Recent theory development on strong QED field in heavy-ion collisions

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Outline

- Strong QED fields generated in heavy ion collisions
- Quantum kinetic theory in strong EB fields
- Lepton pairs photoproduction in Ultra-Peripheral Collisions
- Summary

Strong EB fields in HIC (I)



- Two charged nuclei moving alone z direction generate the EB fields.
- EB fields can be computed by Lienard-Wiechert potential.

$$\vec{E}(\vec{r},t) = \frac{e}{4\pi} \sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{R}_i - R_i \vec{v}_i}{(R_i - \vec{R}_i \cdot \vec{v}_i)^3} (1 - v_i^2),$$
$$\vec{B}(\vec{r},t) = \frac{e}{4\pi} \sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{v}_i \times \vec{R}_i}{(R_i - \vec{R}_i \cdot \vec{v}_i)^3} (1 - v_i^2),$$

Strong EB fields in HIC (II)

Theoretical estimation:

Lienard-Wiechert potential + Event-by-event



A. Bzdak, V. Skokov PRC 2012 ;W.T. Deng, X.G. Huang PRC 2012;V.Roy, SP, PRC 2015; H. Li, X.I. Sheng, Q.Wang, 2016; etc. / review: K. Tuchin 2013

Evolution of EB fields



Quantum kinetic theory in strong EB fields

Chiral magnetic effect



B

spin

UR

mom

- Nonzero axial chemical potential
- Number of Left handed fermions ≠ Number of Right handed fermions



Charge current: charge separation

$$\mathbf{j} = \frac{e^2}{2\pi^2} \mu_5 \mathbf{B},$$

Kharzeev, Fukushima, Warrigna, (08,09), etc.

Connection to the Schwinger pair production



 $\frac{1}{2}\partial_t n_5 =$ Schwinger Pair Production rate

Fukushima, Kharzeev, Warringa, PRL(2010)



We rediscover a non-perturbative method to compute dynamical quantities in strong EB fields.
 >Axial Ward identity, correct mass correction!

$$\partial_{\mu} j_5^{\mu} = \frac{e^2 EB}{2\pi^2} \exp\left(-\frac{\pi m^2}{eE}\right)$$

Mass correction to CME

$$j^{3} = \frac{e^{2}EB}{2\pi^{2}} \coth\left(\frac{B}{E}\pi\right) \exp\left(-\frac{\pi m^{2}}{eE}\right) t$$

> Dynamical chiral condensate

Copinger, Fukushima, SP, PRL(2018)

Also see recent review: Copinger, SP, IJMPA (2020)

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Chiral kinetic equation

• Chiral kinetic theory is a useful tool to study CME.

$$\sqrt{G}\partial_t f + \sqrt{G}\dot{\mathbf{x}} \cdot \nabla_x f + \sqrt{G}\dot{\mathbf{p}} \cdot \nabla_p f = C[f].$$

• Particle's effective velocity:

$$\sqrt{G}\dot{\mathbf{x}} = \frac{\partial\varepsilon}{\partial\mathbf{p}} + \frac{\hbar}{\hbar}\left(\frac{\partial\varepsilon}{\partial\mathbf{p}}\cdot\mathbf{\Omega}\right)\mathbf{B} + \frac{\hbar}{\hbar}\mathbf{E}\times\mathbf{\Omega},$$

• Effective force:

$$\sqrt{G}\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial\varepsilon}{\partial\mathbf{p}} \times \mathbf{B} + \frac{\hbar}{(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega}},$$

• Berry curvature

 $\mathbf{\Omega} = \frac{\mathbf{p}}{2|\mathbf{p}|^3},$

Also see the talk by Mikhail Stephanov

$$\sqrt{G} = 1 + \hbar \mathbf{B} \cdot \mathbf{\Omega},$$

Chiral kinetic theory (massless fermions)

- Hamiltonian formulism, effective theory Son, Yamamoto, PRL, (2012); PRD (2013)
- Path integration

