Next to Eikonal Corrections to the shock wave in QCD

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Forthcoming

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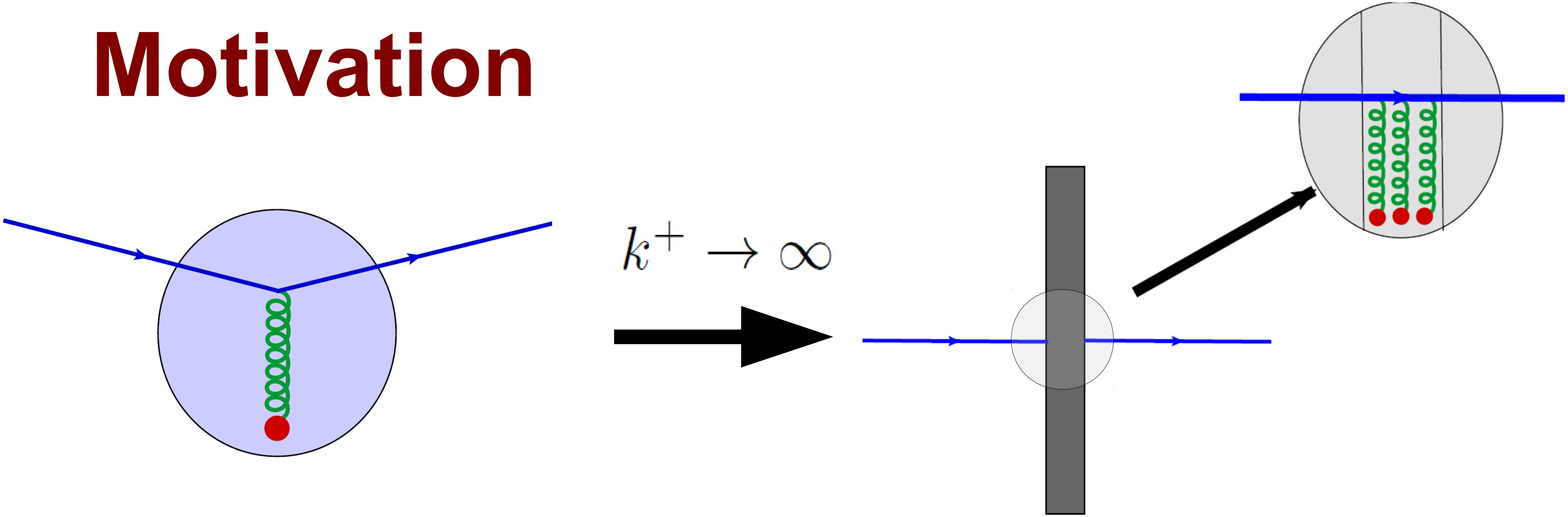
JET Collaboration

The Ohio State University

Motivation

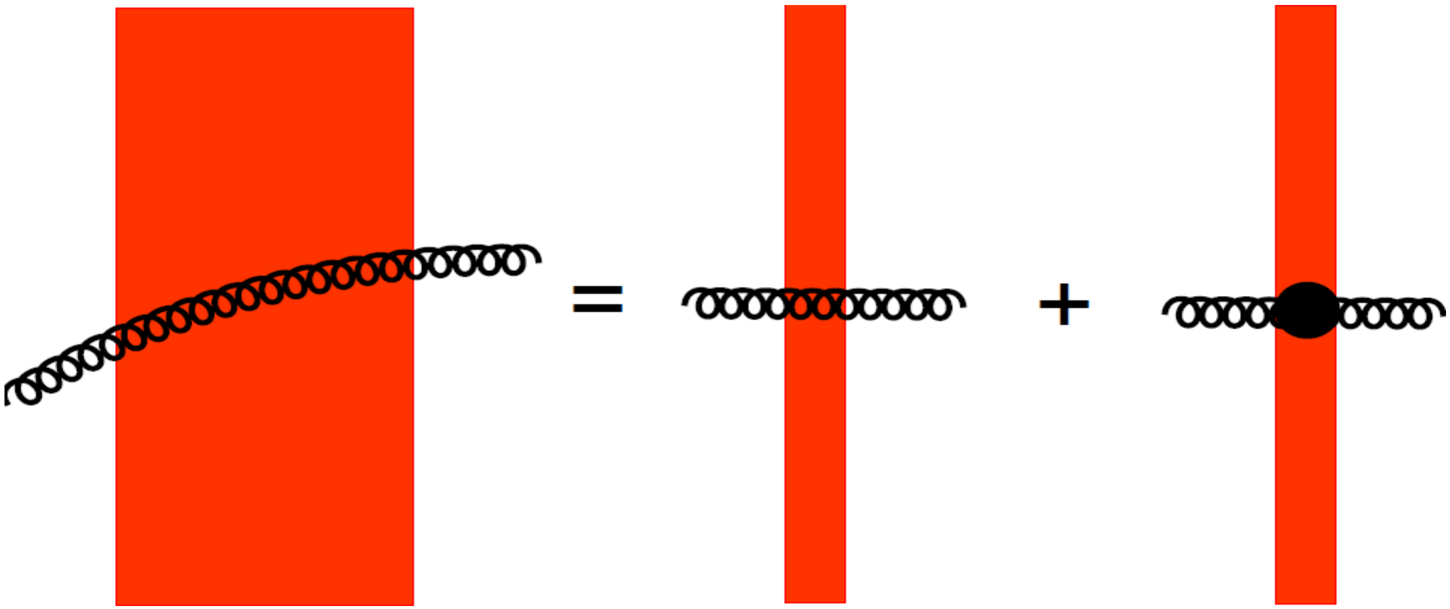


$$\Psi_{\delta\alpha}(\{k_i^+, \vec{x}_{i\perp}\}) \longrightarrow \Psi_{\delta\alpha}(\{k_i^+, \vec{x}_{i\perp}\}) \prod_i U_i(\vec{x}_\perp)$$

$$U_i(\vec{x}_\perp) \equiv T_+ \exp\left[i g_i \int dx^+ \, \mathcal{A}_a^-(x^+, 0, \vec{x}_\perp) t^a \right]$$

