\beta}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\alpha\beta}} \left(\sqrt{-g}\, J^\mu_{\text{on-shell}}[A,g]\right)$$

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

Recall Onsager relations

$$G^R_{TJ} = \eta_T \eta_J G^R_{JT}$$

... equilibrium generating functional

$$\delta W_s[A,g] = \int d^4x \sqrt{-g} \left(\tfrac{1}{2} T^{\mu\nu}_{\text{eq.}} \delta g_{\mu\nu} + J^\mu_{\text{eq.}} \delta A_\mu\right)$$

sources

$$W_{cons} = W_s + \int d^4x \sqrt{-g} \left(c_1 T^2 \Omega\cdot A + c_2 T \left(B\cdot A + \mu\Omega\cdot A\right) + \frac{C}{3}\mu \left(B\cdot A + \tfrac{1}{2}\mu\Omega\cdot A\right)\right)$$

Chiral magnetic & chiral vortical effect in equilibrium

$$W_s = \int d^4x \sqrt{-g} \left(p(T,\mu,B^2) + \sum_{n=1}^{5} M_n(T,\mu,B^2) s_n + O(\partial^2)\right)$$

5 thermodynamic transport coefficients

$$s_2 = \epsilon^{\mu\nu\rho\sigma} u_\mu B_\nu \nabla_\rho B_\sigma$$

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➡ **Hydrodynamic 2-point functions**

[Ammon,Kaminski et al.; JHEP (2017)]

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

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sources

$$W_{cons} = W_s + \int d^4x \sqrt{-g} \left(c_1 T^2 \Omega\cdot A + c_2 T\,(B\cdot A + \mu\Omega\cdot A) + \frac{C}{3}\mu\,(B\cdot A + \tfrac{1}{2}\mu\Omega\cdot A)\right)$$

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5 thermodynamic transport coefficients

$$s_2 = \epsilon^{\mu\nu\rho\sigma} u_\mu B_\nu \nabla_\rho B_\sigma$$

Hydrodynamics - Constitutive relations

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)] *[Ammon, Kaminski et al.; JHEP (2017)]* *[Hernandez, Kovtun; JHEP (2017)]*

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P} \Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

from generating functional

$$\begin{aligned}
\mathcal{E}_{\text{eq.}} = & -p + T p_{,T} + \mu p_{,\mu} + (T M_{5,T} + \mu M_{5,\mu} - 2M_5)\, B\cdot\Omega \\
& + (T M_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} + T^4 M_{3,B^2} - M_1)\, s_1 \\
& + (T M_{2,T} + \mu M_{2,\mu} - M_2)\, s_2 \\
& + \frac{4B^2}{T^4} (M_1 - T M_{1,T} - \mu M_{1,\mu} - 4B^2 M_{1,B^2} - T^4 M_{3,B^2})\, s_3 \\
& + \left(T M_{4,T} + \mu M_{4,\mu} + \frac{4B^2}{T^4} M_{1,\mu} + M_{3,\mu} \right) s_4 \,, \qquad (2.10a)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{\text{eq.}} = & \, p - \tfrac{4}{3} p_{,B^2} B^2 - \tfrac{1}{3}(M_5 + 4M_{5,B^2} B^2) B\cdot\Omega - \tfrac{2}{3} (M_2 + 2B^2 M_{2,B^2})\, s_2 \\
& + \frac{4B^2}{3T^4} (M_1 - T M_{1,T} - \mu M_{1,\mu} - 4B^2 M_{1,B^2} - T^4 M_{3,B^2})\, s_3 \\
& + \frac{4B^2}{3T^4} (M_{1,\mu} - T^4 M_{4,B^2})\, s_4 \,, \qquad (2.10b)
\end{aligned}$$

$$\begin{aligned}
\mathcal{Q}^\mu_{\text{eq.}} = & -M_5 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\sigma B_\rho + (2M_5 - T M_{5,T} - \mu M_{5,\mu}) \epsilon^{\mu\nu\rho\sigma} u_\nu B_\rho \partial_\sigma T / T \\
& - M_{5,B^2} \epsilon^{\mu\nu\rho\sigma} u_\nu B_\rho \partial_\sigma B^2 + (M_{5,\mu} - 2p_{,B^2}) \epsilon^{\mu\nu\rho\sigma} u_\nu E_\rho B_\sigma \qquad (2.10c)
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}^{\mu\nu}_{\text{eq.}} = & \, 2p_{,B^2} (B^\mu B^\nu - \tfrac{1}{3} \Delta^{\mu\nu} B^2) + B^{\langle\mu} B^{\nu\rangle} (M_{5,B^2} B\cdot\Omega + M_{2,B^2} s_2 + (M_{4,B^2} - \tfrac{1}{T^4} M_{1,\mu}) s_4) \\
& + B^{\langle\mu} B^{\nu\rangle} \frac{1}{T^4} (T M_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} - M_1 + T^4 M_{3,B^2})\, s_3 + M_5 B^{\langle\mu} \Omega^{\nu\rangle} \\
& + 2M_2 B^{\langle\mu} \epsilon^{\nu\rangle\rho\sigma\alpha} u_\rho \partial_\sigma B_\alpha + (T M_{2,T} + \mu M_{2,\mu} - M_2)\, B^{\langle\mu} \epsilon^{\nu\rangle\alpha\rho\sigma} u_\alpha B_\rho \partial_\sigma T / T \\
& + M_{2,B^2} B^{\langle\mu} \epsilon^{\nu\rangle\alpha\rho\sigma} u_\alpha B_\rho \partial_\sigma B^2 - M_{2,\mu} B^{\langle\mu} \epsilon^{\nu\rangle\rho\sigma\alpha} u_\rho E_\sigma B_\alpha \,, \qquad (2.10d)
\end{aligned}$$

Hydrodynamics - Constitutive relations

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

[Ammon, Kaminski et al.; JHEP (2017)]
[Hernandez, Kovtun; JHEP (2017)]

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

from generating functional

$$\begin{aligned}\mathcal{E}_{\text{eq.}} = & -p + T p_{,T} + \mu p_{,\mu} + (T M_{5,T} + \mu M_{5,\mu} - 2M_5)\, B{\cdot}\Omega \\ & + (T M_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} + T^4 M_{3,B^2} - M_1)\, s_1 \\ & + (T M_{2,T} + \mu M_{2,\mu} - M_2)\, s_2\end{aligned}$$

➡ **Complicated because of broken symmetries:**

- **~~Chiral symmetry~~ — microscopic chiral anomaly**
- **~~Parity~~ — axial chemical potential**
- **~~Time reversal~~ + ~~Spatial rotation symmetry~~ — strong magnetic field** $B \sim \mathcal{O}(1)$

➡ **Many novel transport effects**

Hydrodynamics - Constitutive relations

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

[Ammon, Kaminski et al.; JHEP (2017)]
[Hernandez, Kovtun; JHEP (2017)]

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$T^{\mu\nu} = T^{\mu\nu}_{eq} + T^{\mu\nu}_{non-eq}$$

from generating functional

from construction of one-point functions