QUANTUM TOMOGRAPHY OF CHIRAL MEDIA

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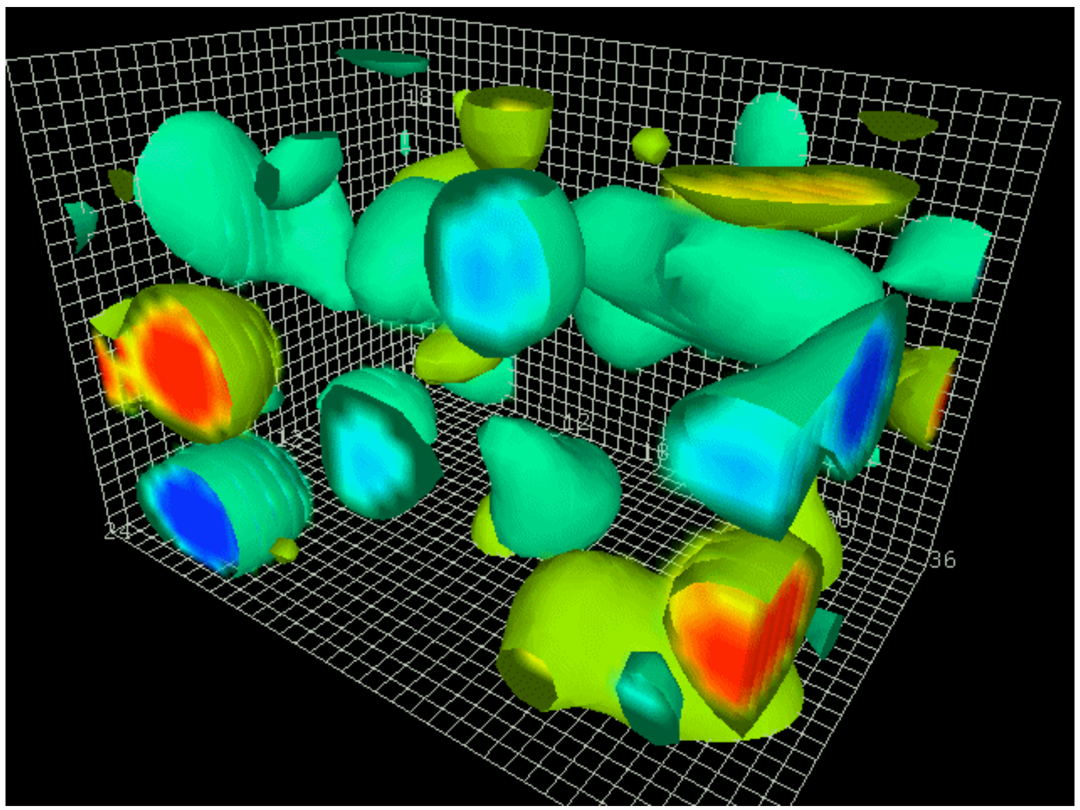
“Chirality, Vorticity and Magnetic field in Heavy-Ion Collisions” @ SUNY Stony Brook

November 5, 2021

CHIRAL MEDIA

Nuclear Matter



D. Leinweber

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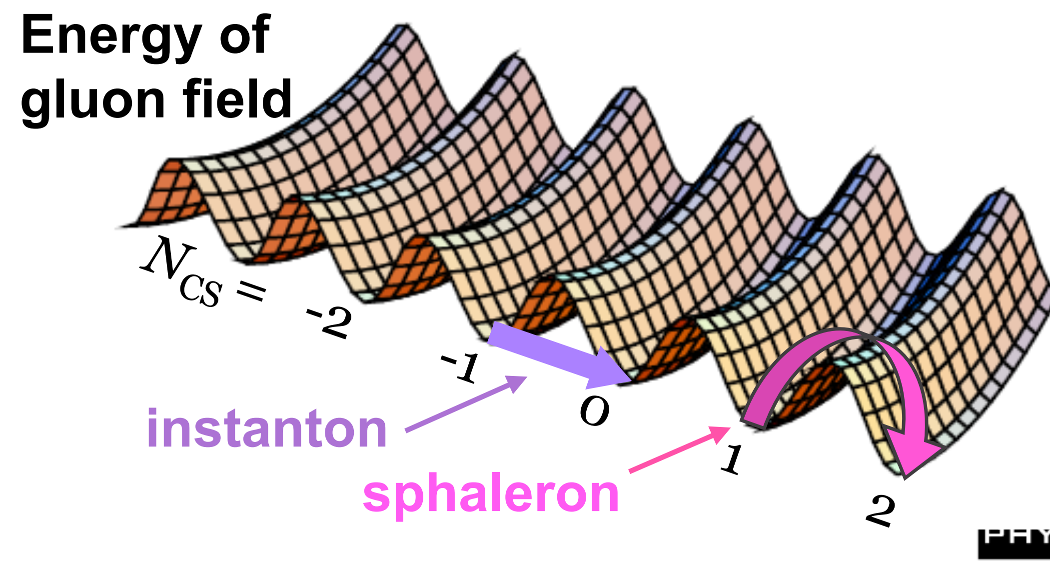


ELSEVIER

Nuclear Physics A 897 (2013) 93–108

www.elsevier.com/locate/nuclphysa

$\mathcal{P}$-odd fluctuations and long range order in heavy ion collisions. Deformed QCD as a toy model

Ariel R. Zhitnitsky

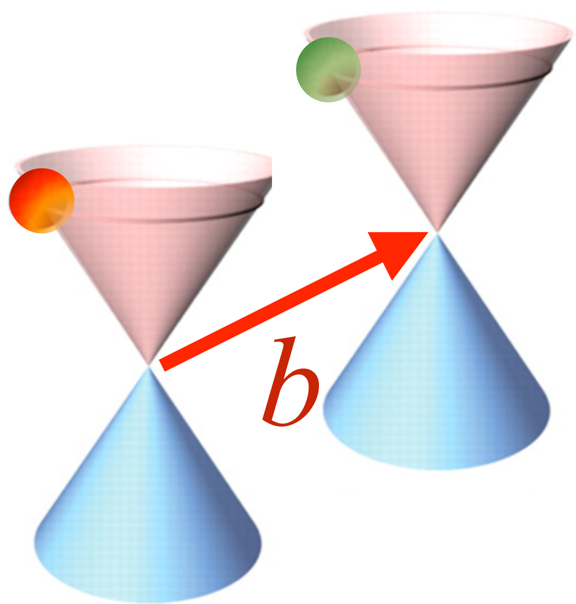
PHYSICAL REVIEW D **68**, 114505 (2003)

Low-dimensional long-range topological charge structure in the QCD vacuum

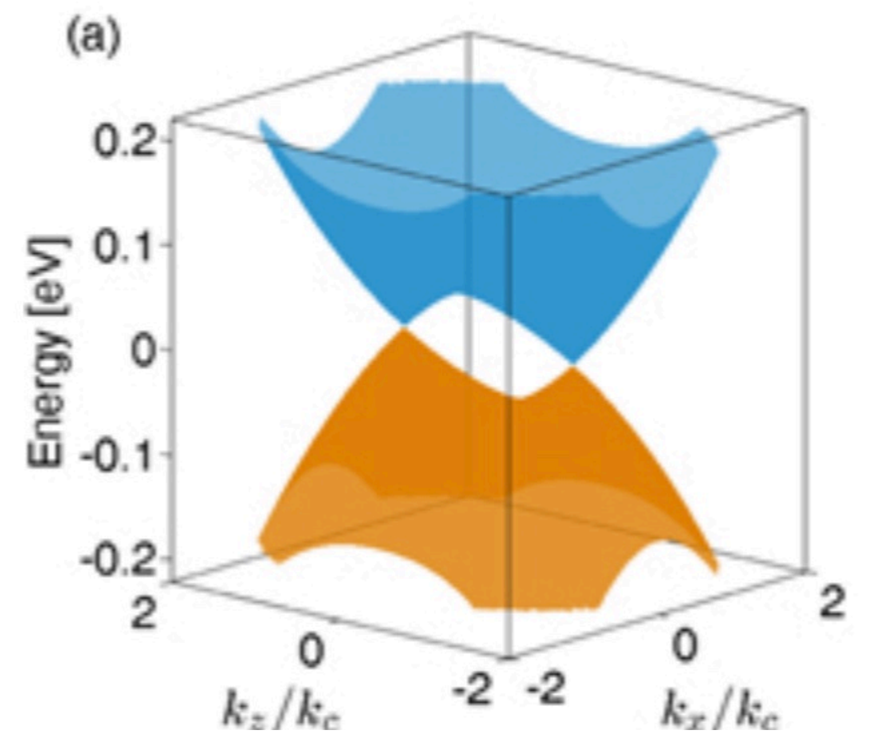
I. Horváth,[1] S. J. Dong,[1] T. Draper,[1] F. X. Lee,[2,3] K. F. Liu,[1] N. Mathur,[1] H. B. Thacker,[4] and J. B. Zhang[5]

