Generalized Balance Functions

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S.P. PRL (2012), arXiv 1304.2442

QGP Chemistry Basics

- 52 ~massless degrees of freedom
- Strongly interacting
- Conserved: up-ness, down-ness, strangeness, color
  Not conserved: quark number
- Lattice measures charge fluctuations:

$$\boxed{\chi_{ab} \equiv \langle Q_a Q_b \rangle / V}$$

Parton gas:

$$\chi_{ab}^{\mathrm{QGP}} = (n_a + n_{\bar{a}})\delta_{ab}$$

a,b = uds

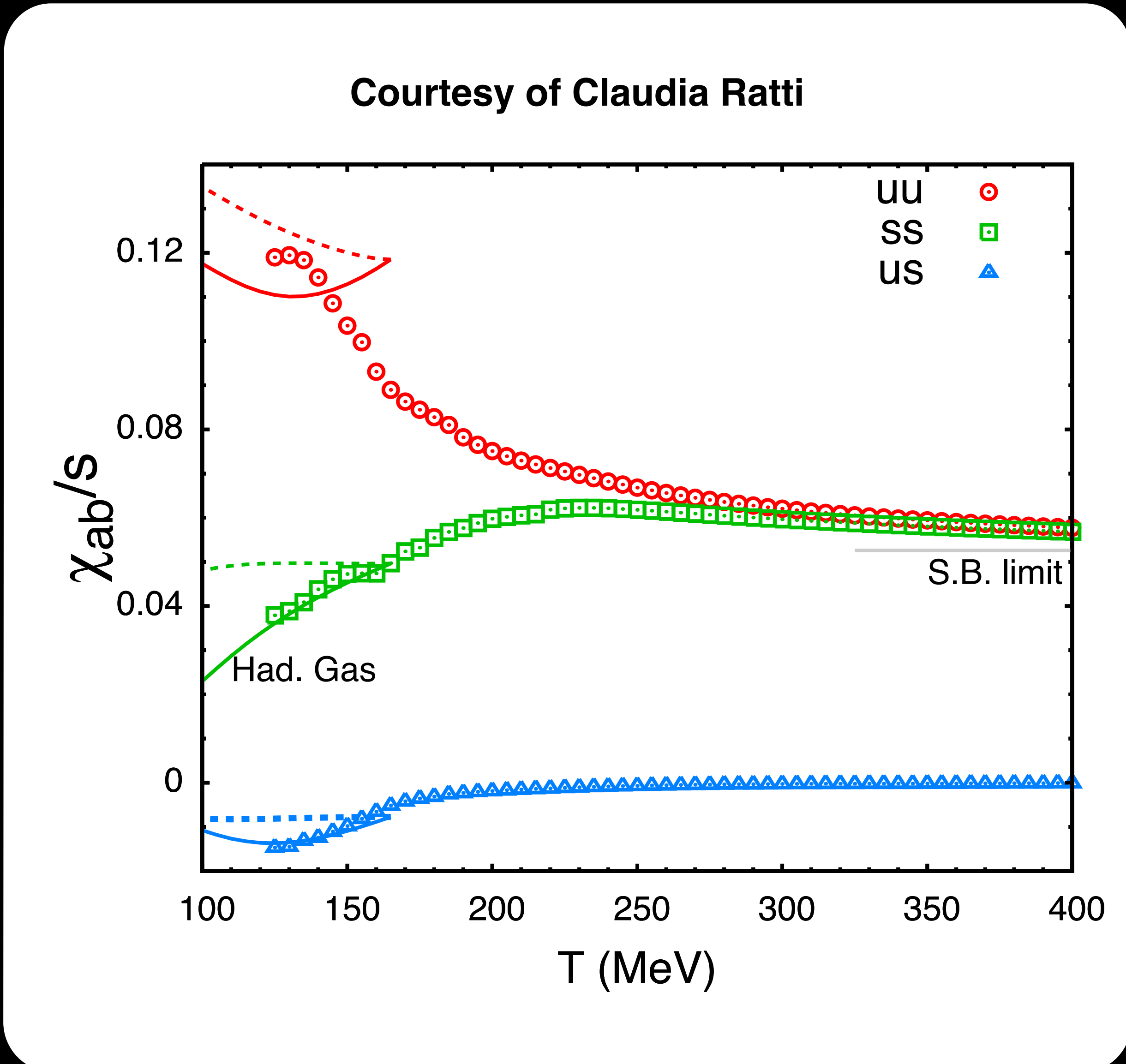
Hadron gas:

$$\chi_{ab}^{HAD} = \sum_{\alpha} n_\alpha q_{\alpha,a} q_{\alpha,b}$$

$\alpha = \pi^+, \pi^-, \pi^0, K^+ \ldots$

Lattice Charge Fluctuations

scaled by entropy



Parton gas:

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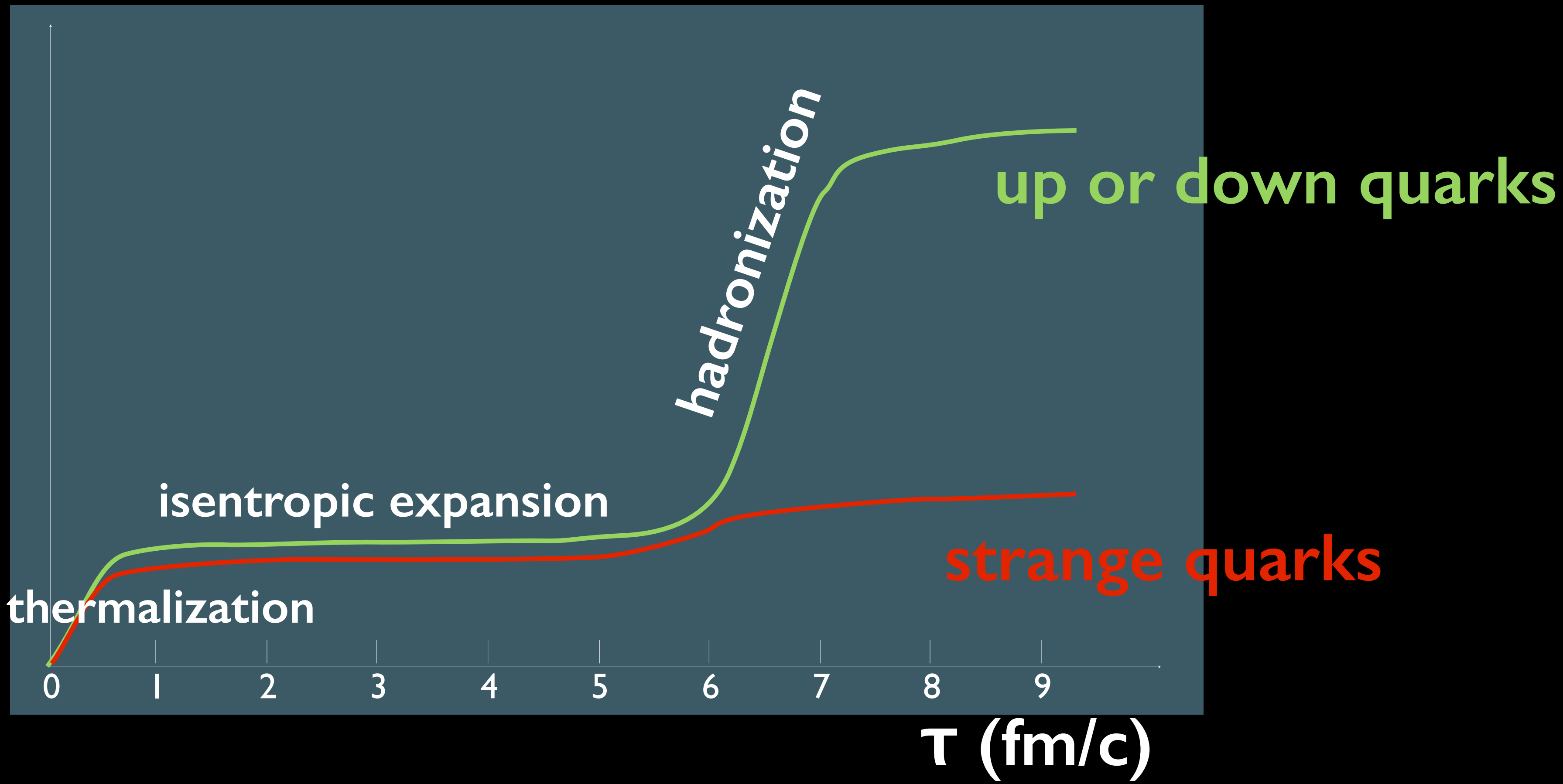
$\alpha=\pi^+,\pi^-,\pi^0,K^+\ldots$

off-diagonal elements (V.Koch, PRL 2005)

Two waves of quark production



Problems with Comparing Experiment to Lattice

1.Lattice = Grand Canonical (Particle Bath)
Experiment = Canonical (net charge = 0)

2. Charge created at hadronization

3. One measures hadrons -- not uds

4. One measures momenta, not positions

Consider charge correlations
$g_{ab}(\Delta\eta)$ is a 3x3 matrix

$$g_{ud}(\Delta\eta) \equiv \langle Q_u(\eta) Q_d(\eta+\Delta\eta) \rangle$$
$$= \left\langle \left[n_u(\eta) - n_{\bar{u}}(\eta) \right] \left[n_d(\eta+\Delta\eta) - n_{\bar{d}}(\eta+\delta\eta) \right] \right\rangle$$