Stephanov, Yin, PRL (2012);

Chen, Son, Stephanov, Yee, Yin, PRL, (2014);

J.W. Chen, J.Y. Pang, SP, Q. Wang, PRD (2014)

- Wigner function (Quantum field theory)
 - hydrodynamics, equilibrium
 - J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);
 - out-of-equilibrium, quantum field theory
 - Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017)
 - Other studies

A.P. Huang, S.Z. Su, Y. Jiang, J.F. Liao, P.F. Zhuang, arXiv:1801.03640

• World-line formulism

N. Muller, R, Venugopalan PRD 2017

Also see recent review:

Gao, Liang, Wang, Int.J.Mod.Phys A 36 (2021), 2130001

Hidaka, SP, D.L. Yang, Q. Wang, invited review, in preparation

Side-jump effects and Lorentz symmetry

• The subgroup for Lorentz group for massless fermions and massive fermions are different.



• From quantum field theory, the distribution function is no longer a scalar.

$$f'(x',p',t') = f(x',p',t') + \hbar N^{\mu} (\partial^x_{\mu} + F_{\nu\mu} \partial^{\nu}_p) f,$$

Infinitesimal Lorentz Transform

$$\delta \mathbf{x} = \hbar \frac{\boldsymbol{\beta} \times \mathbf{p}}{2|\mathbf{p}|},$$

$$\delta \mathbf{p} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|} \times \mathbf{B}$$

Chen, Son, Stephanov, PRL, (2015); Y. Hidaka, SP, D.L. Yang, PRD (2016)

Quantum kinetic theory (massive fermions)

Collision term with quantum corrections

Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD (2019); PRL (2021) Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.

Liu, Mameda, Huang, arXiv:2002.03753.

Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019),

Li ,Yee, PRD100, 056022 (2019)

Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184

Recent reviews:

Gao, Ma, SP, Wang, NST 31 (2020) 9, 90 Gao, Liang, Wang, IJMPA 36 (2021), 2130001 Hidaka, SP, Yang, Wang, in preparation

Also see the talk by Mikhail Stephanov, Nora Weickgenannt, Pengfei Zhuang

Collision kernel

An example for collision kernel of NJL type interactions:

$$\begin{split} & \text{Eq. for Particle} \\ & \text{distribution} \\ & \text{function} \\ & \text{function} \\ & \text{function} \\ & \text{function} \\ \end{split} \\ & = \mathscr{C}_{\text{scalar}} \left(\Delta I_{\text{coll}, \text{qc}}^{(1)} \right) + \mathscr{C}_{\text{scalar}} \left(\Delta I_{\text{coll}, \nabla}^{(2)} \right) - \frac{1}{2\pi\hbar m} \operatorname{Re} \operatorname{Tr} \left(\gamma \cdot \partial_x I_{\text{coll}}^{(1)} \right) \\ & = \mathscr{C}_{\text{scalar}} \left(\Delta I_{\text{coll}, \text{qc}}^{(1)} \right) + \mathscr{C}_{\text{scalar}} \left(\Delta I_{\text{coll}, \nabla}^{(1)} \right) + \mathscr{C}_{\text{scalar}} \left(I_{\text{coll}, \text{PB}}^{(0)} \right) + \mathscr{C}_{\text{scalar}} \left(\partial_x I_{\text{coll}}^{(1)} \right) , \\ & \text{Eq. for Spin} \\ & \text{distribution} \\ & \text{function} \\ & \frac{1}{E_p} p \cdot \partial_x \operatorname{tr} \left[n_j^{(+)\mu} \tau_j^T f^{(1)}(x,p) \right] = \frac{1}{2\pi\hbar m} \int_0^\infty dp_0 \left[\epsilon^{\mu\nu\alpha\beta} p_\nu \operatorname{Im}\operatorname{Tr} \left(\sigma_{\alpha\beta} I_{\text{coll}}^{(2)} \right) + \operatorname{Re} \operatorname{Tr} \left(\gamma^5 \partial_x^\mu I_{\text{coll}}^{(1)} \right) \right] \\ & = \mathscr{C}_{\text{pol}}^\mu \left(\Delta I_{\text{coll}, \text{qc}}^{(1)} \right) + \mathscr{C}_{\text{pol}}^\mu \left(\Delta I_{\text{coll}, \nabla}^{(1)} \right) + \mathscr{C}_{\text{pol}}^\mu \left(I_{\text{coll}, \text{PB}}^{(0)} \right) + \mathscr{C}_{\text{pol}}^\mu \left(\partial_x I_{\text{coll}}^{(1)} \right) . \end{split}$$

