Chiral anomaly and the proton spin: lessons from an exactly solvable model

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November 5, 2021

The 6th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions
Stony Brook University

Outline

- Anomaly contribution to proton spin
- Lessons from an exactly soluble model
- A model with interacting constituent quarks

Anomaly contribution to the proton spin

$$2Ms_\mu g_A^{(0)} = \langle p,s|(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)|p,s\rangle = \langle p,s|J^5_\mu|p,s\rangle$$

$$\partial^\mu J^5_\mu = \frac{\alpha_S N_f}{2\pi}\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu} = 2N_f\partial^\mu K_\mu$$

We could define a conserved current which is not renormalized:

$$J^{con}_{\mu 5} = J^5_\mu - 2N_f K_\mu \Rightarrow \partial^\mu J^{con}_{\mu 5} = 0$$

and decompose

$$g_A^{(0)} = \Sigma' - \frac{\alpha_S N_f}{2\pi}\Delta g$$

where

$$2Ms_\mu\Sigma' = \langle p,s|J^{con}_{\mu 5}|p,s\rangle, \qquad 2Ms_\mu\Delta g = -\frac{4\pi}{\alpha_S}\langle p,s|K_\mu|p,s\rangle$$

Massless Schwinger model

Massless QED_2:

$$S = \int d^2x \left[-\frac{1}{4}(F_{\mu\nu})^2 + \bar{\psi} i \gamma^\mu D_\mu \psi \right]$$

$$D_\mu \psi = \partial_\mu \psi - ieA_\mu \psi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \epsilon_{\mu\nu} E$$

Exactly solvable via bosonization

Many features similar to QCD_4 :

- Confinement : spectrum - bosons with $m^2 = \frac{e^2}{\pi}$
- Spontaneous chiral symmetry breaking: $\langle \bar{\psi} \psi \rangle \neq 0$
- Chiral anomaly

Chiral anomaly in the Schwinger model

$$j_\mu = \bar{\psi}\gamma_\mu\psi, \qquad j_{\mu 5} = \bar{\psi}\gamma_\mu\gamma_5\psi$$

In 1+1d $\qquad \gamma_\mu\gamma_5 = \epsilon_{\nu\mu}\gamma^\nu \quad \Longrightarrow \quad j_{\mu 5} = \epsilon_{\nu\mu} j^\nu$

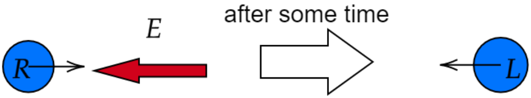
Axial current is anomalous:

$$\partial_\mu j^{\mu 5} = \frac{e}{2\pi}\epsilon_{\mu\nu}F^{\mu\nu} = -\frac{e}{\pi}E \qquad (F_{\mu\nu} = \epsilon_{\mu\nu}E)$$

Or via a topological current:

$$\partial_\mu j^{\mu 5} = \frac{e}{2\pi}\partial_\mu K^\mu, \quad \text{where} \quad K^\mu = 2\epsilon^{\mu\nu}A_\nu$$

A simple interpretation: flipping chirality of particles in E



Bosonization of the massless Schwinger model

- Schwinger model is equivalent to a free massive scalar:

$$S = \int d^2x \left[\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 \right]$$

where $m^2 = \dfrac{e^2}{\pi}$.

- Mapping of operators:

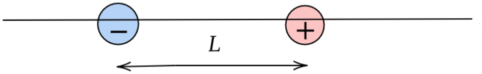
$$\bar{\psi}\psi = c : \cos(2\sqrt{\pi}\phi) : \qquad \bar{\psi}\gamma_5\psi = c : \sin(2\sqrt{\pi}\phi) :$$

$$j_\mu = \frac{1}{\sqrt{\pi}} \epsilon_{\mu\nu} \partial^\nu \phi \qquad j_{\mu 5} = \frac{1}{\sqrt{\pi}} \partial_\mu \phi,$$

- $\partial_\mu j^\mu = 0$ trivially
- Axial anomaly $\Longleftrightarrow m \neq 0$

Analog of polarized parton distribution function

- The goal: to construct some relation in the Schwinger model resembling
$$2Ms_\mu g_A^{(0)} = \langle p, s|J_{\mu 5}|p, s\rangle$$
- We have an axial current but no proton state
- Model the proton as a static external source : two opposite point charges at distance L



