# Helical effects for thermal fermions and polarization asymmetry

Victor E. Ambruș[†], Maxim N. Chernodub[*]

arXiv:1912.11034, arXiv:1912.09977, arXiv:2010.05831

[†] Institute for Theoretical Physics, Goethe University, Frankfurt a. M., Germany
[*] Denis Poisson Institute, University of Tours, France

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CRC-TR 211 Strong-interaction matter under extreme conditions

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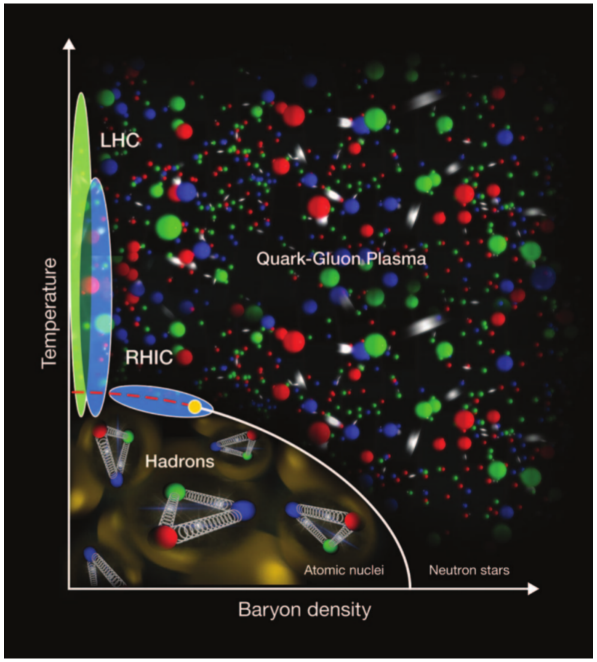
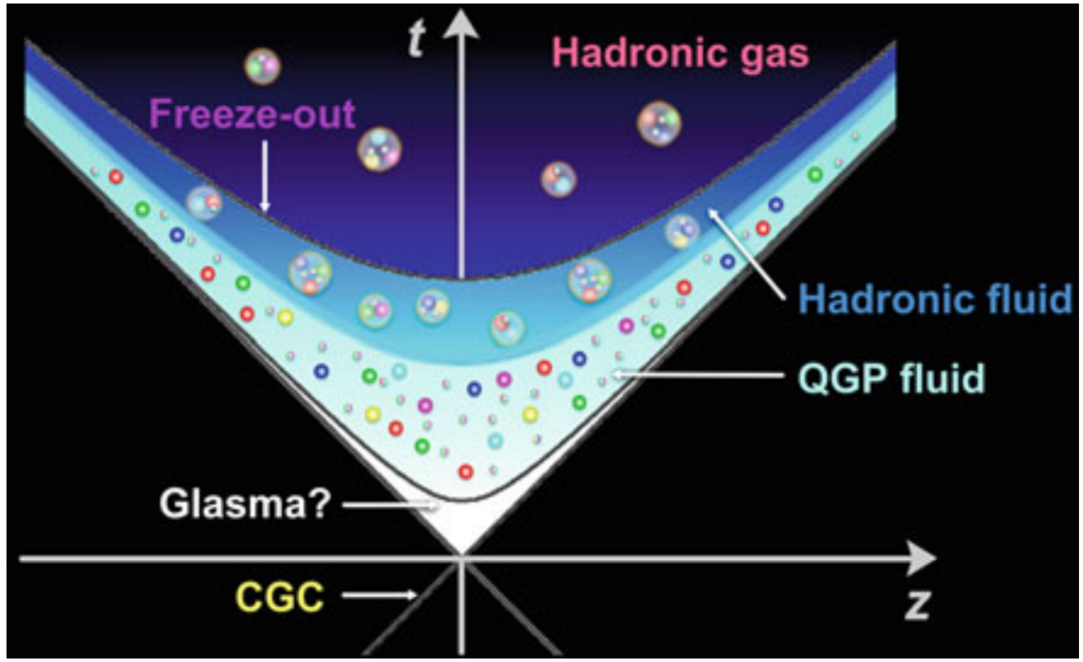
# Outline