- Particle production at high energies is based on the Eikonal Approximation (e.g. BK, JIMWLK Evolution Eqs.)
- Corrections suppressed by the inverse power of the energy are systematically suppressed.
- Phenomenological studies of these evolution equations are applied to scattering processes of large albeit finite energies.
- At which energy power suppressed corrections start to play a role?

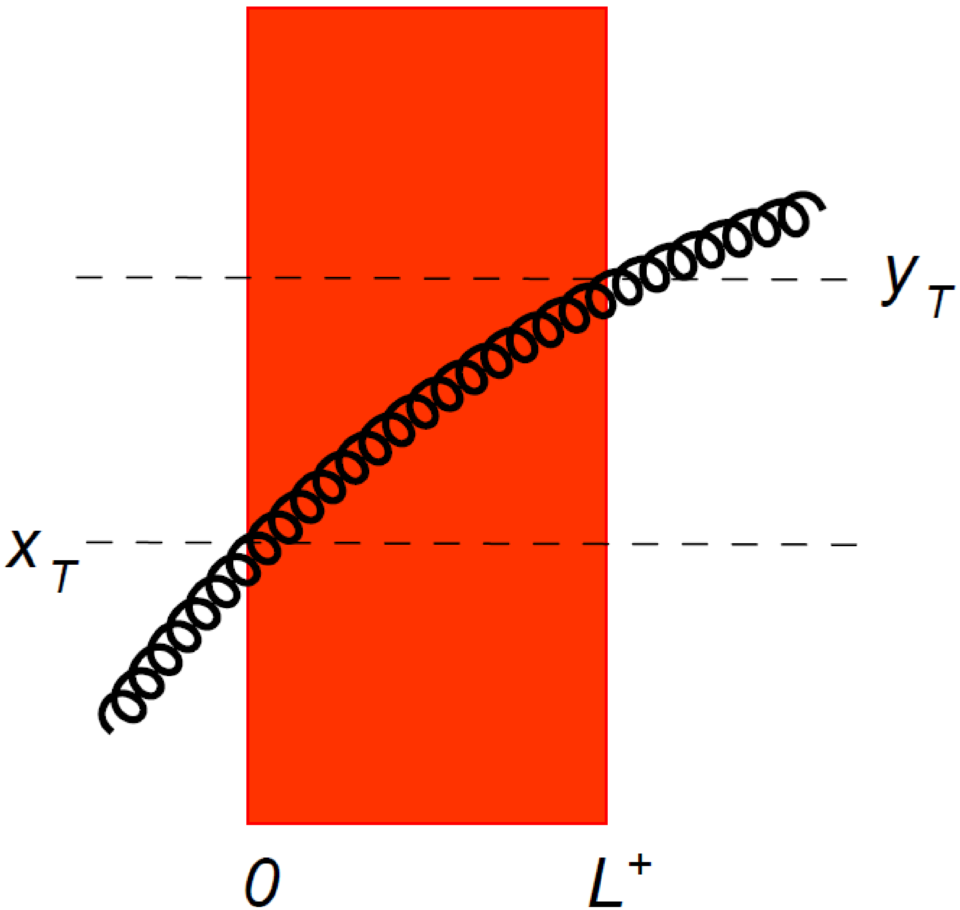
Motivation



In this talk:

- We are interested in power suppressed energy corrections neglected in the high energy limit
- Develop a systematic method to *quantify* the non-eikonal corrections to the shockwave
- Apply this method to single inclusive gluon production in pA collisions as well as single transverse spin asymmetry

Non-Eikonal corrections to the retarded gluon propagator



Gluon propagator in an external background field

Schrödinger like equation in a space-time dependent potential:

$$\left[\delta^{ab}\left(i\partial_{x^+}+\frac{\partial_{\boldsymbol{x}}^2}{2(k^+ + i\epsilon)}\right)+g\left(\mathcal{A}^-(\underline{x})\cdot T\right)^{ab}\right]\mathcal{G}_{k^+}^{bc}(\underline{x};\underline{y}) = i\,\delta^{ac}\,\delta^{(3)}(\underline{x}-\underline{y})$$

Its solution can be written in terms of a path integral

$$\mathcal{G}_{k^+}^{ab}(\underline{x};\underline{y}) = \int_{\boldsymbol{z}(y^+)=\boldsymbol{y}}^{\boldsymbol{z}(x^+)=\boldsymbol{x}} \mathcal{D}\boldsymbol{z}(z^+)\ \exp\left[\frac{ik^+}{2}\int_{y^+}^{x^+} dz^+\ \dot{\boldsymbol{z}}^2(z^+)\right]\mathcal{U}^{ab}\left(x^+, y^+, [\boldsymbol{z}(z^+)]\right)$$