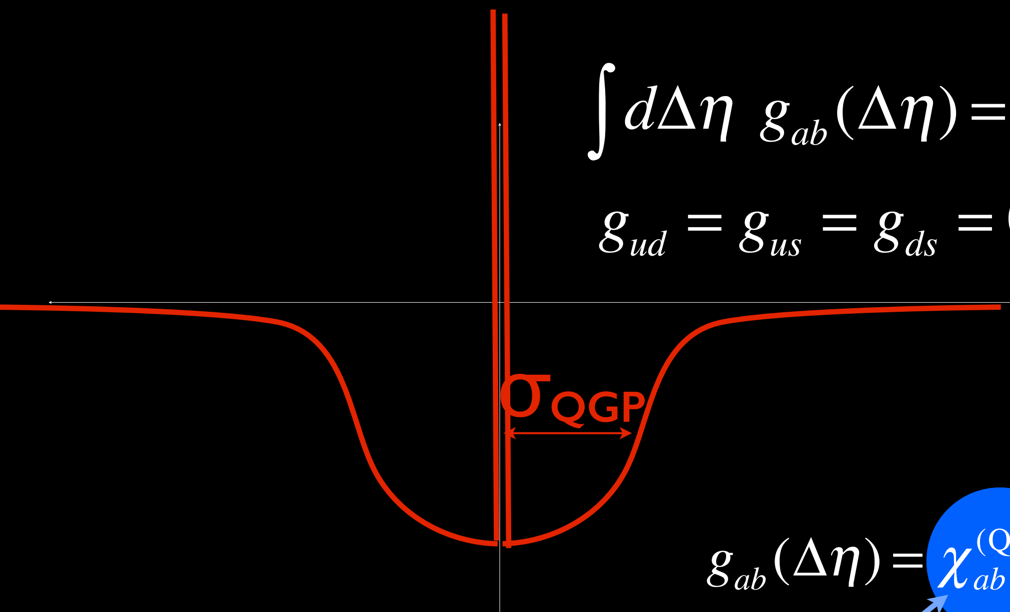
charge conservation

$$\int d\Delta\eta \;\; g_{ab}(\Delta\eta) = 0$$

1. Before hadronization

$$\int d\Delta\eta \;\; g_{ab}(\Delta\eta) = 0$$

$$g_{ud} = g_{us} = g_{ds} = 0$$



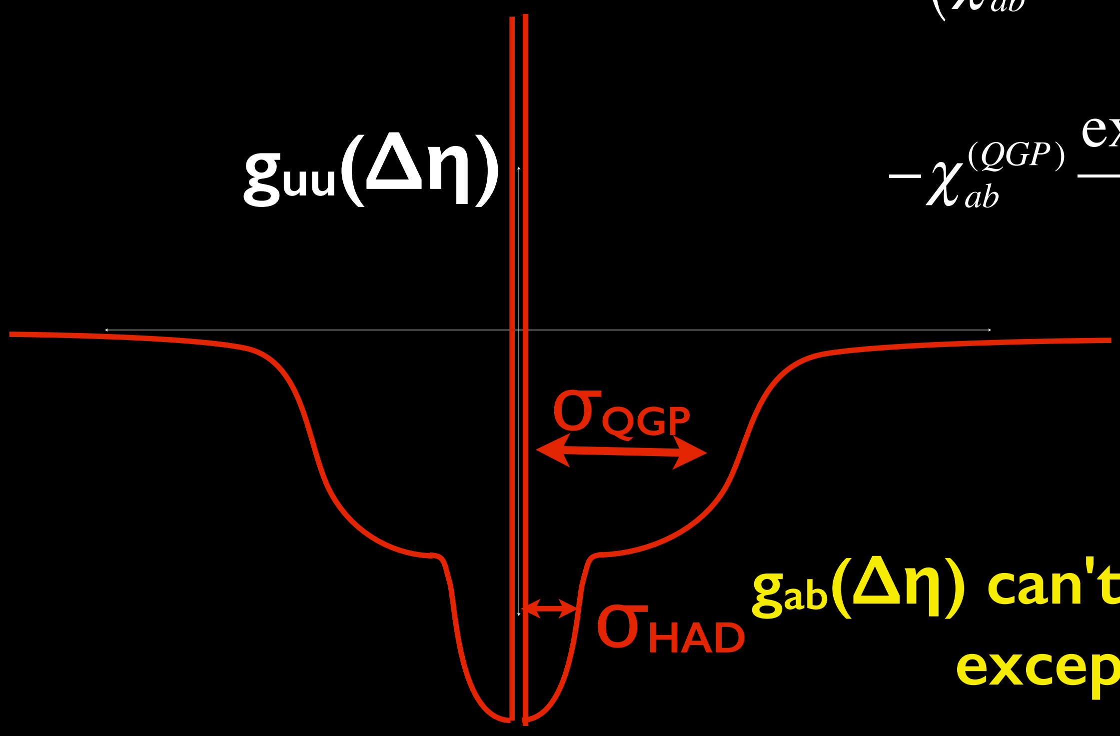
only extra parameter

$$g_{ab}(\Delta\eta) = \chi_{ab}^{(\mathrm{QGP})} \left\{ \delta(\Delta\eta) - \frac{\exp(-\Delta\eta^2 / 2\sigma^2_{(\mathrm{QGP})})}{(2\pi\sigma^2_{(\mathrm{QGP})})^{1/2}} \right\}$$

From lattice!

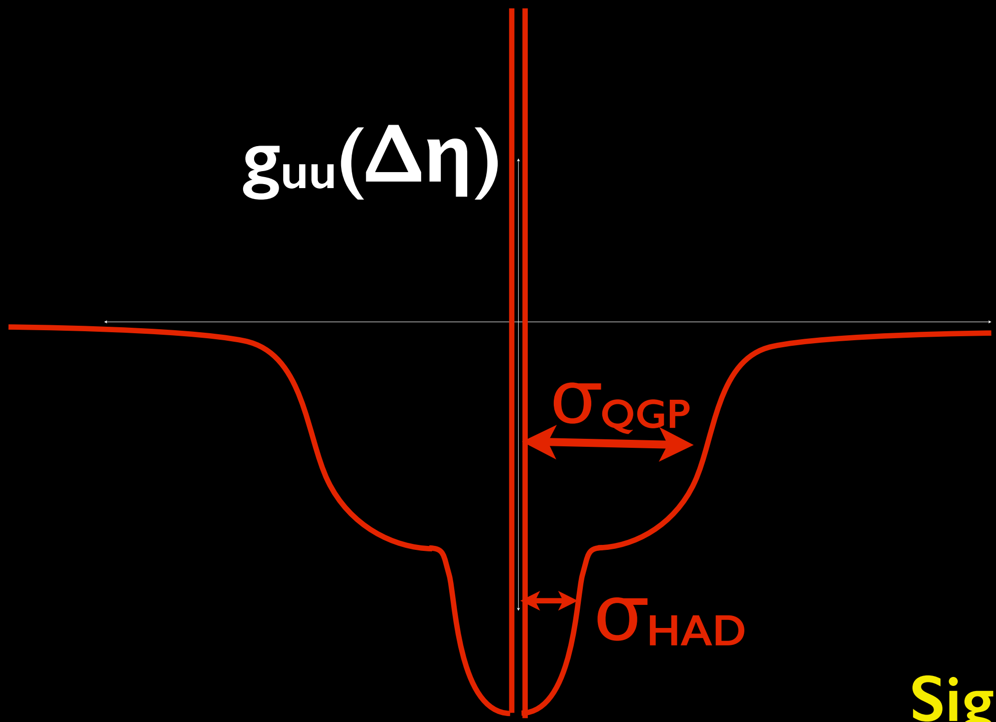
2. Just after hadronization

$$g_{ab}(\Delta\eta) = \chi_{ab}^{(HAD)}\delta(\Delta\eta) - \left(\chi_{ab}^{(HAD)} - \chi_{ab}^{(QGP)}\right)\frac{\exp(-\Delta\eta^2/2\sigma_{(HAD)}^2)}{(2\pi\sigma_{(HAD)}^2)^{1/2}} - \chi_{ab}^{(QGP)}\frac{\exp(-\Delta\eta^2/2\sigma_{(\mathrm{QGP})}^2)}{(2\pi\sigma_{(\mathrm{QGP})}^2)^{1/2}}$$



$g_{ab}(\Delta\eta)$ can't change suddenly except at $\Delta\eta=0$

2nd Bump: Positive or Negative? Crude Expectations



$$g_{ab}(\Delta\eta) = \chi_{ab}^{(HAD)}\delta(\Delta\eta)$$
$$-\left(\chi_{ab}^{(HAD)} - \chi_{ab}^{(QGP)}\right)\frac{\exp(-\Delta\eta^2 / 2\sigma_{(HAD)}^2)}{(2\pi\sigma_{(HAD)}^2)^{1/2}}$$
$$-\chi_{ab}^{(QGP)}\frac{\exp(-\Delta\eta^2 / 2\sigma_{(\mathrm{QGP})}^2)}{(2\pi\sigma_{(\mathrm{QGP})}^2)^{1/2}}$$