# Quark-gluon plasma: hydrodynamic phase




B. V. Jacak, B. Muller, Science **337** (2012) 310.

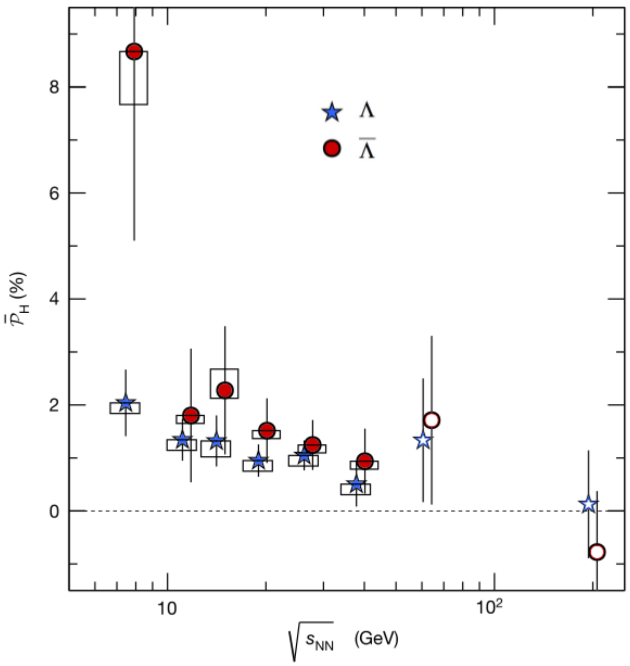



A. Monnai, PhD Thesis (Tokyo, 2014).

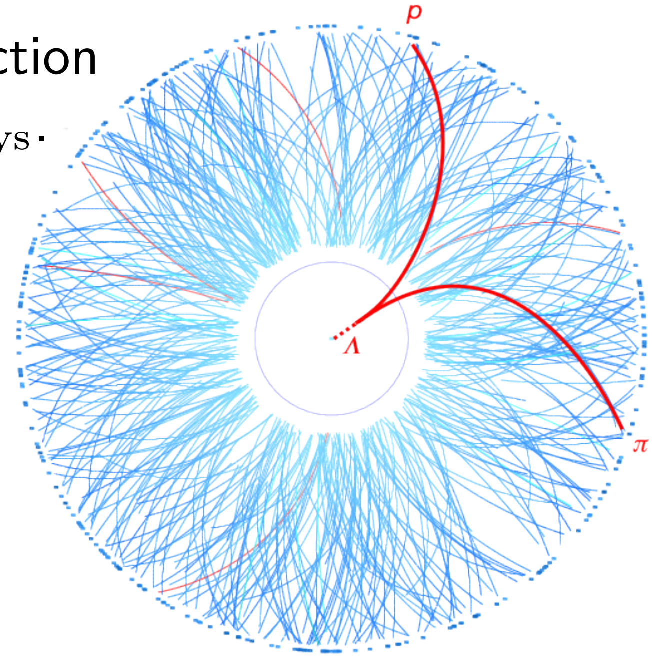
- The QGP produced at RHIC is...
  - the hottest ($k_B T \gtrsim 0.2$ GeV $\Leftrightarrow T \gtrsim 2.3 \times 10^{12}$ K),
  - densest ($p \gtrsim 10\ \mathrm{GeV/fm^3} \simeq 1.6 \times 10^{36}$ Pa)
  - and most vortical ($\omega \simeq 10^{22}\ \mathrm{s}^{-1}$)...

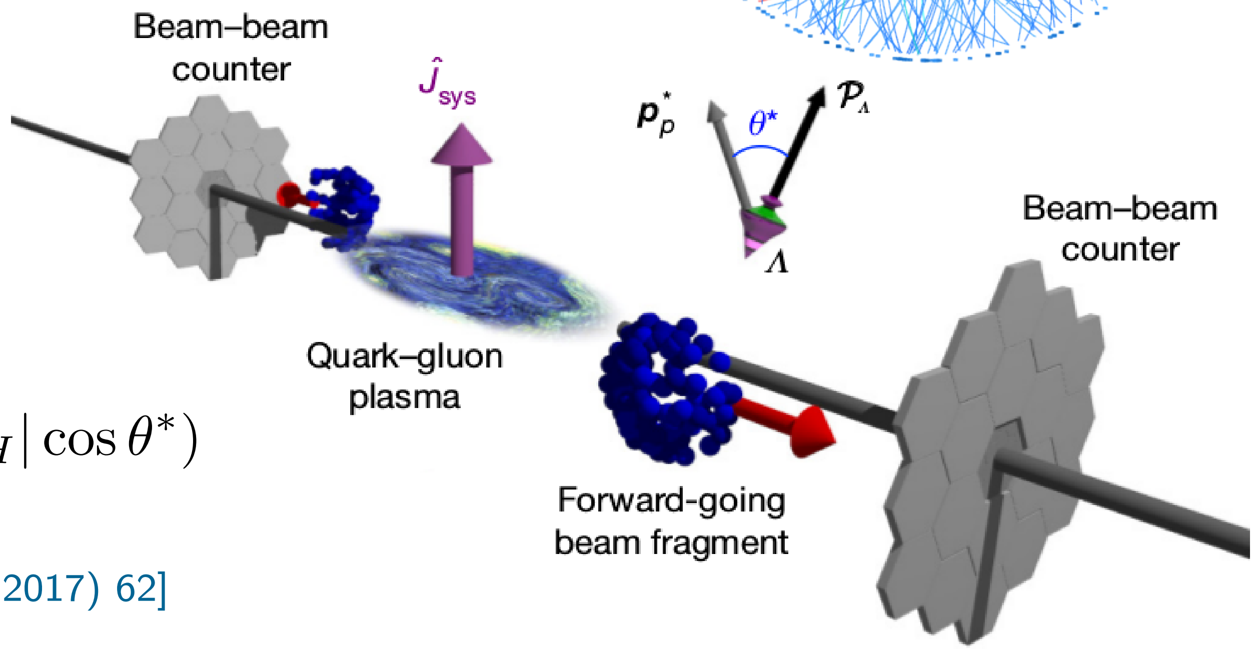
...fluid produced in the laboratory. [STAR Collaboration, Nature **548** (2017) 62]

# QGP: Polarisation of $\Lambda$-hyperons




- $\overline{\mathcal{P}}_H \equiv$ average projection of polarization on $\hat{J}_{\rm sys}$.
- $\Lambda \equiv$ "self-analysing:" proton emitted preferentially along spin.

