Nontrivial holonomy and collisional energy loss

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Motivation: QCD at finite temperature

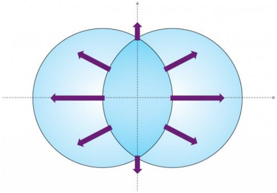
- Commonly accepted language for description of matter in heavy-ion collisions: **finite temperature Quantum ChromoDynamics**
- Method for finite temperature field theory and QCD in particular:
  - Lattice numerical simulations (LQCD): non-perturbative first-principle approach
    Limitations:
    – real-time properties (viscosity, conductivity, etc.) are very difficult to calculate (maximal entropy methods are usually applied),
    – only zero baryon density: sign problem
  - Functional methods (functional renormalization group, Schwinger-Dyson)
    Limitations:
    – based on non-systematic truncations
  - AdS/CFT duality
    Limitations:
    – infinite N_c
    – strong coupling limit only
  - Perturbative methods:
    Limitations:
    – applicable only at large T and μ

Our goal: to include important non-perturbative effects in perturbative QCD.

Hydro

Hydrodynamics is quite successful in describing two-particle correlations in $A + A$ collisions

Elliptic flow



$$v_n \sim \frac{1}{N} \int d\phi \frac{dN}{d\phi} cos(\pi n)$$

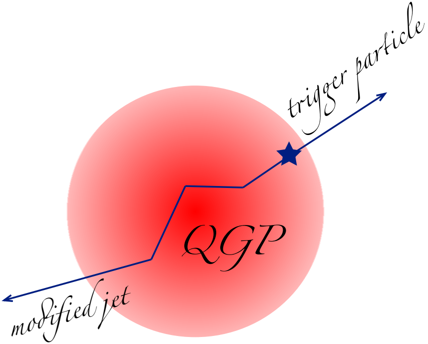
A+A RHIC and LHC: $v_2 \sim 0.1$

This is consistent with an assumption of strongly interacting matter with small viscosity

Small viscosity! Strongly interacting system!

Viscosity is proportional to mean free path (inversely proportional to scattering cross-section)

Heavy-quark/jet energy loss in quar-gluon plasma

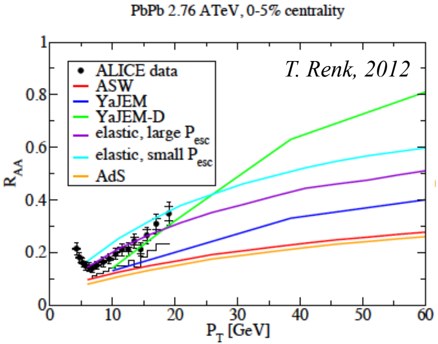


- A particle traveling in the medium scatters on its constituents and loses its energy.
- Small viscosity of the matter implies strong interaction.
- Thus it is expected that the energy loss is dramatically large.
- This is not the case!

AdS/CFT and jet energy loss

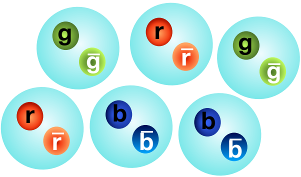
Suppression of single inclusive high $p_\perp$ hadrons

$$R_{AA} = \frac{d^2N/dp_\perp dy}{T_{AA}(0)d^2\sigma^{NN}/dp_\perp dy}$$

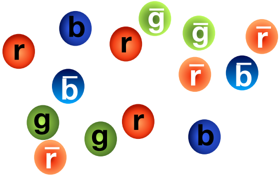


While η/s is close to its “minimal” AdS/CFT value, the energy loss seems to be overestimated in AdS/CFT.