With $\mathcal{U}^{ab}\left(x^+, y^+, [\boldsymbol{z}(z^+)]\right) = \mathcal{P}_+ \exp\left\{ig\int_{y^+}^{x^+} dz^+\, T\cdot\mathcal{A}^-\left(z^+, \boldsymbol{z}(z^+)\right)\right\}^{ab}$

F. Gelis, R. Venugopalan, NPA 743 (2004),13
I. Balitsky, hep-ph/0101042
Y. Mehtar-Tani, PRC 75 (2007) 034908

Gluon propagator in an external background field

In its discretized form, the path integral looks like

$$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \lim_{N\to+\infty} \int \left(\prod_{n=1}^{N-1} \mathrm{d}^2 z_n\right) \left(\frac{-i(k^+ + i\epsilon)N}{2\pi(x^+ - y^+)}\right)^N \left(1 + \mathcal{O}\left(\frac{1}{N}\right)\right)$$

$$\times \exp\left[\frac{i(k^+ + i\epsilon)N}{2(x^+ - y^+)} \sum_{n=0}^{N-1} (z_{n+1} - z_n)^2\right] \mathcal{U}^{ab}(x^+, y^+, \{z_n\})$$

with

$$\mathcal{U}^{ab}(x^+, y^+, \{z_n\}) = \mathcal{P}_+ \left\{ \prod_{n=0}^{N-1} \exp\left[ig \frac{(x^+ - y^+)}{N} \left(\mathcal{A}^-(z_n^+, z_n) \cdot T\right)\right]\right\}^{ab}$$

Altinoluk, Armesto, Beuf, Martinez and Salgado, forthcoming

Expanding the discretized path integral

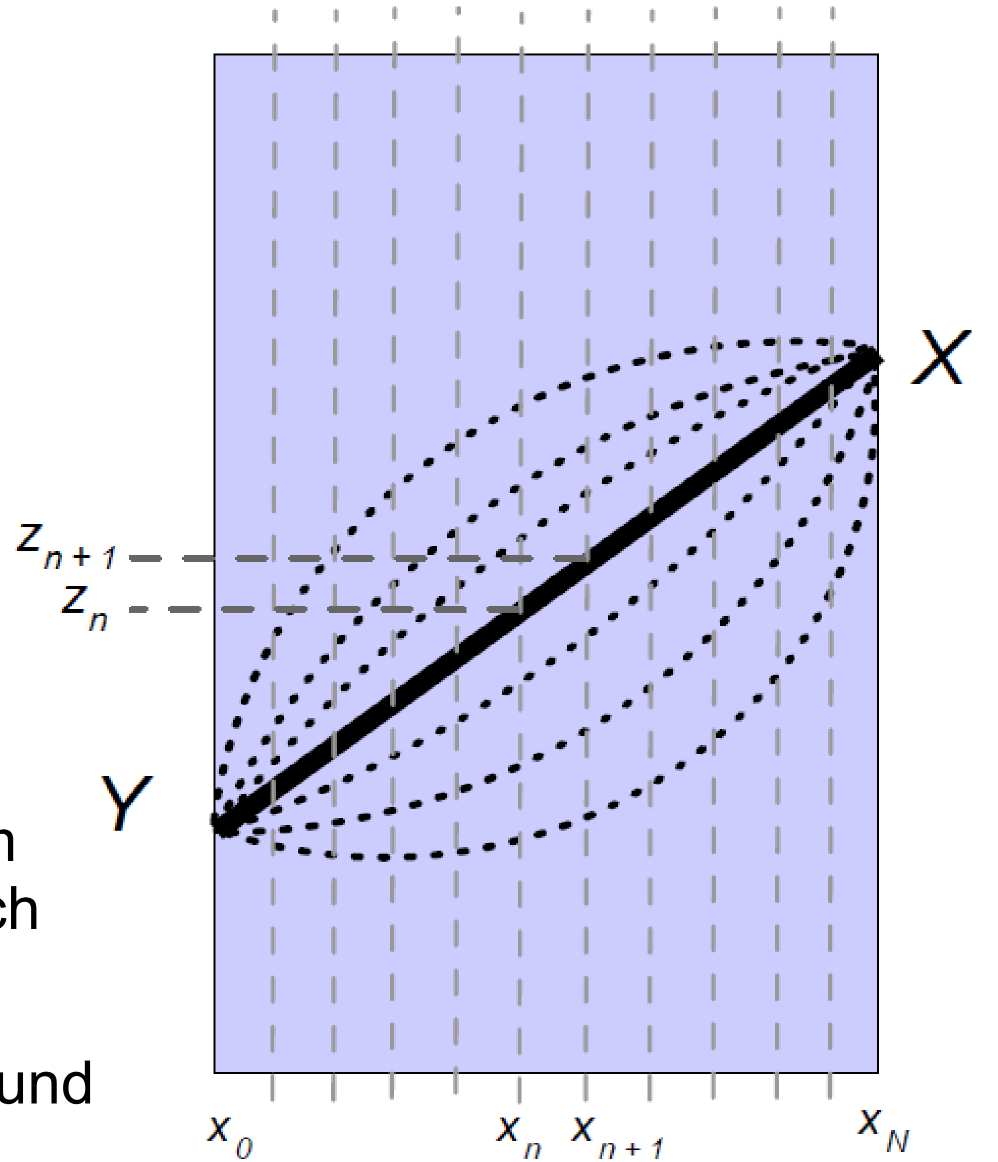
Step 1: If $k^+ \to \infty$

Kinetic term dominates over the potential so:

$$z_n = z_n^{\rm cl} + u_n$$

$$z_n^{\rm cl} = y + \frac{n}{N}(x - y)$$

- Taylor expand the potential term around the classical path at each step contribution.
- Integrate out the fluctuation around the classical path at each step.