CHIRAL MEDIA

Dirac and Weyl semimetals



TaAs
NbAs
NbP
TaP



nature physics

LETTERS

PUBLISHED ONLINE: 8 FEBRUARY 2016 | DOI: 10.1038/NPHYS3648

Chiral magnetic effect in $ZrTe_5$

Qiang Li[1*], Dmitri E. Kharzeev[2,3*], Cheng Zhang[1], Yuan Huang[4], I. Pletikosić[1,5], A. V. Fedorov[6], R. D. Zhong[1], J. A. Schneeloch[1], G. D. Gu[1] and T. Valla[1*]

arXiv:1412.6543 [cond-mat.str-el]

CHIRAL MEDIA

Axionic stars/clumps, dark matter

PHYSICAL REVIEW D **92,** 103513 (2015)

Do dark matter axions form a condensate with long-range correlation?

Alan H. Guth,[1,*] Mark P. Hertzberg,[1,2,†] and C. Prescod-Weinstein[3,‡]

[1] *Center for Theoretical Physics & Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

[2] *Institute of Cosmology & Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155, USA*

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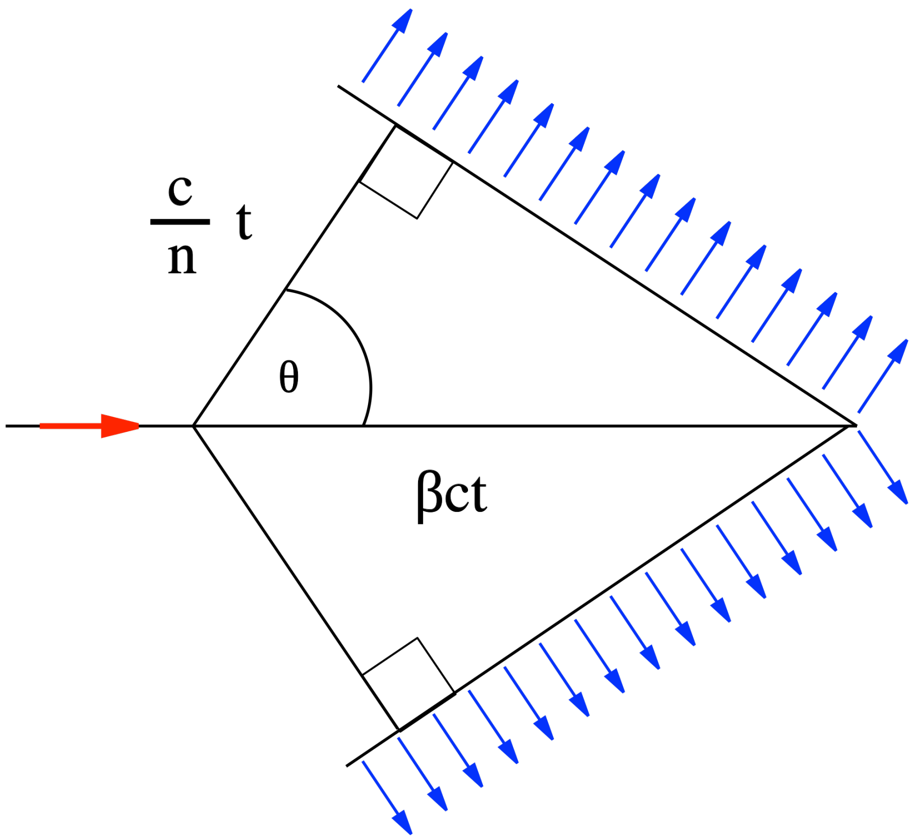
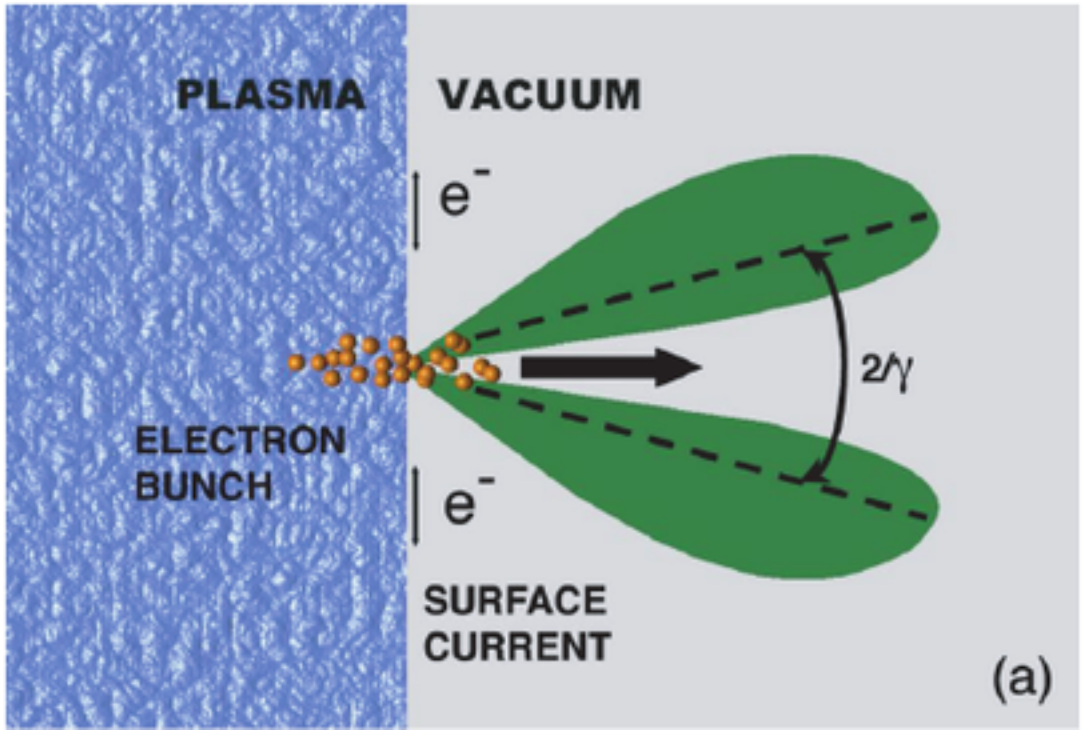
(Received 14 July 2015; published 16 November 2015)

PROBING THE MATTER: CHERENKOV AND TRANSITION RADIATION





Classical Cherenkov radiation is emitted by a charged particle that moves faster than the phase velocity of light: $vn>1$

$$\cos\theta = \frac{1}{\beta\sqrt{\epsilon}} = \frac{1}{\beta n}$$

Classical transition radiation is emitted by a charged particle that moves through inhomogeneous matter.

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33.7. Cherenkov and transition radiation [33,77,78]

A charged particle radiates if its velocity is greater than the local phase velocity of

PHYSICAL REVIEW LETTERS **121,** 182301 (2018)

Transition Radiation as a Probe of the Chiral Anomaly

Xu-Guang Huang[1,2] and Kirill Tuchin[3]
[1]*Physics Department and Center for Particle Physics and Field Theory, Fudan University, Shanghai 200433, China*
[2]*Key Laboratory of Nuclear Physics and Ion-beam Application (MOE), Fudan University, Shanghai 200433, China*
[3]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*