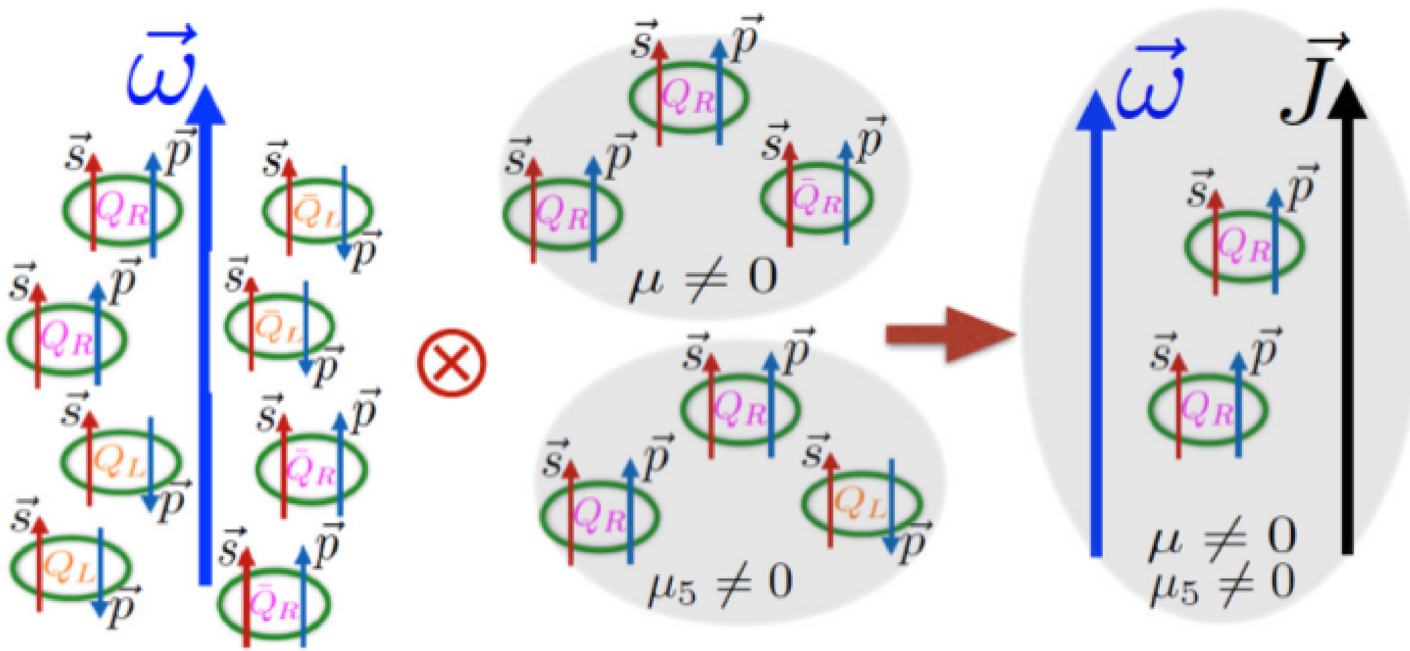



- $|\omega| \approx \frac{k_B T}{\hbar}(\overline{\mathcal{P}}_{\Lambda'} + \overline{\mathcal{P}}_{\overline{\Lambda}'})$.
- $\frac{dN_H}{d\cos\theta^*} = \frac{1}{2}(1 + \alpha_H |\mathcal{P}_H| \cos\theta^*)$

[STAR Collaboration, Nature **548** (2017) 62]

# Known mechanism: Chiral vortical effect (CVE)




$$\boldsymbol{J}_V = \sigma_V \boldsymbol{\omega}, \qquad \boldsymbol{J}_A = \sigma_A \boldsymbol{\omega},$$

$$\sigma_V = \frac{\mu_V \mu_A}{\pi^2}, \qquad \sigma_A = \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}.$$

$\boldsymbol{J}_A \neq 0$ even when $\mu_A = 0$!

[D. E. Kharzeev *et al.*, Nucl. Phys. **88** (2016) 1]

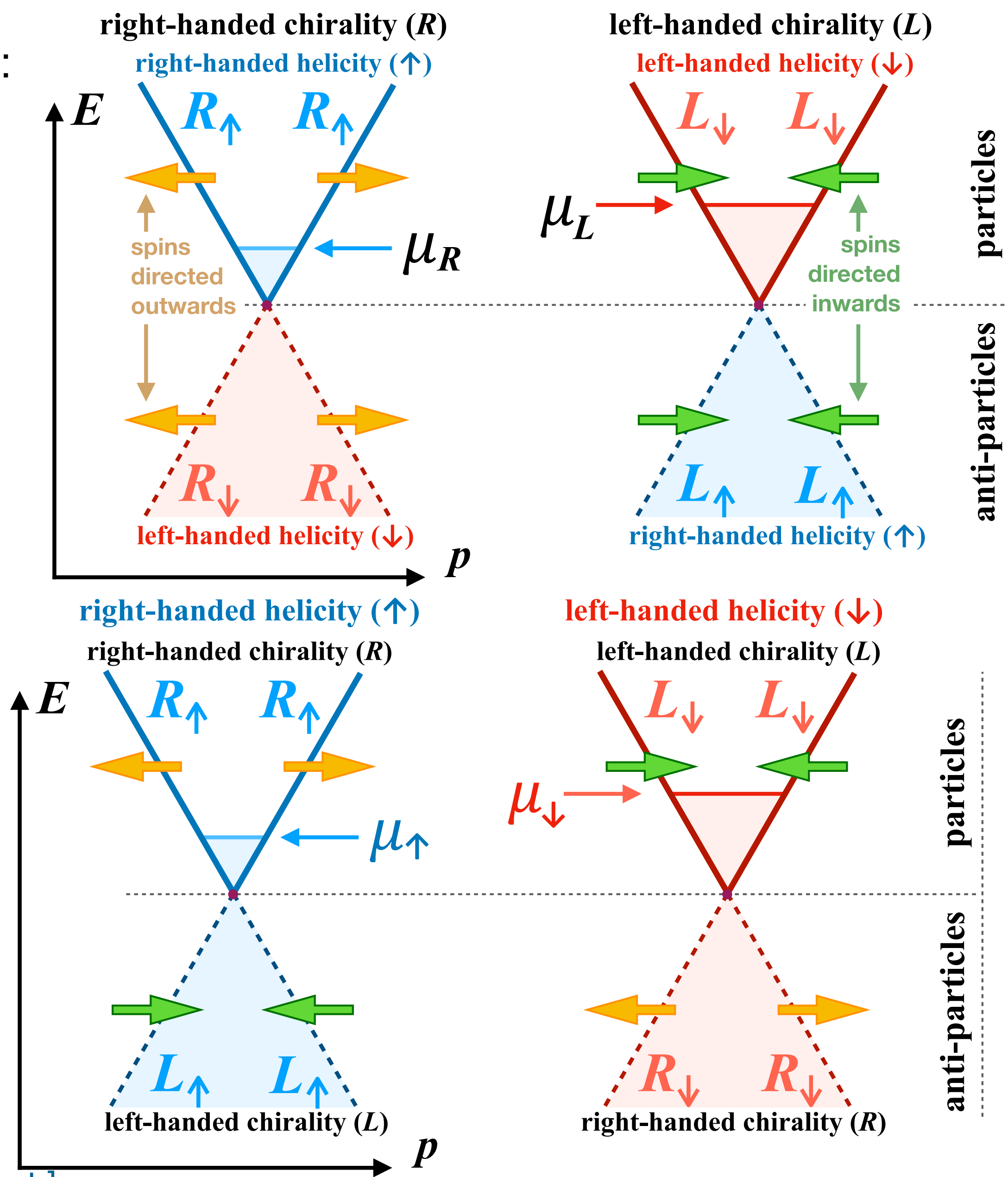
# New mechanism: Helical vortical effect (HVE)

