Viscous hydrodynamics for systems undergoing strongly anisotropic expansion

Dennis Bazow

The Ohio State University

References: arXiv:1311.6720
Collaborators: U. Heinz and M.Strickland

Motivation

- Relativistic fluid dynamics is used to model HIC
- There are large corrections to an ideal fluid due to the rapid longitudinal expansion
- The QGP is an anisotropic plasma
- Can then build in the anisotropies from the beginning to create a more reliable approximation scheme to the QGP matter

Relativistic fluid dynamics

- Canonical way to derive viscous hydrodynamics is to linearize around an isotropic equilibrium distribution function

$$y - y_0 = \delta y \ll 1 \, , \qquad y_0 = \frac{u \cdot p}{T} - \frac{\mu}{T}$$

$$f(y) = \underbrace{f_{\rm eq}(y_0)}_{\equiv f_0} + \underbrace{f_{\rm eq}\left(1 - a f_{\rm eq}\right)\delta y}_{\equiv \delta f} + \mathcal{O}\left(\delta y^2\right)$$

- Particle momentum-space is approximated at leading-order by a sphere
- Isotropic energy-momentum tensor (ignoring bulk viscous pressure)

$$T^{\mu\nu} = (\mathcal{E} + \mathcal{P})\, u^\mu u^\nu - \mathcal{P} g^{\mu\nu} + \pi^{\mu\nu} \, , \qquad R_\pi^{-1} = \sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}/\mathcal{P} \ll 1$$

Viscous hydrodynamic limitations

- Look at Navier-Stokes solution for insights on the momentum-space anisotropies

$$\pi^{\mu\nu}_{\rm NS} = \eta \nabla^{\langle \mu} u^{\nu \rangle}$$

$$\mathcal{P}_\perp = \mathcal{P} + \frac{1}{2}\left(\pi^{xx}_{\rm NS} + \pi^{yy}_{\rm NS}\right) = \mathcal{P} + \frac{2\eta}{3\tau} \qquad \pi^{zz}_{\rm NS} = -2\pi^{xx}_{\rm NS} = -2\pi^{yy}_{\rm NS} = -\frac{4\eta}{3\tau}$$

$$\mathcal{P}_{\rm L} = \mathcal{P} + \pi^{zz}_{\rm NS} = \mathcal{P} - \frac{4\eta}{3\tau}$$

Can recast pressure ratio in terms of the temperature of the system

$$\frac{\mathcal{P}_{\rm L}}{\mathcal{P}_\perp} = \frac{3\tau {\rm T} - 16\bar{\eta}}{3\tau {\rm T} + 8\bar{\eta}}\,, \qquad \bar{\eta} \equiv \eta/\mathcal{S}$$

- Initial conditions at RHIC: ${\rm T}_0 = 400$ MeV, $\tau_0 = 0.5$ fm/c gives $\mathcal{P}_{\rm L}/\mathcal{P}_\perp \leq 0.5$
- Initial conditions at LHC: ${\rm T}_0 = 600$ MeV, $\tau_0 = 0.25$ fm/c gives $\mathcal{P}_{\rm L}/\mathcal{P}_\perp \leq 0.35$
- The longitudinal pressure can become negative for large values of $\bar{\eta}$, early times, or low temperatures

Hydrodynamic expansion revisited: a reorganized approach

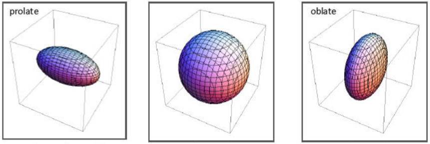
- Hydrodynamic expansion breaks down in far-from-equilibrium situations
  - i.e. the first-order correction becomes of the order of the leading-order piece in the perturbative expansion
- Generalized solution

$$f(x,p) = f_0(x,p) \sum_{\ell,\alpha} a_\alpha(x) P_\alpha^{(\ell)}(p)$$

- f_0 is the LO approximation (arbitrary weight factor)
- In order to obtain the most rapid convergence, choose f_0 such that it is as close as possible to the exact solution f
- The choice of f_0 is guided by general insights into the properties of f for the problem at hand