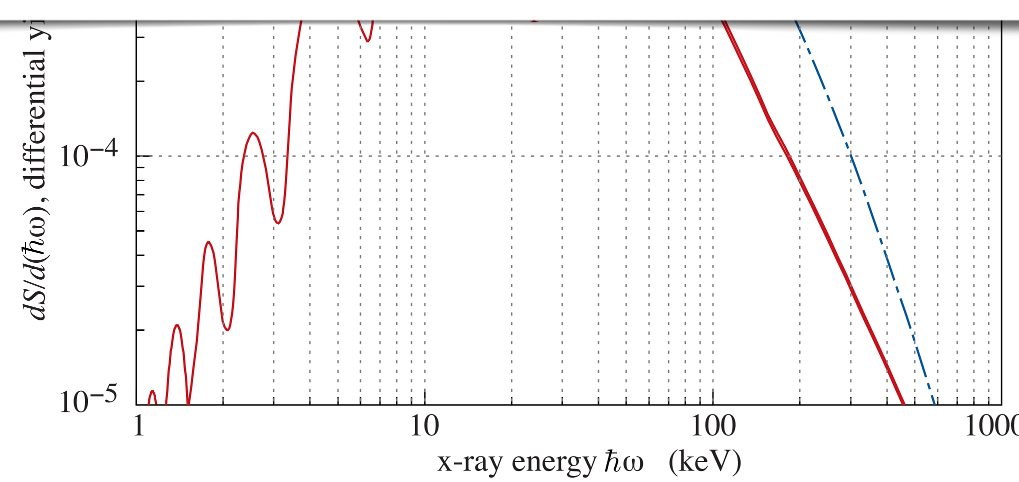


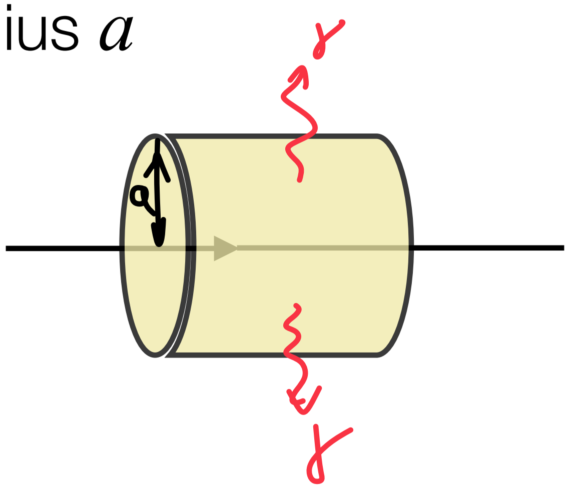
Figure 33.27: X-ray photon energy spectra for a radiator consisting of 200 25 μm thick foils of Mylar with 1.5 mm spacing in air (solid lines) and for a single surface (dashed line). Curves are shown with and without absorption. Adapted from Ref. 88.

FERMI'S MODEL OF COLLISIONAL ENERGY LOSS

Fermi (1940)

The energy loss rate = flux of the Poynting vector out of cylinder of radius a coaxial with the particle path:

$$-\frac{d\varepsilon}{dz} = 2\pi a \int_{-\infty}^{\infty} (E_\phi B_z - E_z B_\phi) dt = 2a \, \mathrm{Re} \int_0^{\infty} (E_{\phi\omega} B^*_{z\omega} - E_{z\omega} B^*_{\phi\omega}) d\omega$$



Maxwell equations $\boldsymbol{\nabla} \times \boldsymbol{B}_\omega = -i\omega \boldsymbol{D}_\omega + j_\omega$ etc.

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

Energy loss: $a \to 0$ UR limit: $-\frac{d\varepsilon}{dz} = \frac{q^2}{4\pi v^2} \omega_p^2 \ln \frac{v}{a\omega_p}$

(small) Cherenkov radiation contribution emerges at $a \to \infty$ if $v > 1/\sqrt{\epsilon(0)}$

QED IN CHIRAL MEDIUM: MAXWELL-CHERN-SIMONS

Sikivie (84), Wilczek (87), Carroll et al (90)

$$\mathcal{L}_{\rm MCS} = \mathcal{L}_{\rm QED} + c_A \theta(x) \vec{E} \cdot \vec{B}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0\,,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho - c\,\boldsymbol{\nabla}\theta \cdot \boldsymbol{B}\,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}\,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j} + c(\partial_t\theta\, \boldsymbol{B} + \boldsymbol{\nabla}\theta \times \boldsymbol{E})\,,$$

Anomalous Hall Effect

Chiral magnetic effect

Kharzeev, McLerran, Warringa (2008)

Fukushima, Kharzeev, Warringa (2008)

$$\boldsymbol{j} = \sigma_\chi \boldsymbol{B}$$

"chiral (magnetic) conductivity"

Let the field θ be homogenous and weekly time-dependent $\dot{\theta} = \text{const}$

EM FIELDS OF A CHARGE IN CHIRAL MEDIUM

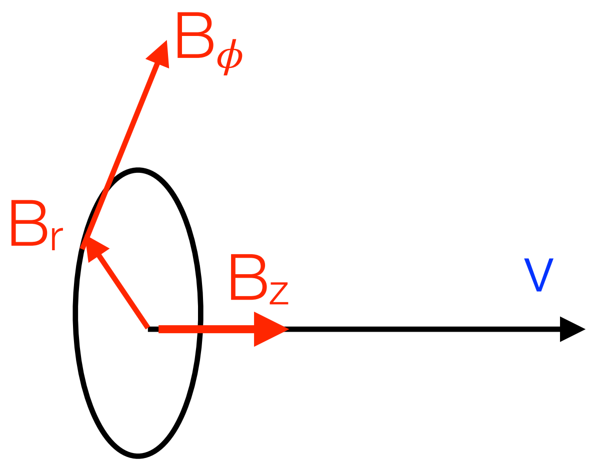
EM field of a point charge

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{D} + \sigma_\chi \boldsymbol{B} + qv\hat{\boldsymbol{z}}\delta(z-vt)\delta(\boldsymbol{b})\,,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = q\delta(z-vt)\delta(\boldsymbol{b})\,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}\,,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0\,,$$



Can be solved for constant chiral conductivity, e.g.

$$B_{\phi\omega}(\boldsymbol{r}) = \frac{q}{2\pi}\frac{e^{i\omega z/v}}{k_1^2-k_2^2}\sum_{\nu=1}^{2}(-1)^{\nu+1}k_\nu(k_\nu^2-s^2)K_1(bk_\nu)$$

$$B_{b\omega}(\boldsymbol{r}) = \sigma_\chi \frac{q}{2\pi}\frac{i\omega}{v}\frac{e^{i\omega z/v}}{k_1^2-k_2^2}\sum_{\nu=1}^{2}(-1)^{\nu}k_\nu K_1(bk_\nu)$$

$$k_\nu^2 = s^2 - \frac{\sigma_\chi^2}{2} + (-1)^\nu \sigma_\chi \sqrt{\omega^2\epsilon + \frac{\sigma_\chi^2}{4}}$$

$$s^2 = \omega^2\left(\frac{1}{v^2} - \epsilon(\omega)\right)$$

High energy approximation:

$$B_\phi = \frac{eb}{8\pi x_-^2} e^{-\frac{b^2\sigma}{4x_-}}\left[\sigma\cos\left(\frac{b^2\sigma_\chi}{4x_-}\right) + \sigma_\chi \sin\left(\frac{b^2\sigma_\chi}{4x_-}\right)\right]$$