Sign depends on $\chi_{ab}^{(HAD)} - \chi_{ab}^{(QGP)}$

Positive or negative (electric charge)

$$\chi_{\text{electric}}^{(HAD)} = n_{\text{charged}}\,,\ \chi_{\text{electric}}^{(QGP)} = \frac{4}{9}n_u + \frac{1}{9}n_d + \frac{1}{9}n_s$$

Narrower bump is same sign, but much taller than wider bump

$\pi^+\pi^-$ BF dominated by narrow peak

Before and after (STRANGENESS)

$$\chi^{(HAD)}_{\text{strange}} = n_K + n_\Lambda + 4n_\Xi + 9n_\Omega, \ \chi^{(QGP)}_{\text{strange}} = n_s$$

Smaller bump is very small

K^+K^- BF dominated by broad peak

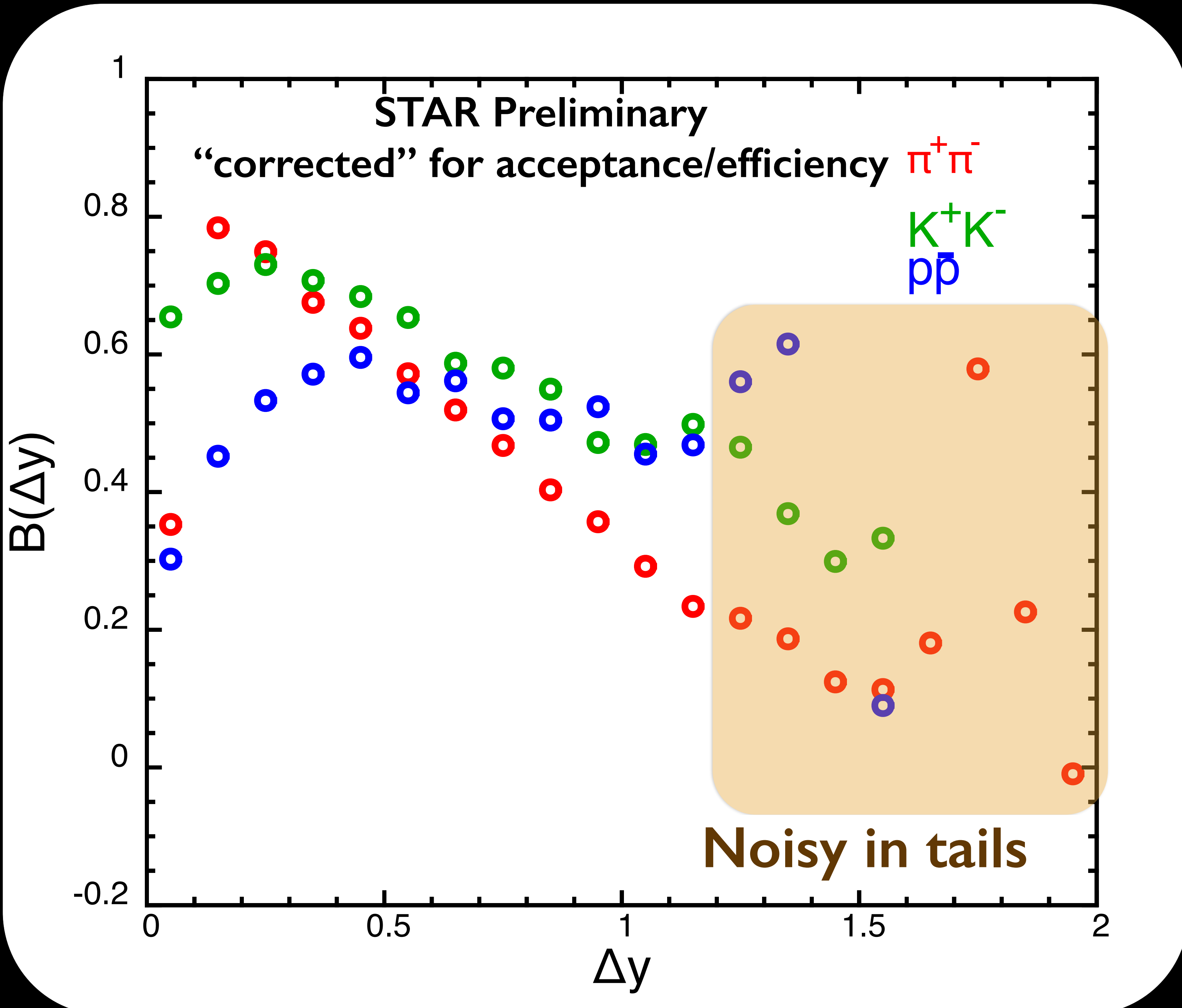
Before and after (BARYON No.)

$$\chi_{\text{baryon}}^{(HAD)} = n_p + n_n + n_\Lambda + n_\Xi + n_\Omega,\ \chi_{\text{baryon}}^{(QGP)} = \frac{1}{9}(n_u + n_d + n_s)$$

Smaller Bump is smaller with opposite sign

p-pbar BF dominated by broad peak, but has dip

STAR results verify this qualitatively



Back to being more exact

Take into account:

- hadrons have multiple charges,
- hyperon decays
- χ has off-diagonal elements…

2. Just after hadronization (uds basis)

$$-g'_{ab}(\Delta\eta) = \chi_{ab}^{(QGP)} \frac{e^{-\Delta\eta^2/2\sigma^2_{(QGP)}}}{\sqrt{2\pi\sigma^2_{(QGP)}}} + (\chi_{ab}^{(HAD)} - \chi_{ab}^{(QGP)}) \frac{e^{-\Delta\eta^2/2\sigma^2_{(HAD)}}}{\sqrt{2\pi\sigma^2_{(HAD)}}}$$

$$\chi_{ab}^{(HAD)} \equiv \sum_{\alpha\in\text{hadrons}} n_\alpha q_{\alpha,a} q_{\alpha,b}$$

$$\chi_{ab}^{(QGP)} \equiv 2n_a \delta_{ab}$$

3. But, we measure $G_{\alpha\beta}$ not g_{ab}!!!

$\alpha,\beta=\pi,p,K\ldots a,b=u,d,s$

$$G_{\alpha\beta}(\Delta\eta) \equiv \langle [n_\alpha - n_{\bar{\alpha}}][n_\beta - n_{\bar{\beta}}] \rangle$$

$$e.g., \quad G_{pK^-} = \langle [n_p - n_{\bar{p}}][n_{K^-} - n_{K^+}] \rangle$$

Generalized Balance Function

(aside from factor of $\langle n_\beta \rangle$)

Analogous problem...