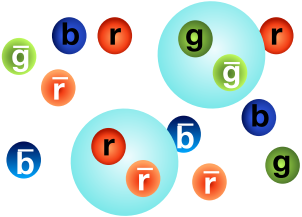
Ionization in QCD plasma



Neutral state $\rightsquigarrow$ confined phase, color neutrality > hadronic scale

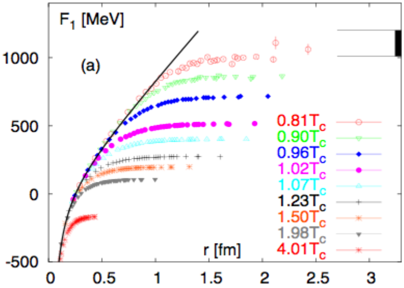


Completely ionized plasma $\rightsquigarrow$ perturbative QGP with freely moving charges



Partially ionized plasma $\rightsquigarrow$ *partial* ionization of color: hadrons and color charges; semi-QGP, nontrivial holonomy

Order parameter for deconfinement phase transition



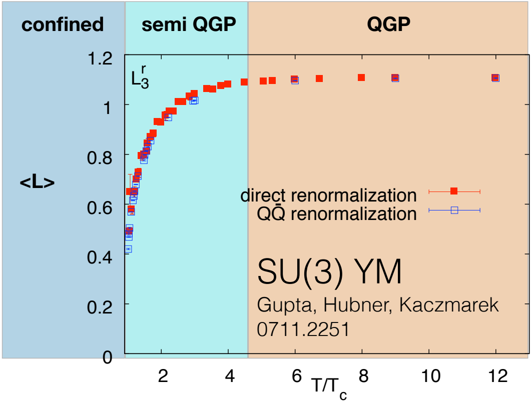
- Wilson line $\mathbb{L} = \exp\left(ig\int_0^{1/T} A_0 d\tau\right)$;
  trace of Wilson line: the Polyakov loop, $L = \mathrm{tr}\mathbb{L}/N_c$, is gauge invariant
- Free energy of static quark, in pure glue $F(T) = F_1(r \to \infty, T)/2$
- Polyakov loop $L = \exp(-F(T)/T)$

Polyakov loop: $\langle L \rangle \sim e^{-F_{\text{test qk}}/T}$

Confined: $F_{\text{test qk}} \to \infty$, $\langle L \rangle \to 0$

Semi QGP: $0 < \langle L \rangle < 1$
$\langle L \rangle$ measures degree of ionization

Perturbative QGP: $F_{\text{test qk}}/T \to 0$, $\langle L \rangle \to 1$

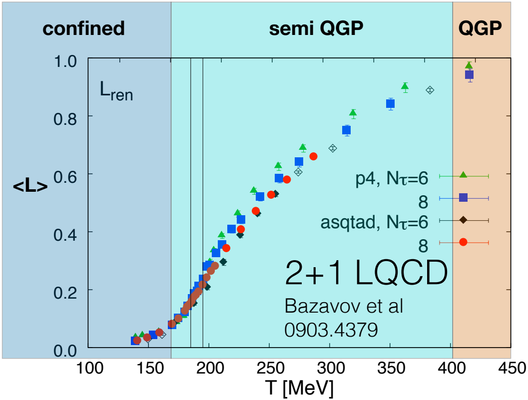


Polyakov loop: $\langle L\rangle \sim e^{-F_{\text{test qk}}/T}$

Confined: $F_{\text{test qk}} \to \infty$, $\langle L\rangle \to 0$

Intermediate regime: $0 < \langle L\rangle < 1$
$\langle L\rangle$ measures degree of ionization

Deconfined: $F_{\text{test qk}}/T \to 0$, $\langle L\rangle \to 1$



Polyakov loop and eigenvalues of Wilson line

Non-trivial holonomy

$$\mathrm{tr}\,\mathbb{L} = \exp\left(ig\int_0^{1/T} A_0 d\tau\right)$$

can be represented by eigenvalues of Wilson line.

In mean-field approximation:

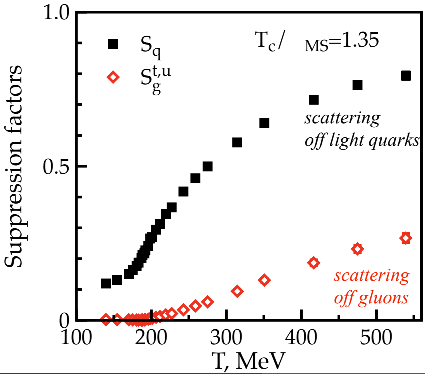
$$A_0^{\mathrm{cl}} = \frac{i}{g}\,\mathrm{diag}(Q^1, Q^2, \cdots, Q^{N_c})$$

The background field A_0^{cl} is manifestly non-perturbative ($\propto 1/g$).