EM FIELDS OF A CHARGE IN CHIRAL MEDIUM

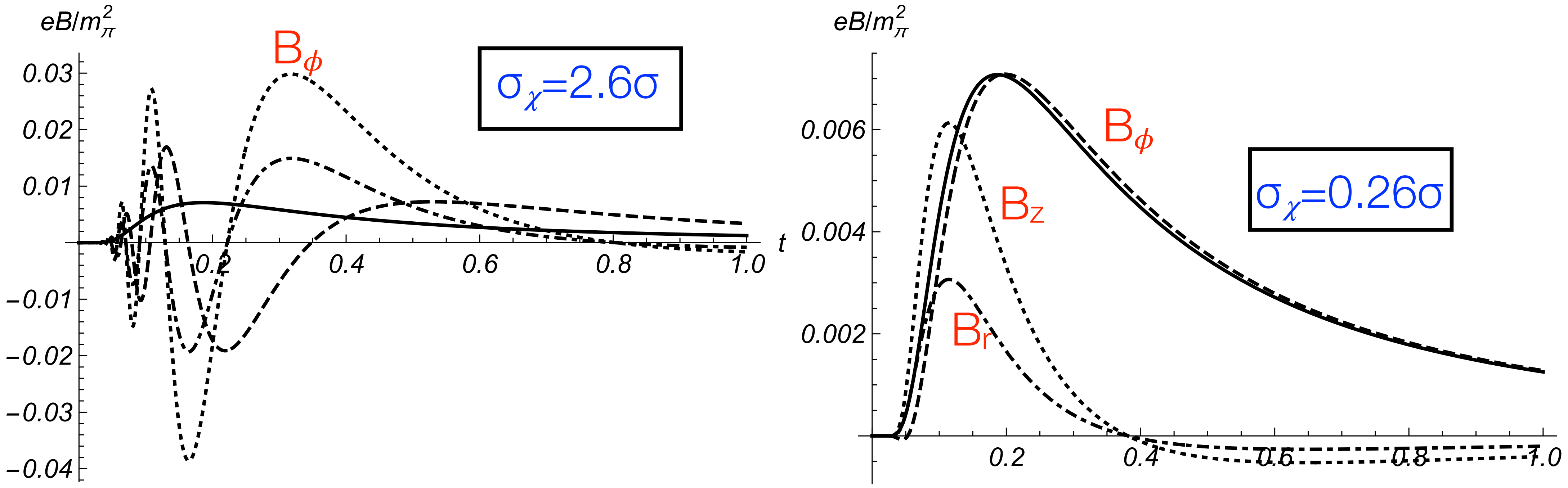
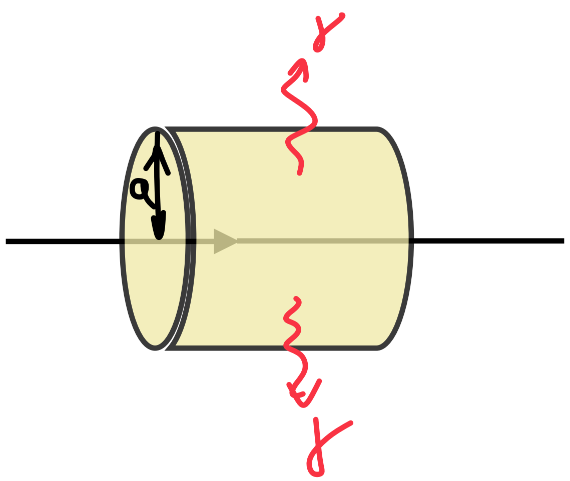


FIG. 2: Magnetic field of a point charge as a function of time t at $z = 0$. (Free space contribution is not shown). Electrical conductivity $\sigma = 5.8$ MeV. Solid line on both panels corresponds to $B = B_\phi$ at $\sigma_\chi = 0$. Broken lines correspond to B_ϕ (dashed), B_r (dashed-dotted) and B_z (dotted) with $\sigma_\chi = 15$ MeV on the left panel and $\sigma_\chi = 1.5$ MeV on the right panel. Note that the vertical scale on the two panels is different.

FERMI'S MODEL WITH ANOMALOUS CURRENT

Hansen, KT (2021)

$$-\frac{d\varepsilon}{dz} = 2\pi a \int_{-\infty}^{\infty} (E_\phi B_z - E_z B_\phi) dt = 2a \,\mathrm{Re} \int_0^\infty (E_{\phi\omega} B^*_{z\omega} - E_{z\omega} B^*_{\phi\omega}) d\omega$$



For simplicity consider $\omega_0 = 0$

UR limit $\gamma \gg 1$ at $a \to 0$ gives energy loss

$$-\frac{d\varepsilon}{dz} = \frac{q^2}{4\pi v^2}\left(\omega_p^2 \ln \frac{v}{a\omega_p} + \frac{1}{4}\gamma^2 \sigma_\chi^2\right)$$

increases as (energy)^2 due to anomaly

Chiral Cherenkov radiation emerges at $a \to \infty$ even if $\epsilon = 1$

$$\frac{dW}{d\omega} = -\frac{d\varepsilon}{dz\omega d\omega}\bigg|_{a\to\infty} = \frac{q^2}{4\pi}\left\{\frac{1}{2}\left(1 - \frac{1}{v^2}\right) + \frac{\sigma_\chi}{2\omega} + \frac{(1+v^2)\sigma_\chi^2}{8v^2\omega^2} + \ldots\right\}, \quad \omega < \sigma_\chi \gamma^2$$

Power of chiral Cherenkov radiation $P = \frac{q^2}{4\pi}\frac{\sigma_\chi^2 \gamma^2}{4}$

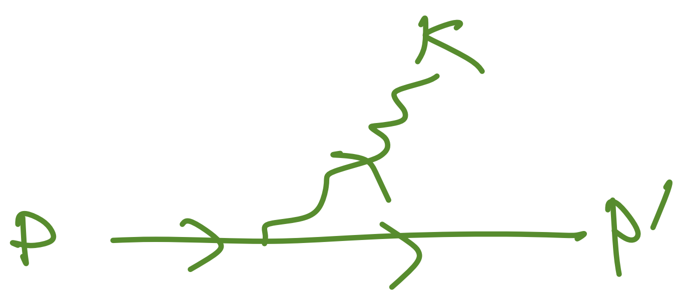
IN THE UR LIMIT, ENERGY LOSS IS DUE TO THE CHIRAL CHERENKOV RADIATION

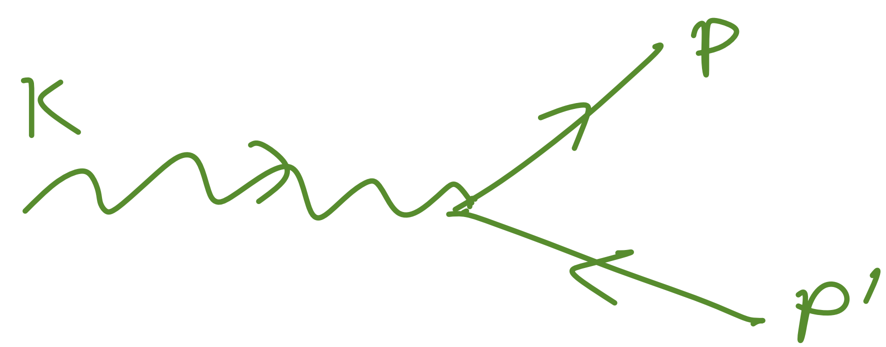
QFT CALCULATION: 1→2 PROCESSES

In radiation gauge: $\nabla^2 \boldsymbol{A} = \partial_t^2 \boldsymbol{A} - \sigma_\chi \boldsymbol{\nabla} \times \boldsymbol{A}$

The dispersion relation $k^2 = -\lambda \sigma_\chi |\boldsymbol{k}|$ → photon becomes space- or timelike

λ =helicity





$k^2 = (p \pm p')^2 = 2m(m \pm \varepsilon)$ forbidden in vacuum, but allowed in chiral medium

Pair production: $k^2 > 0 \Rightarrow \lambda \sigma_\chi < 0$

Photon radiation: $k^2 < 0 \Rightarrow \lambda \sigma_\chi > 0$

UR approx.: $$\boldsymbol{A} = \frac{1}{\sqrt{2\omega V}} \boldsymbol{\epsilon}_\lambda \, e^{i\omega z + i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp - i\omega t} \exp\left\{-i \frac{1}{2\omega} \int_0^z \left[k_\perp^2 - \underbrace{\sigma_\chi(z')\omega\lambda}_{\text{“}m_\gamma^2\text{”}}\right] dz'\right\}$$

QUANTUM CHERENKOV RADIATION

KT, PLB 786 (2018)

$$\mathcal{M} = -eQ\bar{u}(p')\gamma^\mu u(p)\epsilon^*_\mu \times 4\pi\varepsilon x(1-x)\delta(q_\perp^2 + \kappa_\lambda)$$

$$x = \frac{\omega}{\varepsilon}$$

$$\kappa_\lambda(z) = x^2 m^2 - (1-x)x\lambda\sigma_\chi\varepsilon$$

can become negative!