Altinoluk, Armesto, Beuf, Martinez and Salgado, forthcoming

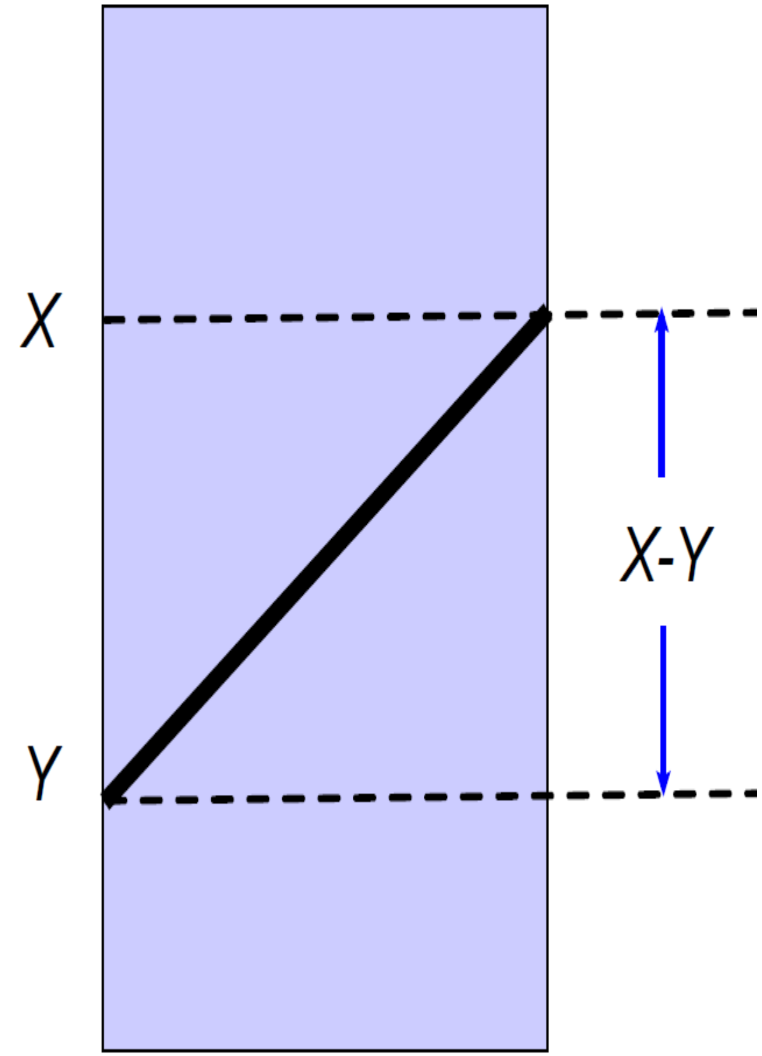
Expanding the discretized path integral

Step 2:

It is expected to have a straight line trajectory of fixed transverse position

X- Y is small

Expand around the difference of the initial and final transverse positions



Altinoluk, Armesto, Beuf, Martinez and Salgado, forthcoming

Non-Eikonal corrections to the gluon propagator

Collecting all the contributions to order $1/k^+$

$$\int d^2\boldsymbol{x}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, \mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+-y^+)\, e^{-i\boldsymbol{k}\cdot\boldsymbol{y}}\, e^{-ik^-(x^+-y^+)}\Bigg\{\mathcal{U}(x^+,y^+,\boldsymbol{y})$$

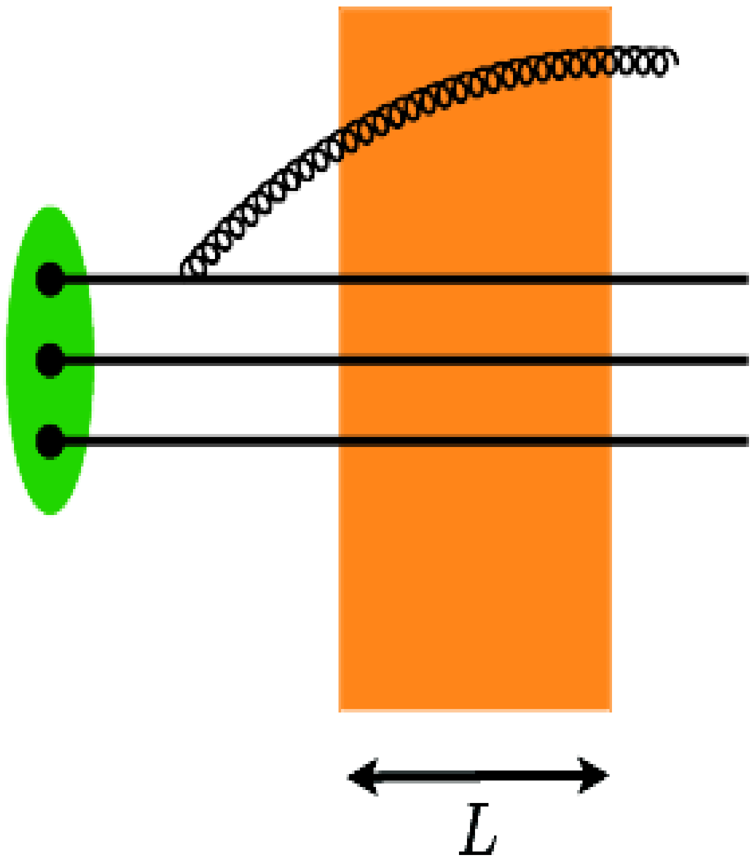
$$+\frac{(x^+-y^+)}{k^+}\boldsymbol{k}^i\, \mathcal{U}^i_{(1)}(x^+,y^+,\boldsymbol{y}) + i\frac{(x^+-y^+)}{2k^+}\mathcal{U}_{(2)}(x^+,y^+,\boldsymbol{y}) + O\left(\frac{1}{(k^+)^2}\right)\Bigg\}^{ab}$$