$$\begin{split} \mathscr{C}_{\text{scalar}} \left(\Delta I_{\text{coll, qc}}^{(1)} \right) & \mathscr{C}_{\text{scalar}} \left(\Delta I_{\text{coll, }\nabla}^{(1)} \right) & \mathscr{C}_{\text{scalar}} \left(\partial_x I_{\text{coll}}^{(1)} \right) & \mathscr{C}_{\text{scalar}} \left(I_{\text{coll, PB}}^{(0)} \right) \\ \mathscr{C}_{\text{pol}}^{\mu} \left(\Delta I_{\text{coll, qc}}^{(1)} \right) & \mathscr{C}_{\text{pol}}^{\mu} \left(\Delta I_{\text{coll, }\nabla}^{(1)} \right) & \mathscr{C}_{\text{pol}}^{\mu} \left(\partial_x I_{\text{coll}}^{(1)} \right) & \mathscr{C}_{\text{pol}}^{\mu} \left(I_{\text{coll, PB}}^{(0)} \right) \end{split}$$

Perturbative Correction to Ordinary terms

Non-local terms related to the space derivatives may be the key to describe the spin-orbital transformation. Also see the talk by Nora Weickgenannt, Pengfei Zhuang

Sheng, Weickgenannt, Speranza, Rischke, Wang PRD (2021)

Problem for numerical simulations

- It is changeling to simulate the QKT:
 - Collison kernel is too complicated, need to be further simplified.
 - One needs to consider the non-local terms.
- Usually, to solve kinetic theory, one can use the cross section
 + MC sampling instead of integrating the collision kernel. But,
 it would fail in quantum kinetic theory with collisions.

• We may need to face the high dimensional integrations in collision kernel.

Relativistic Boltzmann equations on GPU



Relativistic Boltzmann equations on GPU RBG

Basic, but nontrivial.

- We introduce a new numerical framework to derive full solutions of a relativistic BE on GPUs.
 - Full 2->2 collisional term:

high dimensional integrals.

≻High performance:

space 10x10x10,

momentum 30x30x30,

Time steps: 10⁴-10⁶,

on one Nvidia Tesla V100 card costs a few days!

Particle number is strictly conserved.

> Dynamical Debye mass

J.J Zhang, H.Z. Wu, SP, G.Y. Qin, Q. Wang, PRD (2019)

Tests for time evolution of quarks and gluons



Grids: 1 grid (space) ; momentum: 30x30x30=27,000 Phase space size: [-3fm,3fm]³ x [-2GeV,2GeV]³ Time step: dt=0.0005fm ; 100,000 steps Time cost: around 50 hours on a Nvidia Tesla V100 card J.J Zhang, H.Z. Wu, SP, G.Y. Qin, Q. Wang, PRD (2019)

RBG-Maxwell equations and v1, dv1 problem

• We solve the Boltzmann equation coupled to Maxwell equations.

$$egin{aligned} & [p^\mu\partial_\mu+Q_ap_\mu F^{\mu
u}\partial_{p^
u}]f_a(t,\mathbf{x},\mathbf{p})=\mathcal{C}[f_a],\ & \partial_\mu F^{\mu
u}=j^
u_{ ext{ext}}+j^
u_{ ext{med}}, \ ext{(Spectators + Participant)} \end{aligned}$$

QCD 2->2 scattering full collision term

• A good example to show the effects from EB fields. Qualitative consistent with exp. Could help us to understand the dv1 difference for pions and protons.