- Split particles into four groups:
  - $\mu^R_\uparrow$ : particle: $R \Rightarrow\uparrow$
  - $\mu^L_\downarrow$ : particle: $L \Rightarrow\downarrow$
  - $\bar\mu^R_\downarrow$ : anti-particle: $R \Rightarrow\downarrow$
  - $\bar\mu^L_\uparrow$ : anti-particle: $L \Rightarrow\uparrow$
- Charge densities:

$$Q_V \equiv (n^R_\uparrow + n^L_\downarrow) - (\bar n^R_\downarrow + \bar n^L_\uparrow),$$
$$Q_A \equiv (n^R_\uparrow + \bar n^R_\downarrow) - (n^L_\downarrow + \bar n^L_\uparrow),$$
$$Q_H \equiv (n^R_\uparrow + \bar n^L_\uparrow) - (n^L_\downarrow + \bar n^R_\downarrow).$$

- Vortical conductivities:

$$\sigma_V \simeq \frac{2\mu_H T}{\pi^2}\ln 2 + \frac{\mu_V \mu_A}{\pi^2},$$
$$\sigma_A \simeq \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2 + \mu_H^2}{2\pi^2},$$
$$\sigma_H \simeq \frac{2\mu_V T}{\pi^2}\ln 2 + \frac{\mu_H \mu_A}{\pi^2}.$$




VEA, M. N. Chernodub, arXiv:1912.11034 [hep-th]

# Chirality ($\gamma^5$)

- For particles ($U_{R/L}$) and anti-particles ($V_{R/L} = i\gamma^2 U^*_{R/L}$):

$$\begin{aligned} \gamma^5 U_R &= +U_R, & \gamma^5 U_L &= -U_L, \\ \gamma^5 V_R &= -V_R, & \gamma^5 V_L &= +V_L. \end{aligned} \tag{1}$$

- The axial current $J^\mu_A = \bar{\psi}\gamma^\mu\gamma^5\psi$ satisfies (classically)

$$\partial_\mu J^\mu_A = 2im\bar{\psi}\gamma^5\psi,$$

and is hence conserved when $m = 0$.

- $Q_A = \int d^3x\, J^0_A$ can be promoted to a quantum operator:

$$:\widehat{Q}_A := \sum_j \chi_j (\hat{b}^\dagger_j \hat{b}_j + \hat{d}^\dagger_j \hat{d}_j), \qquad \chi_R = +1, \qquad \chi_L = -1, \tag{2}$$

which satisfies $[\widehat{Q}_A, \widehat{H}] = 0$ when $m = 0$.

# Helicity $(h)$

- The polarisation of $\psi$ can be characterised using $h = \dfrac{\boldsymbol{S} \cdot \boldsymbol{P}}{p}$, with

$$hU_\lambda = \lambda U_\lambda, \qquad hV_\lambda = \lambda V_\lambda, \qquad \lambda = \pm\frac{1}{2}. \tag{3}$$

- The helicity current $J_H^\mu = \bar{\psi}\gamma^\mu h\psi + \overline{h\psi}\gamma^\mu\psi$ is conserved $\forall m$:

$$\partial_\mu J_H^\mu = 0.$$

- Comparing Eqs. (1) and Eq. (3) shows that for $m = 0$,

$$2\lambda_j = \chi_j. \tag{4}$$

- $Q_H = \int d^3x\, J_H^0$ can also be represented as a quantum operator:

$$:\widehat{Q}_H := \sum_j 2\lambda_j(\hat{b}_j^\dagger \hat{b}_j - \hat{d}_j^\dagger \hat{d}_j), \tag{5}$$

satisfying $[\widehat{Q}_H, \widehat{H}] = 0$.

# Chirality ($\gamma^5$) vs. helicity ($h$)

- For $m = 0$, $\gamma^5$ and $h = \dfrac{\boldsymbol{S}\cdot\boldsymbol{P}}{p}$ share the eigenmodes $U_j$ and $V_j = i\gamma^2 U_j^*$:

$$\begin{pmatrix} 2h \\ \gamma^5 \end{pmatrix} U_j = 2\lambda_j U_j, \qquad \begin{pmatrix} 2h \\ -\gamma^5 \end{pmatrix} V_j = 2\lambda_j V_j, \tag{6}$$

- $J^\mu_A = \bar\psi\gamma^\mu\gamma^5\psi$ satisfies $\partial_\mu J^\mu_A = 2im\bar\psi\gamma^5\psi$.
- $J^\mu_h = \bar\psi\gamma^\mu h\psi + \overline{h\psi}\gamma^\mu\psi$ satisfies $\partial_\mu J^\mu_H = 0$ (for all $m$).