with

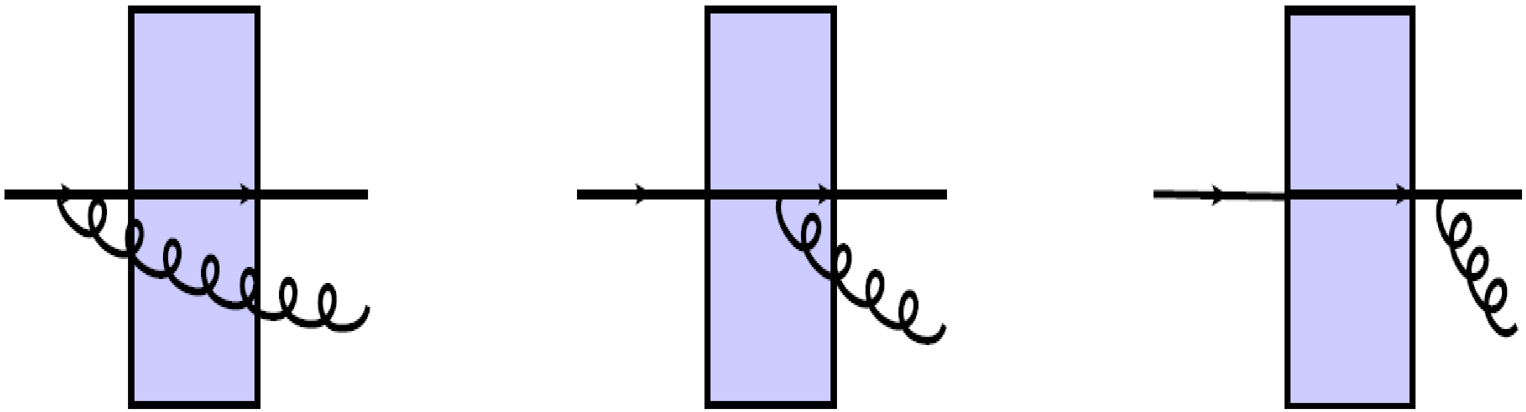
$$\mathcal{U}^{i,ab}_{(1)}(x^+,y^+,\boldsymbol{y}) = \int_{y^+}^{x^+} dz^+ \frac{1}{(x^+-y^+)}\Big\{[\partial_{\boldsymbol{y}^i}\mathcal{U}(x^+,z^+,\boldsymbol{y})]\mathcal{U}(z^+,\boldsymbol{y}^+,\boldsymbol{y})\Big\}^{ab}$$

$$\mathcal{U}^{ab}_{(2)}(x^+,y^+,\boldsymbol{y}) = \int_{y^+}^{x^+} dz^+ \frac{1}{(x^+-y^+)}\Big\{[\partial^2_{\boldsymbol{y}}\mathcal{U}(x^+,z^+,\boldsymbol{y})]\mathcal{U}(z^+,y^+,\boldsymbol{y})\Big\}^{ab}.$$

Gluon production in pA collisions beyond Eikonal approximation



Semi-classical method to calculate gluon production



In a field theory coupled to external sources at LO:

Sum of tree level diagrams $\longleftrightarrow$ Solving classical EOM with retarded boundary conditions

F. Gelis, R. Venugopalan, NPA 776 (2006), 135

Classical Yang Mills Eqs.

Evolution of the gauge field: $\left[D_\mu, F^{\mu\nu}\right] = \mathcal{J}^\nu$

Color charge conservation: $\left[D_\mu, \mathcal{J}^\mu\right] = 0$

Linearizing around a background field: $\mathcal{A}^\mu = A^\mu_{med} + a^\mu$

$$\Box_x a^i - 2ig\left[\mathcal{A}^-_{med}, \partial_- a^i\right] = \mathcal{J}^i - \partial^i\left(\frac{\mathcal{J}^+}{\partial_-}\right)$$ **LC gauge**

Reduction formula: $\mathcal{M}^a_\lambda = \lim_{k^2\to 0} \int d^4x e^{ik\cdot x} \Box_x \mathcal{A}^a_\mu(x) \epsilon^\mu_\lambda(\vec{k})$

Single inclusive gluon spectrum

$$(2\pi)^3 (2k^+) \frac{dN}{dk^+ d^2\mathbf{k}}(B) = \sum_{\lambda\,\mathrm{phys.}} \left\langle \left\langle \left|\mathcal{M}^a_\lambda(\underline{k}, B)\right|^2 \right\rangle_p \right\rangle_A$$