Punchline: transition region (“semi”-QGP): must exhibit partial ionization of color shear viscosity, energy loss. . . must depend upon the degree of ionization

Energy loss

$$S_i = \frac{\text{energy loss in semi−QGP}}{\text{energy loss in perturbative QGP}}$$



- S_q - Coulomb scattering (scattering off of **light quark**, t channel)
- $S_g^{t,u}$ - Compton scattering (scattering off of gluons, t and u channels)

  S_i **increases as color is ionized**

Perturbative vs semi-QGP

Usual argument of kinetic theory

Majumder, Muller and Wang, hep-ph/0703082

Liao and Shuryak, 0810.4116

Asakawa, Bass, and Muller, hep-ph/0603092, 1208.2426

- Viscosity $\eta \sim \rho^2/\sigma$
  - ρ - density of color charges $\rho \sim 1$
  - σ - crossection: $\sigma \sim g^4$, g - coupling
  - $\underline{\text{large } g \rightsquigarrow \text{small } \eta}$

- Energy loss $\frac{dE}{dx} \sim g^2\rho^2$
  - $\underline{\text{large } g \rightsquigarrow \text{large } \frac{dE}{dx}}$

Semi-QGP

Y. Hidaka, R, Pisarski 0912.0940

R. Pisarksi, V. Skokov proceedings of QM2013

- Viscosity $\eta \sim \rho^2/\sigma$
  - ρ - density of color charges, $\rho \sim \langle L\rangle^2$
  - σ - cross section: $\sigma \sim \langle L\rangle^2$
  - $\underline{\eta \sim \langle L\rangle^2\text{, small in semi-QGP}}$

- Energy loss (large N_c)
  - $\underline{\frac{dE}{dx} \sim \langle L\rangle \cdot \frac{dE}{dx}_{\text{on light quarks}}}$
    $\underline{+\langle L\rangle^2 \cdot \frac{dE}{dx}_{\text{off gluons}}}$

Summary for different models

AdS/CFT

- small viscosity (conjectured lowest limit) $\eta/s = 1/4\pi$l
- large energy loss ($\propto l^3$) (here l = length)

Collection of results

- AdS/sQCD: energy loss $\propto l^3$
- pQCD, energy loss $\propto l^2$
- experiment, energy loss $< l^2$
- semi QGP provides suppression of pQCD results

Details

- Collisional energy loss in large N_c limit
- Collisional energy loss due to scattering off of light quark, $N_c = 3$
- Collisional energy loss due to scattering off of gluons, $N_c = 3$
- Outlook: radiative energy loss, photon and dilepton production

Non-zero Polyakov loop $\rightsquigarrow$ non-trivial holonomy

- Finite temperature field theory: $S(1) \times R(3)$
- Polyakov loop $L = \mathrm{Tr}\,\mathcal{P} \exp\left(ig \int_0^{1/T} A_0 d\tau \right)$
- Anzatz for $[A_0]_{ab} = \delta_{ab} \frac{Q^a}{g}$, for the sake of simplicity $Q^a = 2\pi T \cdot q^a$
- Tracelessness tr $\mathrm{A}_0 = 0 \rightsquigarrow \sum_a Q^a = \sum_a q^a = 0$
- Classical approximation: zero action for A_0
- One loop about A_0: Gross, Pisarski, Yaffe '81 (no functional modification to two loop order, see A. Dumitru et al, 2013)

$$U_{\mathrm{pert}} = -2\pi^2 T^4 \left[\frac{N^2 - 1}{45} - \frac{1}{3} \sum_{a,b} (q_a - q_b)^2 \left(1 - |q_a - q_b|\right)^2 \right]$$