- Why is chirality good? Chiral vortical / magnetic / separation / etc. effects
- Why is chirality bad? $m \neq 0$; Axial anomaly ($\partial_\mu J^\mu_A = -\frac{e^2}{8\pi^2} F_{\mu\nu}\widetilde{F}^{\mu\nu}$)

- Why is helicity good? Works at $m \neq 0$; Helical vortical effects
- Why is helicity bad? interactions; anomaly?; ambiguous when $m \neq 0$

# CPT symmetries

| | $Q_V$ | $Q_A$ | $Q_H$ | $\boldsymbol{J}_V$ | $\boldsymbol{J}_A$ | $\boldsymbol{J}_H$ | $\boldsymbol{\omega}$ |
|---|---|---|---|---|---|---|---|
| $\mathcal{C}$ | $-$ | $+$ | $-$ | $-$ | $+$ | $-$ | $+$ |
| $\mathcal{P}$ | $+$ | $-$ | $-$ | $-$ | $+$ | $+$ | $+$ |
| $\mathcal{T}$ | $+$ | $+$ | $+$ | $-$ | $-$ | $-$ | $-$ |

- $J_V^\mu$, $J_A^\mu$ and $J_H^\mu$ form a triad: same $\mathcal{T}$, different $\mathcal{C}$ and $\mathcal{P}$.
- New vortical effects $\boldsymbol{J}_\ell = \sigma_\ell \boldsymbol{\omega}$ allowed by $\mathcal{CPT}$ symmetries:

$$
\begin{aligned}
&(-,-,+) && \sigma_V \simeq \underline{\frac{2\mu_H T}{\pi^2}\ln 2} + \frac{\mu_V \mu_A}{\pi^2}, \\
&(+,+,+) && \sigma_A \simeq \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2 + \mu_H^2}{2\pi^2}, \\
&(-,+,+) && \sigma_H \simeq \underline{\frac{2\mu_V T}{\pi^2}\ln 2} + \frac{\mu_H \mu_A}{\pi^2}.
\end{aligned}
\tag{7}
$$

- Terms in blue survive when $\mu_A = \mu_H = 0 \Rightarrow$ finite $\boldsymbol{J}_A$ and $\boldsymbol{J}_H$ when $\boldsymbol{\omega} \neq 0$.

- Fermions in equilibrium can be described using

$$f_{\sigma,\lambda} = f^{(\text{eq})}(\beta \cdot k - \alpha_{\sigma,\lambda}), \qquad \beta^\mu = \frac{u^\mu}{T}, \qquad \alpha_{\sigma,\lambda} = \frac{\boldsymbol{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}}{T}, \tag{8}$$

where $q_{V/A/H} = (\sigma, 2\lambda, 2\lambda\sigma)$ and $\boldsymbol{\mu} = (\mu_V, \mu_A, \mu_H)$.

- Global equilibrium is achieved when

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0, \qquad \nabla_\mu \alpha_{\sigma,\lambda} = 0. \tag{9}$$

- One possible solution of the *Killing eq.* corresponds to rigid rotation:

$$\beta = \beta_0(\partial_t + \Omega \partial_\varphi), \tag{10}$$

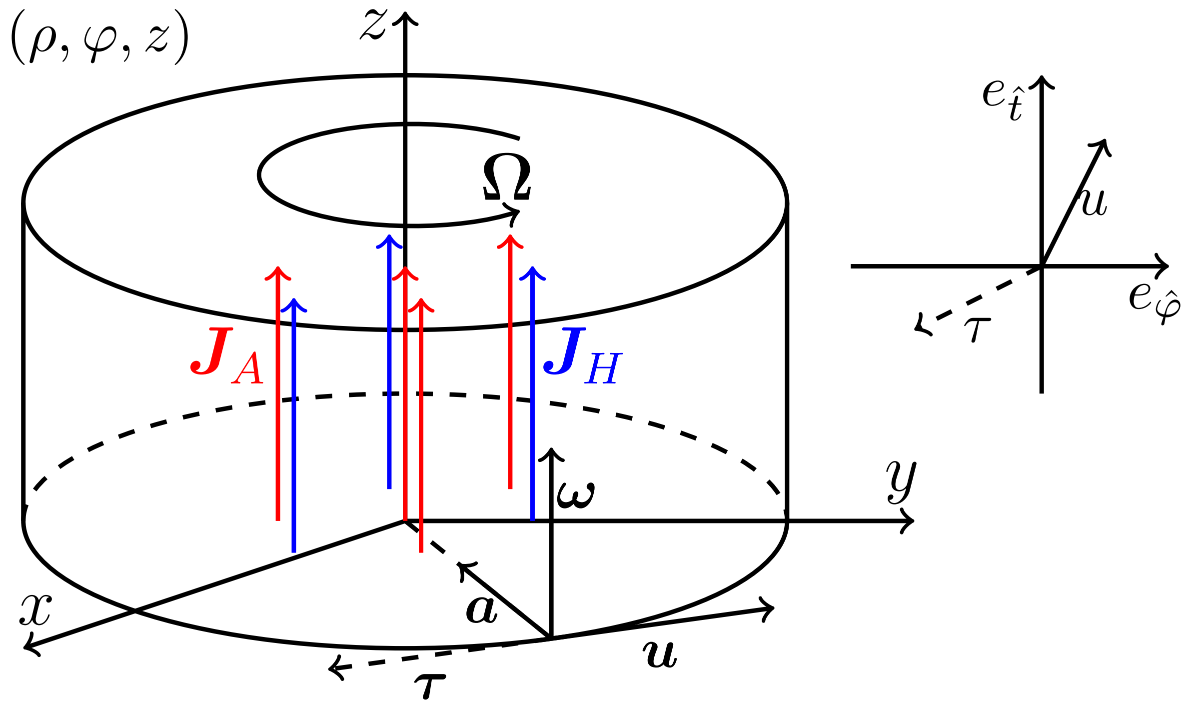
giving rise to

$$u = \Gamma(\partial_t + \Omega\partial_\varphi), \qquad \begin{pmatrix} T \\ \mu \end{pmatrix} = \Gamma \begin{pmatrix} T_0 \\ \mu_0 \end{pmatrix}, \qquad \Gamma = (1 - \rho^2\Omega^2)^{-1/2}. \tag{11}$$

- In equilibrium, $T^{\mu\nu} = (E + P)u^\mu u^\nu - P g^{\mu\nu}$ and $J^\mu_\ell = Q_\ell u^\mu$, where

$$P = -\frac{T^4}{\pi^2} \sum_{\sigma,\lambda} \text{Li}_4(-e^{\boldsymbol{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}/T}), \qquad Q_\ell = \frac{\partial P}{\partial \mu_\ell}. \tag{12}$$