Chiral Cherenkov effect: photon radiation at $\vartheta \sim \sqrt{|\sigma_\chi|/\omega}$

Kappa is negative if $\lambda\sigma_\chi > 0$ and $x < x_0 = \dfrac{1}{1+m^2/(\lambda\sigma_\chi\varepsilon)}$

Photon radiation rate

$$\frac{dW_+}{dx} = \frac{\alpha Q^2}{2\varepsilon x}\left\{\sigma_\chi\varepsilon\left(\frac{x^2}{2} - x + 1\right) - m^2 x\right\}\theta(x_0 - x)$$

$$\frac{dW_-}{dx} = 0\,.$$

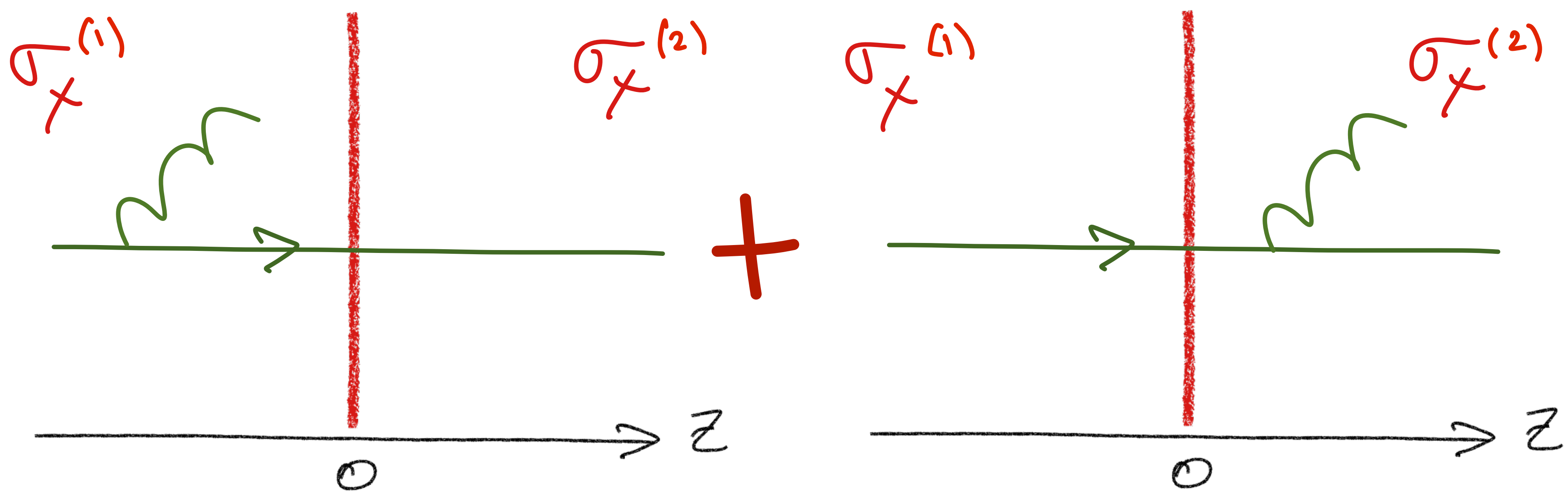
Vanishes as $\hbar \to 0$

Quantum anomaly!

Total rate of energy loss $-\dfrac{d\varepsilon}{dz} = \displaystyle\int_0^1 \frac{dW_+}{dx} x\varepsilon dx = \frac{1}{3}\alpha Q^2 \sigma_\chi \varepsilon$

Thus the recoil reduces $\gamma^2 \to \gamma$

TRANSITION RADIATION



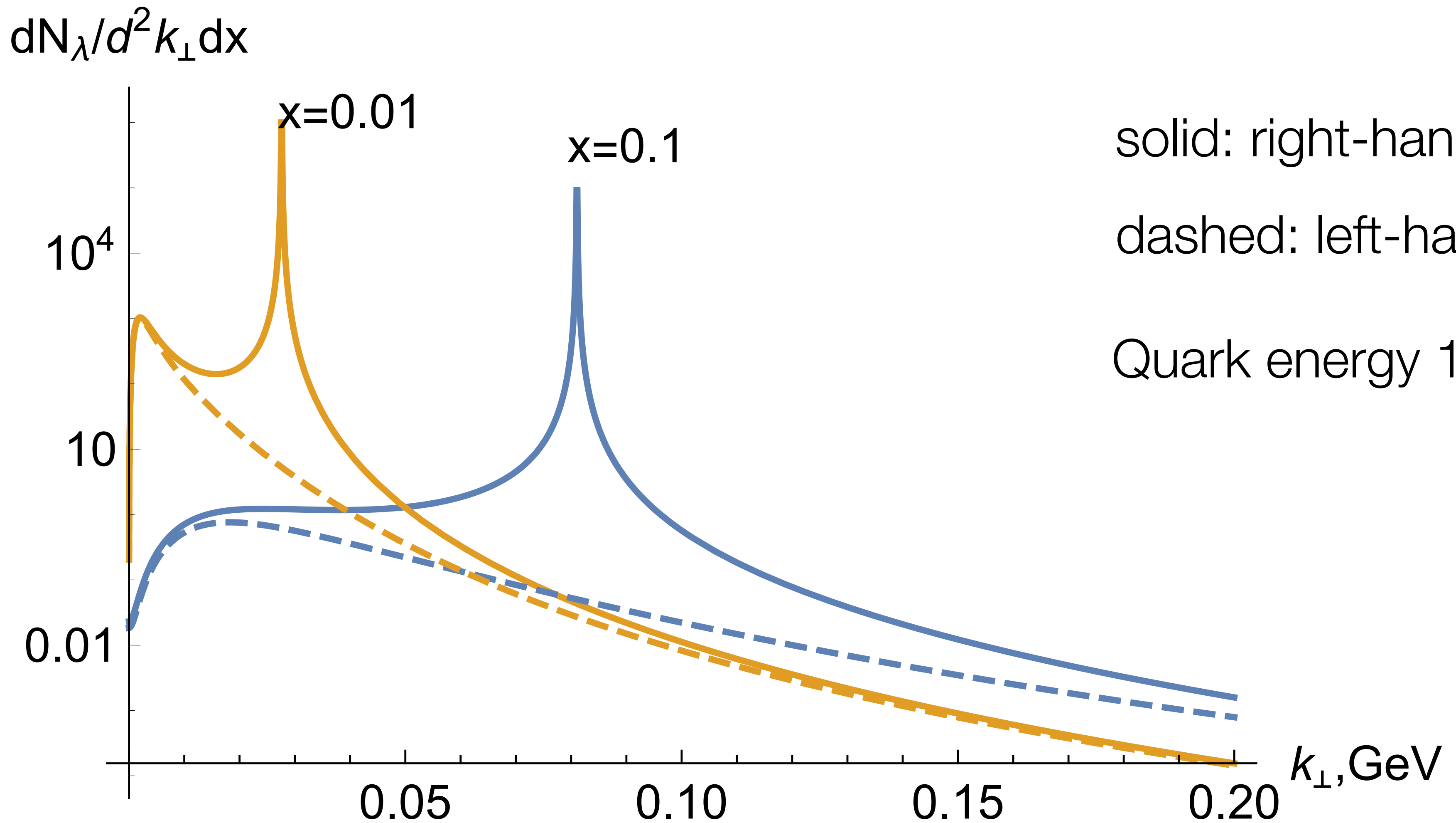
$$\frac{dN}{d^2q_\perp dx} = \frac{\alpha Q^2}{2\pi^2 x}\left\{\left(\frac{x^2}{2} - x + 1\right) q_\perp^2 + \frac{x^4 m^2}{2}\right\}\sum_\lambda \left|\frac{1}{q_\perp^2 + \kappa_\lambda^{(1)} - i\delta} - \frac{1}{q_\perp^2 + \kappa_\lambda^{(2)} + i\delta}\right|^2$$

(Transition radiation in ordinary materials corresponds to $\kappa_{\rm tr} = m^2x^2 + m_\gamma^2(1-x)$ finite at $\hbar\to 0$)

Contribution of the pole at $q_\perp^2 + \kappa_\lambda = 0$ is the chiral Cherenkov radiation.