Anisotropic expansion

- In HIC, rapid longitudinal expansion suggests to use an f_0 distorted along the p_z (beam)-direction with azimuthal symmetry



$-1 < \xi < 0$ $\quad$ $\xi = 0$ $\quad$ $\xi > 0$

$$\xi > 0 \implies \mathcal{P}_{\mathrm{L}} < \mathcal{P}_{\perp}$$

- Expansion around a “local anisotropic equilibrium” distribution function

$$f(x,p) = \underbrace{f_{\mathrm{iso}}\left(\frac{\sqrt{m^2 + p_\perp^2 + (1+\xi(x))\,p_z^2}}{\Lambda(x)}\right)}_{\text{Romatschke-Strickland form in LRF}} + \delta\tilde{f}$$

- $\xi(x)$ is the anisotropy parameter
- Λ is the effective transverse temperature

$$\xi = \frac{\langle p_\perp^2 \rangle}{2\langle p_{\mathrm{L}}^2 \rangle} - 1$$

Hydrodynamic tensor decomposition for anisotropic systems

- Expansion around a spheroidal distribution function

$$f(x,p) = f_{\mathrm{RS}} + \delta\tilde{f}$$

  leads to $\mathcal{P}_x = \mathcal{P}_y \neq \mathcal{P}_z$
- Large portion of dissipative currents caused by spheroidal deformation of particle momentum-space are treated non-perturbatively
- $\delta\tilde{f}$ gives rise to dissipative currents which (mostly) account for viscous effects other than those included in $\delta f = f_{\mathrm{aniso}} - f_{\mathrm{eq}}$

$$j^\mu = \mathcal{N}_{\mathrm{aniso}} u^\mu + \tilde{V}^\mu$$

$$T^{\mu\nu} = \left(\mathcal{E}_{\mathrm{aniso}} + \mathcal{P}_\perp + \tilde{\Pi}\right) u^\mu u^\nu - \left(\mathcal{P}_\perp + \tilde{\Pi}\right) g^{\mu\nu} + (\mathcal{P}_L - \mathcal{P}_\perp)\, z^\mu z^\nu + \tilde{\pi}^{\mu\nu}$$

R. Ryblewski and W. Florkowski, 1103.1260; M. Martinez, R. Ryblewski, and M. Strickland, 1204.1473

Quasi-thermodynamic quantities

$$\mathcal{N}(\xi,\Lambda) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} f_{\mathrm{RS}} = \mathcal{R}_0(\xi)\mathcal{N}_{\mathrm{iso}}(\Lambda)$$
$$\mathcal{E}(\xi,\Lambda) = T^{00} = \mathcal{R}(\xi)\mathcal{E}_{\mathrm{iso}}(\Lambda)$$
$$\mathcal{P}_\perp(\xi,\Lambda) = \frac{1}{2}\left(T^{xx} + T^{yy}\right) = \mathcal{R}_\perp(\xi)\mathcal{P}_{\mathrm{iso}}(\Lambda)$$
$$\mathcal{P}_{\mathrm{L}}(\xi,\Lambda) = T^{zz} = \mathcal{R}_{\mathrm{L}}(\xi)\mathcal{P}_{\mathrm{iso}}(\Lambda)$$

$$\mathcal{R}(\xi) = \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right),$$
$$\mathcal{R}_\perp(\xi) = \frac{3}{2\xi}\left(\frac{1+(\xi^2-1)\mathcal{R}(\xi)}{\xi+1}\right),$$
$$\mathcal{R}_L(\xi) = \frac{3}{\xi}\left(\frac{(\xi+1)\mathcal{R}(\xi)-1}{\xi+1}\right)$$

- The bulk quantities factorize into a product of two functions only in massless limit
- In the $m \neq 0$ case can define an "anisotropic equation of state"

LO aHydro: (0+1)d case

M. Martinez, M. Strickland, 1007.0889

0^{th} moment: $\partial_\mu j^\mu \neq 0$

$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - \frac{6}{\Lambda}\partial_\tau \Lambda = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