KT factorized form of the single gluon inclusive spectrum

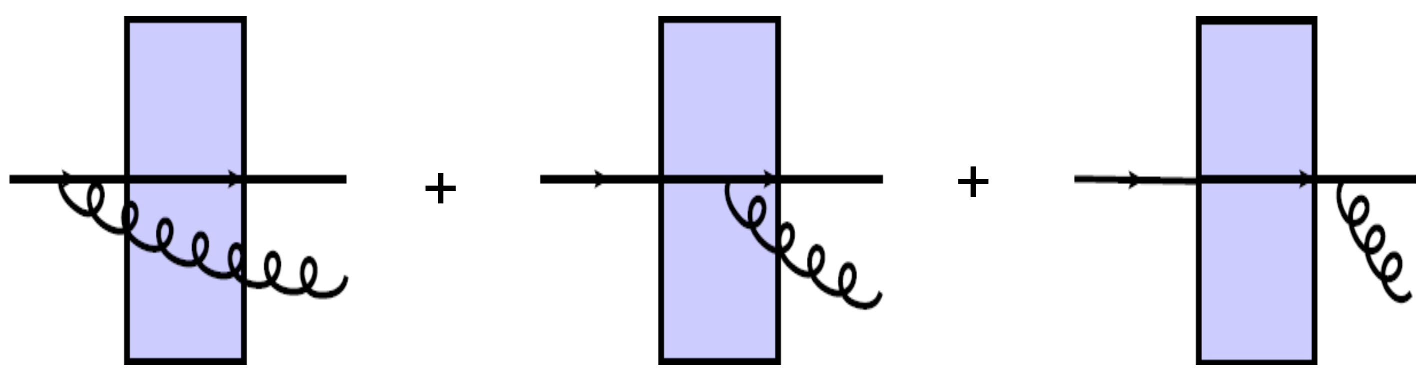
$$x_{\text{cut}} = k^+/P^+$$

$$k^+ \frac{dN}{dk^+\, d^2\boldsymbol{k}}(\boldsymbol{B}) \simeq \int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\, \varphi_p(\boldsymbol{q}; x_{\text{cut}})\, \frac{\boldsymbol{q}^2}{4}\, \frac{1}{N_c^2-1} \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{B}}$$

$$\times \sum_{\lambda\,\text{phys.}} \left\langle \overline{\mathcal{M}}^{ab}_{\lambda}\left(\underline{k}, \boldsymbol{q}-\frac{\boldsymbol{\Delta}}{2}\right)^{\dagger} \overline{\mathcal{M}}^{ab}_{\lambda}\left(\underline{k}, \boldsymbol{q}+\frac{\boldsymbol{\Delta}}{2}\right) \right\rangle_A .$$

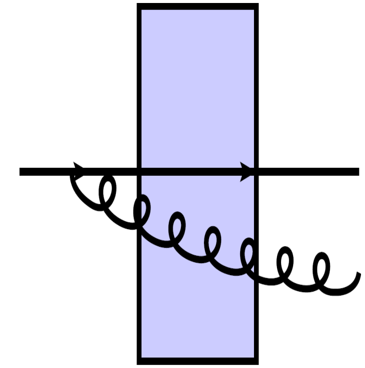
$\varphi_p(\boldsymbol{q}; x_{\text{cut}})$ Unintegrated gluon distribution of the projectile

$$\overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k}, \boldsymbol{q}) =$$

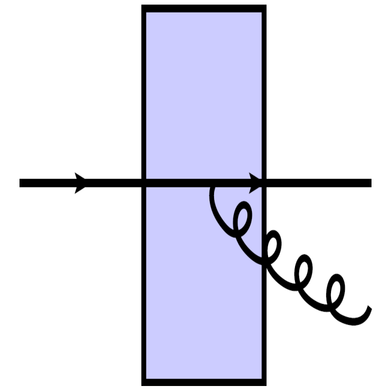


Reduced scattering amplitude

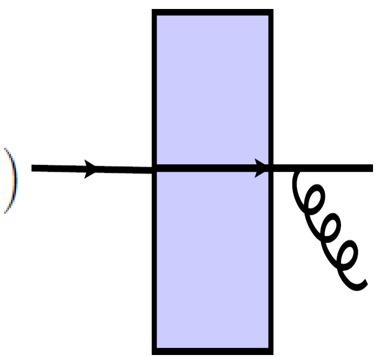
$$\overline{\mathcal{M}}^{ab}_{bef,\lambda}(\underline{k},\boldsymbol{q}) = \varepsilon^{i*}_{\lambda}\, e^{ik^-L^+}\, i\int d^2\boldsymbol{z}\, e^{i\boldsymbol{q}\cdot\boldsymbol{z}}\, (-2)\,\frac{\boldsymbol{q}^i}{\boldsymbol{q}^2}\int d^2\boldsymbol{z}'\, e^{-i\boldsymbol{k}\cdot\boldsymbol{z}'}\, \mathcal{G}^{ab}_{k^+}(L^+,\boldsymbol{z}',0,\boldsymbol{z})$$



$$\overline{\mathcal{M}}^{ab}_{in,\lambda}(\underline{k},\boldsymbol{q}) = \varepsilon^{i*}_{\lambda}\, e^{ik^-L^+}\, i\int d^2\boldsymbol{y}\;\; e^{i\boldsymbol{q}\cdot\boldsymbol{y}}\,\frac{1}{k^+}\int_0^{L^+} dy^+ \int d^2\boldsymbol{z}'\;\; e^{-i\boldsymbol{k}\cdot\boldsymbol{z}'}$$
$$\times\left[\partial_{\boldsymbol{y}^i}\mathcal{G}^{ac}_{k^+}(L^+,\boldsymbol{z}',\underline{y})\right]\;\mathcal{U}^{cb}(y^+,0,\boldsymbol{y})$$



$$\overline{\mathcal{M}}^{ab}_{aft,\lambda}(\underline{k},\boldsymbol{q}) = \varepsilon^{i*}_{\lambda}\, e^{ik^-L^+}\; i\int d^2\boldsymbol{y}\;\; e^{i(\boldsymbol{q}-\boldsymbol{k})\cdot\boldsymbol{y}}\;\; 2\,\frac{\boldsymbol{k}^i}{\boldsymbol{k}^2}\;\mathcal{U}^{ab}(L^+,0,\boldsymbol{y})$$



$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y})$ **captures finite energy/ length corrections**

So the idea is to use the Next to Eikonal Expansion of the gluon propagator derived previously!!!