The rest is the "chiral transition radiation"

CHERENKOV + TRANSITION RADIATION IN QGP



$k_\perp \approx \theta\omega$

- Charged particles traveling through the chiral medium emit electromagnetic radiation sensitive to the chiral anomaly.
- It is circularly polarized and has resonant peaks at angles proportional to the anomaly

APPLICATIONS: WEYL SEMIMETAL

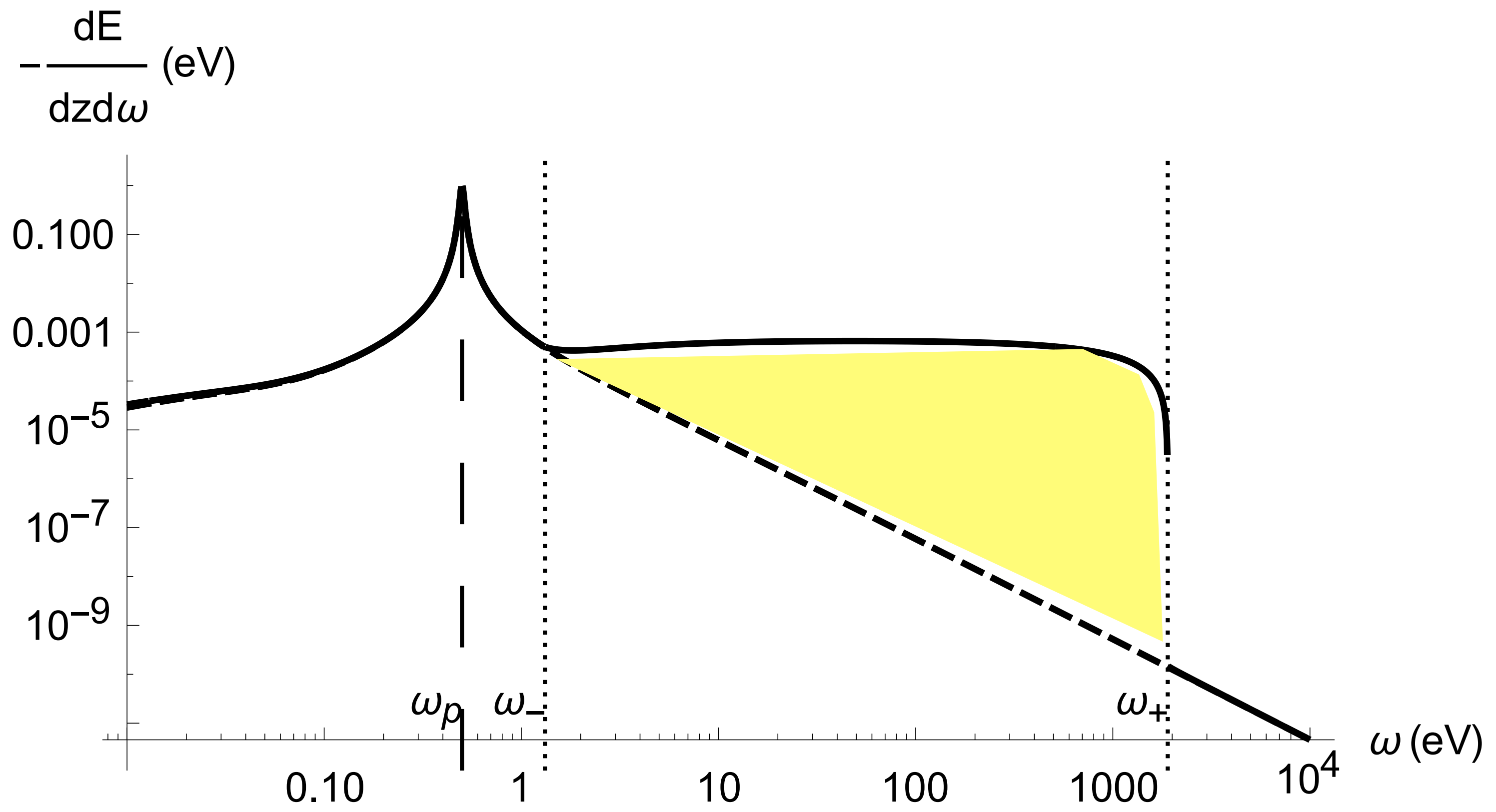
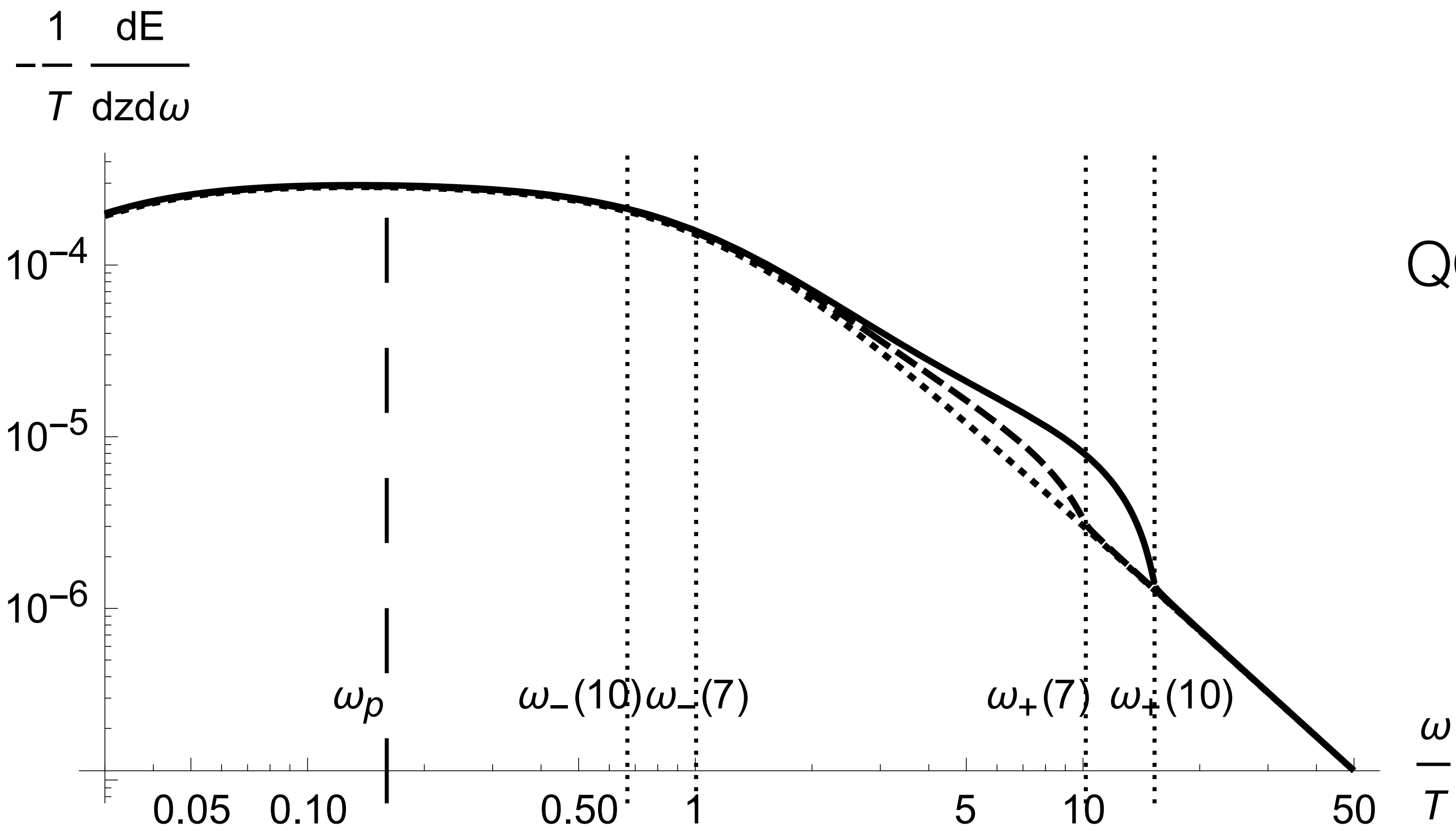


FIG. 2. Collisional energy loss spectrum of electron with $\gamma = 100$ in a semimetal with parameters $\omega_p =$

Very small recoil $\omega_M \lesssim \sigma_\chi \gamma^2 \ll \varepsilon$

Neglecting coherence effects: $\frac{\Delta\varepsilon^{\chi C}}{\Delta\varepsilon^{BH}} \sim \frac{\sigma_\chi}{e^2 T} \sim \frac{\mu_5}{T}$ >>1 in a TaAs at room temp.