  Gives only trivial A_0
- Non-perturbative contribution are modeled by (R. Pisarski et al)

$$U_{\mathrm{non-pert}} = T^2 T_d^2 \left[c_1 \sum_{a,b}^{N} |q_a - q_b| \left(1 - |q_a - q_b|\right) + c_2 \sum_{a,b}^{N} (q_a - q_b)^2 \left(1 - |q_a - q_b|\right)^2 + c_3 \right]$$

- c_i are fixed to get transition at $T = T_d$, and describe lattice data
- three colors: $q_1 = -q_2 = q$, $q_3 = 0$. Confining at $q = 1/3$ and perturbative $q = 0$.

Distribution function for quarks

- Lagrangian for quarks in background field A_0
  $\mathcal{L} = \bar{\psi}\left(\gamma_\mu i D^\mu - m\right)\psi$ with $D^\mu = \partial^\mu + igA^\mu$ and $A^\mu = \delta^{\mu 0}A^0, \quad A_4 = iA_0$
- Logarithm of partition function:

$$\ln Z = \ln \int \mathcal{D}\psi \exp\left(-\int d^4 x_E \mathcal{L}_E\right) = \sum_a \ln\det\left(\gamma^\mu \partial_\mu + 2\pi T q_a \delta^{\mu 4} + im\right)$$

$$= 2\sum_a \ln\det\left[(\partial_4 + 2\pi q_a)^2 - \boldsymbol{\partial}^2 + m^2\right]$$

$$= -2T\sum_a \int \frac{d^3p}{(2\pi)^3}\left[\ln\left(1 + \exp\left(-E_p/T - i2\pi q_a\right)\right)\right] + \text{c.c.}$$

$$= \sum_a \int \frac{d^3p}{(2\pi)^3}\frac{p^2}{E_p} n^{\mathrm{q}}(E_p - 2\pi i T q_a) + \text{c.c.}, \quad E_p = \sqrt{p^2 + m^2}$$

- Quark distributing function

$$n^{\mathrm{q}}(E_p - 2\pi i T q_a) = \frac{1}{\exp(E_p/T - 2\pi i q_a) + 1}$$

Distribution function for quarks & gluons in A_0

Quark and gluon propagator in a background A_0 field: Hidaka, Pisarski 0906.1751

- Distribution function for quarks ($Q_a = 2\pi T q_a$)

$$n^q_a(p, Q) = \left[\exp\left(\frac{p - iQ_a}{T}\right) + 1\right]^{-1}$$

- Compare the previous to usual Fermi-Dirac distribution at finite chemical potential

$$n^q_a(p) = \left[\exp\left(\frac{p - \mu}{T}\right) + 1\right]^{-1}$$

$\mu \to iQ_a$

- Distribution function for gluons

$$n^g_{a,b}(p, Q) = \left[\exp\left(\frac{p - i(Q_a - Q_b)}{T}\right) - 1\right]^{-1}$$

Distribution function for quarks & gluons: limits

- Perturbative QGP (trivial holonomy), $Q = 0$

$$n^g(k, Q=0) = \left[\exp\left(\frac{k}{T}\right) - 1\right]^{-1} \quad n^q(k, Q=0) = \left[\exp\left(\frac{k}{T}\right) + 1\right]^{-1}$$

- In confined regime: $\mathrm{tr}\,\mathbb{L}^n = \sum_a \exp(2\pi n i q_a) = 0$ unless $n = jN_c, j \in \mathbb{Z}$, when $\mathrm{tr}\,\mathbb{L}^n = (-1)^{k(N_c+1)} N_c$ therefore

$$\sum_a n_a{}^q = \sum_a \sum_{n=1}^{\infty} \exp\left(-nk/T\right) \mathrm{tr}\,\mathbb{L}^n = \frac{N_c}{1 + \exp(N_c \beta k)}$$