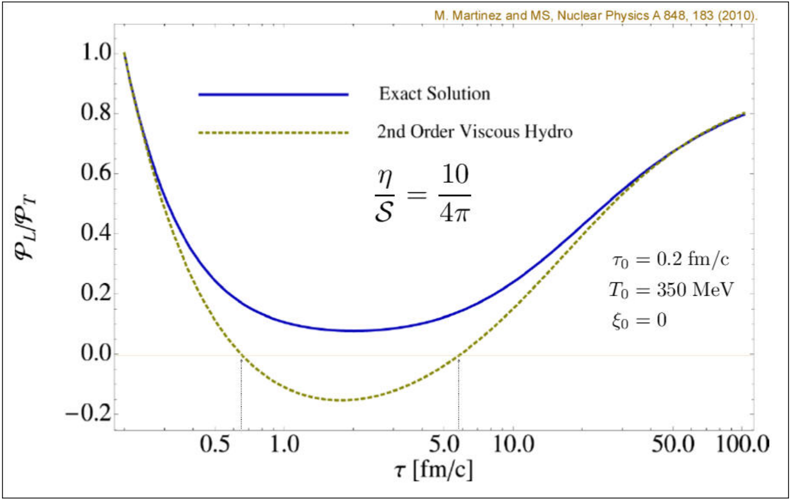
1^{st} moment: $\partial_\mu T^{\mu\nu} = 0$

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + \frac{4}{\Lambda}\partial_\tau \Lambda = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$

Relaxation rate Γ determined by matching to viscous hydrodynamics for small ξ

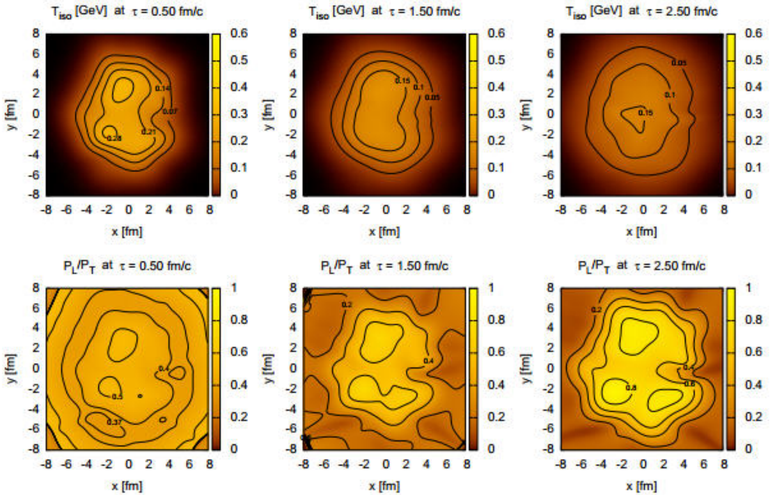
$$\Gamma = \frac{2}{\tau_\pi} = \frac{2\mathrm{T}}{5\bar{\eta}} = \frac{2\mathcal{R}^{1/4}(\xi)\Lambda}{5\bar{\eta}}$$

Pressure anisotropy



Transverse expansion

M. Martinez, R. Ryblewski, and M. Strickland, 1204.1473



$4\pi\eta/\mathcal{S} = 1$, Pb-Pb @ 2.76 TeV: $T_0 = 600\,\text{MeV}$, $\tau_0 = 0.25\,\text{fm/c}$, $b = 7\,\text{fm}$

Macroscopic equations of motion: (2+1)d

0^{th} moment

$$\dot{\mathcal{N}} = -\mathcal{N}\theta - \partial_\mu \tilde{V}^\mu + \mathcal{C}$$