APPLICATIONS: QGP



QCD/QED: $\times \frac{\alpha_s^2}{\alpha_{\text{em}}^2}$

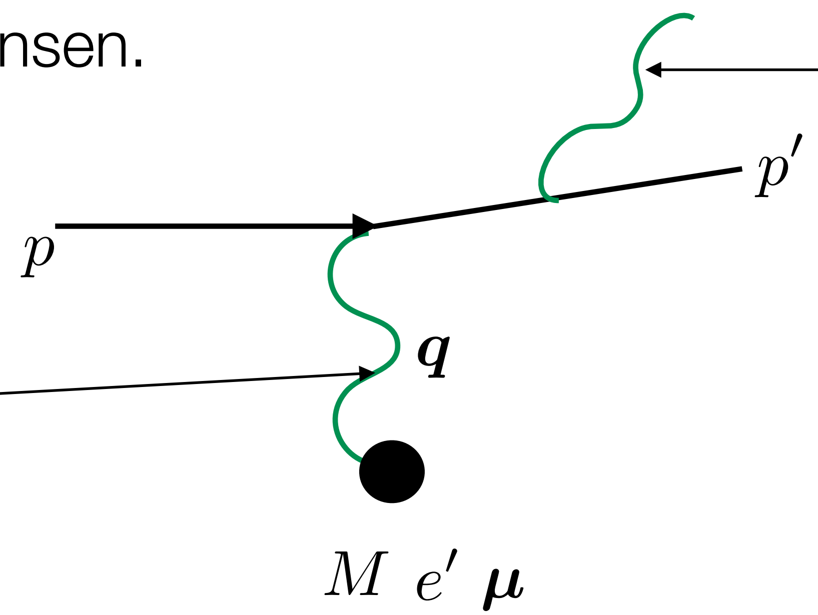
FIG. 1. Electromagnetic part of the collisional energy loss spectrum of a d-quark with $\gamma = 20$ in Quark-Gluon Plasma. Plasma parameters: $\omega_p = 0.16T$, $\Gamma = 1.11T$ [36], $m = T = 250$ MeV. Solid line: $\sigma_\chi = 10$ MeV, dashed line: $\sigma_\chi = 7$ MeV, dotted line: $\sigma_\chi = 0$. $\omega_\pm$ are defined in (13).

The same qualitative picture in QCD (after e→g, including color factors etc.)

$$-\frac{d\varepsilon}{dz}\bigg|_{\text{anom}} = \frac{g^2 C_F}{4\pi}\frac{\tilde{\sigma}_\chi \varepsilon}{3}$$

CHIRAL CONTRIBUTIONS TO BETHE-HEITLER

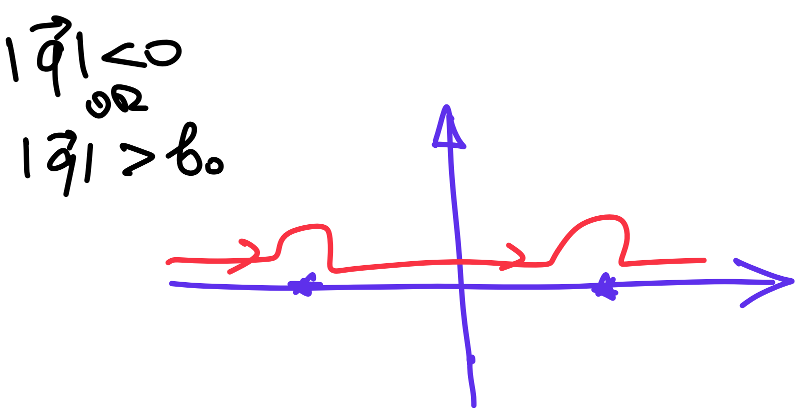
Work in progress with J. Hansen.

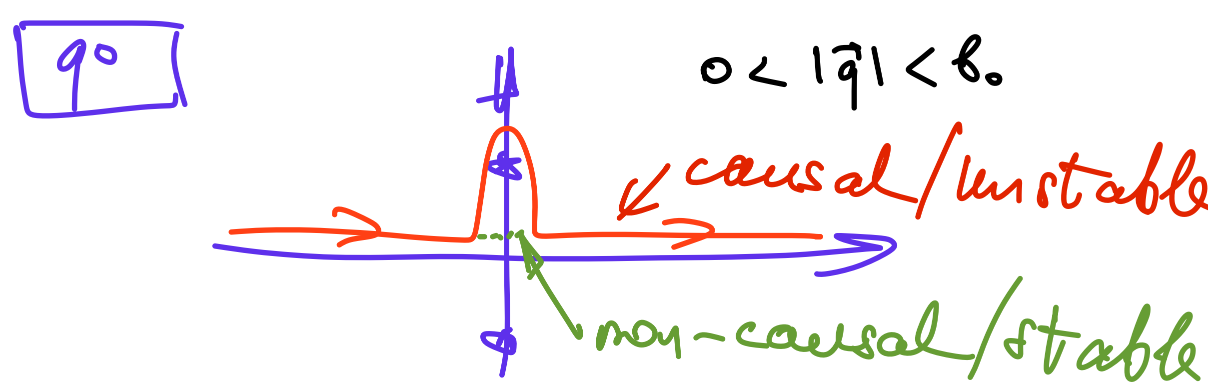


Photon propagator $$D_{\mu\nu}(q) = -i\frac{q^2 g_{\mu\nu} + i\epsilon_{\mu\nu\rho\sigma}b^\rho q^\sigma + b_\mu b_\nu}{q^4 + b^2 q^2 - (b\cdot q)^2} \qquad b^\mu = (\sigma_\chi, \mathbf{0})$$

Lenhert, Potting
Qiu, Cao, Huang

Poles: $(q^0)^2 - \boldsymbol{q}^2 = -\lambda b_0 |\boldsymbol{q}|$ Modes $|\boldsymbol{q}| \leq b_0$ are unstable



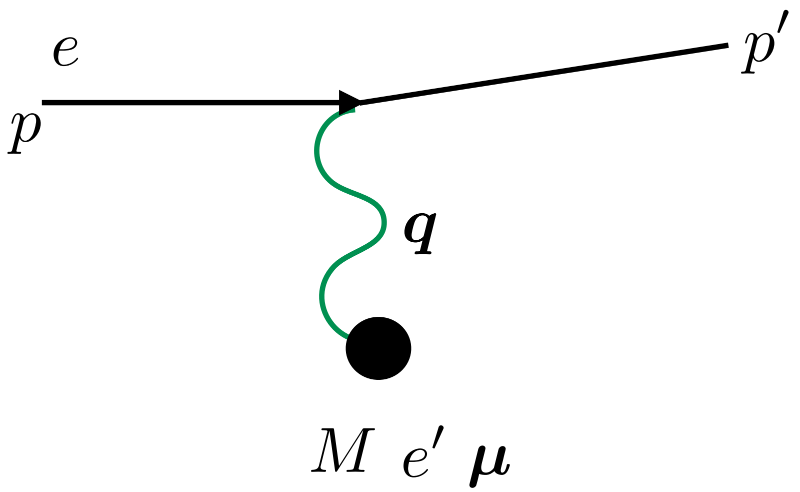


Carroll, Field, Jackiw

At $q^0 \to 0$ there's only one unstable mode $|\boldsymbol{q}| = b_0$

CONTRIBUTION OF ANOMALY TO TRANSPORT

elastic scattering on a heavy "ion"

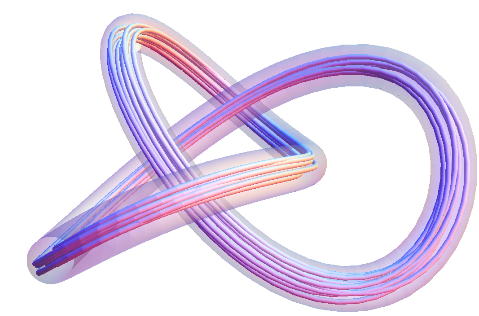


Static limit q_0=0 ($q^0 \ll |\boldsymbol{q}| \;\; q^0 \ll b_0$)

$$D_{00}(\boldsymbol{q}) = \frac{i}{\boldsymbol{q}^2}\,,$$

$$D_{0i}(\boldsymbol{q}) = D_{0i}(\boldsymbol{q}) = 0\,,$$

$$D_{ij}(\boldsymbol{q}) = -\frac{i\delta_{ij}}{\boldsymbol{q}^2 - b_0^2} - \frac{\epsilon_{ijk}q^k}{b_0(\boldsymbol{q}^2 - b_0^2)} + \frac{\epsilon_{ijk}q^k}{b_0\boldsymbol{q}^2}$$



Current of an ion with charge e' and magnetic moment $\boldsymbol{\mu}$:

$$J^0(\boldsymbol{x}) = e'\delta(\boldsymbol{x})\,, \qquad \boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{\nabla} \times (\boldsymbol{\mu}\delta(\boldsymbol{x}))$$

Produces the potentials $A^0(\boldsymbol{q}) = e'/\boldsymbol{q}^2 \qquad A^\ell(\boldsymbol{q}) = -\frac{1}{\boldsymbol{q}^2 - b_0^2}\left[i(\boldsymbol{\mu}\times\boldsymbol{q})^\ell + \frac{b_0}{\boldsymbol{q}^2}(\boldsymbol{\mu}\cdot\boldsymbol{q}q^\ell - \boldsymbol{q}^2\mu^\ell)\right]$