Number of quarks in confined state ($k \to N_c k$)

$$N^q_{\mathrm{conf}} = \int [dk] \sum_a n_a{}^q = \frac{1}{N_c^{d-1}} \int [dk] \frac{1}{1 + \exp(\beta k)},$$

where d is the number of spatial dimensions.
To be compared with number of particles in deconfined regime N_{dc}

$$N^q_{\mathrm{dc}} = \sum_a \int [dk] n^q(Q=0) = N_c \int [dk] \frac{1}{1 + \exp(\beta k)}$$

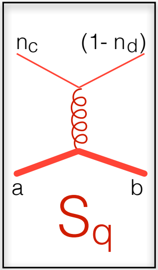
In the large N_c limit, $N_{\mathrm{conf}} \to 0$ unless $d < 2$.

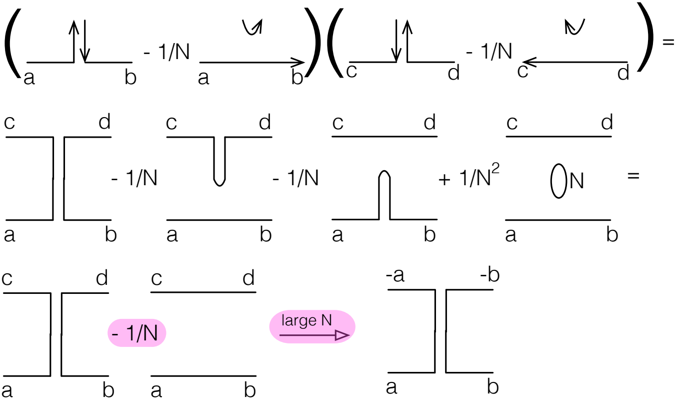
Collisional energy loss

- Heavy, energetic particle in the initial state: $E \gg M \gg T$.
- Collisional energy loss per unit length is proportional to the density of particles. Scattering off of light quarks:

$$\frac{dE}{dx} \sim \sum_{a} \int_{k} n(k + 2\pi i T q_{a}) \times \text{Matrix element}$$

Large N limit for scattering off light quark: birdtracks





Generators of $SU(N_c)$: **'t Hooft's double line basis** (N_c^2 generators)

$$(t^{ab})_{cd} = \frac{1}{\sqrt{2}}\left(\delta^a_c \delta^b_d - \frac{1}{N_c}\delta^{ab}\delta_{cd}\right) \qquad \mathrm{Tr}\ t^{ab} t^{cd} = \frac{1}{2} P^{(ab)(cd)}$$

$$\frac{dE}{dx} \propto \sum_{a,b}^{N} \int [dk][dk'][dp']f(p,k,k',p')n(E_k + iQ_a)\left[1 - n(E_{k'} + iQ_b)\right]$$

$$\sum_a n(E_k + iQ_a) = \sum_a \left[\exp\left(\beta E_k + i\beta Q_a\right) + 1\right]^{-1}$$

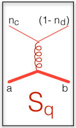
$$= \sum_{n=1}^{\infty} (-1)^n \exp\left(-n\beta E_k\right) \sum_a \exp(in\beta Q_a) = \sum_{n=1}^{\infty} (-1)^n \exp\left(-n\beta E_k\right) \mathrm{tr}\, L^n$$

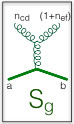
for small tr L scattering off **light quarks**

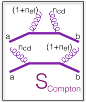
$$\frac{dE}{dx} \propto \mathrm{tr}L \cdot \left(\frac{dE}{dx}\right)_{\text{pert.}}$$

Similar argument for scattering off **gluons** gives

$$\frac{dE}{dx} \propto (\mathrm{tr}\, L)^2 \cdot \left(\frac{dE}{dx}\right)_{\text{pert.}}$$







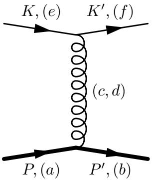
N=3

As in perturbative QCD, but taking into account modification of distribution function in final/initial state.