Given $\delta\rho_a$ and n_α, find δn_α

Solution: assign chemical potential

$$\delta n_\alpha = \langle n_\alpha \rangle \left(e^{\mu_a q_{\alpha,a}/T} - 1 \right)$$

$$\delta\rho_a = \sum_\alpha \delta n_\alpha q_{\alpha,a} \approx \sum_{\alpha b} \langle n_\alpha \rangle q_{\alpha a} q_{\alpha b} \frac{\mu_b}{T} = \sum_b \chi_{ab}^{(\mathrm{had})} \frac{\mu_b}{T}$$

$$\frac{\mu_a}{T} = \sum_b (\chi^{-1})_{ab} \delta\rho_b$$

$$\delta n_\alpha = \langle n_\alpha \rangle \sum_b q_{\alpha a} (\chi^{-1})_{ab} \delta\rho_b$$

3. Apply to our problem

$$\left\langle \delta n_\alpha(0)\delta n_\beta(\Delta\eta)\right\rangle = \left\langle n_\alpha\right\rangle\left\langle n_\beta\right\rangle \sum_{abcd} q_{\alpha a}\,\chi_{ac}^{(HAD)-}\boxed{g_{cd}^{'}(\Delta\eta)}\chi_{db}^{(HAD)-1}\,q_{\beta b}$$

3. Putting this together

$$-G'_{\alpha\beta}(\Delta\eta) = w_{\alpha\beta}^{(\mathrm{QGP})} \frac{e^{-\Delta\eta^2/2\sigma^2_{(QGP)}}}{\sqrt{2\pi\sigma^2_{(QGP)}}} + w_{\alpha\beta}^{(\mathrm{HAD})} \frac{e^{-\Delta\eta^2/2\sigma^2_{(HAD)}}}{\sqrt{2\pi\sigma^2_{(HAD)}}}$$

$$w_{\alpha\beta}^{(\mathrm{QGP})} = -2\sum_{abcd} \langle n_\alpha \rangle q_{\alpha,a} \chi_{ab}^{-1(HAD)} \chi_{bc}^{(QGP)} \chi_{cd}^{-1(HAD)} \langle n_\beta \rangle q_{\beta,d}$$

$$w_{\alpha\beta}^{(HAD)} = -2\sum_{ab} \langle n_\alpha \rangle q_{\alpha,a} \chi_{ab}^{-1(HAD)} \langle n_\beta \rangle q_{\beta,b} - w_{\alpha\beta}^{(QGP)}$$

prefactors depend only on yields and χ_{ab} from lattice

3. Prefactors...

	p	Λ	Σ^+	Σ^-	Ξ^0	Ξ^-	Ω^-	π^+	K^+
$\bar{p}$	0.441,-0.066	0.485,-0.162	0.491,-0.146	0.479,-0.178	0.535,-0.242	0.529,-0.258	0.578,-0.338	0.006, 0.016	-0.044, 0.096
$\bar{\Lambda}$	0.183,-0.061	0.242,-0.094	0.242,-0.094	0.242,-0.094	0.302,-0.128	0.302,-0.128	0.361,-0.161	0.000,-0.000	-0.059, 0.033
$\bar{\Sigma}^-$	0.074,-0.022	0.097,-0.038	0.099,-0.033	0.095,-0.043	0.122,-0.049	0.120,-0.054	0.144,-0.064	0.002, 0.005	-0.023, 0.016
$\bar{\Sigma}^+$	0.072,-0.027	0.097,-0.038	0.095,-0.043	0.099,-0.033	0.120,-0.054	0.122,-0.049	0.144,-0.064	-0.002,-0.005	-0.025, 0.011
$\bar{\Xi}^0$	0.046,-0.021	0.069,-0.029	0.070,-0.028	0.069,-0.031	0.093,-0.036	0.092,-0.038	0.115,-0.045	0.001, 0.001	-0.023, 0.008
$\bar{\Xi}^+$	0.046,-0.022	0.069,-0.029	0.069,-0.031	0.070,-0.028	0.092,-0.038	0.093,-0.036	0.115,-0.045	-0.001,-0.001	-0.023, 0.007
$\bar{\Omega}^+$	0.009,-0.005	0.015,-0.007	0.015,-0.007	0.015,-0.007	0.021,-0.008	0.021,-0.008	0.027,-0.009	-0.000,-0.000	-0.006, 0.001
π^-	0.119, 0.318	0.000,-0.000	0.239, 0.636	-0.239,-0.636	0.119, 0.318	-0.119,-0.318	-0.000,-0.000	0.239, 0.636	0.119, 0.318
K^-	-0.175, 0.384	-0.627, 0.352	-0.603, 0.417	-0.651, 0.288	-1.055, 0.385	-1.079, 0.321	-1.507, 0.354	0.024, 0.064	0.452, 0.031

(QGP,HAD)

prefactors completely determined by χ_{QGP} and final-state hadronic yields
(hadron yields from thermal model with B-reduction)

4. Use blast-wave to go from
coordinate space η to momentum-space rapidity
(Monte Carlo + decays)
Use STAR parameters fit to spectra (T and $u_{\perp}$)