- The direction of electric field plays the role of a polarization
- Electric field is a topological density in (1+1) so we are bringing topology to the problem

Axial current from an external dipole source

An external current is added to the Schwinger model:

$$S = S_{QED_2} + \int d^2x A_\mu J^\mu_{ext}$$

Writing $J^\mu_{ext} = \epsilon^{\mu\nu}\partial_\nu D$ one could get the bosonized action ([1]):

$$S = \int d^2x \left[\frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\left(\phi + \frac{\sqrt{\pi}}{e}D\right)^2\right]$$

$$\boxed{\langle ext|j_{x5}(x)|ext\rangle = \frac{m}{2}(e^{-m|x+L/2|} - e^{-m|x-L/2|})}$$

- Changes sign if we change the sign of D
- The effect comes from the topology of electric field
- If $L \ll \frac{1}{m}$ we get almost 0, can't resolve at such distances
- If $L \gg \frac{1}{m}$ we get two screened external charges

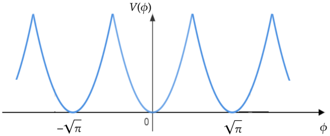
[1] Coleman, S., Jackiw R., Susskind L., Ann. of Phys. 93 (1975)

How can we understand this result in terms of constituent quarks?

Modifying the potential

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

ϕ is periodic:



$$\sum_{branches} \frac{m^2}{2}\phi^2$$

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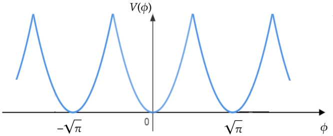
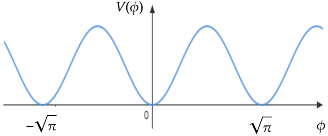




$$\sum_{branches} \frac{m^2}{2}\phi^2 \quad \Rightarrow \quad \frac{m^2}{4\pi}[1 - \cos(2\sqrt{\pi}\phi)]$$

Analogous to dilute instanton gas approximation in QCD

Free constituent quarks

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{4\pi}[1 - \cos(2\sqrt{\pi}\phi)]$$

It is the Lagrangian of the sine-Gordon model

$$\mathcal{L}_{SG} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\alpha}{\beta^2}\cos(\beta\phi) + \gamma$$

which is dual to the massive Thirring model ([1]):

$$\mathcal{L}_T = \bar{\psi} i \gamma^\mu \partial_\mu \psi - m' \bar{\psi}\psi - \frac{g}{2} j_\mu j^\mu$$

with the identification

$$\frac{4\pi}{\beta^2} = 1 + \frac{g}{\pi} = 1 \text{ (since } \beta = 2\sqrt{\pi}\text{)} \implies \boxed{g = 0}$$

[1] Coleman, S., Phys. Rev. D 11, 2088 (1975)

How to introduce interaction between constituent quarks?

Dilute instanton gas $\longrightarrow$ Chiral perturbation theory

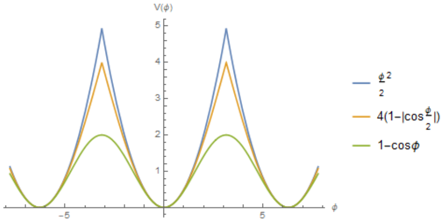
Another modification of the potential

The QCD vacuum energy in chiral perturbation theory ([1]):

$$E_0(\theta) = -m_\pi^2 f_\pi^2 \left|\cos\frac{\theta}{2}\right|$$

We make a corresponding replacement:

$$\frac{m^2}{2}\phi^2 \to \frac{m^2}{\pi}\left[1 - \left|\cos(\sqrt{\pi}\phi)\right|\right]$$



Introduces interaction between constituent quarks

[1] di Cortona, G.G., Hardy, E., Vega, J.P. et al. J. High Energ. Phys. 2016, 34 (2016)

Conclusion

- Schwinger model is exactly soluble and can help clarify the relation between the axial anomaly and the proton spin.
- In Schwinger model the axial anomaly screens the axial current of external quarks (similar to the smallness of quark spin contribution).
- Constituent quark models emerge as modifications of the Schwinger model. Spin composition of hadrons in these models is under ongoing investigation.