N=3: Scattering off light quarks

$$\frac{dE}{dx} = \sum_i \int_k \frac{n_i(k)}{2k} \int_{k'} \frac{1 \pm n_i(k')}{2k'} \int_{p'} \frac{(2\pi)^4 \delta^{(4)}(P+K-P'-K')\,|\mathcal{M}_i|^2 \omega}{4EE'd}$$

$$\frac{1}{d}\sum_i |\mathcal{M}_i|^2 = \frac{8N_\mathrm{f} g^4}{N_c} \frac{2(s-M^2)^2 + (u-M^2)^2 + 2M^2 t}{t^2} \times \frac{N_c^2-1}{4N_c}$$



Keeping only leading log terms

$$\left.\frac{dE(A_0^{\mathrm{cl}})}{dx}\right|_\mathrm{q} = S_q(A_0^{\mathrm{cl}}) \left.\left(\frac{dE}{dx}\right)\right|_q^{\mathrm{pert}}; \quad \left.\left(\frac{dE}{dx}\right)\right|_q^{\mathrm{pert}} = \alpha_\mathrm{s}^2 T^2 N_\mathrm{f} \frac{N_c^2-1}{12N_c} \pi \ln \frac{ET}{m_D^2}$$

with semi QGP suppression factor

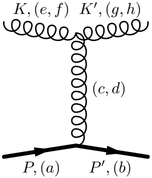
$$S_q(A_0^{\mathrm{cl}}) = \frac{12}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \frac{\mathrm{tr}\mathbb{L}^n}{N_c}$$

- pert. limit $A_0 = 0$: $S_q(A_0^{\mathrm{cl}} = 0) = 1$
- confining limit: $S_q(A_0^{\mathrm{cl}}) = 1/N_c^2$

$$S_i = \left(\frac{dE}{dx}\right)_i \Big/ \left(\frac{dE}{dx}\right)_{i,\text{pert.}}$$

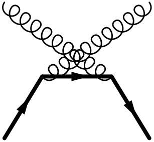
- t channel

$$S^t_{\rm g}(A_0^{\rm cl}) = \frac{1}{N_{\rm c}^2 - 1}\left(\frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{|{\rm tr}\mathbb{L}^n|^2}{n^2} - 1\right); \qquad \left.\frac{dE}{dx}\right|_{\rm g}^{t,\text{pert}} = \alpha_s^2 T^2 (N_{\rm c}^2 - 1)\frac{\pi}{6}\ln\frac{ET}{m_D^2}$$



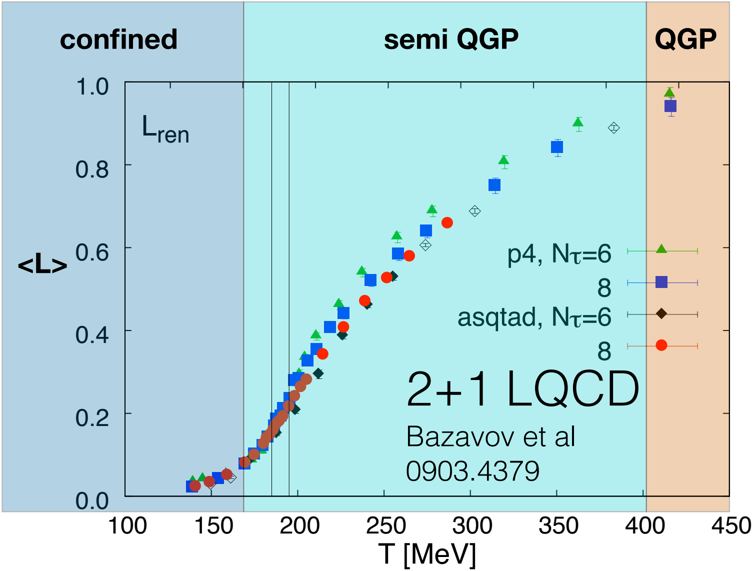
- u channel, the same suppression factor

$$\left.\frac{dE}{dx}\right|_{\rm g}^{u,\text{pert}} = \alpha_s^2 T^2 C_F^2 \frac{\pi}{6}\ln\frac{ET}{M^2}, \qquad C_F = (N_c^2 - 1)/(2N_c)$$