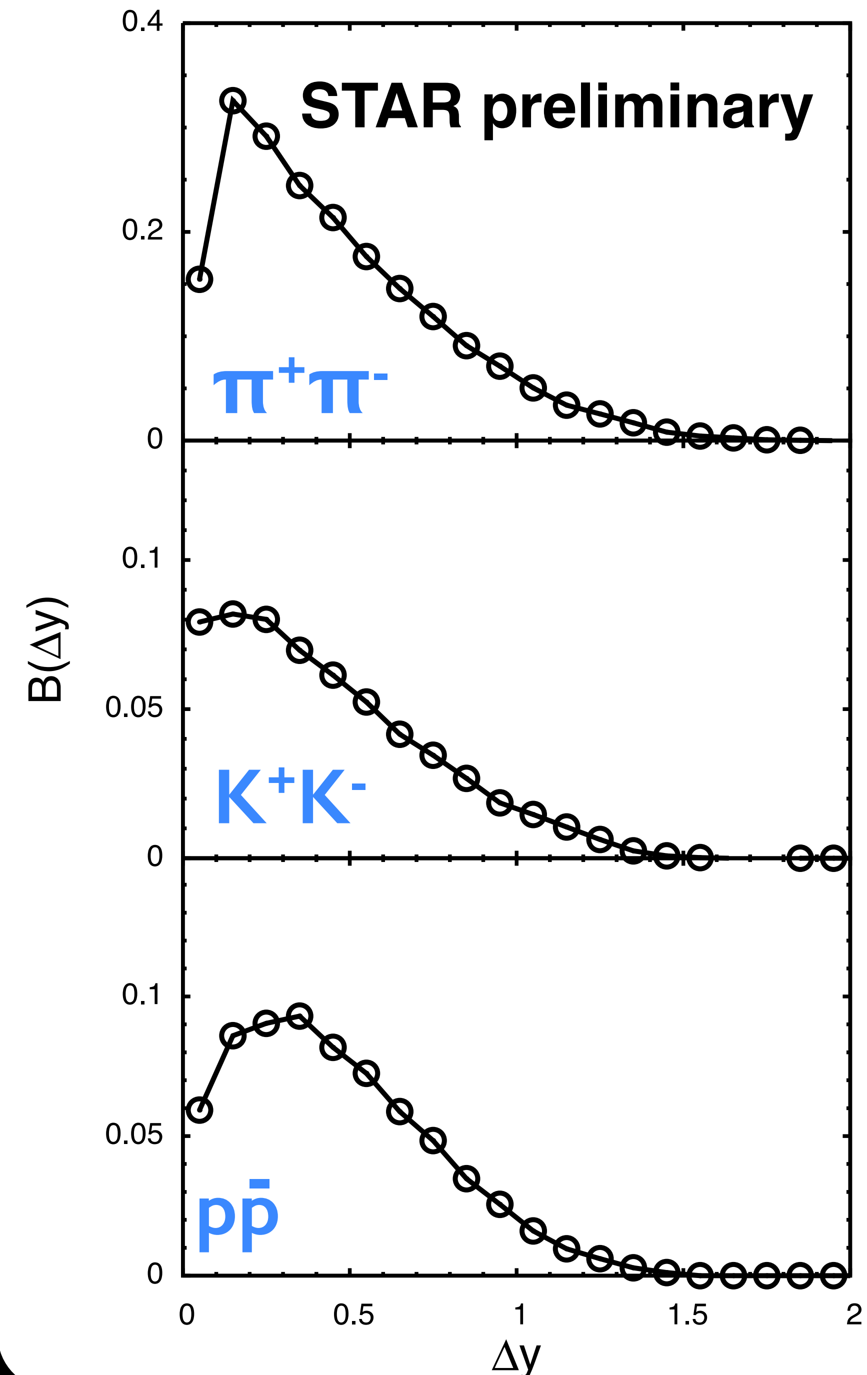
4 Parameters:

1. (quarks in QGP / hadrons in FS)
2. s/(ud) in QGP
3. σ_{qgp}
4. σ_{had}

from STAR (balance functions)

thesis of Hui Wang (2012)

Single source won't work
KK broader than ππ!
p$\bar{p}$ broader than both
-- and has dip !!