1^{st} moment

$$\begin{aligned}
\dot{\mathcal{E}} + (\mathcal{E}+\mathcal{P}_\perp+\tilde{\Pi})\theta + (\mathcal{P}_{\mathrm{L}}-\mathcal{P}_\perp)\frac{u_0}{\tau} + u_\nu\partial_\mu\tilde{\pi}^{\mu\nu} &= 0 \\
(\mathcal{E}+\mathcal{P}_\perp+\tilde{\Pi})\dot{u}_x + \partial_x(\mathcal{P}_\perp+\tilde{\Pi}) + u_x(\dot{\mathcal{P}}_\perp+\dot{\tilde{\Pi}}) + (\mathcal{P}_\perp-\mathcal{P}_{\mathrm{L}})\frac{u_0 u_x}{\tau} - \Delta^{1\nu}\partial^\mu\tilde{\pi}_{\mu\nu} &= 0 \\
(\mathcal{E}+\mathcal{P}_\perp+\tilde{\Pi})\dot{u}_y + \partial_y(\mathcal{P}_\perp+\tilde{\Pi}) + u_y(\dot{\mathcal{P}}_\perp+\dot{\tilde{\Pi}}) + (\mathcal{P}_\perp-\mathcal{P}_{\mathrm{L}})\frac{u_0 u_y}{\tau} - \Delta^{2\nu}\partial^\mu\tilde{\pi}_{\mu\nu} &= 0
\end{aligned}$$

- Need more equations for $\tilde{V}^\mu$, $\tilde{\Pi}$, and $\tilde{\pi}_{\mu\nu}$
- Treated perturbativly like in viscous hydro (14-moment approximation), leads to "anisotropic tansport" equations

(2+1)d EOM for the anisotropic degrees of freedom

$$\frac{\dot{\xi}}{1+\xi} - 6\frac{\dot{\Lambda}}{\Lambda} - 2\theta = 2\Gamma\left(1 - \sqrt{1+\xi}\,\mathcal{R}^{3/4}(\xi)\right)$$

$$\mathcal{R}'\dot{\xi} + 4\mathcal{R}\frac{\dot{\Lambda}}{\Lambda} = -\left(\mathcal{R}+\frac{1}{3}\mathcal{R}_\perp\right)\theta_\perp - \left(\mathcal{R}+\frac{1}{3}\mathcal{R}_L\right)\frac{u_0}{\tau} + \frac{\tilde{\pi}^{\mu\nu}\sigma_{\mu\nu}}{\mathcal{E}_0(\Lambda)}$$

$$[3\mathcal{R}+\mathcal{R}_\perp]\,\dot{u}_\perp = -\mathcal{R}'_\perp\partial_\perp\xi - 4\mathcal{R}_\perp\frac{\partial_\perp\Lambda}{\Lambda} - u_\perp\left(\mathcal{R}'_\perp\dot{\xi}+4\mathcal{R}_\perp\frac{\dot{\Lambda}}{\Lambda}\right)$$
$$-u_\perp(\mathcal{R}_\perp-\mathcal{R}_L)\frac{u_0}{\tau} + \frac{3}{\mathcal{E}_0(\Lambda)}\left(\frac{u_x\Delta^1{}_\nu + u_y\Delta^2{}_\nu}{u_\perp}\right)\partial_\mu\tilde{\pi}^{\mu\nu}$$

$$[3\mathcal{R}+\mathcal{R}_\perp]\,u_\perp\dot{\phi}_u = -\mathcal{R}'_\perp D_\perp\xi - 4\mathcal{R}_\perp\frac{D_\perp\Lambda}{\Lambda} - \frac{3}{\mathcal{E}_0(\Lambda)}\left(\frac{u_y\partial_\mu\tilde{\pi}^{\mu 1} - u_x\partial_\mu\tilde{\pi}^{\mu 2}}{u_\perp}\right)$$