So after taking the square of the reduced amplitude:

$$\frac{1}{N_c^2-1}\sum_{\lambda\,\text{phys.}}\left\langle \overline{\mathcal{M}}_\lambda^{ab}(\underline{k},\boldsymbol{q})^\dagger\,\overline{\mathcal{M}}_\lambda^{ab}(\underline{k},\boldsymbol{q})\right\rangle_A = \frac{1}{\boldsymbol{k}^2\,\boldsymbol{q}^2}\int d^2\boldsymbol{b}\int d^2\boldsymbol{r}\;e^{-i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{r}}$$

$$\times\Bigg\{4\,(\boldsymbol{k}-\boldsymbol{q})^2 S_A(\boldsymbol{r},\boldsymbol{b}) + 2\frac{L^+}{k^+}\left[(\boldsymbol{k}-\boldsymbol{q})^2\,\boldsymbol{k}^j+\boldsymbol{k}^2(\boldsymbol{k}^j-\boldsymbol{q}^j)\right]\left[\mathcal{O}^j_{(1)}(\boldsymbol{r},\boldsymbol{b})+\mathcal{O}^j_{(1)}(-\boldsymbol{r},\boldsymbol{b})\right]$$

$$+2i\frac{L^+}{k^+}\boldsymbol{k}\cdot(\boldsymbol{k}-\boldsymbol{q})\left[\mathcal{O}_{(2)}(\boldsymbol{r},\boldsymbol{b})-\mathcal{O}_{(2)}(-\boldsymbol{r},\boldsymbol{b})\right]+O\left(\left(\frac{L^+}{k^+}\right)^2\right)\Bigg\}\,.$$

where

$$S_A(r,b)=\frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;b-\frac{r}{2}\right)\mathcal{U}\left(L^+,0;b+\frac{r}{2}\right)\right]\right\rangle_A\,,$$

→ Dipole amplitude

$$\mathcal{O}^j_{(1)}(r,b)=\frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;b-\frac{r}{2}\right)\mathcal{U}^j_{(1)}\left(L^+,0;b+\frac{r}{2}\right)\right]\right\rangle_A\,,$$

$$\mathcal{O}_{(2)}(r,b)=\frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;b-\frac{r}{2}\right)\mathcal{U}_{(2)}\left(L^+,0;b+\frac{r}{2}\right)\right]\right\rangle_A\,.$$

→ Non-Eikonal Contributions

So the single inclusive gluon cross section in pA reads

$$\frac{d\sigma}{dk^+\, d^2\boldsymbol{k}} = \int d^2B\ \frac{dN}{dk^+\, d^2\boldsymbol{k}}(B)$$

$$= \frac{1}{\boldsymbol{k}^2}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\,\varphi_p(\boldsymbol{q};x_{\rm cut})\ (\boldsymbol{k}-\boldsymbol{q})^2\int d^2\boldsymbol{b}\int d^2\boldsymbol{r}\, e^{-i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{r}}\ S_A(\boldsymbol{r},\boldsymbol{b})$$

$$+O\left(\left(\frac{L^+}{k^+}\right)^2\right).$$

- We recover the usual kT factorization formula as the eikonal contribution.
- The first subleading Non Eikonal Corrections cancel due to rotational symmetry in the transverse plane around the center of the nucleus.

Transverse single spin asymmetry:
Polarized Target

Transverse Single Spin Asymmetry (SSA)

Consider the process $p + A^{\uparrow} \to g + X$

SSA is defined as

$$A_N = \frac{k^+ \frac{d\sigma^{\uparrow}}{dk^+ \, d^2\boldsymbol{k}} - k^+ \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\boldsymbol{k}}}{k^+ \frac{d\sigma^{\uparrow}}{dk^+ \, d^2\boldsymbol{k}} + k^+ \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\boldsymbol{k}}} = \frac{k^+ \frac{d\sigma^{\uparrow}}{dk^+ \, d^2\boldsymbol{k}} - k^+ \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\boldsymbol{k}}}{2k^+ \frac{d\sigma}{dk^+ \, d^2\boldsymbol{k}}}$$

Warning

- **Transverse polarization of the target is unknown. See Sievert & Kovchegov, arXiv:1310.5028**
- **Let us assume dependence on the transverse spin vector $\vec{s}$ in the probability distribution for the background field**