Outlook for simulation of QKT

- Remarkably, we may need to integrate (high dimensional) collision directly kernel to keep the non-local effects instead of other widely used methods.
- Future Plan:
 - Simplify collision kernel
 - Simulations for the non-local collision kernel



Lepton pairs photoproduction in Ultra-Peripheral Collisions

Ultra-Peripheral Collisions



- Ultra-Peripheral Collisions (UPC): the impact parameter is larger than 2 times the radius of a nucleus
- Since the QCD effects are higher orders and QED effects are enhanced by the Ze, UPC provides a nice platform to study the strong EB effects.

 $Z\alpha \approx 1 \rightarrow$ High photon density Magnetic field strength B $\approx 10^{12} - 10^{14}$ T

Also see the talk by Daniel Brandenburg

Generation matter directly from lights

Scientists Generate Matter Directly From Light – Physics Phenomena Predicted More Than 80 Years Ago

TOPICS: Antimatter Atomic Physics Brookhaven National Laboratory DOE Popular By BROOKHAVEN NATIONAL LABORATORY JULY 30, 2021



Abstract energy concept illustration.

J. Adam *et al.* (STAR Collaboration), Measurement of e+e- Momentum and Angular Distributions from Linearly Polarized Photon Collisions, *Phys. Rev. Lett* 127, 052302

Collisions of Light Produce Matter/Antimatter from Pure Energy

Study demonstrates a long-predicted process for generating matter directly from light – plus evidence that magnetism can bend polarized photons along different paths in a vacuum

July 28, 2021



Making matter from light: Two gold (Au) ions (red) move in opposite direction at 99.995% of the speed of light (v, for velocity, = approximately c, the speed of light). As the ions pass one another without colliding, two photons (v) from the electromagnetic cloud surrounding the ions can interact with each other to create a matter-antimatter pair: an electron (e⁻) and positron (e⁺).

Vacuum birefringence

 Vacuum birefringence: Index of refraction for photon interaction with B field depends on relative polarization angle



• The difference of linear polarization between probe and observed lights leads to $\sim \cos(n\phi)$ type correction to differential cross section.

$$\Delta \phi = \Delta \phi[(e^+ + e^-), (e^+ - e^-)]$$

$$\approx \Delta \phi[(e^+ + e^-), e^+]$$

Li, Zhou, Zhou, PLB 795, 576 (2019)



Strong QED fields studies: Hattori, Itakura, Annals.Phys. (2013)

Dilepton (photo)production



Equivalent photon approximation (EPA)

- A. J. Baltz, Y. Gorbunov, S. R. Klein and J. Nystrand, PRC 80, 044902 (2009)
- W. Zha, L. Ruan, Z. Tang, Z. Xu and S. Yang, PLB 781, 182 (2018)
- W. Zha, J. D. Brandenburg, Z. Tang and Z. Xu, PLB 800 (2020) 135089

Based on QED calculations

- C. Li, J. Zhou and Y. J. Zhou, Phys. Lett. B 795, 576 (2019) ; arXiv:1911.00237 [hep-ph]].
- Klein, Muller, Xiao, Yuan, PRL 122 (2019) 13, 132301; PRD 102 (2020) 9, 094013
- W. Zha, J. D. Brandenburg, Z. Tang and Z. Xu, PLB 800 (2020) 135089
- Xiao, Yuan, Zhou, PRL 125 (2020) 23, 232301
- R.J. Wang, SP, Q. Wang, PRD (2021)

Our theoretical framework

Need to describe the space and momentum of photons.



• We start from the wave-package description and derive the general expression of the cross section including the space and momentum dependent photon distribution function.