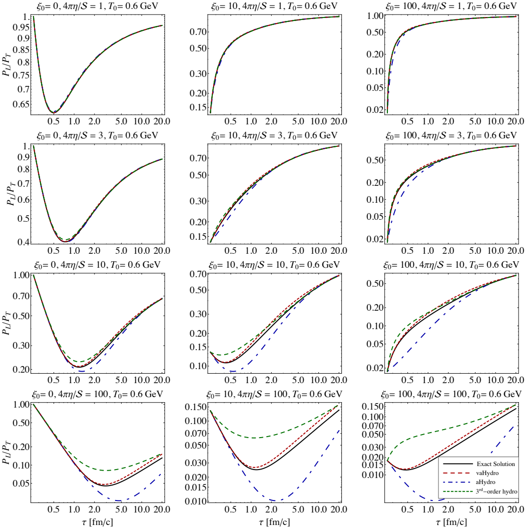
$$\dot{\tilde{\pi}}^{\mu\nu} = -2\dot{u}_\alpha\tilde{\pi}^{\alpha(\mu}u^{\nu)} - \Gamma\Big[(\mathcal{P}-\mathcal{P}_\perp)\Delta^{\mu\nu} + (\mathcal{P}_\mathrm{L}-\mathcal{P}_\perp)z^\mu z^\nu + \tilde{\pi}^{\mu\nu}\Big] + \mathcal{K}_0^{\mu\nu} + \mathcal{L}_0^{\mu\nu}$$
$$+\mathcal{H}_0^{\mu\nu\lambda}\dot{z}_\lambda + \mathcal{Q}_0^{\mu\nu\lambda\alpha}\nabla_\lambda u_\alpha + \mathcal{X}_0^{\mu\nu\lambda}u^\alpha\nabla_\lambda z_\alpha - 2\lambda^0_{\pi\pi}\tilde{\pi}^{\lambda\langle\mu}\sigma^{\nu\rangle}_\lambda + 2\tilde{\pi}^{\lambda\langle\mu}\omega^{\nu\rangle}_\lambda - 2\delta^0_{\pi\pi}\tilde{\pi}^{\mu\nu}\theta,$$

Testing vaHydro

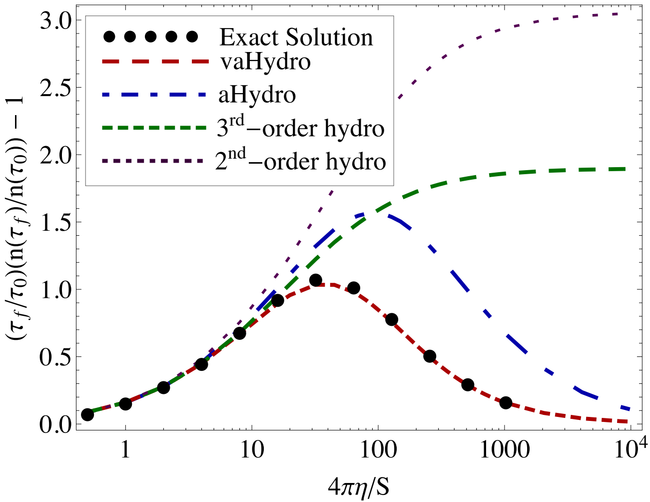
- Is there a way to test the accuracy of various approximation methods?
- Exact (numerical) solution to the Boltzmann equation in RTA exists for (0+1)d systems [W. Florkowski, R. Ryblewski, M. Strickland 1304.0665, 1305.7234]
- Relaxation rate is determined by matching at asymtotically late time to the exact solution

$$\Gamma = \frac{\mathcal{R}^{1/4}(\xi)\Lambda}{5\bar{\eta}}$$

Pressure anisotropy



Particle production

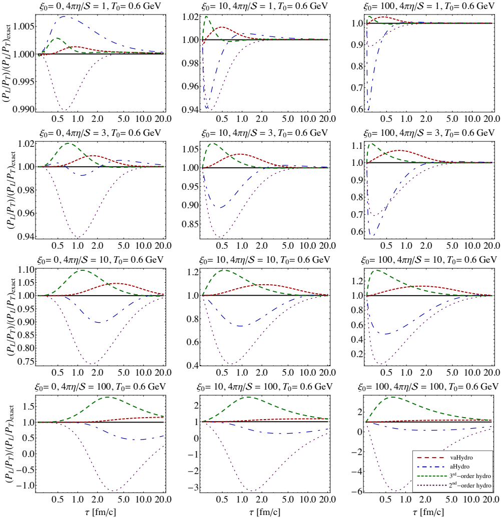


Conclusion

- The anisotropic hydrodynamics framework is a more efficient way to solve the relativistic hydrodynamics equations for HIC
- Second-order anisotropic hydrodynamics allows for corrections to the spheroidal form
- Expect it to improve the validity of viscous hydrodynamics for HIC especially at early times, for large $\eta/\mathcal{S}$, or near the transverse edge

Backup slides

Relative error of pressure ratio



Relative error of effective temperature