4 Parameter MCMC

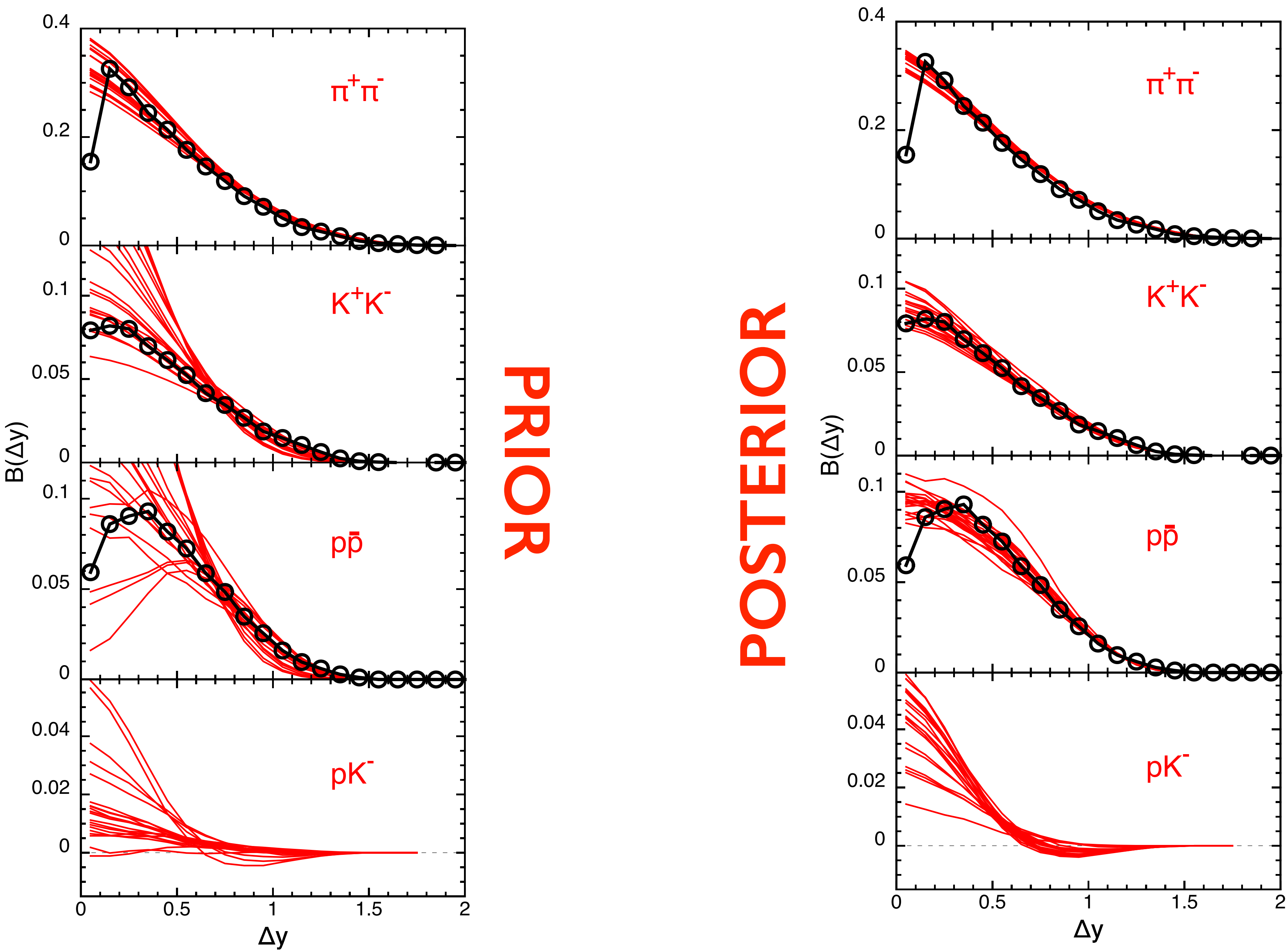
$0.3 < \sigma_{qgp} < 1.5$

$0 < \sigma_{had}/\sigma_{qgp} < 1$

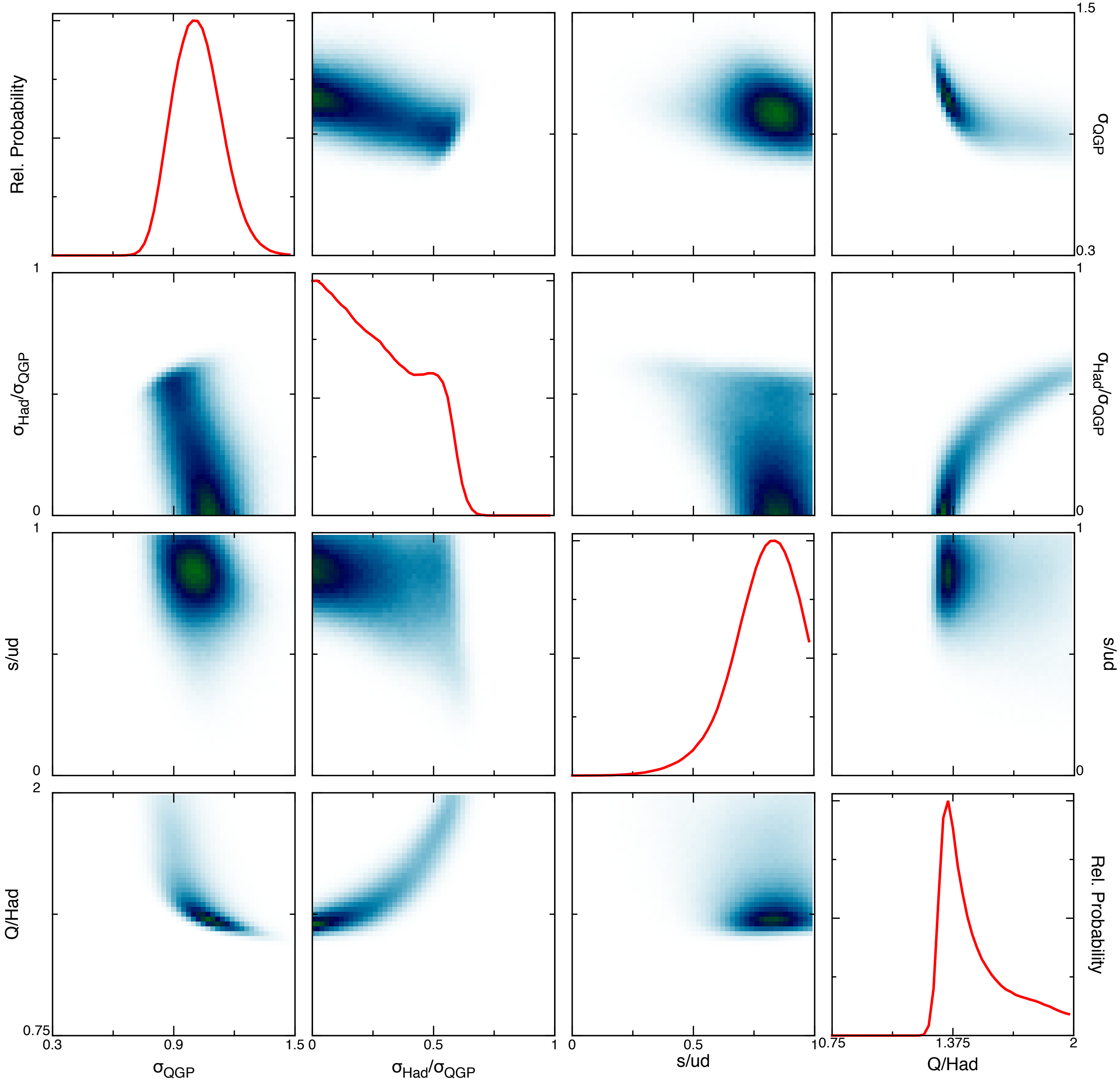
$0.75 <$ quark/hadron < 2

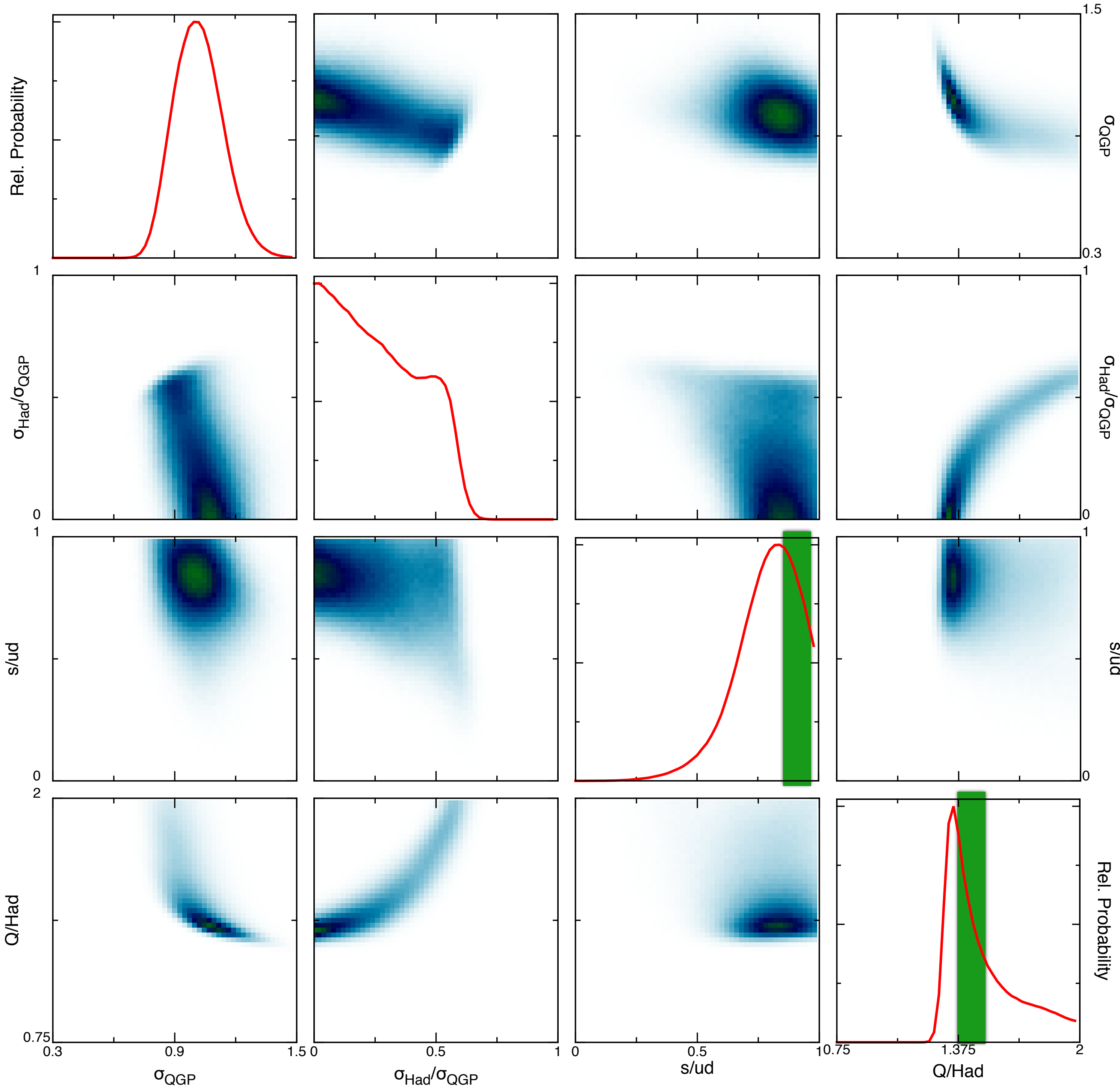
$0 <$ s/(ud) < 1

Charge Balance Functions (STAR)



MCMC Results





MCMC Results

- $\sigma_{qgp} \approx 0.9$
- $\sigma_{had} \approx 0.25$
- ud/hadron and s/ud ratios consistent with equilibrated QGP

Conclusions

- STAR results validate 2-wave picture