Cross section averaged over the magnetic moment directions:

$$\left\langle \frac{d\sigma}{d\Omega'} \right\rangle = \frac{e^2}{8\pi^2}\left\{ \frac{2E^2e'^2}{\boldsymbol{q}^4}\left(1 - \frac{\boldsymbol{q}^2}{4E^2}\right) + \frac{2\mu^2}{3(\boldsymbol{q}^2 - b_0^2)^2}\left(1 + \frac{b_0^2}{\boldsymbol{q}^2}\right)\left[(\boldsymbol{p}\times\boldsymbol{q})^2 + \frac{\boldsymbol{q}^4}{2}\right]\right\}$$

Coulomb **Anomaly**

SPIN AVERAGE CROSS-SECTION

KT, PLB 808 (2020)

Transport cross section $\sigma_T = \frac{e^2}{16\pi \boldsymbol{p}^4}\left(4E^2 e'^2 L + \frac{2\mu^2}{3} 4\boldsymbol{p}^4 \mathcal{I}\right)$

Coulomb **Anomaly**

$$\mathcal{I} = 1 + \frac{1}{\epsilon}\left[2a(1+a) - \epsilon^2\right]\left(\arctan\frac{1-a}{\epsilon} + \arctan\frac{a}{\epsilon}\right) + \frac{1}{2}(1+3a)\ln\left(1 + \frac{1-2a}{a^2+\epsilon^2}\right)$$

$$a = \frac{b_0^2}{4\boldsymbol{p}^2}\,, \qquad \epsilon = \frac{\Gamma^2}{4\boldsymbol{p}^2}$$

Γ is width $\frac{\boldsymbol{q}^2}{(\boldsymbol{q}^2 - b_0^2)^2 + \Gamma^4}$

due to processes that tame the instability

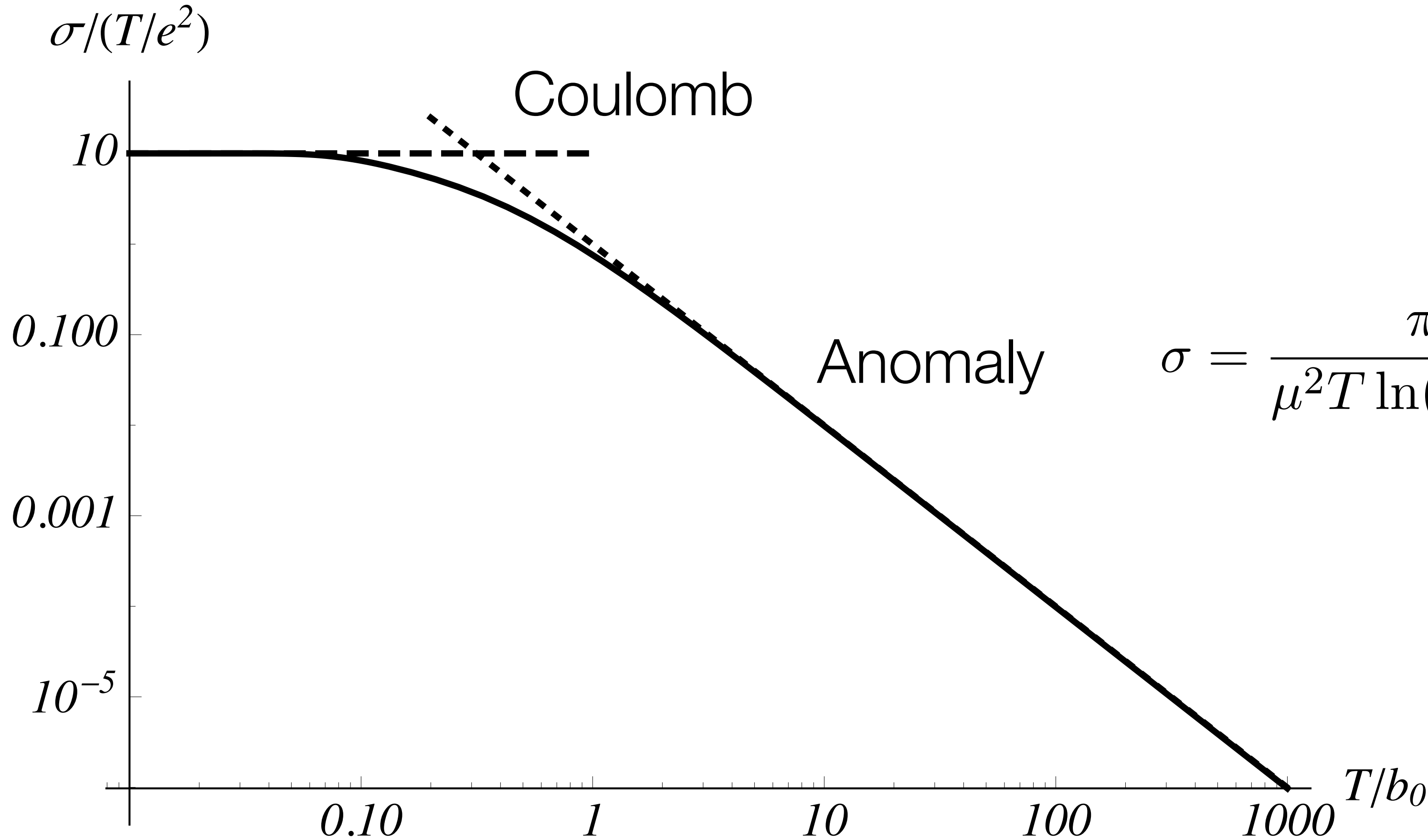
At large momenta $\sigma_T \approx \frac{e^2\mu^2}{6\pi}\ln\frac{4\boldsymbol{p}^2}{b_0^2}$ $\Rightarrow$ anomaly dominates Coulomb

Large σ_T $\Rightarrow$ small m.f.p. $\Rightarrow$ suppression of transport coefficients

ANOMALOUS CONTRIBUTION TO CONDUCTIVITY

Electrical conductivity $\sigma = \frac{e^2}{3T} \int f_0 \frac{1}{n\sigma_T} d^3p$

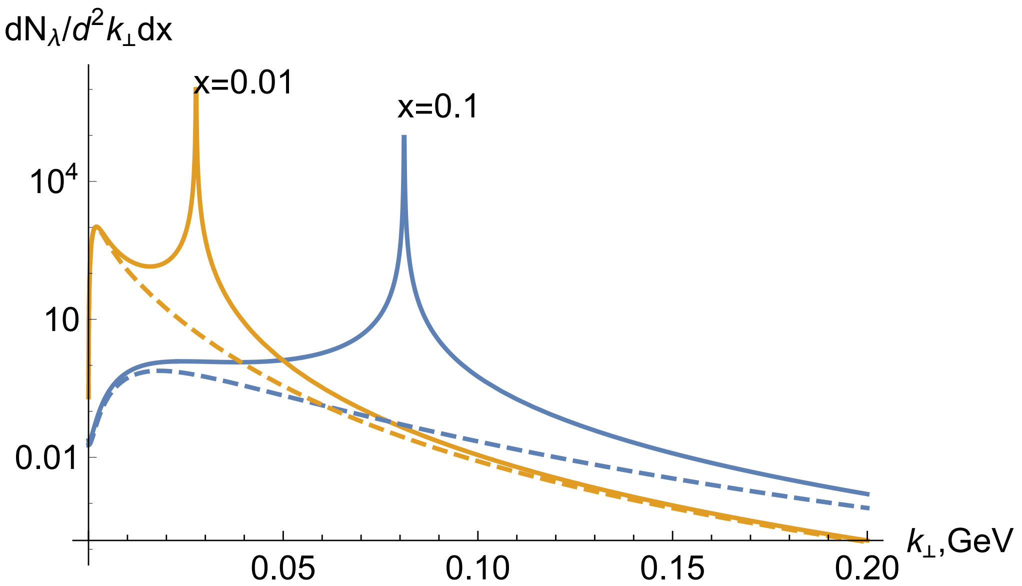
(anomaly contribution to f_0 is neglected for simplicity)



$$\sigma = \frac{\pi}{\mu^2 T \ln(T/b_0)}$$

SUMMARY

- Radiation by a fast particle is a powerful tool to study the properties of chiral media such as the quark-gluon plasma, Weyl semimetals, axion dark matter, primordial magnetic fields etc.



- Great opportunity for ambitious experimentalists.