- Limits: perturbative: $S_{\rm g}(A_0^{\rm cl} = 0) = 1$
  confining background field: $S_{\rm g} = 0$

Polyakov loop: LQCD



Polyakov loop as a function of T: perturbative corrections

- One needs to extract the eigenvalues of the Wilson line, q_a
- One can use models or first principle lattice QCD calculations
- Lattice QCD: not only classical field contributions to Polyakov Loop. Polyakov loop receives the perturbative corrections (Burnier, Laine, Veplalainen 2009)

- Perturbative corrections exponentiate, i.e. for the case of the non-trivial holonomy

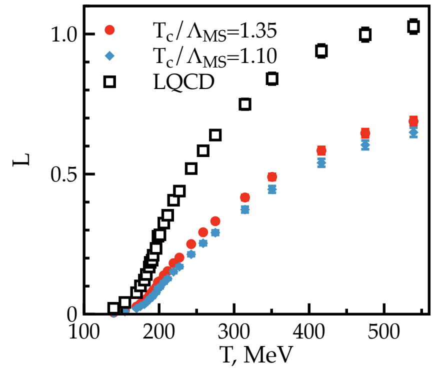
$$L_{\mathrm{w/pert}}(A_0^{\mathrm{cl}}) = L(A_0^{\mathrm{cl}}) \exp\left[\delta L_{\mathrm{pert}}(A_0^{\mathrm{cl}})\right]$$

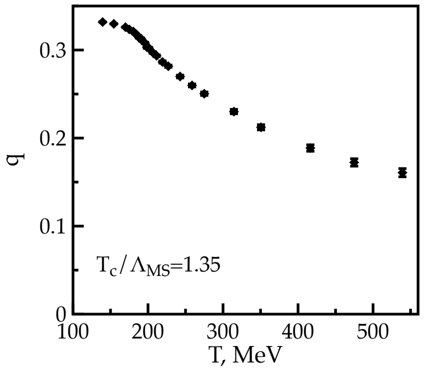
The dependence of the perturbative corrections on the background field A_0

$$\delta L_{\mathrm{pert}}(A_0^{\mathrm{cl}}) \approx \delta L_{\mathrm{pert}}(0)$$

.

Polyakov loop as a function of T: $N_c = 3$





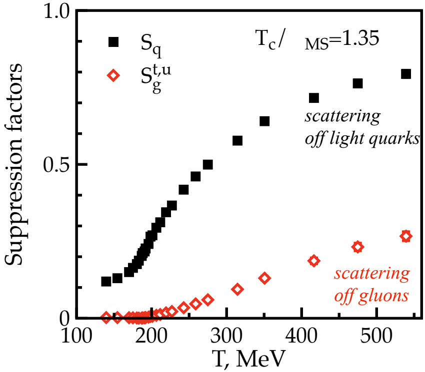
S. Lin, R. Pisarski and V.S. arXiv:1312.3340

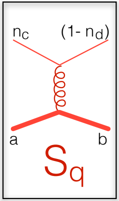
$$\boldsymbol{q} = (-q, 0, q); \quad L = \frac{1}{3}\left[1 + 2\cos(2\pi q)\right]$$

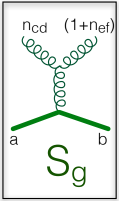
tr$\mathbb{L}$ is known, tr$\mathbb{L}^n$ are needed

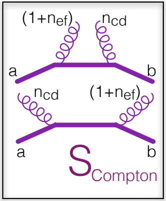
Lattice: Karsch et al

$$\mathrm{tr}\mathbb{L}^n = 1 + 2\cos(2\pi n q)$$









$$S_i = \left(\frac{dE}{dx}\right)_i \Bigg/ \left(\frac{dE}{dx}\right)_{i,\text{pert.}}$$

For three colors analytical result:

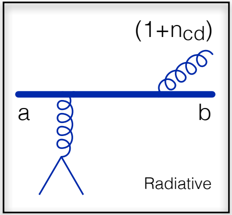
$$S_q^{N_c=3} = 1 - 8\,q^2$$

$$S_g^{N_c=3} = 1 - 3\,q\,(2 - 3\,q)$$

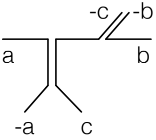
- Different processes are suppressed differently
- Processes with gluons are suppressed stronger then those with quarks
- for quarks: $S_q \to 1/N_c^2$ at low temperatures

S. Lin, R. Pisarski and V.S. arXiv:1312.3340

Radiative energy loss: preliminary large N result



Large N limit



Suppressed by

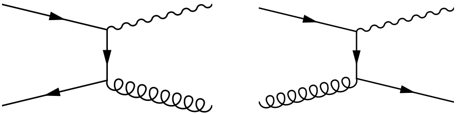
$$n_{-a}(1 - n_c)(1 + n_{-c,-b}) \sim \mathrm{tr}\, L$$

Correct treatment includes ladder diagram resumption, work in progress...

Photon production

Based on very similar arguments

Naive calculations include only



Suppression: tr L

Suppression: at given T the density of quarks is smaller.

However, complete leading order results include collinear emission, nonlinear quark annihilation and LPM effects, Arnold Moore Yaffe (AMY) '00

AMY in nontrivial background A_0: work in progress...

Dilepton production

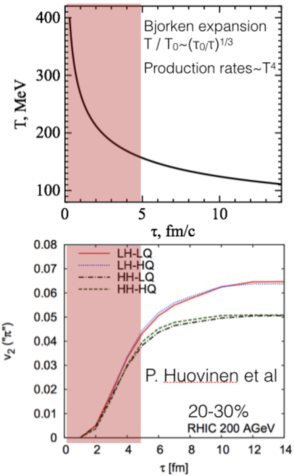
Color singlet in the final state, no suppression at leading order.

Therefore high T photons are suppressed, but not high T dileptons.

Photon azimuthal anisotropy

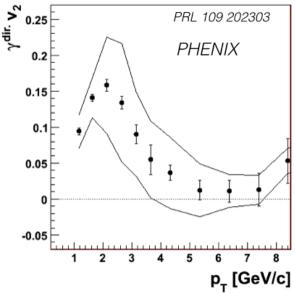
Naive argument:

- photon emission $\propto T^4$ and thus the highest in quark-gluon phase
- the flow of matter is the smallest in initial state
- thus, it was expected that photon azimuthal anisotropy is smaller then hadronic one (from final state) by a factor of ~ 5

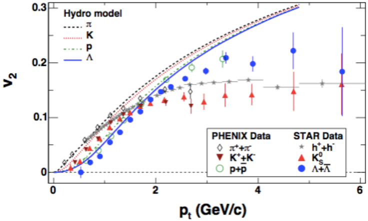


Photon azimuthal anisotropy

- Direct photons:



- Hadrons:



- experiment (PHENIX at RHIC and ALICE at LHC) showed that hadron and photon azimuthal anisotropy is approximately the same
- there is either an alternative mechanism for photon azimuthal anisotropy (e.g. conformal anomaly and magnetic field G. Bazar, D. Kharzeev and V.S. Phys.Rev.Lett. 109 (2012) 202303) or there is a suppression of photon production in QGP state

Our mechanism provides this suppression for photon, but not for dileptons!

Dileptions: no suppression! As a consequence $v_2^{l^+l^-} < v_2^{\gamma}$

Conclusions: perturbative QCD meets heavy-ion phenomenology

- semiQGP: nontrivial holonomy as an instrument to include non-perturbative confinement effects into perturbative QCD
- calculations in a non-trivial background classical fields: quite tedious, but doable
- collisional energy loss: suppression factor for scattering off of quarks is proportional to the Polyakov loop, scattering off of gluons is proportional to the second power of the Polyakov loop
- this approach may help to solve two puzzles of HIC phenomenology:
  – strong-coupling paradox: small viscosity and moderate energy loss
  – large azimuthal anisotropy of photons

Thank you!