- uds densities of QGP appear close to lattice values

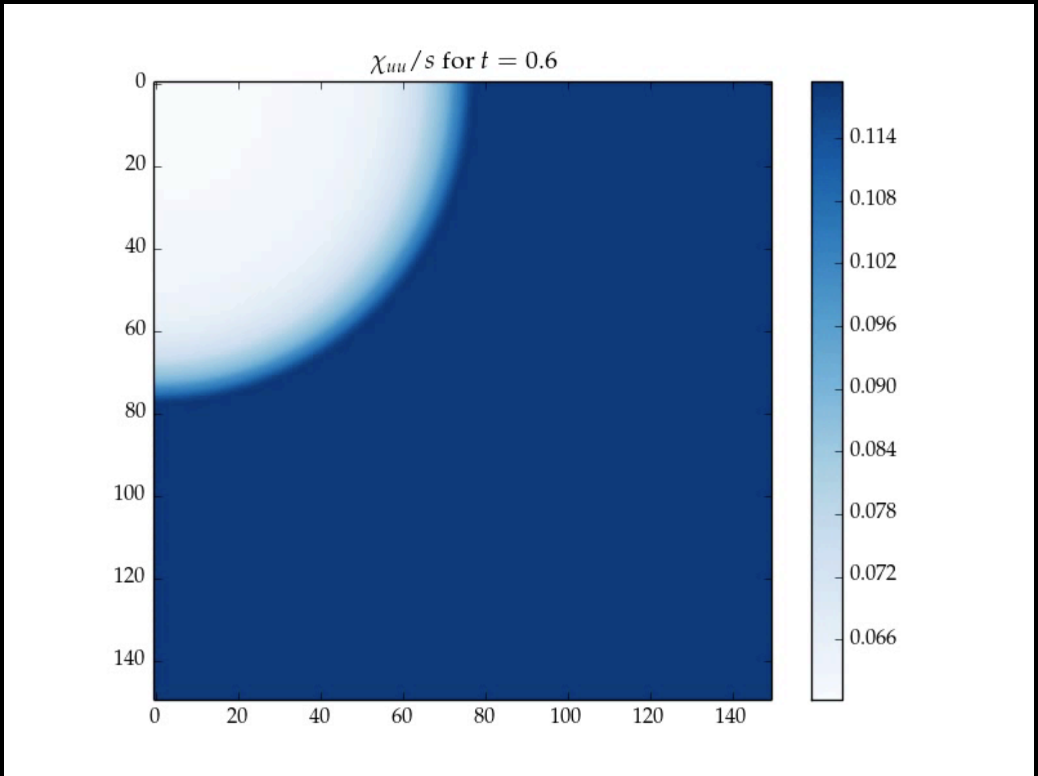
To-Do List: (Experiment)

- Other charge combinations, e.g. pK
- σ_ϕ vs σ_η
- $B(Q_{out},Q_{side},Q_{long})$ binned by k_t , reaction plane
- pp, pA collisions

To-Do List: (Theory)

- **Continuous creation/annihilation**

$$g'_{ab}(x_1,x_2) = \int_0^\tau d^4x' D_a(x',x_1) D_b(x',x_2) \frac{d(\chi_{ab}/s)}{d\tau'}(x')$$



- **non-zero baryon density**
- **pp/e^+e^- collisions need theory**
- **How do quarks arise from gluons/string/fluxtubes?**