# Kinematic frame for rigid rotation




The *kinematic tetrad* is given by: [Becattini, Grossi, PRD 2015]

$$\text{Velocity}: \quad u = \Gamma(e_{\hat{t}} + \rho\Omega e_{\hat{\varphi}}), \qquad \Gamma = (1 - \rho^2\Omega^2)^{-1/2},$$

$$\text{Acceleration}: \quad a = \nabla_u u = -\rho\Omega^2\Gamma^2 e_{\hat{\rho}},$$

$$\text{Vorticity}: \quad \omega = \frac{1}{2}\varepsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\sigma}} e_{\hat{\alpha}} u_{\hat{\beta}} (\nabla_{\hat{\gamma}} u_{\hat{\sigma}}) = \Gamma^2\Omega e_{\hat{z}},$$

$$\text{Fourth vector}: \quad \tau = \varepsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\sigma}} e_{\hat{\alpha}} u_{\hat{\beta}} a_{\hat{\gamma}} \omega_{\hat{\sigma}} = -\rho\Omega^3\Gamma^5(\rho\Omega e_{\hat{t}} + e_{\hat{\varphi}}).$$

# Quantum rigidly-rotating thermal states

- Computing $J^\mu_{V/A/H} = \langle \widehat{J}^\mu_{V/A/H} \rangle$ using $[Z = \mathrm{Tr}(\hat{\varrho})]$

$$\langle \hat{A} \rangle = Z^{-1} \mathrm{Tr}(\hat{\varrho} \hat{A}), \qquad \hat{\varrho} = \exp\left[-\frac{1}{T_0}\left(\widehat{H} - \Omega \widehat{M}^z - \sum_\ell \mu_{\ell;0} \widehat{Q}_\ell\right)\right],$$

the charge currents can be seen to deviate from the perfect fluid form,

$$J^\mu_\ell = Q_\ell u^\mu + \sigma^\omega_\ell \omega^\mu + \sigma^\tau_\ell \tau^\mu, \tag{13}$$

where at leading order w.r.t. $\Omega$ we have:

$$\begin{aligned} Q_\ell &= \frac{\partial P}{\partial \mu_\ell} = -\frac{T^3}{\pi^2} \sum_{\sigma,\lambda} q^\ell_{\sigma,\lambda} \mathrm{Li}_3\left[-\exp\left(\frac{\boldsymbol{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}}{T}\right)\right], \\ \sigma^\omega_\ell &= \frac{1}{2} \frac{\partial^2 P}{\partial \mu_A \partial \mu_\ell} = -\frac{T^2}{2\pi^2} \sum_{\sigma,\lambda} q^A_{\sigma,\lambda} q^\ell_{\sigma,\lambda} \mathrm{Li}_2\left[-\exp\left(\frac{\boldsymbol{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}}{T}\right)\right], \\ \sigma^\tau_\ell &= \frac{1}{12} \frac{\partial^3 P}{\partial^2 \mu_A \partial \mu_\ell} = \frac{T}{12\pi^2} \sum_{\sigma,\lambda} q^\ell_{\sigma,\lambda} \ln\left[1 + \exp\left(\frac{\boldsymbol{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}}{T}\right)\right]. \end{aligned} \tag{14}$$

where the circular term is suppressed since $\tau = -\Omega^3 \Gamma^5 (\rho^2 \Omega \partial_t + \partial_\varphi)$.

# Axial/helical vortical effects: Constitutive relations

- For high temperatures, $\sigma^\tau_\ell = \mu_\ell / 6\pi^2$ and

$$\sigma^\omega_V = \frac{2\mu_H T}{\pi^2} \ln 2 + \frac{\mu_A \mu_V}{\pi^2} + O(T^{-1}),$$
$$\sigma^\omega_A = \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2 + \mu_H^2}{2\pi^2} + O(T^{-1}),$$
$$\sigma^\omega_H = \frac{2\mu_V T}{\pi^2} \ln 2 + \frac{\mu_A \mu_H}{\pi^2} + O(T^{-1}). \qquad (15)$$

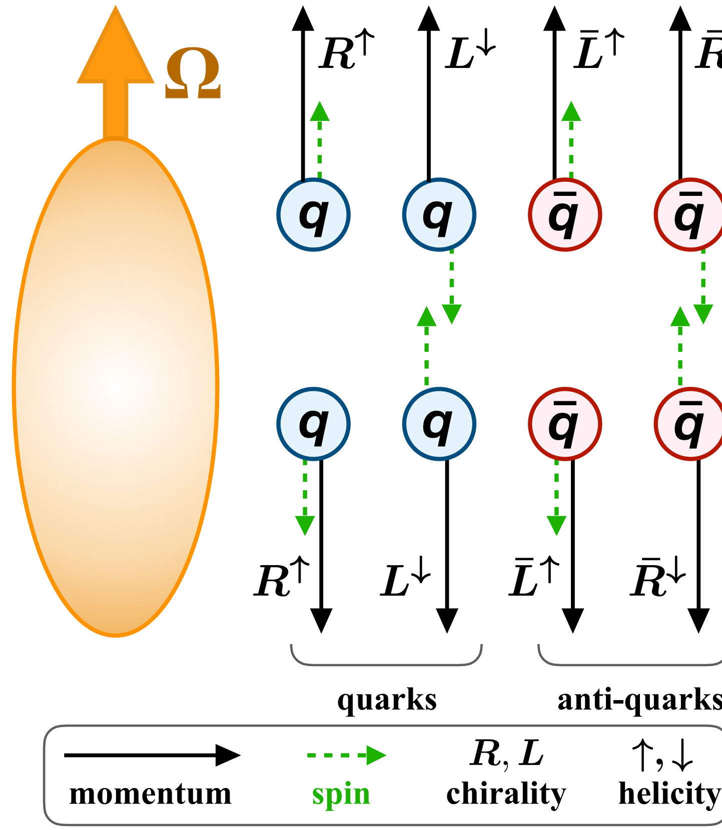
- At finite $T$ and $\mu_V$, $\boldsymbol{\omega}$ generates both $\boldsymbol{J}_A$ **and** $\boldsymbol{J}_H$!