R.J. Wang, SP, Q. Wang, PRD 2021

Differential cross section

• Our general expression for differential cross section is as follows.

$$\frac{d\sigma}{d^{3}k_{1}d^{3}k_{2}} \approx \frac{1}{32(2\pi)^{6}} \frac{1}{E_{k1}E_{k2}} \int d^{2}\mathbf{b}_{T}d^{2}\mathbf{b}_{1T}d^{2}\mathbf{b}_{2T} \int d^{4}p_{1}d^{4}p_{2}$$
Fluctuations from wave
package: we neglect
this part contributions
so far.
Photon Wigner function:
provides information of
space and momentum
for photons

$$S_{\sigma\mu}(P,p) \equiv \int \frac{d^{4}y}{(2\pi)^{4}}e^{ip\cdot y} \langle P|A_{\sigma}^{\dagger}(0)A_{\mu}(y)|P\rangle,$$
Transverse Momentum Dependent (TMD) Photon Distribution
Klein, Muller, Xiao, Yuan, PRL (2019)

Connection to other theories

• If we consider A as background fields and take the virtuality expansion, we can reproduce the Equivalent photon approximation (EPA).

$$\sigma_0(A_1A_2 \to l\bar{l}) = \int d\omega_1 d\omega_2 n_{A1}(\omega_1) n_{A2}(\omega_2) \sigma_{\gamma\gamma \to l\bar{l}}(\omega_1, \omega_2),$$

- If we integrate over the space dependence of photon Wigner function, we will reduce to the formulism introduced by Greiner etc. [Vidovic, Greiner, Best, Soff, PRC (1993)] and used by W. Zha, Z.B. Tang, X.B. Xu's group.
- If we use the light-cone coordinates and take a twist expansion, then we can reproduce the formulism from TMD community (J. Zhou's and B.W. Xiao's group).

$$\frac{d\sigma_{\text{twist}2}}{d^{3}k_{1}d^{3}k_{2}} = \frac{1}{2(2\pi)^{10}} Z^{4} \alpha^{2} v \int d\omega_{1} d^{2} \mathbf{p}_{1T} d\omega_{2} d^{2} \mathbf{p}_{2T} \frac{1}{E_{k1}E_{k2}} \frac{p_{1T}^{\sigma} p_{1T}^{\mu} p_{2T}^{\rho} p_{2T}^{\nu}}{\omega_{1}^{2} \omega_{2}^{2}} \left| \frac{F(-p_{1}^{2})}{-p_{1}^{2}} \right|^{2} \left| \frac{F(-p_{2}^{2})}{-p_{2}^{2}} \right|^{2} \\ \times L_{\mu\nu;\sigma\rho}(p_{1}, p_{2}; p_{1}, p_{2}; k_{1}, k_{2})(2\pi)^{4} \delta^{4}(p_{1} + p_{2} - k_{1} - k_{2}).$$

$$\sigma_{\text{twist}\,n} = \frac{Z^{4} \alpha^{2} v}{8\pi^{4}} \int \frac{d^{3}k_{1}}{(2\pi)^{3} 2E_{k1}} \frac{d^{3}k_{2}}{(2\pi)^{3} 2E_{k2}} \frac{d\omega_{1}}{\omega_{1}^{2}} \frac{d\omega_{2}}{\omega_{2}^{2}} d^{2} p_{1T} d^{2} p_{2T} \left| \frac{F(-p_{1}^{2})}{-p_{1}^{2}} \right|^{2} \left| \frac{F(-p_{2}^{2})}{-p_{2}^{2}} \right|^{2} (2\pi)^{4} \delta^{4}(p_{1} + p_{2} - k_{1} - k_{2}) \mathcal{I}$$

Total cross section and some differential cross section

TABLE I. The total cross sections from STAR measurements and theoretical models. The numerical integration errors are labeled as "int."

Data or models	Total cross sections
STAR data [67]	$0.261 \pm 0.004(\text{stat.}) \pm 0.013(\text{sys.})$
	± 0.034 (scale) mb
STARLight [73]	0.22 mb
Zha et al.'s gEPA [77]	0.26 mb
Zha et al.'s work [77]	0.26 mb
This work Eq. (20)	0.252 ± 0.0016 (int.) mb
gEPA Eq. (30)	0.256 ± 0.0030 (int.) mb

- Our results agree with experimental data.
- Differences between our results and data may come from the higher order corrections.