Due to the rotational symmetry around the center of the target one gets

$$
k^+\left(\frac{d\sigma^{\uparrow}}{dk^+\,d^2\boldsymbol{k}}-\frac{d\sigma^{\downarrow}}{dk^+\,d^2\boldsymbol{k}}\right)=\frac{2}{\boldsymbol{k}^2}\frac{L^+}{k^+}\int\frac{d^2\boldsymbol{q}}{(2\pi)^2}\,\varphi_p(\boldsymbol{q};x_{\rm cut})
$$
$$
\times\Bigg\{\Big[(\boldsymbol{k}-\boldsymbol{q})^2\,\boldsymbol{k}^j+\boldsymbol{k}^2(\boldsymbol{k}^j-\boldsymbol{q}^j)\Big]\int d^2\boldsymbol{r}\,\cos\Big(\boldsymbol{r}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big)\int d^2\boldsymbol{b}\,\mathcal{O}^j_{(1)}(\boldsymbol{r},\boldsymbol{b},\boldsymbol{s})
$$
$$
+\boldsymbol{k}\cdot(\boldsymbol{k}-\boldsymbol{q})\int d^2\boldsymbol{r}\,\sin\Big(\boldsymbol{r}\cdot(\boldsymbol{k}-\boldsymbol{q})\Big)\int d^2\boldsymbol{b}\,\mathcal{O}_{(2)}(\boldsymbol{r},\boldsymbol{b},\boldsymbol{s})\Bigg\}+O\left(\left(\frac{L^+}{k^+}\right)^2\right)
$$

- **Eikonal corrections cancel exactly**
- **First subleading Non-Eikonal corrections turn out to be the dominant terms!!!!**

Conclusions

- **We develop a method to calculate Next to Eikonal corrections to the gluon propagator in an external background field**
- **We apply this method to gluon production in pA as well as to Single transverse spin asymmetry**
- **First subleading order of the Non-Eikonal corrections are negligible for the case of single gluon inclusive spectrum in pA**
- **For single transverse spin asymmetry, the Non-Eikonal corrections are the dominant contributions**
- **This method can be applied to other processes, e.g., DIS and radiative energy loss in pA**

Backup slides

Some definitions

Unintegrated Gluon Distributions

$$\left\langle \tilde{\rho}^a(\boldsymbol{q})^* \, \tilde{\rho}^b(\boldsymbol{q}) \right\rangle_p = \frac{\delta^{ab}}{N_c^2 - 1} \, \frac{(2\pi)^3}{2} \, \boldsymbol{q}^2 \, \varphi_p(\boldsymbol{q}; x_{\rm cut})$$

$$xG(x, \mu_F^2) = \int d^2\boldsymbol{q} \;\; \theta(\mu_F^2 - \boldsymbol{q}^2) \;\; \varphi_p(\boldsymbol{q}; x_{\rm cut}) \qquad \text{for} \quad x < x_{\rm cut}$$

Properties of the decorated operators I

$$
\begin{aligned}
S_A(\boldsymbol{r},\boldsymbol{b}) &= \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;\boldsymbol{b}-\frac{\boldsymbol{r}}{2}\right)\mathcal{U}\left(L^+,0;\boldsymbol{b}+\frac{\boldsymbol{r}}{2}\right)\right]\right\rangle_A\,,\\
\mathcal{O}^j_{(1)}(\boldsymbol{r},\boldsymbol{b}) &= \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;\boldsymbol{b}-\frac{\boldsymbol{r}}{2}\right)\mathcal{U}^j_{(1)}\left(L^+,0;\boldsymbol{b}+\frac{\boldsymbol{r}}{2}\right)\right]\right\rangle_A\\
\mathcal{O}_{(2)}(\boldsymbol{r},\boldsymbol{b}) &= \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;\boldsymbol{b}-\frac{\boldsymbol{r}}{2}\right)\mathcal{U}_{(2)}\left(L^+,0;\boldsymbol{b}+\frac{\boldsymbol{r}}{2}\right)\right]\right\rangle_A
\end{aligned}
$$

In the adjoint representation, Wilson lines are real so:

$$
\begin{aligned}
S_A(-\boldsymbol{r},\boldsymbol{b}) &= S_A(\boldsymbol{r},\boldsymbol{b})\\
\mathcal{O}^j_{(1)}(\boldsymbol{r},\boldsymbol{b}) &= \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^{j\dagger}_{(1)}\left(L^+,0;\boldsymbol{b}+\frac{\boldsymbol{r}}{2}\right)\mathcal{U}\left(L^+,0;\boldsymbol{b}-\frac{\boldsymbol{r}}{2}\right)\right]\right\rangle_A\\
\mathcal{O}_{(2)}(\boldsymbol{r},\boldsymbol{b}) &= \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^{\dagger}_{(2)}\left(L^+,0;\boldsymbol{b}+\frac{\boldsymbol{r}}{2}\right)\mathcal{U}\left(L^+,0;\boldsymbol{b}-\frac{\boldsymbol{r}}{2}\right)\right]\right\rangle_A
\end{aligned}
$$

Properties of the decorated operators II

When considering STSA:

$$
\begin{aligned}
S_A(-\boldsymbol{r}, -\boldsymbol{b}, -\boldsymbol{s}) &= S_A(\boldsymbol{r}, \boldsymbol{b}, \boldsymbol{s}), \\
\mathcal{O}^j_{(1)}(-\boldsymbol{r}, -\boldsymbol{b}, -\boldsymbol{s}) &= -\mathcal{O}^j_{(1)}(\boldsymbol{r}, \boldsymbol{b}, \boldsymbol{s}), \\
\mathcal{O}_{(2)}(-\boldsymbol{r}, -\boldsymbol{b}, -\boldsymbol{s}) &= \mathcal{O}_{(2)}(\boldsymbol{r}, \boldsymbol{b}, \boldsymbol{s}) \ .
\end{aligned}
$$

$$S_A(-\boldsymbol{r}, \boldsymbol{b}, \boldsymbol{s}) = S_A(\boldsymbol{r}, \boldsymbol{b}, \boldsymbol{s})$$