# Particle/anti-particle polarisation from $J_A \pm J_H$

- Considering now a system with $\boldsymbol{\Omega}_{\rm sys} = \boldsymbol{n}_{\rm sys} |\boldsymbol{\Omega}_{\rm sys}|$ and $J_\ell \equiv \boldsymbol{J}_\ell \cdot \boldsymbol{n}$, we can identify:

$$J_V = J_\uparrow + J_\downarrow - \bar{J}_\uparrow - \bar{J}_\downarrow,$$
$$J_A = J_\uparrow + \bar{J}_\uparrow - J_\downarrow - \bar{J}_\downarrow,$$
$$J_H = J_\uparrow + \bar{J}_\downarrow - J_\downarrow - \bar{J}_\uparrow,$$

where $(\uparrow, \downarrow) \equiv$ (right-, left-)handed helicity, while $\bar{\ }$ $\equiv$ anti-particles.




- The net helicity current of particles and anti-particles can be obtained as

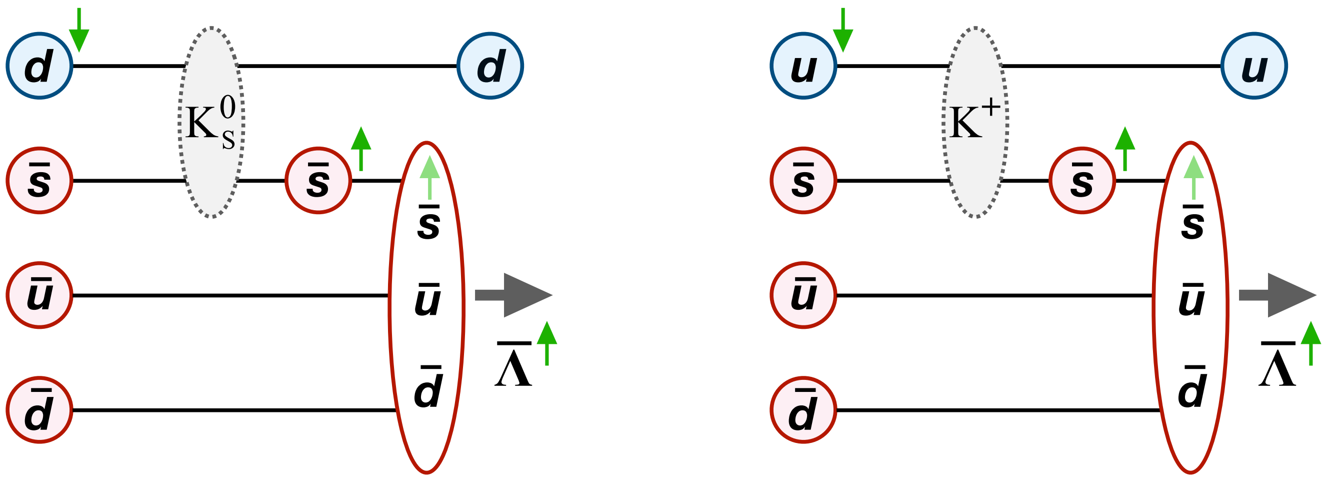
$$J_\uparrow - J_\downarrow = \frac{J_A + J_H}{2}, \qquad \bar{J}_\uparrow - \bar{J}_\downarrow = \frac{J_A - J_H}{2}. \tag{16}$$

- The polarisation of (light flavour) quarks / anti-quarks can be related to the above via

$$\mathcal{P}_q = \kappa_{qj} (J_\uparrow - J_\downarrow), \qquad \mathcal{P}_{\bar{q}} = \kappa_{\bar{q}\bar{j}} (\bar{J}_\uparrow - \bar{J}_\downarrow), \tag{17}$$

where $\kappa_{qj} = \kappa_{\bar{q}\bar{j}}$ are (C-even) kinematical factors.

# (Anti-)hyperon polarisation from $q/\bar{q}$




(a) (b)

- The discussion above applies to $q = (u, d)$. [strange-neutrality requires $\mu_s = 0$]
- $\mathcal{P}_\Lambda$ comes predominantly from $\mathcal{P}_s$. [QCDSF Collaboration, PLB **545** (2002) 112.]
- $\mathcal{P}_q$ can be transferred to $\mathcal{P}_{\bar{s}}$ via intermediate $K_S^0$, $K^+$ states:

$$\mathcal{P}_s = \kappa_{s\bar{q}} \mathcal{P}_{\bar{q}}, \qquad \mathcal{P}_{\bar{s}} = \kappa_{\bar{s}q} \mathcal{P}_q, \qquad \kappa_{s\bar{q}} = \kappa_{\bar{s}q}. \tag{18}$$

- The intermediate Kaons donate their polarised $\bar{s}$ quarks to the antihyperon:

$$\mathcal{P}_\Lambda = \kappa_{\Lambda s} \mathcal{P}_s, \qquad \mathcal{P}_{\bar{\Lambda}} = \kappa_{\bar{\Lambda}\bar{s}} \mathcal{P}_{\bar{s}}, \qquad \kappa_{\Lambda s} = \kappa_{\bar{\Lambda}\bar{s}}. \tag{19}$$

# Freezeout calculation

| $a$, GeV | $b$, GeV | $c$, GeV | $d$, GeV | $f$, GeV$^{-1}$ |
|---|---|---|---|---|
| $0.166(2)$ | $0.139(16)$ | $0.053(21)$ | $1.308(28)$ | $0.273(8)$ |

- Applying the vortical effects for $\mathcal{P}_{q/\bar{q}}$, we get

$$\mathcal{P}_\Lambda = \frac{1}{2}\kappa_{\Lambda s}\kappa_{s\bar{q}}\kappa_{\bar{q}\bar{j}}(\sigma_A^\omega - \sigma_H^\omega)\omega, \qquad \mathcal{P}_{\bar{\Lambda}} = \frac{1}{2}\kappa_{\bar{\Lambda}\bar{s}}\kappa_{\bar{s}q}\kappa_{qj}(\sigma_A^\omega + \sigma_H^\omega)\omega. \tag{20}$$