Parameter dependence

We find that the differential cross section strongly depends on the choice of $P(b_{\perp})$ and the radius of nuclei R=1.1-1.2A^{1/3}.



Centrality dependence



In collaboration with R.J. Wang, S. Lin, Y.F. Zhang, Q. Wang, etc. in preparation

Angle distribution related to vacuum birefringence



In collaboration with R.J. Wang, S. Lin, Y.F. Zhang, Q. Wang, etc. in preparation

PT broadening



- We need to consider the higher order corrections related to the PT broadening effect.
- One well-known effect comes from the Sudakov factor in TMD community.

Klein, Mueller, Xiao, Yuan, PRL(2019); PRD (2020) Li, Zhou, Zhou, PLB (2019); PRD(2020) Hatta, Xiao, Yuan, Zhou, PRD (2021)

In collaboration with S. Lin, R.J. Wang, Q. Wang, etc. in preparation

Sudakov factor



• We learn how to resum the soft photon radiation and derive

$$\frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} = \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{ir_{\perp} \cdot q_{\perp}} e^{S(r_{\perp})} \int d^2 q'_{\perp} e^{-ir_{\perp} \cdot q'_{\perp}} \frac{d\sigma(q'_{\perp})}{d\mathcal{P}.\mathcal{S}.}$$
Leading order
d $\mathcal{P}.\mathcal{S} = dp_{\perp} \cdot dp_{\perp} \cdot du_{\perp} du_{\perp} du_{\perp} d^2 h_{\perp}$
section

 $d\mathcal{P}.\mathcal{S}. = dp_{1\perp}dp_{2\perp}dy_1dy_2d^2b_{\perp}$

Hatta, Xiao, Yuan, Zhou, PRD (2021)

• Note that S(r) is different with the ordinary double-log type Sudakov factor.

$$S(r_{\perp}) = -rac{lpha}{\pi^2} \ln rac{Q^2}{m^2} \left(\ln rac{r_{\perp}^2 Q^2}{c_0^2}
ight)$$

No colinear divergence due to the mass of leptons.

In collaboration with S. Lin, R.J. Wang, Q. Wang, etc. in preparation

Summary

Summary

- The quantum matter under strong QED fields is a still rapid developing area.
- Quantum kinetic theory is a tool to study the quantum effects in strong EB fields. It is still changeling to simulate the QKT with the collision term due to the non-local terms and other quantum corrections.
- Ultra-peripheral collisions with excellent measurements from STAR, etc. provide us a nice platform to study the strong QED effects. Many famous but never been discovered effects, e.g. the generation matter directly from lights and vacuum birefringence etc, may be measured from experiments.

Thank you for your time!

Backup

Analytic solutions for MHD

- 1+1 D ideal MHD Bjorken flow
 V.Roy, SP, L. Rezzolla, D. Rischke, Phys.Lett. B750, 45
 SP, V. Roy, L. Rezzolla, D. Rischke, Phys.Rev. D93, 074022
- 2+1 D ideal MHD Bjorken flow (perturbative) SP, Di-Lun Yang, Phys.Rev. D93, 054042
- Background Magnetic field: contribution to v2 V.Roy, SP, L. Rezzolla, D. Rischke, Phy.Rev. C96, 054909
- MHD + CME + Chiral anomaly Siddique, R.J. Wang, SP, Q. Wang, PRD 2019

Optical birefringence

• Optical birefringence:

Different index of refraction for light polarized parallel vs. perpendicular to material's ordinary axis



Figures from the talks given by Daniel Brandenburg and Zhangbu Xu

Our primary result (IV)



Collisional term via ZMCintegral

• 5 dimensional integral on each phase space grid:

Wu, Zhang, Pang, Q. Wang, Computer Physics Communications (2019)



Symmetric Sampling

We ensure that