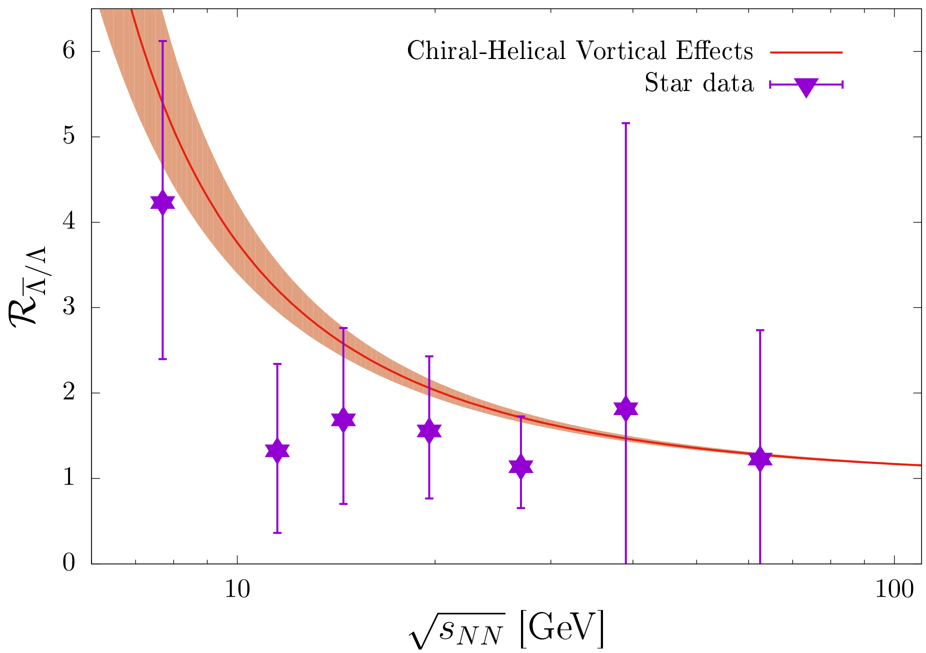
- At freezeout, [Cleymans, Oeschler, Redlich, Wheaton, PRC **73** (2006) 034905]

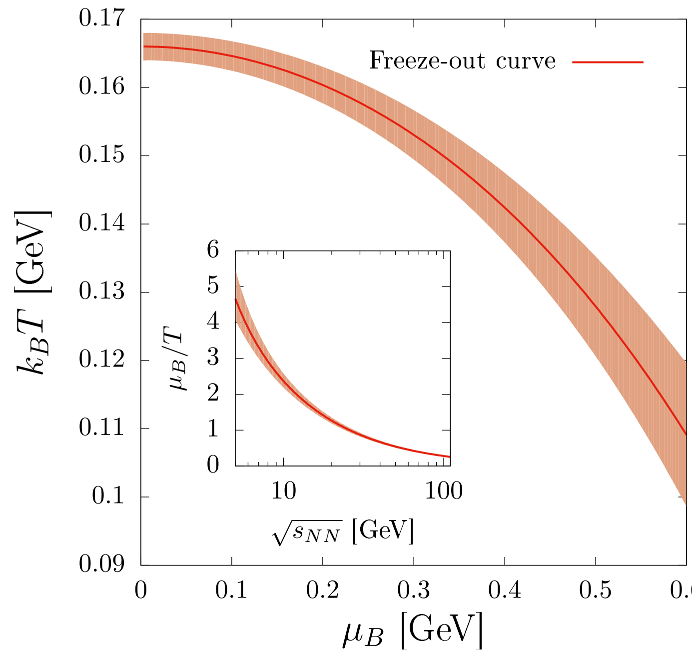
$$T \equiv T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \qquad \mu_B(\sqrt{s}) = \frac{d}{1 + f\sqrt{s}}. \tag{21}$$

- The total polarisation can be obtained by integrating $\mathcal{P}$ over the FO hypersurface:

$$\mathcal{P}_{q/\bar{q}} = \frac{1}{2}\kappa_{qj}(\sigma_A^\omega \pm \sigma_H^\omega)\int d\Sigma_\mu \omega^\mu. \tag{22}$$

# Result




- The anti-hyperon / hyperon polarisation ratio becomes simply

$$\mathcal{R}_{\bar{\Lambda}/\Lambda} = \frac{\mathcal{P}_{\bar{\Lambda}}}{\mathcal{P}_{\Lambda}} = \frac{\mathcal{P}_q}{\mathcal{P}_{\bar{q}}} = \frac{\sigma_A^\omega + \sigma_H^\omega}{\sigma_A^\omega - \sigma_H^\omega} = 1 + \frac{8 \ln 2}{\pi^2} \frac{\mu_B}{T} + O(\mu_B^2/T^2). \qquad (23)$$

# Conclusion

- The $(V, A, H)$ triad uncovers the *helical vortical effects* (HVE).
- $\boldsymbol{J}_A$ generated at finite $T$ and/or finite $\mu_V$, even when $\mu_A = \mu_H = 0$.
- $\boldsymbol{J}_H$ generated at finite $T$ and $\mu_V$, even when $\mu_A = \mu_H = 0$.
- Polarisation of light quarks /antiquarks can be expressed via $J_A \pm J_H$.
- Assuming $\mathcal{P}_{q/\bar{q}} \to \mathcal{P}_{\bar{s}/s} \to \mathcal{P}_{\bar{\Lambda}/\Lambda}$, it is easy to derive $\mathcal{R}_{\bar{\Lambda}/\Lambda} \simeq 1 + \frac{8 \ln 2}{\pi^2} \frac{\mu_B}{T}$.

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THANK YOU FOR YOUR ATTENTION!